Charles University in Prague
Faculty of Mathematics and Physics

BACHELOR THESIS

Jiří Procházka

Elastic hadron scattering at high energies

Institute of Particle and Nuclear Physics

Supervisor: RNDr. Vojtěch Kundrát, DrSc.
Institute of Physics of the ASCR, v.v.i.

Field of study: General Physics

Prague 2007

First of all I would like to thank to my supervisor Dr. Vojtěch Kundrát for perfect collaboration during working on this theses. I would like to thank also to my family for permanent support and extraordinary helpful atmosphere.

I declare that I completed my bachelor thesis myself and with use of cited literature only. I agree with using this thesis.

Prague, May 29, 2007 (corrected version)

Jiří Procházka

Contents

Introduction

The first proton-proton (pp) interactions at higher energies were observed in the CERN Intersecting Storage Rings (ISR), the world's first hadron collider, in 1971. A lot of interesting experiments have been performed since that time. The purpose of all these experiments is to learn more about structure of fundamental particles. A number of remarkable properties were discovered. In the following the attention will be devoted mainly to elastic proton interactions. Two of four fundamental interactions (forces) play significant role in such a case - the electromagnetic and the strong interactions.

First of them, the electromagnetic, is also called the Coulomb interaction. Protons, as charged particles with electric charge $+e$, are repelled mutually by this interaction. The second interaction, the strong interaction, is attractive and is called hadronic or nuclear because this strong interaction stick together different nucleons (i.e. protons and neutrons) in a nucleus. This force, nuclear interaction, is stronger than other known forces so that the protons and the neutrons are bound together in a nucleus. The strong interaction is short-ranged in contrast to the Coulomb interaction which is regarded as long-ranged. Hadronic force overcomes the electric repulsion between protons in the nucleus.

Elastic scattering is in principle the most simple scattering where the same incident particles come out after the collision. Characteristics of these processes such as cross section depend only weakly on energy. On the other hand the average number of produced particles in inelastic collisions and corresponding cross section depend strongly on energy. The elastic processes represent always a significant part of processes at any energy. There is, however, a smaller part of inelastic processes that exhibit similar energy behavior. They are denoted as diffractive processes. One assumes that one particle or both the incident particles are brought to an excited state in the collision that decay then into several secondary particles, all moving in original directions. One speaks about single or double diffraction. The majority of the rest of collision processes are denoted as non-diffractive inelastic. In contrast to diffractive ones, they are strong energy dependent and the final state particles have large transverse momenta.

In spite of the fact that since 1971 many experimental data have been gathered there is no reliable theory of pp diffractive processes. There are only several different models which more or less well describe present experimental data on phenomenological grounds. To learn more about the structure of fundamental particles it is necessary to improve and extend present experimental data of diffractive processes. New hadron collider LHC (Large Hadron Collider) is being built now at CERN and should be finished at the end of this year. It will provide high intensity proton collisions with center of mass energy up to 14 TeV.

The five experiments, with huge detectors, will study what will occur if the corresponding LHC beams collide. One of them is TOTEM (TOTal Elastic and diffractive cross section Measurement) that will be devoted mainly to study of elastic processes. As discussed in Letter of Intent in 1997 [1], the TOTEM experimental programme was proposed to measure (see also [2] and [3])

- **the total proton-proton cross section** with an absolute error of 1 mb by using the luminosity independent method. It requires simultaneous measurement of *pp* elastic scattering at low momentum transfer and of the total inelastic rate.

- **elastic scattering** in the largest possible interval of four-momentum transfer from the Coulomb region $-t \approx 10^{-3}\,GeV^2$ up to the nuclear region $-t \approx 10\,GeV^2$

- the **diffractive dissociation**, including single and double diffraction

Until now all experiment elastic data have been analysed with the help of West and Yennie's formula that have been shown to be not sufficiently general. Some corresponding problems will be studied and demonstrated in the present thesis by analysing previous experimental elastic proton-proton data (at energy of 53 GeV) with use of one more general formula under different additional conditions involved in West-Yennie formula.

Chapter 1

Two-Body Elastic Scattering

1.1 Conventions

In this thesis, we will employ commonly used formalism of four-momentum to describe kinematics of particles (see [4] or [5])

$$P = (E, \boldsymbol{p}) \tag{1.1}$$

where E is total energy of a particle and $\boldsymbol{p}$ is its three-momentum. We will use natural units $\hbar = c = 1$. Scalar product of these four-momenta is defined as follows

$$P^2 = g_{\mu\nu}\, p^\mu p^\nu = E^2 - \boldsymbol{p}^2 \tag{1.2}$$

where the metric tensor $g_{\mu\nu}$ is

$$g_{\mu\nu} = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{1.3}$$

Due to our choice of natural units $\hbar = c = 1$ the energy E and magnitude of $\boldsymbol{p}$ (which we will denote by p) have the same units GeV.

1.2 Kinematics

A special case of two-body reaction is the elastic scattering

$$1 + 2 \to 1' + 2' \tag{1.4}$$

where the two scattering particles remain in the same state but in a different kinematic configuration. The four-momentum of the ith incoming (outgoing) particle is denoted by P_i (P_i') for $i = 1, 2$ (see Fig. 1 in the case of center of mass system (CMS)).

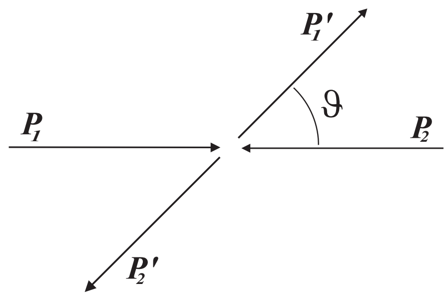

Figure 1: Two-body elastic scattering in center of mass system (CMS).

Due to (see Ref. [4]):

- *the conservation of four-momentum* (which is the same as conservation of energy and three-momentum)
$$P_1 + P_2 = P_1' + P_2'; \tag{1.5}$$
- *the mass shell conditions*
$$P_i^2 = m_i^2, \tag{1.6}$$
$$P_i'^2 = m_i^2, \tag{1.7}$$
for elastic scattering, the mass m_i ($i = 1, 2$) is invariant mass of the ith-particle;
- *fixation of a reference frame,*

the kinematics of the given process is fully described by two independent variables. It is possible to choose these two parameters among the three Mandelstam variables, defined as

$$s = (P_1 + P_2)^2, \tag{1.8}$$
$$t = (P_1 - P_1')^2, \tag{1.9}$$
$$u = (P_1 - P_2')^2. \tag{1.10}$$

From these relations we can easily derive identity

$$s + t + u = \sum_{i=1}^{4} m_i^2. \tag{1.11}$$

Only two of three Mandelstam variables are thus independent. We shell use s and t as it is common. It is further convenient to choose CMS, because the variable s is the square of the total center of mass energy of colliding particles and the variable t is the squared momentum transfer in this reference frame. In case of pp elastic scattering, which we are interested in, the magnitudes of three-momenta of incoming and outgoing particles in CMS are the same (denote them by p, resp. p') and they have equal masses m. The relations between the CMS scattering angle ϑ and three-momentum p (see Fig. 1) and variables s and t are

$$s = 4(p^2 + m^2) \tag{1.12}$$

$$t = -2p^2(1 - \cos\vartheta). \tag{1.13}$$

We see from the last relation (1.13) that

$$-4p^2 \leq t \leq 0, \tag{1.14}$$

i.e. the value of t is not positive and its minimal value may be $-4p^2$, in contrast to s which is always positive because it is the total center of mass energy of colliding particles.

In high energy limit (i.e. for $s \to \infty$) the mass m can be neglected in comparison to p^2 in formula (1.12) and it yields

$$s \approx 4p^2. \tag{1.15}$$

1.3 Scattering amplitude

The motion of two spinless particles is two-body problem which is equivalent to two independent one-body problems. The easy one is translation motion of the centre of mass of these two particles and the second one, more complicated, is relative motion of these two particles in CMS. This relative motion may be solved as motion of a particle with reduced mass $\mu = \frac{m_1 m_2}{m_1+m_2}$ in an interacting potential $V(\boldsymbol{r})$ of the two particles. In the case of pp scattering μ equals to the half of mass of one proton. This modified rotation motion is standardly described with use of time independent part of Shrödinger equation as follows, see [4] or [6],

$$-\frac{\hbar^2}{2\mu}\triangle\psi(\mathbf{r}) + V(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r}) \tag{1.16}$$

where E is energy of the particle with reduced mass μ. One of two asymptotic solution of this equation, i.e. solution for $r \to \infty$, without the normalisation condition, is

$$e^{i\boldsymbol{k}.\boldsymbol{r}}. \tag{1.17}$$

This wave function corresponds to initial state of incoming particle in the time $t \to -\infty$ with reduced mass μ interacting via the potential $V(\boldsymbol{r})$. Similarly the second asymptotic solution of Eq. (1.16)

$$f(\boldsymbol{k}, \boldsymbol{k}')\frac{e^{ikr}}{r} \tag{1.18}$$

correspond to final state of scattered particle in the time $t \to \infty$. The vectors $\boldsymbol{k}$, resp. $\boldsymbol{k}$', in Eqs. (1.17) and (1.18) are the wave vectors of the incoming, resp. outgoing, particle and k^2 is defined as $\frac{2\mu}{\hbar^2}E$. Energy conservation requires $\boldsymbol{k}^2 = \boldsymbol{k}'^2 = k^2$ for elastic scattering. Function $f(\boldsymbol{k}, \boldsymbol{k}')$ in Eq. (1.18) is the so called scattering amplitude and it is a complex function. Differential cross section is in quantum mechanics commonly defined with use of this scattering amplitude as

$$\frac{d\sigma}{d\Omega} = |f(\boldsymbol{k}, \boldsymbol{k}')|^2 \tag{1.19}$$

where $d\Omega$ is differential space angle. In the relativistic theory of the two particle scattering process the equation for differential cross section (1.19) is commonly transformed to Mandelstam variables s and t as follows (see [5])

$$\frac{d\sigma(s,t)}{dt} = \frac{\pi}{sp^2}|F(s,t)|^2 \tag{1.20}$$

where we have introduced now scattering amplitude $F(s,t)$ in s and t variables. The complex scattering amplitude $F(s,t)$ can be characterized by two real functions, the modulus $|F(s,t)|$ and the phase $\zeta(s,t)$, in this way

$$F(s,t) = i|F(s,t)|e^{-i\zeta(s,t)}. \tag{1.21}$$

1.4 Eikonal model and impact parameter space

The total elastic scattering amplitude of two charged and spinless nucleons may be defined with the help of Fourier-Bessel transformation according to Glauber [6] or Islam [7] as

$$F(s,t=-q^2) = \frac{s}{4\pi i}\int\limits_{\Omega_b} e^{i\vec{q}.\vec{b}}[e^{2i\delta(s,b)} - 1]d^2b \tag{1.22}$$

where Ω_b represents the two-dimensional Euclidean space of the impact parameter $\vec{b}$, the vector $\vec{q}$ is defined as $\boldsymbol{k} - \boldsymbol{k'}$ and eikonal $\delta(s,b)$ is proportional to

$$\delta(s,b) \sim \int\limits_b^\infty \frac{V(s,r)rdr}{\sqrt{r^2-b^2}}. \tag{1.23}$$

Potential *V(s,r)* corresponds to potential between particles at individual corresponding positions during their motions. Relation (1.23) holds for energy-dependent spherically symmetric potential *V(s,r)* that might be generally complex. The distribution of elastic processes in the impact parameter space is given then by

$$D(s,b) = |h_{el}(s,b)|^2 \tag{1.24}$$

where the amplitude $h_{el}(s,b)$ for $b \geq 0$ is given by the Fourier-Bessel transformation with respect to finite region of kinematically allowed region of $t \in (t_{min}, 0)$

$$h_{el}(s,b) = \frac{1}{16\pi p\sqrt{s}}\int\limits_{t_{min}}^0 F^N(s,t)J_0(b\sqrt{-t})dt \tag{1.25}$$

where $J_0(x)$ is Bessel function of zeroth order

$$J_0(x) = \frac{1}{2\pi} \int_0^{2\pi} e^{ixcos\varphi} d\varphi. \tag{1.26}$$

If the distribution of elastic processes $D(s, b)$ has (for given s value) its maximum at impact parameter $b = 0$, i.e. for head-on collisions of two particles, we speak about central behavior of elastic collision. In the case when this distribution has maximum at some $b > 0$ we denote this situation as peripheral behavior of elastic collision.

Chapter 2

Interference between the Coulomb and hadronic scattering

High-energy elastic scattering of nucleons is realized not only due to the strong hadronic interaction but in the case of charged hadrons also as the result of the Coulomb interaction. Coulomb and nuclear scattering can be characterized by common differential cross section that may be measured. Elastic differential cross section of charged nucleons is being then currently described with help of total elastic scattering amplitude $F^{C+N}(s,t)$, according to Eq. (1.20).

2.1 West-Yennie formula

The total amplitude $F^{C+N}(s,t)$ is commonly written as the sum of hadronic amplitude $F^N(s,t)$ and Coulomb amplitude $F^C(s,t)$ known from QED which are mutually bound with the help of relative *real* phase $\alpha\phi(s,t)$ as follows

$$F^{C+N}(s,t) = F^C(s,t)e^{i\alpha\phi(s,t)} + F^N(s,t) \tag{2.1}$$

where $\alpha = 1/137.036$ is the fine structure constant. The total amplitude $F^{C+N}(s,t)$ is not therefore mere sum of hadronic and Coulomb amplitude. West and Yennie [8] applying the method of Feynman diagram technique (one photon exchange) derived further in the case of charged point-like particles and of high energy limit (i.e. for $s \to \infty$) for the phase function $\phi(s,t)$ the formula being now used in the form

$$\phi(s,t) = \mp \left[\ln\left(\frac{-t}{s}\right) - \int_{-4p^2}^{0} \frac{dt'}{|t-t'|}\left(1 - \frac{F^N(s,t')}{F^N(s,t)}\right)\right]. \tag{2.2}$$

It means that at the given energy the t-dependence of the relative phase between the Coulomb and hadronic amplitudes is determined practically by the t-dependence of hadronic component $F^N(s,t)$ entering into the integrand of Eq. (2.2).
Assuming some assumptions concerning the t-dependence of hadronic amplitude $F^N(s,t)$ and adding two electric form factors $f_1(t)$ and $f_2(t)$ describing the same electric structure of the both colliding hadrons into the Coulomb amplitude, i.e.

$$F^C(s,t) = \pm\, \frac{\alpha s}{t}, \tag{2.3}$$

the mentioned authors were able to calculate integral (2.2) analytically. Thus for the total amplitude (2.1) with relative phase given by (2.2) only then derived the simplified formula

$$F^{C+N}_{WY}(s,t) = \pm\, \frac{\alpha s}{t} f_1(t) f_2(t) e^{i\alpha\Phi} \;+\; \frac{\sigma_{tot}}{4\pi} p\sqrt{s}(\rho + i) e^{Bt/2} \tag{2.4}$$

which seemed to be more convenient for the analysis of experimental data then (2.1) and the original WY formula (2.2). In the relation p is the value of the momentum in CMS, the parameter B is called diffraction slope, σ_{tot} is the total cross section and ρ is the ratio of the real part to the imaginary part of the hadronic amplitude which is assumed to be independent of t as well as the diffraction slope B. The form factors $f_1(t)$ and $f_2(t)$ correspond to the electric structure of two colliding nucleons. The phase $\phi(s,t)$ in Eq. (2.4) equals then

$$\phi(s,t) = \mp \left[\ln\left(\frac{-Bt}{2}\right) + \gamma \right] \tag{2.5}$$

where $\gamma = 0.577215$ is Euler's constant. The upper (lower) sign in Eqs. (2.2), (2.3), (2.4) and (2.5) corresponds to the scattering of particles with the same (opposite) charges. The simplified WY formula contains a dominant imaginary part of hadronic amplitude at all values of momentum transfer t because the parameter ρ is regarded as small, see Table 1 in Chapter 3. Formulae (2.5) for the relative phase $\phi(s,t)$ and (2.4) for the total elastic scattering amplitude $F^{C+N}_{WY}(s,t)$ contain only three free, unknown, parameters, i.e. ρ, B and σ_{tot}. The formula (2.4) is assumed to hold for small values of absolute value of momentum transfer t.

The second term on the right hand side of equation (2.4) represents the hadronic amplitude $F^N_{WY}(s,t)$ which was derived under the following three assumptions

(i) the influence of spins of all the particles involved can be neglected;

(ii) the t-dependence of the modulus of the elastic hadron amplitude is purely exponential in the *whole kinematically allowed region of momentum transfer*, i.e.

$$|F^N_{WY}(s,t)| \sim e^{Bt/2}; \tag{2.6}$$

(iii) the real and imaginary parts of the elastic hadron amplitude exhibit the same t-dependence at *all kinematically allowed values of* t, i.e. the quantity ρ is t-independent.

The first assumption (i) has been discussed in [9] or [10]. According to these papers the spins effects have negligible effect in the case of forward elastic hadron pp scattering in ISR energy range of $\sqrt{s}$, i.e. from $23.5\,GeV$ to $62.5\,GeV$, and in all high-energy elastic hadron scattering with non-polarized hadrons. The first assumption (i) seems, therefore, not to represent practically any important limitation.

On the other hand the second and third assumptions (ii) and (iii) are much more important. The purely t-dependence of hadronic amplitude $|F^N_{WY}(s,t)|$ determines according to Eq. (1.20) the corresponding differential cross section $\frac{d\sigma^N_{WY}}{dt}$ and approximately corresponds to observed experimental data for the pp elastic scattering at the ISR energies (where the diffractive structure - existence of diffractive minimum - has been experimentally confirmed) only for t running from the forward direction to diffractive minimum. The purely experimental t-dependence is without any doubt in contradiction to high energy elastic nucleon experimental data. The purely exponential t-dependence at *all kinematically allowed values of momentum transfer* was assumed and thus simplified West-Yennie's formula (2.4) has been derived in an inconsistent way [11].

In spite of this fact the simplified WY formula is currently used for the analysis of differential cross section data in the interference region only (i.e. in the case of elastic nucleon scattering for $|t| \leq 0.01\,GeV^2$ at present high energies). For higher values of $|t|$ it is believed that the influence of Coulomb scattering can be - on the basis of the WY simplified formula - completely neglected and another formulae with *different* t dependence of hadronic amplitude $F^N(s,t)$ (non pure exponential dependence) are commonly used. However, let as stress again that the theoretical assumptions used in the derivation of simplified WY amplitude are not fulfilled by the experimental data and its current application in this way can be hardly justified. And this is the reason why the more general formula removing these discrepancies should be used for the analysis of data at *all* values of t *simultaneously*.

Simplified West and Yennie's formula can be used only as a rough approximation at very small $|t|$ and thus this formula can not be used, of course, to test these two assumptions in the whole kinematically allowed region of t. We will use more general approach which should hold in the whole kinematically allowed region of momentum transfers, see next section, to test the assumptions involved in West and Yennie's approach. The analysis will be then done in Chapter 3.

2.2 More general formula

The deficiencies from the experimental as well as theoretical point of view may be removed if one starts from the eikonal formula shown in Sect. 1.4. Due to Eq. (1.22) the total amplitude $F^{C+N}(s,t)$ is fully determined by the total eikonal $\delta^{C+N}(s,b)$. Taking into account the relation (1.23) for eikonal the total eikonal $\delta^{C+N}(s,b)$ is given by the sum of individual eikonals $\delta^C(s,b)$ and $\delta^N(s,b)$ for Coulomb and hadronic eikonals

$$\delta^{C+N}(s,b) = \delta^C(s,b) + \delta^N(s,b). \tag{2.7}$$

Eq. (2.7) is in agreement with the additivity of potentials, i.e. Coulomb and hadronic potential, and with linearity of potential $V(s,r)$ in expression (1.23). The Coulomb and hadronic elastic scattering (the elastic differential cross section) is thus fully determined by the total potential which is simply given by the sum of both the potentials. The 'interference' between Coulomb and hadronic scattering follows thus from the mere sum of corresponding potentials. More general formula for total elastic amplitude of charged hadrons was then derived in the framework of the eikonal model by Kundrát and Lokajíček [12]:

$$F^{C+N}_{KL}(s,t) = \pm\, \frac{\alpha s}{t} f_1(t) f_2(t) \; + F^N(s,t)[1 \mp i\alpha G(s,t)] \tag{2.8}$$

where

$$G(s,t) = \int\limits_{t_{min}}^{0} dt' \left\{ \ln\left(\frac{t'}{t}\right) \frac{d}{dt'}[f_1(t')f_2(t')] + \frac{1}{2\pi}\left[\frac{F^N(s,t')}{F^N(s,t)} - 1\right] I(t,t') \right\} \tag{2.9}$$

and

$$I(t,t') = \int\limits_{0}^{2\pi} d\Phi'' \frac{f_1(t'')f_2(t'')}{t''}; \tag{2.10}$$

here $t'' = t + t' + 2\sqrt{tt'}\cos\Phi''$. The minimal kinematically allowed value t_{min} in (2.9) equals $-s + 4m^2$ according to relations (1.12) and (1.14). The upper (lower) sign in (2.8) corresponds again to the scattering of particles with the same (opposite) charges. Formulae (2.8),(2.9) and (2.10) should hold generally for any s and t with the accuracy up to terms linear in α in contrast to West and Yennie's formula (2.4) which could be used only as a rough approximation at very small $|t|$. The expression in the last bracket of Eq. (2.8) may be so regarded as the first term in the Taylor series expansion of the exponential $e^{\mp i\alpha G}$ and one can write within the same precision

$$F^{C+N}_{KL}(s,t) = F^C(s,t) + F^N(s,t)e^{\mp i\alpha G(s,t)}. \tag{2.11}$$

This is analogical formula to formula (2.1). The relative phase $\alpha\phi(s,t)$ in (2.1) is real but the function $G(s,t)$ may be now generally complex.

To express proton form factors $f_1(t) = f_2(t)$ in the case of pp scattering in the large region of t we can employ t-dependent Borkowski's electric proton form factors

$$f_1(t) = f_2(t) = \sum_{j=1}^{4} \frac{g_j}{w_j - t} \tag{2.12}$$

where parameters g_j and w_j entering into the form factors have been extracted from the measured electron-proton elastic scattering cross sections (see [13]). The integral $I(t,t')$ defined by Eq. (2.10) can be now determined analytically (see [9] or [12]) as follows

$$I(t,t') = \sum_{j,k=1}^{4} g_j g_k W_{jk} I_{jk} \tag{2.13}$$

where for $j \neq k$

$$
\begin{aligned}
I_{jk} = 2\pi \Bigg[& \frac{(P_j-1)^2}{\sqrt{P_j}(P_j-P_k)(P_j-U)} + \frac{(P_k-1)^2}{\sqrt{P_k}(P_k-P_j)(P_k-U)} + \\
& + \frac{(U-1)^2}{\sqrt{U}(U-P_j)(U-P_k)} \Bigg]
\end{aligned}
\tag{2.14}
$$

and

$$
I_{jj} = 2\pi \left[\frac{(P_j-1)(3P_j+P_j^2-U-3P_jU)}{2P_j^{3/2}(P_j-U)^2} + \frac{(U-1)^2}{\sqrt{U}(U-P_j)^2} \right]. \tag{2.15}
$$

It holds further that

$$
P_j = \frac{w_j + (\sqrt{-t}+\sqrt{-t'})^2}{w_j + (\sqrt{-t}-\sqrt{-t'})^2}, \qquad U = \left(\frac{\sqrt{-t}+\sqrt{-t'}}{\sqrt{-t}-\sqrt{-t'}} \right)^2 \tag{2.16}
$$

and

$$
W_{jk} = \frac{1}{[w_j + (\sqrt{-t}-\sqrt{-t'})^2][w_k + (\sqrt{-t}-\sqrt{-t'})^2][\sqrt{-t}-\sqrt{-t'}]^2}. \tag{2.17}
$$

Formula (2.8) for the total elastic scattering amplitude is convenient for any t dependence of hadronic amplitude $F^N(s,t)$ without any limitation. It can be either used for the analysis of differential cross section data at *all* values of t *simultaneously* in a unique way if the hadronic amplitude $F^N(s,t)$ is suitably parametrized. On the contrary to simplified WY formula it does not contain the characteristic quantities σ_{tot}, ρ and B explicitly. But they can be derived easily from the hadronic amplitude as it will be shown for ρ and B in the next chapter.

Or, the general formula (2.8) can be also used for the determination of the diff. cross section data at any values of t if the hadronic amplitude $F^N(s,t)$ is specified within the framework of another phenomenological model description.

Chapter 3

Analysis of experimental data under different conditions

Until now the most analysis of elastic experimental data have been based on the basis of West-Yennie formula only. In the following we should like to demonstrate some differences when different formulae are made use of. We shall go back to pp experimental data obtained earlier at energy of 53 GeV that is represented by the measured total elastic differential cross section. We will start from formula (see (1.20))

$$\left.\frac{d\sigma(s,t)}{dt}\right|_{KL} = C_{norm}\frac{\pi}{sp^2}|F^{C+N}_{KL}(s,t)|^2 \tag{3.1}$$

where the formula (2.8) for the total elastic amplitude $F^{C+N}_{KL}(s,t)$ will be applied to. The parameter C_{norm} corresponds to one percent systematic error of measured cross section. This parameter will be fitted within the bounds from 0.99 to 1.01 and it will not be explicitly mentioned in the following. We shall fit experimental data of the measured elastic differential cross section in region of $-t$ from 0.00126 GeV^2 to 9.75 GeV^2 (243 points) taken from [14] and [15]. We will add some limiting conditions involved in West-Yennie's formula to more general formula (2.8).

The goal will consist in establishing the t-dependence of function $F^{C+N}_{KL}(s,t)$ at $\sqrt{s} = 53\,GeV$ under different conditions when the influence of individual limiting assumptions contained in West-Yennie approach will be tested in comparison to general case.

To determine function $F^{C+N}_{KL}(s,t)$ we shell use t-dependent Borkowski's electric proton form factors (2.12), with parameters g_j and w_j taken from [13], for the form factors $f_1(t) = f_2(t)$ involved in (2.8). The integral $I(t,t')$ in (2.8) will be analytically computed with use of formula (2.13). Now, if we have expression for proton form factors, the only unknown function in the r.h.s. of Eq. (2.8) for the total amplitude $F^{C+N}_{KL}(s,t)$ is the elastic hadronic amplitude $F^N(s,t)$. Because there is not any reliable theory of elastic processes, the only way how to establish the hadronic amplitude is to parameterize it and then fit it to measured data. Since the hadronic amplitude is complex function we will have to parameterize two real functions, the phase $\zeta^N(s,t)$ and the modulus $|F^N(s,t)|$, see Eq. (1.21) which will be introduced later.

To perform the necessary comparison we will assume in the general case that both the corresponding quantities, i.e. ratio ρ of the real part to the imaginary part of the hadronic amplitude and diffractive slope B, are t-dependent quantities. To show the difference between differential cross section computed with use of more general formula under different condition and differential cross section computed with use of the simple West-Yennie's formula we will calculate the t-dependence of ratio

$$R(t) = \left| \frac{\frac{d\sigma}{dt}\Big|_{KL} - \frac{d\sigma}{dt}\Big|_{WY}}{\frac{d\sigma}{dt}\Big|_{WY}} \right|. \tag{3.2}$$

To compute this quantity we need to know parameters σ_{tot}, ρ and B involved in West-Yennie's formula (2.4). We will use the values of these parameters derived in [16] from experimental data, see Table 1.

Parameter	Value
σ_{tot} $[mb]$	42.38 ± 0.15
ρ	0.077 ± 0.009
B $[GeV^{-2}]$	12.87 ± 0.14

Table 1: The values of the parameters involved in West-Yennie's formula, taken from [16].

We will present here the results of the analysis with some values of free parameters taken from Table 1 which specify the hadronic amplitude $F^N(s,t)$. Similar results have never been published yet.

An estimation of the values of all free parameters specifying the hadronic amplitude has been performed by the modified programs developed in Ref. [12]. The numerical minimization of the χ^2 values in all cases has been performed with the help of the MINUIT program [17]. The corresponding statistical errors of the free parameters were determined by the HESSE procedure.

3.1 Generalization of quantities B and ρ

The diffractive slope B and quantity ρ, i.e. the ratio of the real part to the imaginary part of the hadronic amplitude, are supposed to be constant in the whole region of kinematically allowed momentum transfer t in the case of simplified West and Yennie formula (2.4) while these quantities exhibit important t dependencies. In general case the t-dependent quantity $\rho(s,t)$ may be defined simply as (see, e.g., [12])

$$\rho(s,t) = \frac{\Re F^N(s,t)}{\Im F^N(s,t)}. \tag{3.3}$$

It is also possible to define the t-dependent diffractive slope B as

$$B(s,t) = \frac{2}{|F^N(s,t)|} \frac{d}{dt} |F^N(s,t)|. \tag{3.4}$$

In the West-Yennie formula two assumptions are contained that both these quantities are constant in the whole interval of all t values. We will study in the following how the final fit is influenced by one or the other assumptions. We will do this in two ways. We will fit these constant quantities with other free parameters to experimental data and then we will also fix them at the values from Table 1 and fit the rest of free parameters.

As constant quantity we will denote t-independent quantity in the next sections.

3.2 Standard phase

First of all we will analyse experimental data using more general formula (2.8) on condition of standard hadronic phase

$$\zeta^N = \arctan \frac{\rho_0}{1 - \left|\frac{t}{t_{diff}}\right|} \tag{3.5}$$

which is commonly used in West-Yennie formula (see, e.g., [9] or [12]) even thought constant hadronic phase (constant quantity ρ, see Sect. 2.1) have been assumed to derive this formula. More general formula do not require this kind of parameterization of hadronic phase. The parameter t_{diff} will be fixed on value -1.375 GeV^2 and so only one parameter ρ_0 determines hadronic phase. It only remains to parametrize the modulus of hadronic amplitude $|F^N(s,t)|$ for analysis of experimental data. The modulus of hadronic phase $|F^N(s,t)|$ can be parametrized as

$$|F^N(s,t)| = (a_1 + a_2 t)e^{b_1 t + b_2 t^2 + b_3 t^3} + (c_1 + c_2 t)e^{d_1 t + d_2 t^2 + d_3 t^3}. \tag{3.6}$$

This choice of parameterization should be sufficiently flexible to describe experimental data. The fitted values of free parameters describing hadronic phase given by parameterizations (3.5) and (3.6) are shown in Table 2 (Fit I). Quantity $R(t)$ corresponding to Fit I is in Fig. 3. Calculated t-dependent diffractive slope $B(t)$ defined by Eq. (3.4) is in Fig. 2 and finally corresponding differential cross section for pp scattering given by Eq. (3.1) is in Fig. 4 (green line).

3.3 Constant quantity ρ

In this section we will analyse experimental data using the more general formula (2.8) under the assumption that the quantity ρ is independent of momentum transfer t. According to Eqs. (1.21) and (3.3) one can also write for the hadronic phase $\zeta^N(s,t)$

$$\zeta^N(s,t) = \arctan \rho(s,t). \tag{3.7}$$

It yields the hadronic phase ζ^N is constant if ρ is constant. Constant ρ is thus parameter which fully describe the phase $\zeta^N(s,t)$. The modulus $|F^N(s,t)|$ will be parametrized again as (3.6). The values of parameters fitted to experimental data are in Table 2 (Fit II). The case when the parameter ρ is fixed on the value from Table 1 and the others free parameters are left free is in Table 2, see Fit III. Quantities $R(t)$ corresponding to these fits

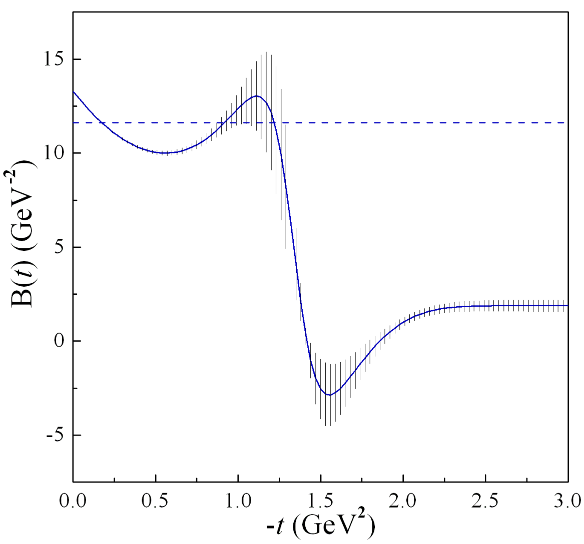

Figure 2: t-dependence of the diffractive slope defined by Eq. (3.4) and corresponding to Fit I (full line) and to Fit IV (dashed line), i.e. to the fits with standard phase, see Sect. 3.2, and to the fit with standard phase and constant diffractive slope, see Sect. 3.4.

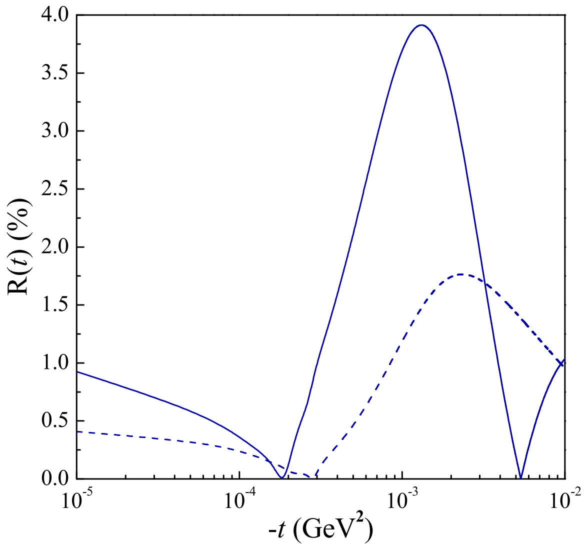

Figure 3: t-dependence of the quantity $R(t)$ defined by Eq. (3.2) corresponding to Fit I (full line) and to Fit IV (dashed line), i.e. to the fit with standard phase, see Sect. 3.2, and to the fit with standard phase and constant diffractive slope, see Sect. 3.4.

are in Fig. 7. The diffractive slope corresponding to Fit II is in Fig. 5 and the diffractive slope corresponding to Fit III is in Fig. 6. Finally corresponding differential cross section given by Eq. (3.1) is in Fig. 4 (green line).

3.4 Constant diffractive slope B

In the case of purely exponential modulus of hadronic amplitude (see (2.6))

$$|F^N(s,t)| = a_1 e^{b_1 t} \tag{3.8}$$

t-independent diffractive slope (3.4) equals $2b_1$ and it is thus constant. On the other hand if the diffractive slope $B(s,t)$ is constant then we can determine modulus of hadronic amplitude from Eq. (3.4). This equation is now differential equation with constant coefficients for t-dependent hadronic modulus $|F^N(s,t)|$. The solution is $|F^N(s,t)| = \tilde{a}_1 e^{Bt/2}$. The parameters a_1, b_1 and $\tilde{a}_1$ are t-independent but they may be s dependent. They are constant for fixed s. In other words constant diffractive slope B is equivalent to purely exponential modulus of hadronic amplitude.

To analyse experimental data under assumption of constant diffractive slope B only we thus use parameterization (3.8) for modulus of hadronic amplitude. To parameterize fully hadronic phase ζ^N we use standard phase (3.5). Obtained fit (Fit IV) is in Table 2. Quantity $R(t)$ corresponding to this fit is in Fig. 3. The diffractive slope corresponding to this fit is in Fig. 2 and finally corresponding differential cross section given by Eq. (3.1) is in Fig. 4 (red line).

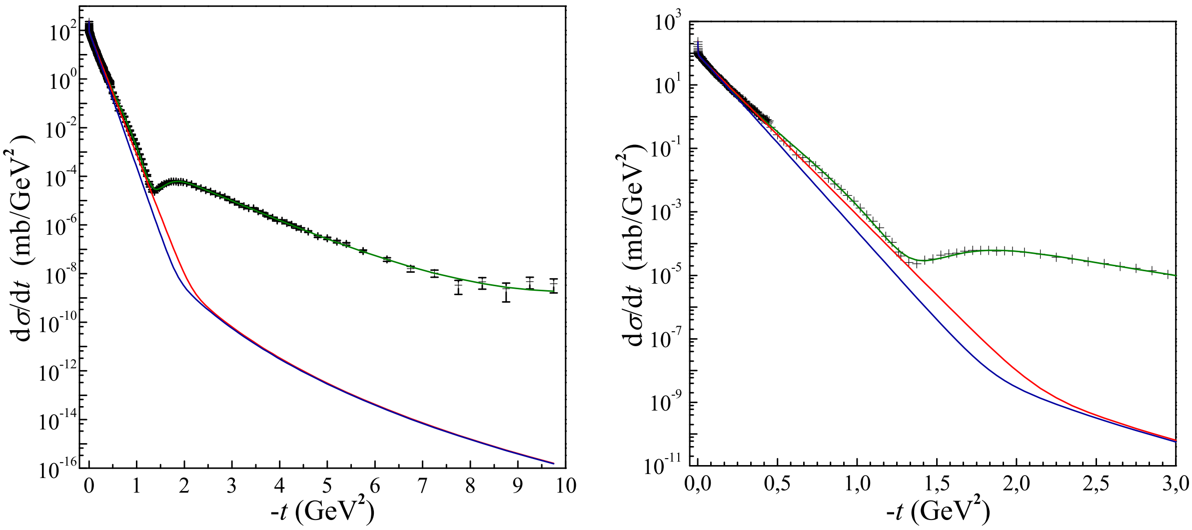

Figure 4: Elastic differential cross section for pp scattering at energy of 53 GeV. Green line corresponds to Fit I, Fit II and Fit III (non important differences are between these fits) i.e. to non-constant but t-dependent diffractive slope defined by Eq. (3.4). Red line corresponds to Fit IV and Fit V (non important differences between these fits again). Blue line corresponds to Fit VI. These last three fits contain assumption of constant diffractive slope (3.4). Right figure is left figure zoomed into region of -t up to 3 GeV^2.

3.5 Constant B and ρ

In the case of constant diffractive slope B and quantity ρ we use parameterization (3.8) for hadronic amplitude and (3.7) for hadronic phase. The parameters fitted to experimental data are in Table 2 (Fit V). Fit VI corresponds to constant parameters B and ρ fixed on the values taken from Table 1. Quantities $R(t)$ corresponding to these fits are in Fig. 8. The constant diffractive slope corresponding to Fit V resp. Fit VI is in Fig. 5 (red full line) resp. in Fig. 6 (red dashed line). Differential cross sections given by Eq. (3.1) and corresponding to these two fits are in Fig. 4 (red line for Fit V and blue line for Fit VI).

Elastic differential cross sections corresponding to non-constant diffractive slope B, see Fig. 4, are for large values of -t practically given by the hadronic interaction. The Coulomb interaction has neglectable effect. On the contrary, in the case of fits with constant diffractive slope (see again Fig. 4) the effect of hadronic interaction may be neglected for large values of -t compared to the Coulomb interaction. It is of course in disagreement with our assumption that the hadronic interaction is short-ranged and much more stronger then the Coulomb interaction; the elastic differential cross section for large values of momentum transfer -t should be given only by the strong interaction.

In Sec. 1.2 we have mentioned that the momentum transfer -t must be lower then kinematically allowed maximum value. The maximum value of momentum transfer -t may be limeted further on the basis of the fact that elastic collisions are always accompagnied by inelastic processes. They are only inelastic processes for lower values of impact parameter that would correspond to high values of -t (in the case of high energies of colliding particles which we are interested in). One should ask how much the later limitation differs from the formel and also how the size of colliding protons affect the physically allowed region of momentum transfer.

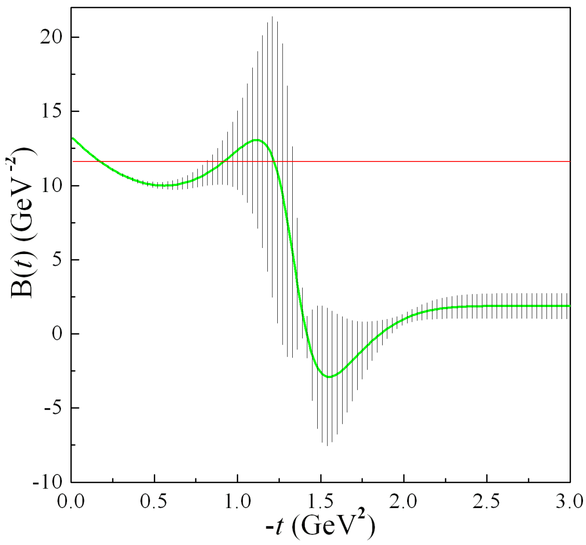

Figure 5: t-dependence of the diffractive slope defined by Eq. (3.4). Green full line corresponds to Fit II i.e. to the fit with free constant quantity ρ, see Sect. 3.3. Red full line corresponds to Fit V i.e. to the fit with free constant both constant diffractive slope B and ρ, see Sect. 3.5.

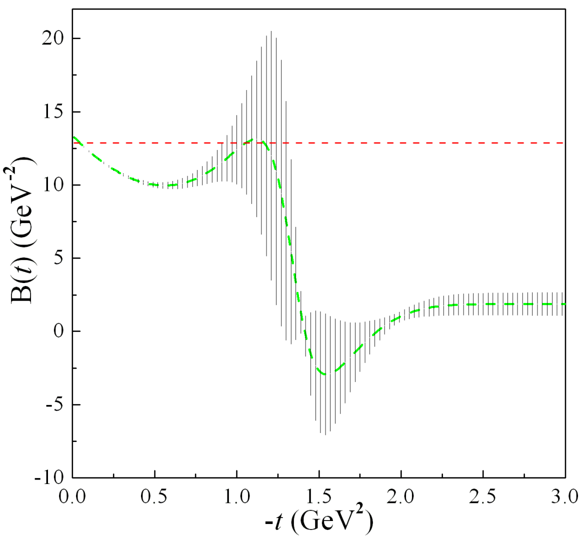

Figure 6: t-dependence of the diffractive slope defined by Eq. (3.4). Green dashed line corresponds to Fit III i.e. to the fit with constant quantity ρ taken from Table 1, see Sect. 3.3. Red dashed line corresponds to Fit VI i.e. to the fit with both constant diffractive slope B and ρ taken from Table 1, see Sect. 3.5.

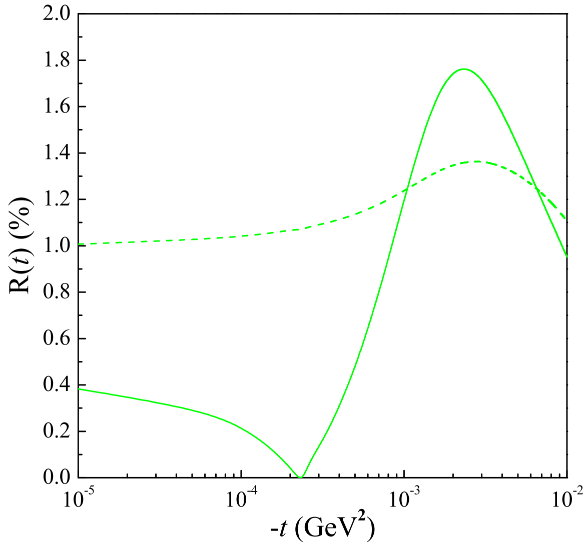

Figure 7: t-dependence of the quantity $R(t)$ defined by Eq. (3.2) corresponding to Fit II (full line) and to Fit III (dashed line), i.e. to the fit with free constant ρ and to the fit with constant ρ taken from Table 1, see Sect. 3.3.

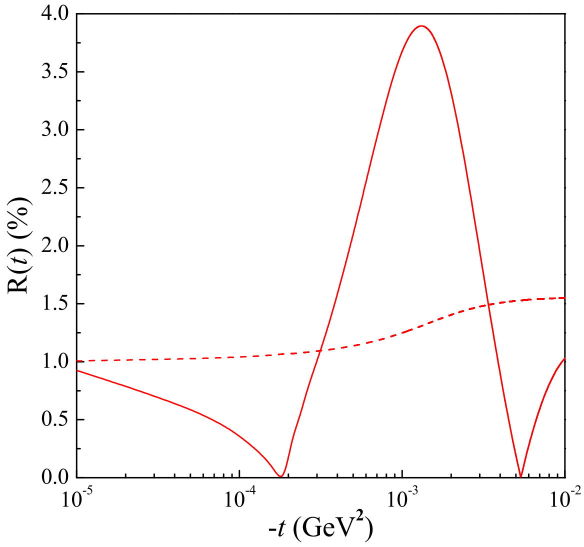

Figure 8: t-dependence of the quantity $R(t)$ defined by Eq. (3.2) corresponding to Fit V (full line) and to Fit VI (dashed line), i.e. to the fit with free constant both B and ρ and to the fit with constant both B and ρ taken from Table 1, see Sect. 3.5.

Quantity	Fit I	Fit II	Fit III	Fit IV	Fit V	Fit VI
ρ	-	0.0648 ± 0.0017	0.077 ± 0.009	-	0.0211 ± 0.0016	0.077 ± 0.009
ρ_0	0.0646 ± 0.0017	-	-	0.0208 ± 0.0016	-	-
$t_{diff}\,[GeV^2]$	-1.375	-	-	-1.375	-	-
a_1	12142 ± 28	12140 ± 14	12096 ± 11	11928.2 ± 7.5	11928.8 ± 7.5	12153.1 ± 4.2
$a_2\,[GeV^{-2}]$	10809 ± 50	10814 ± 90	10738 ± 84	-	-	-
$b_1\,[GeV^{-2}]$	5.786 ± 0.012	5.787 ± 0.017	5.828 ± 0.015	5.8077 ± 0.0077	5.8085 ± 0.0078	6.435 ± 0.070
$b_2\,[GeV^{-4}]$	3.127 ± 0.040	3.130 ± 0.050	3.217 ± 0.047	-	-	-
$b_3\,[GeV^{-6}]$	1.328 ± 0.026	1.329 ± 0.035	1.375 ± 0.034	-	-	-
c_1	60.6 ± 5.1	61 ± 10	57.13 ± 9.3	-	-	-
$c_2\,[GeV^{-2}]$	-0.71 ± 0.0040	-0.35 ± 16.48	-0.36 ± 13.81	-	-	-
$d_1\,[GeV^{-2}]$	0.840 ± 0.082	0.84 ± 0.28	0.80 ± 0.25	-	-	-
$d_2\,[GeV^{-4}]$	-0.044 ± 0.018	-0.043 ± 0.033	-0.051 ± 0.031	-	-	-
$d_3\,[GeV^{-6}]$	-0.0056 ± 0.0015	-0.0056 ± 0.0024	-0.0062 ± 0.0023	-	-	-
χ^2/df	672.020/231	673.843/232	687.965/233	11730.1/240	11724.7/240	16112.6/242

Table 2: Values of free parameters. Fit I corresponds to the fit with standard hadronic phase, see Sect. 3.2. Fit II, resp. Fit III, corresponds to the fit with free, resp. fixed, quantity ρ, see Sect. 3.3. Fit IV corresponds to the fit with standard hadronic phase and constant free diffractive slope B, see Sect. 3.4. Fit V, resp. Fit VI, corresponds to the fit with constant free, resp. fixed, both diffractive slope B and quantity ρ, see Sect. 3.5. Symbol χ^2/df means total chi-square divided by number of degrees of freedom.

3.6 Luminosity at the LHC

The luminosity L is a constant quantity which in elastic processes bounds together a counting rate $\Delta N(t)$, the number of counts per unit of time in a small interval around momentum transfer Δt with the corresponding differential cross Sect. [18]

$$\Delta N(t) = L\frac{d\sigma}{dt}. \tag{3.9}$$

The normalization factor L has units of $(\text{area})^{-1}(\text{time})^{-1}$. The differential cross section defined by Eq. (1.20) is determined with the help of the total elastic scattering amplitude $F^{C+N}(s,t)$, which can be calculated at small values of t either according to the WY simplified approach or by the eikonal more precise approach. Experimentally, the luminosity at the LHC are planed to be determined at the t lying inside the interference region. Our results plotted in Figs. 3, 7 and 8 show that if such an approach of luminosity determination, based on the estimation of simplified WY total elastic amplitude, is used then the luminosity would be burned by a systematic error approaching the value of $4 \div 5$ %, similarly as in [19] which overcomes the planed luminosity determination at the LHC planned to 1 - 1.5 %, see [1], [2] and [3].

3.7 Interference term

Eq. (2.11) may be rewritten into the form

$$F^{C+N}_{KL}(s,t) = F^C(s,t) + F^N(s,t) + F^I(s,t) \tag{3.10}$$

where we have introduced interference term $F^I(s,t)$. The importance of this term for elastic differential cross section given by Eq. (3.1) may be represented by fraction

$$f(s,t) = \frac{|F^{C+N}_{KL}(s,t)|^2 - |F^C(s,t)|^2 - |F^N(s,t)|^2}{|F^N(s,t)|^2}. \tag{3.11}$$

Fits I, II and III match best to experimental elastic differential cross section of all the six performed fits, see Fig. 4. These fits do not contain assumption of constant but t-dependent diffractive slope B defined by Eq. (3.4). Relative contribution of interference term to elastic cross section represented by fraction $f(s,t)$ corresponding to Fit I, Fit II and Fit III is plotted in Fig. 9. Differences between these fractions $f(s,t)$ are significant mainly before and around region of diffractive dip ($-t \sim 1.3\, GeV^2$).

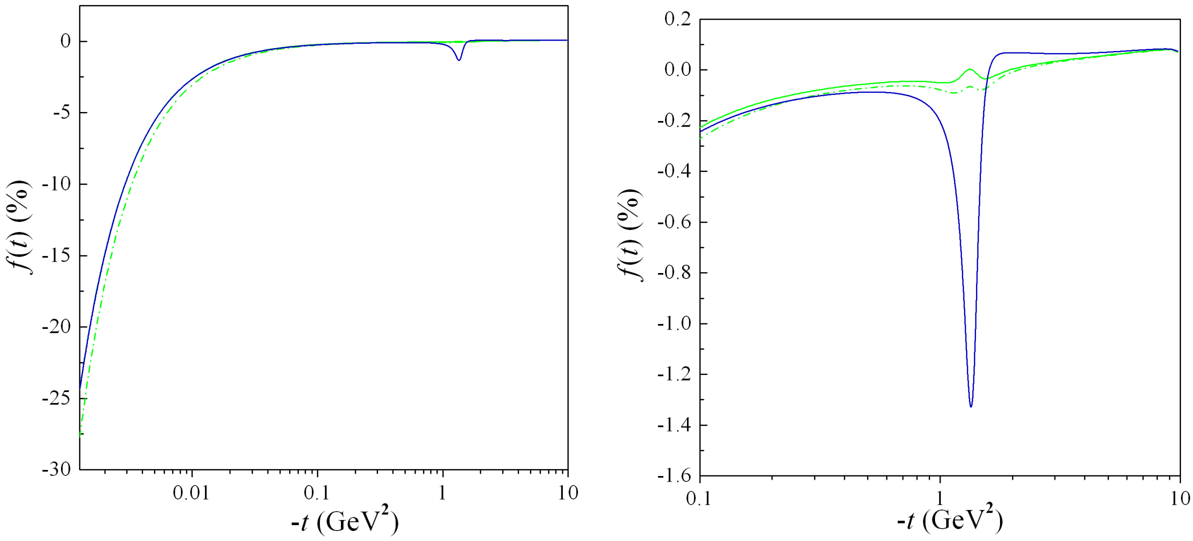

Figure 9: Relative contribution of the interference term for pp scattering at energy of 53 GeV. Blue full line corresponds to Fit I, see Sect. 3.2. Green full resp. dashed line corresponds to Fit II resp. Fit III, see Sect. 3.3. Right figure is left figure zoomed into the region before and around diffractive dip, i.e. in the region of $-t \sim 1.3\,GeV^2$.

Chapter 4

Conclusion

Until now practically all performed analysis of elastic high-energy hadron scattering have been based on West-Yennie simplified formula. In the presented analysis we have used experimental data obtained for 53 GeV proton-proton collisions and demonstrated the differences caused by individual assumptions on which the given simplified West-Yennie formula has been based.

Fit I and Fit II (and also Fit III) have the same parameterization of modulus of hadronic amplitude but they have different expression for hadronic phase. Fit I contains standard t-dependent hadronic phase (3.5) and Fit II involves constant hadronic phase; nevertheless, corresponding chi-squares are similar, see Table 2.

The fits with assumption of constant diffractive slope B, which is involved in West-Yennie formula, i.e. Fit IV, V and VI have much worse chi-square compared to Fit I, II and III (with t-dependent diffractive slope), see Table 2.

The purely exponential t-dependence of hadronic modulus, which is equivalent to constant diffractive slope B, corresponds to observed experimental data for the pp elastic scattering only for t running from the forward direction to diffractive minimum. The purely exponential t-dependence of hadronic modulus is without any doubt in contradiction to high-energy elastic nucleon experimental data for higher absolute values of momentum transfer $|t|$, see Fig. 4.

Choice of parameterization of modulus of hadronic amplitude is thus much more significant in describing experimental data of differential cross section compared to parameterization of phase of hadronic amplitude.

The fractions $f(s,t)$ for these fits have an anomaly at $-t \sim 1.3\ GeV^2$ (before and around the diffractive dip) as we can see in Fig. 9. This effect shows that the influence of Coulomb scattering at higher $|t|$ values can be hardly neglected.

Our results also show that the luminosity can be burdened by $4 \div 5$ % systematic error if determined by the standard methods based on the application of simplified WY total scattering amplitude.

References

[1] The TOTEM collaboration, Letter of Intent, CERN/LHCC 97-49, LHCC/I 11 (1997).

[2] The TOTEM collaboration, Technical Proposal, CERN/LHCC 99-7, LHCC/P5 (1999).

[3] The TOTEM collaboration, Technical Design Report, CERN-LHCC-2004-002 (2004).

[4] V. Barone and E. Predazzi, High-Energy Particle Diffraction (Springer, 2002).

[5] H. Pilkuhn, The Interaction of Hadron (North-Holldand, 1967).

[6] R. J. Glauber, in Lectures in theoretical physics Vol.1 (Interscience Publishers N.Y, 1959).

[7] M. M. Islam, in Proceedings of the Theoretical Physics Institute, Univ. of Colorado, sumer 1967, p.97.

[8] G. B. West and D. R. Yennie, Phys. Rev. 172 (1968) 1413.

[9] V. Kundrát, Elastic hadron scattering at high energies, Doctor of Mathematical and Physical sciences Thesis, Institute of Physics AV ČR, 1996.

[10] V. Kundrát and M. Lokajíček, Mod. Phys. Lett. A 11 (1996) 2241.

[11] V. Kundrát and M. Lokajíček, Phys. Lett. B 611 (2005) 102.

[12] V. Kundrát and M. Lokajíček, Z. Phys. C 63 (1994) 619.

[13] F. Borkowski et al., Nucl. Phys. B 93 (1975) 461.

[14] J. Bystricky et al., in Nucleon-nucleon and kaon-nucleon scattering edited by H. Schopper (Landolt-Börnstein Series, Vol. 1), (Springer, Berlin, 1980).

[15] M. K. Carter, P. D. B. Collins and M. R. Whalley, Compilation of Nucleon-Nucleon and Nucleon-Antinucleon Elastic Scattering Data , RAL-86-002, preprint.

[16] N. Amos et al., Nucl. Phys. B 262 (1985) 689.

[17] F. James and M. Roos, Comput. Phys. Commun. 10 1975 343.

[18] M. M. Block and R. N. Cahn, Rev. Mod. Phys. 57 (1985) 563.

[19] V. Kundrát and M. Lokajíček, 'Coulomb and hadronic scattering in elastic high-energy nucleon collisions'; Proceedings of the XVIIth Rencontres de Blois, France, May 15 - 20 (2005), p. 115.