Posudek vedoucího pro diplomovou práci Moniky Fornůskové

Jan M. Swart

26. července 2019

Autor:	Monika Fornůsková
Typ práce:	diplomová
Název práce:	The Stigler-Luckock model for a limit order book
Jméno vedoucího:	Jan M. Swart

Způsob pracování

Monika Fornůsková started working on the thesis in April 2018. The time until the summer was spent mainly on reading the relevant literature. Monika spent the second half of 2018 in Australia, where she originally planned to work on the thesis, but in the end did not find very much time to do this. After her return, she started working intensively on the thesis from early January on. The thesis consists of two parts: a practical part with original work, which is mainly based on numerical simulations, and a theoretical part which puts the results into the framework of the existing literature. In the work on the theoretical part she showed herself capable of independently reading and in part also finding the relevant literature. In the practical part, she programmed the simulations very efficiently and without help. She needed some help with the theoretical explanation of the numerical results and with making the link between the practical model and more abstract mathematical concepts from, e.g., game theory, but in the end she also managed this well. While I think certain things could in places be explained better and more concisely, I am happy that the text is really the text of the author and she never just copied what I suggested without properly understanding it. The thesis is written in English, which I had to correct from time to time (in particular the articles) and which still leaves room for improvement, but is easy to read and understand.

K samotné práci

The aim of the work was to improve a model for the evolution of an order book on a stock market originally developend by Stigler (1964) and Luckock

(2003), further studied by Kelly and Yudovina (2018), Swart (2018), and extended in Peržina and Swart (2018). In the original model, limit orders of unit size arrive according to independent Poisson processes. The frequencies of buy limit orders below a given price level, respectively sell limit orders above a given level, are described by fixed demand and supply functions. Buy (respectively, sell) limit orders that arrive above (respectively, below) the current ask (respectively, bid) price are immediately matched to the best available opposite order. As Luckock (2003) showed, the model is rather unrealistic in the sense that the bid and ask prices do not converge to the equilibrium price predicted by classical theory going back to Walras (1874) but instead keep fluctuating in a "competitive window" of considerable size.

This observed behavior (which has partially been backed up by rigorous statements) opens an opportunity for market makers who place both buy and sell limit orders with the aim of making a profit from the spread, and in doing so provide liquidity to the market. As shown in Peržina and Swart (2018), the addition of such market makers can make the model more realistic in the sense that for a suitable choice of parameters it can lead to the bid and ask prices converging to the predicted Walrasian equilibrium price. Nevertheless, the model in Peržina and Swart (2018) is still rather unrealistic as far as the behavior of the market makers is concerned.

The aim of the present thesis is to come up with more realistic trading strategies for market makers in a modified Stigler-Luckock model, where, in contrast to the original model, "normal" traders who are not market makers do not place limit orders but instead take the best available suitable offer, if there is one, and do nothing otherwise. In the thesis, the author tries to find "good" strategies by letting market makers with limited resources trade on a market with a fixed set of 100 possible prices.

If there is just a single market maker, then, unsurprisingly, the optimal strategy from the point of the market maker is to trade with a rather large spread, since the profit of the market maker per trade is proportional to the size of the spread. If several market makers compete with each other, however, then the normal traders will only trade with the market maker offering the smallest spread, and hence in such a scenario the optimal strategy consists of offering the smallest possible spread, i.e., one tick size. Although these results were obtained by numerical simulation, it seems they should be amenable to mathematical proof.

A much more interesting situation arises when supply and demand, and as a result the Walrasian equilibrium price, are not constant in time. In the thesis, the author considers a hidden Markov model where the state of the market switches between two states, one with high demand and low supply,

and another one with low demand and high supply. If the model switches frequently between the two states, then it is not very different from the previous model, but in the thesis the parameters are chosen in such a way that the market makers do not have sufficient resources to keep trading at a fixed price and hence are forced to detect and follow the changes in the equilibrium price due to the nonconstant supply and demand.

This model appears to me much more difficult than the model with constant supply and demand, and a full mathematical description of the optimal strategy for market makers may well be out of reach. Nevertheless, as the author shows, with the help of numerical simulations trading strategies can be found that appear to be better than others and important insight can be gleaned.

F. Kelly, and E. Yudovina. A Markov model of a limit order book: thresholds, recurrence, and trading strategies. *Math. Operat. Res.* 43 (2018), 181–203.

H. Luckock. A steady-state model of the continuous double auction. *Quant. Finance* 3 (2003), 385–404.

V. Peržina and J.M. Swart. How much market making does a market need?. *J. Appl. Probab.* 55(3) (2018), 667–681.

G.J. Stigler. Public regulation of the securities markets. *J. Business* 37 (1964), 117–142.

J.M. Swart. Rigorous results for the Stigler-Luckock model for the evolution of an order book. *Ann. Appl. Probab.* 28(3) (2018), 1491–1535.

L. Walras. *Éléments d' Économie politique pure; ou, Théorie de la Richesse Sociale.* Corbaz, Lausanne, 1874.

Závěr

In general, the thesis is well written. I think that in some places the theoretical explanations could be more precise and more mathematical, and the numerical simulations could perhaps in some places have been done more systematically. For this reason, I believe the results as they stand are not solid enough to be publishable in a scientific journal. Nevertheless, the model is interesting and the text certainly fulfills the requirements of a Master thesis.

Doporučuji práci uznat jakou diplomovou.