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Faculty of Social Sciences
Institute of Economic Studies

MASTER'S THESIS

Are realized moments useful for stock market returns analysis?

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Declaration of Authorship

The author hereby declares that he compiled this thesis independently; using only the listed sources and literature, and the thesis has not been used to obtain a different or the same degree.

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Prague, December 24, 2018

Signature

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Abstract

This thesis analyzes the use of realized moments in asset pricing. The analysis is done using dataset containing logreturns for 29 of the most traded stocks and covering 10 years of data. The dataset is split into training set covering 7 years and test set covering 3 years of data. For each of the stocks a separate time series model is estimated. In evaluation of the quality of the models, metrics such as RMSE, MAD, accuracy in forecasting the sign of future returns, and returns achievable by executing trades based on the recommendations from the model are used. Even though the inclusion of realized moments does not provide significant improvements in terms of RMSE, it is found that realized skewness and kurtotis significantly contribute to explaining the returns of individual stocks as they lead to consistent improvements in identifying future positive, as well as negative, returns. Moreover, the recommendations from the models using realized moments can help us achieve significantly higher returns from trading stocks. Inclusion of the interaction terms for variance and returns, skewness and returns, and kurtosis and variance, provides additional improvement of forecasting accuracy, as well as improvements in returns achievable by executing transactions based on recommendations from the model.

JEL Classification	C18, C58, G15
Keywords	realized moments, skewness, kurtosis, asset pricing, stock market
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Abstrakt

Tématem této práce je analýza využití realizovaných momentů k oceňování aktiv. K analýze je použit dataset obsahující data o devětadvaceti z nejvíce likvidních veřejně obchodovaných akciových spolecností. Tento dataset pokrýva období deseti let a je dále rozdělen na trénovací a testovací dataset, kde první obsahuje data za 7 let a druhý data za zbývající 3 roky. Pro každou z 29 společností je odhadnut vlastní model. K posouzení kvality jednotlivých modelů jsou využity metriky, jako jsou RMSE, MAD, přesnost v předpovídání kladných a záporných výnosů a výnosy dosažitelné díky obchodování na základě předpovědí modelu. Ačkoliv zahrnutí realizovaných momentů nevede k výraznému snížení RMSE, výsledky ukazují, že realizovaná skloněnost a špičatost přispívají k vysvětlení výnosů akcií jednotlivých společností. Tento přínos je zachycen skrze zvýšenou schopnost modelu správně předpovídat, jestli budoucí výnos bude kladný, nebo záporný. Dále využití realizovaných momentů může investorům umožnit dosažení výrazně vyšších výnosů z obchodování v porovnání s pasivním držením diveryifikovaného portfolia akcií. Zahrnutí součinu rozptylu a výnosů, skloněnosti a výnosů, a špičatosti a rozptylu, přináší další zlepšení v přesnosti modelu a dosažitelných výnosů z obchodování s akciemi.

Klasifikace	C18, C58, G15
Klíčová slova	Realizované momenty, skloněnost, špičatost, oceňování aktiv, akciové trhy

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Acronyms

TQ Top Quartile

PP Positive Predictions

RMSE Root Mean Squared Error

MAD Mean Absolute Deviation

Master's Thesis Proposal

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Defense Planned:	June 2018

Proposed Topic:

Are realized moments useful for stock market returns analysis?

Motivation:

It has been a never-ending struggle to understand the properties of stock returns and be able to forecast their future development. From simple AR and MA models, through GARCH and their fractionally integrated versions, to intra-day data based models such as HAR, researchers and investors alike have been trying to analyze and forecast the behavior of returns. The question of stock return properties is of the utmost importance when it comes to portfolio or risk management.

This thesis aims to gain further understanding of stock returns on two different levels: 1) Further understand distributional properties of returns – so far, there have been various attempts to forecast future returns and future volatility. This thesis aims to explore whether it is possible to predict also realized skewness and kurtosis and gain additional insights into the distribution of returns. 2) Establish, whether current realized skewness or kurtosis can be used in forecasts of future returns or volatility, and whether disregarding skewness and kurtosis has a detrimental effect on the quality of such predictions.

The intuition behind the use of skewness and kurtosis in the financial market modelling is, that volatility should not be the only measure of risk on the market. Two distributions with same mean and variance might have completely different kurtosis: thus, when speaking about returns, the two distributions also carry a different amount of risk as a high kurtosis increases the uncertainty about the actual returns. Moreover, the distribution of returns need not be symmetric, thus making positive returns more or less likely than negative returns. Thus, additional knowledge about two moments might be very helpful in investors' decision making.

The current literature - such as Amaya et al (2015) or Chang et al (2013) - mostly focused on cross-sectional analysis of returns and their realized moments. It is the goal of this thesis to address also the time-series forecasting of future returns using the realized moments.

Furthermore, Corsi and Reno (2009) use HAR with heterogeneous leverage and jumps to model volatility. It seems natural to ask, whether inclusion of realized skewness and kurtosis in the model would have a significant positive impact on the said model performance.

Hypotheses:

1. Hypothesis #1: realized moments can be used to forecast future returns.
2. Hypothesis #2: current realized moments are not independent of past realized moments
3. Hypothesis #3: higher kurtosis should be compensated by higher expected return
4. Hypothesis#4: realized moments improve HAR's performance in 1 day ahead volatility forecasting
5. Hypothesis #5: realized moments improve HAR's performance in multi-period volatility forecasts

Methodology:

The current plan is to use 5-minute intraday data for 29 most liquid US companies spanning the years 2005-2015. This dataset includes information about the prices in each of the time intervals. From this information, log returns will be calculated and subsequently used to calculate the realized measures.

The above-mentioned measures will be used for the analysis of stock market returns as described in the Motivation section of this proposal. The individual time series will be modelled using these measures and the validity of the models will be evaluated using the out of sample forecast together with a benchmarking against some of the other techniques used in financial time-series modelling, such as ARIMA or HAR for log-returns and GARCH or HAR for volatility. Where applicable, robust statistical methods shall be used to verify the estimation results. While the robust methods in time-series setting might not be wide-spread, there have been some applications even in the volatility modelling (Croux et al. (2011))

One additional possible technique to model the joint fluctuation of returns, volatility, skewness and kurtosis might be a VAR model, but I will have to explore this option further in the process of working on the thesis.

Expected Contribution:

This thesis hope to provide further insights into the behavior of stock market returns. If the results of the research suggest that the realized measures are useful for the analysis of stock returns, these measures might be used for investment decisions and risk management. Otherwise, it will be evident that some different approaches should be explored.

Outline:

1. Introduction
2. Literature Review
3. Data
4. Methodology
5. Results
6. Conclusion

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Author **Supervisor**

1 Introduction

The statistical properties of a price process are crucial to understanding the behavior of financial markets. The volatility has been already studied for decades and the current literature focuses mostly on realized volatility estimators. In recent years, estimators of higher moments, such as skewness and kurtosis, have been of an increasing interest. This thesis adds to the existing literature by examining the usability of realized moments in individual stocks' returns analysis. Moreover, sensitivity of the obtained results to changes in estimators or model specification is analyzed. The quality of the individual models is determined based on extensive analysis of their performance on the test set. The following paragraphs outline an introduction to the field of asset pricing, provide a reasoning on why risk is thought to be so important in determining the price of an asset and how realized moments can be used to approximate the factors describing the consumption risk.

The asset pricing theory aims to understand the process leading to prices of uncertain payments, or in other words, understand why one asset gives higher return than another. In theory, the stock market should balance so that price and the expected discounted return of an asset are identical. Using the notation used by Cochrane (2009), we can summarize this idea by the following formulas:

$$p_t = E[m_{t+1} * x_{t+1}] \tag{1.1}$$

$$m_{t+1} = f(data, parameters) \tag{1.2}$$

With p_t being the asset price, x_{t+1} the payoff and m_{t+1} the stochastic discount factor. Equation (1.1) summarizes the relation between the expected discounted payoff and price, while Equation (1.2) represents some general form of the stochastic discount factor. In the classic consumption-based model, the stochastic discount factor m_{t+1} can be written as a product of the individual discount factor and the marginal utilities from consumption in two consecutive time periods:

$$m_{t+1} = \beta * \frac{u'(c_{t+1})}{u'(c_t)} \tag{1.3}$$

Thus, a stochastic discount factor, which is also sometimes called a pricing kernel, can be understood as a rate at which the investor is willing to exchange consumption between the two periods. The pricing kernel is nothing more than a valuation operator creating a link between risk and risk premium.

The goal of the empirical literature is then to find the form of this pricing kernel. Several approaches to approximating the kernel can be found in the literature on asset pricing. One of the methods would be to use the factor pricing models, such as CAPM. These models use a linear model instead of the marginal utility growth used in the consumption model. Once again using the notation of Cochrane (2009), the form of the linear model is given by the following equation:

$$m_{t+1} = a + b' * f_{t+1} \tag{1.4}$$

Where a and b are parameters, while f stands for the factors. The variables used as factors would usually be those indicating occurrence of an event unpleasant to the investor.

The availability of high-frequency data allows us to approximate the pricing kernel using realized moments, such as variance, skewness and kurtosis. As Merton (1980) noted: the volatility of the price process becomes observable as frequency of returns increases. Moreover, skewness and kurtosis can be also observed, and over time various realized moments estimators have been developed. These estimators leveraging high-frequency data can be used to obtain ex-post estimates on the actual levels of risk faced by the investors. Based on the observed levels of risk, we can in turn form our expectations regarding the required risk premium.

The realized variance, skewness and kurtosis provide an intuitive way to describe the risks faced by the investor, because all of them are related to events or states of the world which the investor would rather avoid. Variance measures the uncertainty of the payoffs. The intuitive argument for using variance is that, with increasing uncertainty of the payoff, the investor should demand higher risk premium as a compensation for taking on the risk to his future level of consumption. That being said, while variance has been established as a solid determinant of required risk premium, it doesn't fully describe the risk faced by the investor. While two assets can have the same variance, they might have different likelihoods of individual payoffs, which in turn might result in one asset being more likely to produce negative payoff and the other to produce a positive payoff. Therefore, realized skewness is used as a determinant of the required risk premium. On top of the intuitive argument, there are several theoretical arguments creating the link between assets' skewness and return. They conclude that, in theory, there should be a negative relationship between asset's skewness and return: under the cumulative prospect theory assets with higher skewness should yield lower returns, as was shown Barberis and Huang (2008); Brunnermeier, Gollier, and Parker (2007) and Mitton & Vorkink (2007) derived similar results based on different assumptions. While the author of this thesis is not aware of comparable theoretical results for the relationship between returns and kurtosis, there is at least an intuitive argument for using it: it is possible that investor would like to avoid extremely bad payoffs. At the same time, one can argue that an asset can achieve given level of realized variance either through a sequence of moderate fluctuations or through some small and then one extreme. By including the realized kurtosis in the model, we distinguish between these scenarios.

Using a dataset on 29 frequently traded companies covering years 2005 – 2015, a simple pricing model using past returns, realized variance, skewness and kurtosis as determinants of risk premium is constructed. For each company, a separate estimation is done on a training set containing 70% of the observations. Subsequently, the model's performance is analyzed on the test set which contains the remaining 30% of the observations. The model's ability to explain stock market returns is compared to the AR1 model. Moreover, several alternative estimators of realized moments are used and multiple model extensions and restrictions are considered. All of these models are evaluated based on common accuracy metrics such as RMSE, MAD and accuracy in correctly identifying the sign of the asset return. Furthermore, the models are used to create trading strategies, and the returns achievable by following those strategies are analyzed.

It is found that every single one of the realized moments – variance, skewness and kurtosis – contribute to the model performance in terms of achievable returns as well as the model's ability to correctly identify the direction of future asset price movements. On the other hand, these achievements do not translate into corresponding improvements in standard accuracy measures, such as RMSE or Diebold-Mariano test, which might lead to incorrect conclusions on the usability of realized moments had we failed to dig deeper in analyzing the results. Use of realized moments in asset pricing resulted in 60-120% higher return compared to passively holding stocks over the entire time period. While this comparison disregards the transaction costs, it is clear that with sufficient scale a trader would benefit from the use of model based on realized returns even in face of relatively steep fixed transaction costs imposed upon small agents. Furthermore, it has been found that use of robust estimators of moments can improve the results further. Moreover, accounting for interaction terms between past return and variance, past return and skewness, and past variance and kurtosis can provide almost 30% additional increase in expected returns and lift the accuracy of predicting the sign of future returns from 50.7% to well over 51%.

The rest of the thesis is organized as follows. First, the literature on the realized moments is reviewed. Included are important theoretical and empirical results relevant to the topic of this thesis. In the following chapter, the methodology is outlined and the analyzed dataset is described. The chapter includes information on the used estimators, the analyzed models and the evaluation criteria for determining the model quality. Next, results are discussed. Last chapter concludes this thesis.

2 Literature review

The thesis builds on large literature on realized volatility and significant amount of current research on higher realized moments. Amaya et al (2015) provide comprehensive review of the state of empirical research on this topic. Daily returns have been used to analyze economic drivers of the secular variation in market volatility by Schwert (1989) and Paye (2012). The use of intraday trading data was pioneered by Hsieh (1991) in his work on nonlinear dynamics in financial markets. Individual assets' realized volatility was studied by Andersen, Bollerslev, Diebold & Ebens (2001). The idea of using realized volatility in portfolio allocation was explored by Fleming, Kirby & Ostdiek (2003).

According to Merton (1980), increased sampling frequency allows for unlimited precision of volatility measurement. This finding was a precursor to development of daily volatility measures based on intraday returns. The currently most popular measure of realized volatility, which was famously used in Andersen and Bollerslev (1998), is sum of squared intraday log returns. This measure has a model-free property, which makes it useful in volatility analysis. For details see Andersen et al. (2001). Nielsen et al. (2010) proposed realized semi-variance as a measure of downside risk. They argue that negative and positive shocks might have different impact on future volatility and therefore volatility stemming from these shocks should be measured separately. Next, they also discuss the possibility to use realized bipower downward variation (BPDV) which can be used to analyze jumps in the data. The BPDV is an extension to bipower variation as mentioned in Barndorff-Nielsen and Shephard (2004) and the argument for its use is equivalent to that used for semi-variance.

One theoretical result for relation of skewness and returns is provided by Barberis and Huang (2008) who show that assuming investors make decisions according to cumulative prospect theory implies assets with greater skewness have lower returns.

Amaya et al. (2015) use realized skewness measure defined as sum of cubed short-period log returns. Once again, this measure is calculated from high frequency returns and yields an unbiased estimator of longer periods skewness. As in case of realized variance, realized skewness has the model-free property. Recent work by Neuberger & Payne (2017) shows that the above mentioned realized skewness measure represents only skewness of short-horizon returns. The catch is skewed short-horizon returns might yield symmetric long-horizon returns. They show that skewness of long-horizon returns has two components: 1) skewness of short horizon returns, 2) covariance of current period variance and lagged return. Therefore, the frequent empirical finding that realized skewness might be insignificant determinant of future returns or volatility might be purely due to a fact that researchers are using incorrect measure of realized skewness. As the measure proposed by Neuberger & Payne (2017) is technically a linear combination of the two factors, skewness of short horizon return and covariance of current period variance and lagged return, the use of the original measure employed by Amaya et al (2015) would be equivalent to incorrect model specification leading to an omitted variable bias in the estimates. It seems reasonable to assume that skewness of long-horizon returns would be more important determinant of future returns than skewness of, say, 5-minute returns.

The definition of realized kurtosis used by Amaya et al (2015) is defined as a sum of short-period log-returns raised to the power of 4. As in the case of skewness, Neuberger & Payne (2017) argue that this does not capture the kurtosis of long-period returns. Therefore, they propose a measure based on three components: 1) kurtosis of short-horizon returns, 2) lagged high-frequency returns and cubed returns covariance, 3) covariance of squared short-horizon returns and lagged squared returns.

As mentioned in Neuberger & Payne (2017) it is worth pointing out that the usual standardized measures of realized skewness and kurtosis are unbiased estimators of the respective variables only if skewness and kurtosis are uncorrelated with volatility.

The relationship between skewness and returns has been examined by several studies in recent years. The research of Zhang (2006) shows a negative relation between skewness and stock returns. Boyer, Mitton & Vorkink (2010) find stocks with higher expected idiosyncratic skewness to have lower future returns. As discussed by Amaya et al (2015) options-based skewness measures were analyzed with mixed results. On one hand, works of Xing, Zhang & Zhao (2010) and Rehman & Vilkov (2010) find a positive relation between skewness and returns. On the other hand, Conrad, Dittmar & Ghysels (2013) performed similar analysis with the opposite result.

Amaya et al (2015) find that realized skewness and kurtosis are significant determinants of future returns even in regression including other frequently used variables such as number of analysts following the firm suggested by Arbel and Strebel (1982), market beta and lagged return used in Jegadeesh (1990) and Lehmann (1990), book-to-market ratio used by Fama & French (1993) and others.

There are two main approaches one might want to explore in order to make the estimation more robust and predictions more accurate: 1) use measures robust to outliers as in Amaya et al (2015) who examines several options, such as drift-adjusted realized moments, realized skewness and kurtosis scaled by jump robust realized variance measure, subsampling and Bowley skewness; or more traditional robust measures of skewness and kurtosis, such as was done by Bonato (2011) who uses skewness measures proposed by Bowley (1920), Groeneveld & Meeden (1984) and the Pearson coefficient as described in Kendall & Stuart (1977). The empirical analysis by Bonato draws significantly on the work of Kim & White (2004), who argue that the standard, average-based measures of skewness and kurtosis are likely to perform badly in presence of outliers and they suggest that non-standard measures of skewness and kurtosis can provide deeper insights into market returns behavior. The traditional measure of kurtosis measures the spread of a distribution around mean +- standard deviation, and so kurtosis can be high if majority of the data is concentrated around mean or in thick tails (Moors 1988). Obviously, this might cause some problems in using kurtosis as a measure of tail risk, because we can't distinguish between the above mentioned two cases and yet they represent very different circumstances in financial data. Following the example of Bonato (2011), I also explore the use of quantiles-based kurtosis measure proposed by Crown and Siddiqui (1967). Using these measures, Bonato (2011) concludes that they perform better for distributions with thick tails even if they have finite higher moments; 2) the other option is to use robust estimation methods which put lower weight on the outlying observations, such as least weighted squares, weighted least squares, trimmed least squares or least trimmed squares. For details on outlier robust time series regressions see e.g. Rousseeuw (2005).

3 Methodology

3.1 Data

The original raw data has been obtained from the company TICK data. The dataset contains data on 29 most liquid companies within S&P 500 index and covers the period from July 2005 to December 2015. The prices are tracked at 5 minute's intervals, which means that each day contains approximately 380 observations.

From the data, different measures of realized variance, skewness and kurtosis at daily frequency are calculated. These measures are subsequently analyzed.

The reason for using and comparing multiple measures of each moment is, as explained later on in the thesis, their different properties. The approach most frequently used in recent empirical literature (i.e. calculating realized measures from log returns raised to 2nd, 3rd and 4th power for variance, skewness and kurtosis respectively), gained popularity thanks to the work of Andersen, Bollerslev and Diebold (1998), Andersen, Bollerslev, Diebold, and Labys (2003) in case of realized variance as well as Neuberger (2012).

For the purpose of the analysis, the data set is split into a training set and a test set. The training set spans data from the 1st of July 2005 to 16th of July 2013. The test set spans the subsequent period from 17th of July 2013 to the 31st of December 2015. Thus, the results might be affected by a potential structural break as training set covers the period of unprecedented financial crisis of 2008 and the test period covers the relatively calm and prosperous times. The statistical summary of the training set can be found in Table 21 and Table 22 in the appendix. Table 21 summarizes the underlying 5-minute log returns used in the calculation of realized moments and Table 22 summarizes the daily log returns used as the explained variable in the pricing model. Equivalent information for the test set is presented in Table 23 and Table 24 respectively.

3.2 Measure definitions

The measures used throughout the analysis are discussed below. First, measures of scale are discussed, then skewness and kurtosis. The choice of robust measures is mostly based on work of Kim & White (2004) and the empirical analysis by Bonato

(2011). All measures of higher moments are calculated based on multiple definitions to ensure that the results are not affected by microstructure noise and that tail fatness and asymmetry are calculated properly. First, the classic measures of realized moments as used by Amaya et al. (2015) are considered. Second, measures using jump-robust estimates of realized volatility are used in standardizing the higher moments. Finally, the robust measures of higher moments, relying on percentiles from the distribution of high-frequency returns, are presented. The measures of volatility, skewness and kurtosis recently proposed by Neuberger & Payne (2018) are discussed separately from the other measures as they need to be discussed together. These measures are included mostly to make sure that all recent literature is considered, yet they are used only as inspiration for alternative model specification, rather than being used directly in the analysis. All measures are calculated based on observations made within one trading day.

For calculation of the measures, it is first necessary to establish the intraday log-returns for each of the stocks as:

Definition 1: ith intraday log-return on day t

$$r_{t,i} = p_{t,\frac{i}{N}} - p_{t,\frac{i-1}{N}}$$

Where the notation was taken over from Amaya et al (2015), and thus p stands for natural logarithm of price. The N represents number of observations recorded on a trading day t. The daily log-return is then given by:

Definition 2: log return on day t

$$r_t = \sum_{i=1}^{N} \left(p_{t,\frac{i}{N}} - p_{t,\frac{i-1}{N}} \right)$$

These relatively simple measures are the cornerstones of the thesis as daily log-returns are the dependent variable in the model and all the explanatory variables used in the model are functions of the intraday log-returns.

3.2.1 Measures of scale

The first measure of realized variance, which has gained popularity since publication of Andersen and Bollerslev (1998) and is now widely used in literature, is sum of squares of the intraday log-returns, as is denoted by the following definition:

Definition 3: Realized Variance

$$RV_t^{RV} = \sum_{i=1}^{N} r_{t,i}^2$$

This definition assumes the mean of the 5-minute return to be zero. Amaya et al (2015) performed robustness checks adjusting the definition for drift or demeaning the data and found that the assumption of zero mean does not have significant negative impact on the performance of the estimator. Intuitively, the high-frequency return variance dominates the mean. The above measure has several noteworthy properties: it is additive – in order to get from short period variance to long period variance, we just sum the observed former variances over the later; it is model-free (2001); it converges to a quadratic variation as sampling period gets shorter.

Summary of the limiting properties of realized moments is again provided by Amaya et al (2015). Andersen et al. (2003) show, that under the following assumptions:

1. Log-price of asset at time T is given by:

$$p_T = \int_0^T \mu_s ds + \int_0^T \sigma_s dW_s + J_T$$

2. μ is a locally bounded predictable drift process
3. σ is a positive càdlàg process
4. J is a pure jump process
5. $p_0 = 0$

Then for $N \to \infty$:

$$RM_2 = \sum_{i=1}^{N} \left(p_{\frac{Ti}{N}} - p_{\frac{T(i-1)}{N}} \right)^2 \xrightarrow{p} \int_0^T \sigma_s^2 ds + \sum_{0<s\leq T} (\Delta p_s)\text{^}2$$

Thus, even as $N \to \infty$ both integrated variance and squared jumps contribute to the limit. The realized variance measure includes both of these elements as well.

The above definition of realized variance has a drawback of being significantly influenced by possible jumps in the data, and therefore several definitions of jump robust measures are presented. These measures are intended to only measure the integrated variance as $N \to \infty$. Barndoff-Nielsen and Shephard (2004) suggested the bipower variation measure:

Definition 4: Bipower Variation

$$RV_t^{BPV} = \frac{\pi}{2} * \frac{N}{N-1} * \sum_{i=1}^{N-1} |r_{t,i+1}| * |r_{t,i}|$$

The bipower variation converges to integrated variance even when jumps are present in the data. On the other hand, when N is finite and jumps are relatively large, the bipower variation still suffers from upward bias. For the purpose of eliminating the bias caused by jumps, Andersen, Dobrev and Schaumburg (2012) propose different jump-robust measures: the minimum realized variance and median realized variance. The minimum realized variance attempts to fix the upward bias in BPV by utilizing the second power of the minimum of two consecutive returns, whereas median realized variance uses the second power of the median of three consecutive returns in calculation of the variance. Their exact definitions follow:

Definition 5: Minimum Realized Variance

$$RV_t^{MinRV} = \frac{\pi}{\pi-2}\left(\frac{N}{N-1}\right) * (\sum_{i=1}^{N-1} \min(|r_{t,i}|, |r_{t,i+1}|)\text{^}2$$

Definition 6: Median Realized Variance

$$RV_t^{MedRV} = \frac{\pi}{6-4*\sqrt{3}+\pi} * \left(\frac{N}{N-2}\right) * \sum_{i=2}^{N-1} median(|r_{t,i-1}|, |r_{t,i}|, |r_{t,i+1}|)\text{^}2$$

As $N \to \infty$, these measures approach the integrated variance, however, as the authors of those measures demonstrated, they tend to be less distorted by jumps for finite N.

Moreover, any of those measures can be further used to separate the effect of jump on quadratic variation by defining the following relationships:

Definition 7: Jump Estimators

$$J_t^{BPV} = RV_t^{RV} - RV_t^{BPV}$$

$$J_t^{MinRV} = RV_t^{RV} - RV_t^{MinRV}$$

$$J_t^{MedRV} = RV_t^{RV} - RV_t^{MedRV}$$

Barndoff-Nielsen and Shephard (2004) further suggest improving the consistency of jump estimator by imposing a non-negativity criterion:

Definition 8: Consistent Jump Estimator

$$J_t^{BPV} = \max(0, J_t^{BPV})$$

While this adjustment makes the estimator biased, it provides for a reconciliation with the intuitive understanding that jumps' contribution should be always nonnegative. The above-mentioned authors found this adjusted estimator to perform significantly better compared to simply taking $J_t^{BPV} = RV_t^{RV} - RV_t^{BPV}$. Another approach in analyzing the effects of jumps is to use a statistical test to determine their presence and based on the test construct a threshold bipower variation as was done by Corsi & Reno (2009). However, the impact of jumps on prices is mentioned here purely for completeness of discussion and is not part of the analysis presented further in the thesis.

When determining the tradeoffs between risk and returns, it is natural to consider the possibility that variance coming from negative returns should be priced differently than variance driven by positive returns. Equivalently, contemporary negative jumps might have different impact on both, future returns and variance compared to their positive counterparts. This idea was put forward by Barndorff-Nielsen et al. (2010) who also proposed the two realized semi-variance measures. The negative and positive semi-variance measures on trading day t are defined as:

Definition 9: Negative Semi-Variance

$$RSV_t^- = \sum_{i=1}^{N} r_{t,i}^2 | r_{t,i} \leq 0$$

Definition 10: Positive Semi-Variance

$$RSV_t^+ = \sum_{i=1}^{N} r_{t,i}^2 | r_{t,i} \geq 0$$

Once again, the t denotes the trading day, so $r_{t,i}$ is the ith high frequency return on day t and N is the total number of observed returns on day t. Barndorff-Nielsen et al (2010) found these measures to improve the performance of models trying to establish link between current and future variance, however this thesis follows the approach taken by Amaya et al (2015) in not separating the two. As in case of jumps, these measures are included only to provide reader with an idea on what are some of the possible extensions of the topic covered by the thesis and to highlight that future research in the field is needed.

3.2.2 Measures of skewness

The baseline measure of realized skewness is defined as:

Definition 11: Realized Skewness

$$RS_t^{RV} = \frac{\sqrt{N} \sum_{i=1}^{N} r_{t,i}^3}{(RV_t^{RV})^{\frac{3}{2}}}$$

and attains positive values when right tail is thicker than left tail, while being negative when left tail is thicker than right tail. The realized variance is used only for standardization. This measure has, similar to realized variance, the model free property and has the advantage of being additive before standardization. Thus, being based on 2 additive measures, the realized skewness has certain appeal to empirical researches as once you have non-standardized realized skewness and realized variance for any time period, it’s very easy to obtain measures for weekly, monthly, annual or any other periods as well. On the other hand, regardless of its’ popularity and appeal for empirical work, it has several important drawbacks that have to be understood before the measure can be used for research. First, for the measure to be unbiased, the skewness and variance have to be uncorrelated. Second, as explained in Neuberger & Payne (2018), this measure might not be very informative for forecasting over longer time periods, since the low frequency returns tend to be mostly predicted by low frequency skewness, however the skewness of high frequency returns does not necessarily imply skewness in low frequency returns. In other words, when calculating realized skewness from 5-minute returns, you do not learn much about distribution of annual returns. Third, the measure is sensitive to outliers, and one extreme observation in a trading day can significantly affect the results. The following paragraphs discuss these issues and their possible remedies in more detail.

The limiting properties of realized skewness are further discussed in Amaya et al (2015). As they point out, the work of Barndorff-Nielsen et al (2010) and Jacod (2012) yields a following result for $N \to \infty$:

$$RM_3 = \sum_{i=1}^{N} \left(p_{\frac{Ti}{N}} - p_{\frac{T(i-1)}{N}} \right)^3 \xrightarrow{p} \sum_{0<s\leq T} (\Delta p_s)^3$$

Thus, contrary to the case of realized variance, third moment captures only jump contribution to cubic variation with continuous part being separated out. As Amaya et al (2015) stress out, the third realized moment does not capture the skewness related to correlation between high frequency return and low frequency return. As Neuberger & Payne (2018) point out, the long period skewness is mostly caused by this so-called leverage effect, while the contribution from skewness in high-frequency returns is limited as we aggregate up from high to low frequencies. The remedy to the above described issue proposed by Neuberger & Payne is further discussed in a separate section of the thesis. Another consequence of the above result is that companies with upward (downward) jumps have generally positive (negative) realized third moment.

A robustness exercise undertaken by Amaya et al (2015) involves replacing the realized variance denominator in definition of realized skewness by the jump-robust estimators. Such an adjustment leads us to the following definitions:

Definition 12: Realized Skewness Scaled by Bipower Variation

$$RS_t^{BPV} = \frac{\sqrt{N}\sum_{i=1}^{N} r_{t,i}^3}{(RV_t^{BPV})^{\frac{3}{2}}}$$

Definition 13: Realized Skewness Scaled by Minimum Realized Variance

$$RS_t^{MinRV} = \frac{\sqrt{N}\sum_{i=1}^{N} r_{t,i}^3}{(RV_t^{MinRV})^{\frac{3}{2}}}$$

Definition 14: Realized Skewness Scaled by Median Realized Variance

$$RS_t^{MedRV} = \frac{\sqrt{N}\sum_{i=1}^{N} r_{t,i}^3}{(RV_t^{MedRV})^{\frac{3}{2}}}$$

Use of the above measures in the analysis should allow us to verify whether the obtained results are in any way due to a specific choice of variance estimator.

Moreover, the robust estimators of skewness should be also used in order to make the results robust to any outliers in the data as well as more robust to violation of our assumptions. It is possible to argue that while using the jump-robust variance estimator in the denominator, the numerator is still very sensitive to outliers in the data and thus a limited number of extreme observations or errors in the data can completely highjack the results and invalidate the entire work and hence estimators brought forth by robust statistics literature should be considered in order to validate the results. Furthermore, the robust estimators might provide us with more accurate forecasts and thus higher returns achieved by trading strategies built on top of the pricing models.

Next, three robust estimators of skewness are presented and later used in an attempt to validate the results and potentially improve on the performance of the models. The first robust measure was suggested by Bowley (1920):

Definition 15: Bowley Skewness

$$RS_t^{Bowley} = \frac{(Q_3 - Q_2) + (Q_1 - Q_2)}{Q_3 - Q_1} = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

The proposed measure is based on quartiles of the underlying distribution of high frequency returns, where Q_i stands for *ith* quartile of the intraday return. For symmetric distributions the coefficient attains value of zero. For right-skewed distributions the measure goes up to one, while the minimum attainable by the measure is negative one. The measure maintains a skewness ordering of two distributions, as is discussed by van Zwet (1968). One possible disadvantage of the above measure is that it does not have the dimensionless property and thus two distributions of data in different units of measurement can't be compared using Bowley skewness. This, however, is not a problem here as first, the units are all the same across companies, and second, comparison of distributions of returns for individual companies is not necessarily the goal of the thesis.

Second robust measure comes from generalization of Bowley skewness done by Hinkley (1975):

Definition 16: Hinkley Skewness

$$RS_{t(\alpha)}^{Hinkley} = \frac{F^{(-1)}(1-\alpha) + F^{-1}(\alpha) - 2Q_2}{F^{-1}(1-\alpha) - F^{-1}(\alpha)}$$

The above measure is defined for any value of alpha between 0 and 0.5. However, it is not clear which level should be used. Hence, Groeneveld & Meeden (1984) take the approach of integrating the alpha out:

Definition 17: Groeneveld-Meeden Skewness

$$RS_t^{G\&M} = \frac{\int_0^{0.5}(F^{-1}(1-\alpha) + F^{-1}(\alpha) - 2Q_2)\, d\alpha}{\int_0^{0.5}\left(F^{-1}(1-\alpha) - F^{-1}(\alpha)\right) d\alpha} = \frac{E\left[r_{t,i}\right] - Q_2}{E\left|r_{t,i} - Q_2\right|}$$

Which results in a measure that attains 0 for symmetric distribution, 1 for extremely right skewed and -1 for extremely left skewed distributions. As the denominator can be considered a dispersion measure, it is natural to consider replacing it with standard deviation. Such an adjustment results in the Pearson skewness due to Kendall & Stuart (1977):

Definition 18: Pearson Skewness

$$RS_t^{Pearson} = \frac{E[r_{t,i}] - Q_2}{\sigma} = \frac{\mu - Q_2}{\sigma}$$

Unlike Bowley or Groeneveld & Meeden skewness, the last measure requires the existence of finite second moment. Groeneveld & Meeden (1984) put forth following requirements on properties of skewness coefficient γ, assuming random variable X to have continuous c.d.f. and corresponding differentiable density function $f(x) > 0, x \in (a, b);\ a, b \in R$:

i) $A\ scale\ location\ change\ for\ a\ random\ variable\ does\ not\ alter\ \gamma. In\ other\ words$:
$$\forall a > 0, b \in R\ \&\ Y = aX + b: \gamma(X) = \gamma(Y)$$
ii) $If\ distribution\ is\ symmetric\ then\ \gamma = 0$
iii) $Y = -X => \gamma(Y) = -\gamma(X)$
iv) $Let\ F\ and\ G\ be\ c.d.f.s\ for\ X\ and\ Y\ respectivelly\ and\ F <_c G => \gamma(X) \leq \gamma(Y)$

And they prove those properties are satisfied by both Bowley and Groeneveld-Meeden skewness. However, as has been proven by van Zwet (1968), the Pearson skewness coefficient does not preserve the ordering of two distributions (fails to satisfy property iv). For more details on robust estimators see e.g. Huber & Ronchetti (2009).

3.2.3 Measures of Kurtosis

In order to better understand the outliers in returns the realized kurtosis measure is defined by the following equation:

Definition 19: Realized Kurtosis

$$RK_t^{RV} = \frac{N \sum_{i=1}^{N} r_{t,i}^4}{(RV_t^{RV})^2}$$

The N is used merely as a scaling factor so that the measure corresponds in scale to daily kurtosis, however for the results of the analysis it is inconsequential. This measure has again the additive property before scaling by variance and is model-free. On the other hand, for the kurtosis estimator to be unbiased we have to assume the numerator and denominator are uncorrelated. Moreover, as is explained later, the additive property doesn't mean that we can use this estimator to estimate low-frequency kurtosis from high-frequency data. The limiting properties of the fourth realized moment have been analyzed in the works of Barndorff-Nielsen & Shephard (2004) and Jacod (2012) with the following result for $N \to \infty$:

$$RM_4 = \sum_{i=1}^{N} \left(p_{\frac{Ti}{N}} - p_{\frac{T(i-1)}{N}} \right)^4 \xrightarrow{p} \sum_{0<s\leq T} (\Delta p_s)^4$$

As in case of the third moment, only the jump component is captured, whereas the continuous component of quartic variation is not. As Amaya et al (2015) point out, variance of variance could be one of the sources for the continuous component. Similar to the second realized moment, only the magnitude is captured, while the sign is not. A natural extension thus might be negative and positive semi-kurtosis. This extension is, however, not examined in detail in this thesis.

The above limits imply that realized skewness and kurtosis can't be used to measure cubic and quartic variation with increasing efficiency as sampling frequency increases. Rather, the estimators will measure different things and achieve different results when sampling frequency changes. As has been shown, the measures of skewness and kurtosis presented here do not capture total cubic and quartic variation. Bakshi, Kapadia & Madan (2003) propose using options to obtain risk-neutral moments, an approach later taken by Neuberger (2012) or Conrad, Dittmar & Ghyseis (2013). Alternatively, long-period returns or a method recently proposed by Neuberger & Payne (2018) can be used. The later approach is briefly discussed in the subsequent chapter. However, it is used in the thesis as an inspiration for alternative model specification and the measures proposed by Neuberger & Payne are not used directly.

Following the example of Amaya et al (2015), as in case of skewness, jump robust measures of realized variance are used for scaling the realized kurtosis. The three resulting definitions follow:

Definition 20: Realized Kurtosis Scaled by Bipower Variation

$$RK_t^{BPV} = \frac{N \sum_{i=1}^{N} r_{t,i}^4}{(RV_t^{BPV})^2}$$

Definition 21: Realized Kurtosis Scaled by Minimum Realized Variance

$$RK_t^{MinRV} = \frac{N \sum_{i=1}^{N} r_{t,i}^4}{\left(RV_t^{MinRV}\right)^2}$$

Definition 22: Realized Kurtosis Scaled by Median Realized Variance

$$RK_t^{MedRV} = \frac{N \sum_{i=1}^{N} r_{t,i}^4}{\left(RV_t^{MedRV}\right)^2}$$

In the following part of the text, two more robust measures of kurtosis are discussed. The first measure was proposed by Moors (1988) and uses octiles to estimate kurtosis. The second measure is due to Crown & Siddiqui (1967). Let $E_{i,t}$ be the ith octile of intraday log returns on trading day t, then the Moors kurtosis is defined as:

Definition 23: Moors Kurtosis

$$RK_t^{Moors} = \frac{\left(E_{7,t} - E_{5,t}\right) + \left(E_{3,t} - E_{1,t}\right)}{E_{6,t} - E_{2,t}}$$

The Moors' original argument for this measure was based on following idea: if lot of probability mass is located near 2nd and 6th octile, then both terms in brackets in the numerator are small. This is in its logic similar to the kurtosis measure measuring the dispersion around $\mu \pm \sigma$. The measure can be centered by subtracting 1.23. The denominator is a scaling constant which ensures the measure is invariant under linear transformations. In other words, the estimator *is constant over any class of distributions determined by a location-scale parameter* (1988). Moreover, it is more robust to extreme observations than the standard realized kurtosis measure. Finally, the measure might be valid even in cases when no moments exist.

The last robust measure of kurtosis which is considered in this thesis is based on quantiles and has been used by Bonato (2011), Brys et al (2006), Schmid & Trede (2003), and Kim & White (2004). Bonato attributes the origin of the measure to Crow & Siddiqui (1967) and in general it takes the form:

Definition 24: Crow & Siddiqui kurtosis – general case

$$RK_{t,\alpha}^{C\&S} = \frac{F^{-1}(1-\alpha) - F^{-1}(\alpha)}{F^{-1}(1-\beta) - F^{-1}(\beta)}$$

Various choices for alpha have been used by the above-mentioned authors. Bonato, as well as Kim & White, use $\alpha = 0.025\ and\ \beta = 0.25$. Schmid & Trede propose choices of $\alpha = 0.125\ and\ \beta = 0.25$ or alternatively $\alpha = 0.025\ and\ \beta = 0.125$, where the former results in breakdown point of 12.5% whereas the later results in breakdown point of 2.5%. This thesis sticks to the values used by Bonato, which means the resulting measure is defined as:

Definition 25: Crow & Siddiqui kurtosis

$$RK_{t}^{C\&S} = \frac{F^{-1}(0.975) - F^{-1}(0.025)}{F^{-1}(0.75) - F^{-1}(0.25)}$$

The measure can be centered by subtracting 2.91 from the resulting value.

3.2.4 Neuberger & Payne measures

The measures recently proposed by Neuberger & Payne (2018) are discussed here as they inspired the extended model presented in the Robustness to model specification section. That being said, they do not enter into the analysis directly. Moreover, it seems necessary to mention them as they potentially represent an important development in research of realized moments. These moments, according to their authors, should help to better estimate the low frequency moments from high frequency data.

Neuberger and Payne (2018) argue that while we can use daily realized volatilities to get easily to monthly realized volatility, we cannot do the same for skewness and kurtosis. They claim to have found a way to use high-frequency returns to make more precise estimates of skewness and kurtosis compared to both, the standard and the robust measures. They argue that long-horizon skewness calculated from high-frequency returns has 2 components, the skewness of short-horizon returns, and the so-called leverage effect: the covariance of current variance and lagged returns. The long-horizon kurtosis has 3 components: the kurtosis of high-frequency returns, the covariance of cubed high-frequency returns and lagged returns, and what the authors call the GARCH effect: the covariance of squared high-frequency returns and lagged squared returns.

They found that 1) low-frequency skewness and kurtosis are mainly determined by the leverage and GARCH effects, while skewness and kurtosis of high-frequency returns has only marginal impact, 2) monthly skewness can be used to forecast US index returns at both, monthly and annual frequencies.

The definitions proposed by Neuberger & Payne are much more complex compared to the usual realized moments measures. The measures use standard returns instead of log return, and all definitions necessary to calculate the measures follow:

Definition 26: Low-frequency realized measures proposed by Neuberger & Payne

$$r_t = \frac{P_t}{P_{t-1}}\ , high - frequency\ return, and\ R_t = \frac{P_t}{P_{t-T}}, low - frequency\ return;$$

$$var^L[r] \coloneqq E\big[x^{(2,L)}(r)\big], where\ x^{(2,L)}(r) \coloneqq 2 * (r - 1 - \ln(r));$$

$$var^E[r] \coloneqq E\big[x^{(2,E)}(r)\big], where\ x^{(2,E)}(r) \coloneqq 2 * (r * ln(r) + 1 - r);$$

$$x^{(3)}(r) \coloneqq 6 * \big((r+1) * \ln(r) - 2 * (r-1)\big);$$

$$x^{(4)}(r) \coloneqq 12 * \big((\ln(r))^2 + 2 * (r+2) * \ln(r) - 6 * (r-1)\big);$$

$$y_{t-1} \coloneqq \sum_{u=1}^{T} \frac{\frac{P_{t-1}}{P_{t-u}} - 1}{T},\ \ an\ average\ return\ over\ the\ period\ ;$$

$$z_{t-1} \coloneqq \sum_{u=1}^{T} 2 * \frac{\frac{P_{t-1}}{P_{t-u}} - 1 - \ln\left(\frac{P_{t-1}}{P_{t-u}}\right)}{T};$$

$$rx_{t,w}^{(2,L)} \coloneqq \sum_{s=t+1}^{w} x^{(2,L)}(r_s), \qquad the\ realized\ variance\ over\ a\ period\ (t,w);$$

$$rx_{t,w}^{(3)} \coloneqq \sum_{s=t+1}^{w} x^{(3)}(r_s) + 3 * y_{s-1} * x^{(2,E)}(r_s),$$

$$the\ realized\ 3rd\ moment\ over\ (t,w);$$

$$rx_{t,w}^{(4)} \coloneqq \sum_{s=t+1}^{w} x^{(4)}(r_s) + 4 * (y_{s-1} * x^{(3)}(r_s) + 6 * z_{s-1} * x^{(2,L)}(r_s),$$

$$realized\ 4th\ moment;$$

As long as P is a martingale process, the average conditional moments of long-horizon returns can be estimated by scaling the daily returns by T/(w-t). As Neuberger & Payne (2018) point out, while T is explicitly used as a scaling factor, it is also included in the definitions of y and z; the low-frequency third and fourth moments depend on covariances of high-frequency variance and skewness with returns and variances over the long period.

That being said, it seems reasonable to ask whether it is not easier to use lower frequency data to calculate the long-horizon moments. The answer obviously depends on whether we can extract comparable quality information about long-horizon skewness and kurtosis from daily, monthly or annual returns. Moreover, these measures, when calculated from high frequency returns (e.g. 5-minute frequency), are significantly more computationally expensive compared to the standard measures. For large datasets including several years of data and many firms, it might be better to use a database system to calculate these measures. Furthermore, it's yet to be seen how the above measures perform for thick-tailed distributions or outliers in the data. Sadly, it's beyond the scope of my thesis to answer these questions.

As mentioned above, these measures are not explicitly used in the thesis, however they do inspire an alternative model specification presented in the *Robustness to model specification* chapter. Based on research of Neuberger & Payne (2018), this thesis explores the use of interaction term between variance and kurtosis, skewness and log return, and variance and log return in the model specification.

3.3 The models

Once calculated, the above measures are subsequently used in the model estimation. As the main purpose of this thesis is to determine the usefulness of realized moments in time series analysis, a separate estimation is done for each of the 29 companies. In other words, the goal is to use one model specification for all companies and enable estimation from data for a single company. On top of using various estimators, several model specifications are used in order to: 1) determine whether each of the realized moments contributes positively to the performance of the model, 2) establish whether extending the model by interaction terms in order to account for the factors such as covariance between return and variance, or skewness and return, or variance and kurtosis does improve the out of sample performance or significantly changes the estimated coefficients. Moreover, the models are tested against a benchmark model – AR1 – as well as against the strategy of passively holding a long position in the entire portfolio of stocks.

The estimated models can be split into three categories: 1) Models using standard realized moments estimators and their jump-robust versions, 2) Models using skewness and kurtosis estimators proposed by the robust statistics literature, and 3) Restricted and Extended models.

3.3.1 The core model & benchmark

Next, let's discuss the individual model specifications in more detail. This subchapter covers the models using the standard realized moments estimators and their jump-robust versions. Let $r_{i,t}$ be the return on stock of company *i* on day *t*, then assume the price of company *i* is determined by the following equation $for\ \forall\ i\ =\ 1, \ldots, 29$:

$$\begin{aligned} r_{i,t+1} = \ & \alpha_{i,0}^{(M)} + \alpha_{i,1}^{(M)} * r_{i,t} + \alpha_{i,2}^{(M)} * RV_{i,t}^{(M)} + \\ & \alpha_{i,3}^{(M)} * RS_{i,t}^{(M)} + \alpha_{i,4}^{(M)} * RK_{i,t}^{(M)} + \epsilon_{i,t+1}^{(M)} \end{aligned} \tag{3.1}$$

Where $M \in \{RV, BPV, MinRV, MedRV\}$ and $RV_{i,t}^{RV}, \ldots, RV_{i,t}^{MedRV}$ correspond to the realized volatility estimators presented in *definition 3 to definition 6,* while $RS_{i,t}^{RV}, \ldots, RS_{i,t}^{MedRV}$ are realized skewness estimators defined in *definition 11 to definition 14,* and finally $RK_{i,t}^{RV}, \ldots, RK_{i,t}^{MedRV}$ are realized kurtosis estimators covered by *definition 19* to *definition 22* as presented in the chapter on measure definitions.

Upon the estimation of this model, we get $for\ \forall\ i, M$ the following column vector of the coefficients: $\theta_i^{(M)} = \left(\alpha_{i,0}^{(M)}, \alpha_{i,1}^{(M)}, \alpha_{i,2}^{(M)}, \alpha_{i,3}^{(M)}, \alpha_{i,4}^{(M)}\right)^T$. From those vectors, a following matrix is constructed: $\theta^{(M)} = \left(\theta_1^{(M)}, \ldots, \theta_{29}^{(M)}\right)$. Thus, for each measure definition given by M we get a matrix θ^M which stores the coefficients for each of the companies. Throughout the results section *Model* θ^M is used to talk about what is in fact a set of N time series models.

Since the performance of the model on the testing set of data is taken as the most important determinant of the quality of the model, some benchmarking in needed. This is done in two ways: 1) the model is compared to AR1 model on set of metrics outlined in the following chapter, 2) the performance is compared to a passive investment scenario where the investor buys all stocks at the beginning of the testing period and sells them at the end. In the above specification of the model, this thesis deviates from the approach frequently taken in the literature, and from that of Amaya et al. (2015), by including the lagged return. The impact of decision to include or exclude this term is examined in more detail in the *Robustness to model specification* section.

For each company *i* the benchmark AR1 model is defined as:

$$r_{i,t+1} = \alpha_{i,0} + \alpha_{i,1} * r_{i,t} + \epsilon_{i,t+1} \tag{3.2}$$

The reasoning for the choice of AR1 as the benchmark model is fivefold: 1) it is well established in the quantitative finance literature and is used as benchmark in other empirical work, 2) it is fantastic in its' simplicity, 3) it performs relatively well in terms of deviations, 4) it fits the narrative that returns tend to be negatively correlated, 5) it is a nested model of the $Model\ \theta^{RV}$, as it merely imposes the restriction: $\alpha_{i,2} = \alpha_{i,3} = \alpha_{i,4} = 0$.

3.3.2 The models with robust estimators of skewness and kurtosis

Second set of models uses the robust estimators of skewness and kurtosis, together with Median Realized Variance when pricing the risk. As before, we have a model for each company *i.* In the general form the model can be written as follows: $for\ \forall\ i = 1, \dots, 29$:

$$\begin{aligned} r_{i,t+1} = \alpha_{i,0}^{(M,P)} + \alpha_{i,1}^{(M,P)} * r_{i,t} + \alpha_{i,2}^{(M,P)} * RV_{i,t}^{MedRV} + \\ \alpha_{i,3}^{(M,P)} * RS_{i,t}^{(M)} + \alpha_{i,4}^{(M,P)} * RK_{i,t}^{(P)} + \epsilon_{i,t+1}^{(M,P)} \end{aligned} \tag{3.3}$$

Where $M \in \{Bowley, G\&M, Pearson\}$ and $P \in \{Moors, C\&S\}$. $RS_{i,t}^{Bowley}, \dots, RS_{i,t}^{Pearson}$ refer to the robust skewness estimators defined in *definition 15, definition 17* and *definition 18.* $RK_{i,t}^{Moors}\ and\ RK_{i,t}^{C\&S}$ refer to robust kurtosis estimators proposed by Moors and Crow & Siddiqui respectively, they are presented above in definition 23 and definition 25.

Thus, $for\, \forall\, i, M, P$ we get a column vector of the coefficients: $\theta_i^{(M,P)} = \left(\alpha_{i,0}^{(M,P)}, \alpha_{i,1}^{(M,P)}, \alpha_{i,2}^{(M,P)}, \alpha_{i,3}^{(M,P)}, \alpha_{i,4}^{(M,P)}\right)^T$. From those vectors, a following matrix is constructed: $\theta^{(M,P)} = \left(\theta_1^{(M,P)}, \dots, \theta_{29}^{(M,P)}\right)$. As we have 29 companies, 3 robust estimators of skewness and 2 robust estimators of kurtosis, the total number of estimated models is actually equal to 29*3*2, however, one specification is usually evaluated for all 29 companies at once, which makes the amount of information presented more comprehensible. As before, throughout the results section, for each combination of M and P, the resulting set of 29 time series models will be addressed as Model $\theta^{(M,P)}$. Let's now move on to the sets of extended and restricted models.

3.3.3 The extended and restricted models

The analysis of the alternative model specifications can be seen as the most important part of the thesis as it sheds light on whether the specification used by Amaya et al (2015) is the right one. The specifications described in this section can be split into two groups: restricted versions of model described by (3.1) with M = RV and extended versions of model described by (3.1) with M = RV. The restricted models answer a question what is the cost of omission of individual variables and what is the impact on the estimates. As is discussed in the corresponding results chapter, decision whether lagged return should be included is particularly consequential. The extended models aim to explore the possibility of accounting for leverage and GARCH effects, as detailed in Neuberger & Payne (2018), with the use of realized moments estimated from high frequency log returns. In this section, the analysis relies solely on the standard estimators of realized moments. Neither jump-robust estimators of variance, nor robust skewness and kurtosis estimators are used.

The main purpose of the restricted models is to verify whether a particular sign of coefficient estimate on one of the variables could be due to inclusion or omission of a particular variable. The question could be: is the different sign of the skewness coefficient, compared to Amaya et al (2015), due inclusion of the lagged return variable in the model specification? Similarly, omission of the realized moments serves the purpose of verifying whether we wouldn't be better off without one of them.

These extended models are inspired by the frequent critique that the estimators of realized moments based on log returns do not account for the leverage and GARCH effects when estimating the skewness and kurtosis. Based on the Neuberger & Payne (2018) paper, this thesis uses the interaction terms between kurtosis & variance (where the variance contains the low-frequency information), skewness and log return (where log return also captures some of the low-frequency properties), and variance and log return. Since the thesis uses different measure definitions compared to Neuberger & Payne, it is necessary to verify the contribution of the interaction terms empirically.

Let's once again have a general form of the model for each company and combinations of variables. Let $B = (b_1, b_2, b_3, b_4, b_5, b_6, b_7)$, where $b_1, \dots, b_7 \in \{0,1\}$. Then, as before, $for\ \forall\ i = 1, \dots, 29$:

$$\begin{aligned} r_{i,t+1} = \alpha_{i,0}^{(B)} + \alpha_{i,1}^{(B)} * b_1 * r_{i,t} + \alpha_{i,2}^{(B)} * b_2 * RV_{i,t}^{RV} + \\ \alpha_{i,3}^{(B)} * b_3 * RS_{i,t}^{RV} + \alpha_{i,4}^{(B)} * b_4 * RK_{i,t}^{RV} + \\ \alpha_{i,5}^{(B)} * b_5 * RV_{i,t}^{RV} * RK_{i,t}^{RV} + \alpha_{i,6}^{(B)} * b_6 * RS_{i,t}^{RV} * r_{i,t} + \\ \alpha_{i,7}^{(B)} * b_7 * RV_{i,t}^{RV} * r_{i,t} + \epsilon_{i,t+1}^{(B)} \end{aligned} \tag{3.4}$$

Thus, $for\ \forall\ i, B$ we get a column vector of the coefficients:
$\theta_i^{(B)} = \left(\alpha_{i,0}^{(B)}, \alpha_{i,1}^{(B)}, \alpha_{i,2}^{(B)}, \alpha_{i,3}^{(B)}, \alpha_{i,4}^{(B)}, \alpha_{i,5}^{(B)}, \alpha_{i,6}^{(B)}, \alpha_{i,7}^{(B)}\right)^T$.
From those vectors, a following matrix is constructed: $\theta^{(B)} = \left(\theta_1^{(B)}, \dots, \theta_{29}^{(B)}\right)$. As before, in the results section the naming convention *Model* $\theta^{(B)}$ is used when results for the above model are discussed. To give reader a better understanding how the notation works, let's look at some examples: The set of N models (one for each company) with lagged return, variance, skewness and kurtosis, but no interaction terms, would be called *Model* $\theta^{(1,1,1,1,0,0,0)}$, while set of models including lagged return and all 3 interaction terms, but no realized variance, skewness and kurtosis on their own, would be called *Model* $\theta^{(1,0,0,0,1,1,1)}$.

3.3.4 The coefficient averages and medians

Furthermore, this thesis looks at whether the companies can be priced using an aggregation of the coefficients from the individual models. In ideal world, the coefficients wouldn't be too different for different companies and thus a pricing done using, say, average of the coefficients from the N models should perform relatively well. For this reason, row-wise mean and median of matrices $\theta^{(M)}, \theta^{(M,P)}$ and $\theta^{(B)}$ are calculated. When presenting the predictive performance and summary statistics of the coeficient averages and medians, the following notation is used: $\overline{\theta^{(M)}}, \overline{\theta^{(M,P)}}$ and $\overline{\theta^{(B)}}$ for the row-wise means, and matrices $\widetilde{\theta^{(M)}}, \widetilde{\theta^{(M,P)}}$ and $\widetilde{\theta^{(B)}}$for the row-wise medians.

This wraps up all the models whose behavior is examined throughout the thesis. The methods for model evaluation are discussed next.

3.4 The evaluation criteria

This chapter outlines the evaluation criteria for the models. In this thesis, the models' performance on the test set is considered to be a crucial indicator of the model's quality. The test set for each company contains trading dates from 17th July 2013 to the 31st of December 2015. This translates into over 600 observations for each of the 29 companies. All models are evaluated and compared on the set of metrics calculated from the test set residuals. Furthermore, it is examined whether, in terms of predicting the sign of future return, the models are significantly better compared to random guessing. Moreover, the returns achievable by making short-term trades based on the information provided by the models are studied. The returns achievable by trading based on the model recommendation are perhaps the most important criterion in determining the quality of the model. In order to fully evaluate the models, it is necessary to establish a benchmark. This thesis considers two benchmarks. One of them is the AR1 model presented in the previous chapter, the other one is a strategy of investing into an equally weighted portfolio of all stocks and passively holding it throughout the entire period covered by the test set.

3.4.1 Metrics based on forecast residuals

First, the models are compared based on out of sample prediction errors. Three metrics are used for this purpose. Perhaps the most famous and frequently used is the root mean squared error:

Definition 27: Root mean squared error

$$RMSE_i = \sqrt{\frac{\sum_{t=1}^{N}\left(r_{i,t} - \widehat{r_{i,t}}\right)^2}{N}}$$

Where $\widehat{r_{i,t}}$ is the predicted log return for company *i* at time *t* and N is the number of test set observations for company *i*. While frequently used in the literature, the above statistic is known to be strongly affected by any large outliers in the data. Thus, it might easily happen that a model which would permanently predict 0 returns would be considered superior to a model which fits the returns well overall, but produces couple of large prediction errors. Therefore, two other statistics are used in order to measure the deviations. These put equal weight on large and small errors and are defined as:

Definition 28: Mean absolute deviation

$$MeanAD_i = \frac{\sum_{t=1}^{N} |r_{i,t} - \widehat{r_{i,t}}|}{N}$$

Definition 29: Median absolute deviation

$$MedianAD_i = Median\left|r_{i,t} - \widehat{r_{i,t}}\right|$$

The first two measures are closely tied to a Diebold-Mariano test for comparing the accuracy of two non-nested models. While once can consult Diebold and Mariano (1995) for exact details, intuitively, for company i, the test works as follows: Let's have a loss differential of two returns forecasts for company *i* given by:

$$d_{i,t} = g\left(e_{1,i,t}\right) - g(e_{2,i,t})$$

Where g(.) is the error function satisfying following criteria:

$$1)\ g(.) = 0\ when\ no\ error\ is\ made$$

$$2)\ \forall e: g(e) \geq 0$$

$$3)\ g(.)\ is\ increasing\ in\ |e|$$

Then the models have the same accuracy if and only if for $\forall t\colon E\left[d_{i,t}\right] = 0$. Therefore, we would like to test:

$$H_0\colon \forall t\colon E\left[d_{i,t}\right] = 0$$

$$H_1\colon \exists t\colon E\left[d_{i,t}\right] \neq 0$$

Let:

$$\bar{d_\iota} = \frac{\sum_{t=1}^{T} d_{i,t}}{T}$$

$$\hat{\gamma}_{d_i}(k) = \frac{1}{T}\sum_{t=|k|+1}^{T} \left(d_{i,t} - \bar{d_\iota}\right) * \left(d_{i,t-|k|} - \bar{d_\iota}\right)$$

$$M = T^{\frac{1}{3}}$$

Then the DM statistic is defined as:

Definition 30: Diebold-Mariano test statistic

$$DM = \frac{\bar{d}}{\sqrt{\sum_{k=-M}^{M} \frac{\widehat{\gamma_{d_\iota}(k)}}{T}}}$$

and under the null hypothesis it asymptotically follows standard normal distribution. In the *Robustness to model specification* chapter of the results section, the Diebold-Mariano test is applied to the out of sample predictions for each of the 29 stocks. The accuracy measures are analyzed for individual companies as well the entire dataset together.

However, as this thesis demonstrates in the following paragraphs, looking at the size of the residuals doesn't seem to suffice for analysis of the realized moments and models leveraging them. For this reason, models' ability to forecast the sign of future returns is also presented. The accuracy of the model for company I can be defined as:

Definition 31: Sign accuracy

$$Accuracy_i = \frac{\sum_{t=1}^{N}(sign(r_{i,t}) == sign(\widehat{r_{i,t}}))}{N}$$

Moreover, this thesis also presents the true positive, true negative, false positive and false negative measures and statistics derived from them, such as precision, negative predictive value, sensitivity and specificity.

3.4.2 The returns achievable by trading

The most important measurement of model quality, presented in this thesis, is perhaps the return achievable by relying on information from the model. For this purpose, two trading strategies are designed. First strategy relies on buying stocks in the top quintile of forecasted returns. More specifically, on day t, predictions for the returns on stock of individual companies at time $t + 1$ are made. These predicted returns are then sorted and a trading agent chooses to buy all stocks with predicted return in the top quintile. By assumption, equal weighting of assets is used in constructing the portfolio. The second strategy focuses on buying any stocks with positive predicted return. Thus-achieved daily returns are subsequently summarized using mean over the entire period, quartiles, variance, skewness and kurtosis. The minimum requirement for the models is to beat the AR1 model on expected return. Moreover, the model should be able to beat a passive strategy of holding stocks over the entire time period covered by the test set. Despite the transaction costs of active trading it is possible to beat the market return as long as the expected return from the strategy based on the model exceeds the market return and the trading agent's scale is sufficiently large. This is implied by the fixed transaction costs faced by small agents. In practice, larger entities should be able to extract significantly more favorable terms.

4 Results

This chapter of the thesis summarizes the empirical results of the research. It is divided into two sections: 1) Analysis using standard realized measures as given by *definitions 3,11 and 19*, 2) Robustness analysis. In the first section, the main focus is on the analysis of Model θ^{RV}, its' performance, comparison with the AR1 benchmark model and discussion of the relation to the results achieved by Amaya et al (2015) and other researchers. The robustness analysis has 4 subchapters which discuss the following topics: jump-robust estimators of realized moments; model averaging; estimators proposed by the robust statistics literature; robustness to model specification.

4.1 Standard realized measures

Let's start with examining how realized variance, skewness and kurtosis can be used to significantly improve the performance of a pricing model. First, a benchmark AR1 model is estimated and its' properties explored. Using this benchmark, the performance of the Model θ^{RV} is analyzed. It is found that the Model θ^{RV} significantly outperforms random guessing in determining whether future returns will be positive or not. Moreover, use of realized variance, skewness and kurtosis allows a trading agent to achieve higher returns compared to passively holding stocks or relying on information from the AR1 model. However, the predictions from Model θ^{RV} were found to occasionally overshoot the actual returns and as a result the model is not able to outperform AR1 in terms of accuracy measures such as RMSE or MAD. Throughout the next several paragraphs, the results for the benchmarking model and Model θ^{RV} are discussed.

4.1.1 The benchmark

Let's start by reviewing the performance of the benchmark model. Out of several simple options – AR, MA, ARMA, model with just an intercept, 0 forecast, historical average, … - AR has been chosen as the best performing benchmark case.

The Figure 1 presented below shows for each of the 29 companies the approximate 95% confidence interval for the coefficient on lagged returns as estimated by the AR1

Model. The estimates are based on a training set spanning the period from 1st of July 2005 to 15th of July 2013. In creation of the confidence interval, the heteroskedasticity and autocorrelation robust standard error have been used (HAC sandwich estimator is used, the theoretical details can be found in Zeileis (2004) and the R implementation used in this thesis is detailed in Zeileis (2006)). The coefficient is estimated to be negative most of the time, and for 8 of the companies the robust 95% confidence interval doesn't straddle the 0 mark. While for 3 of the companies the point estimate is positive, it is not significantly different from 0. Thus, we can see that the AR1 model is relatively consistent in its' estimates and the estimated relation is negative. Thanks to this consistency, the AR1 model seems to be a good starting point for modelling the returns of individual companies. The estimated coefficients are moreover summarized in a boxplot in *Figure 2* below. The overall summary statistics for this model are presented in *Table 26*Models AR1 and $\theta^{(M)}$ in the Appendix.

The three companies for which AR1 has the unexpected sign of the first AR coefficient are: Citigroup (ticker C), IBM, and Home Depot (ticker HD). It is worth noting that Citigroup stock price fell from over \$500 in 2007, to less than \$30 in 2009, this steep drop lasting several months is likely to have influenced the estimate.

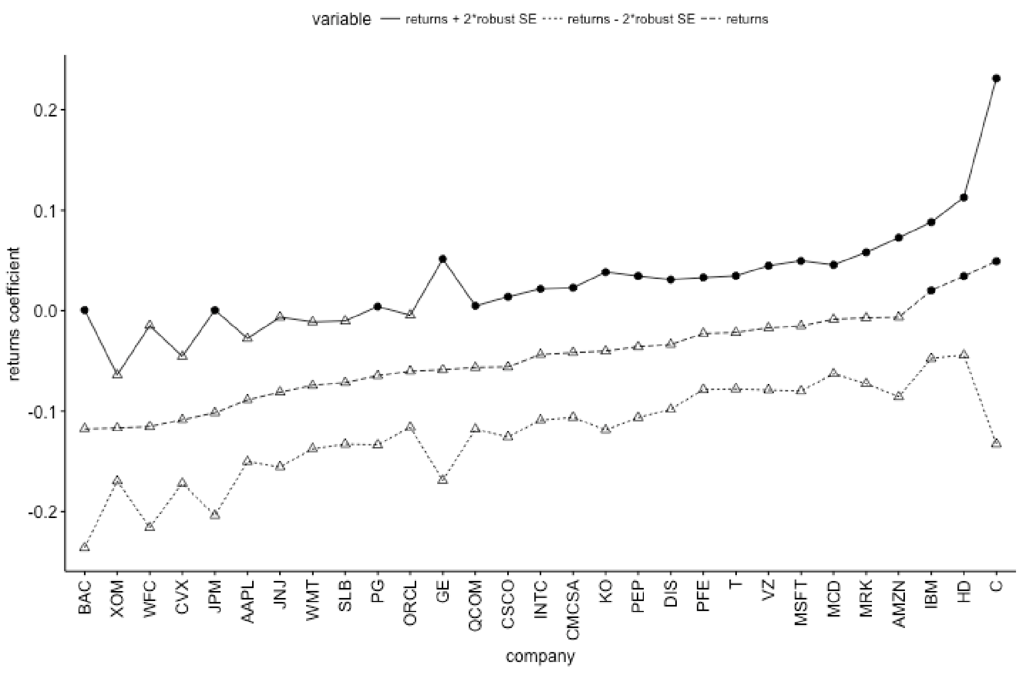

Figure 1: Estimated coefficients for lagged returns in the AR1 model

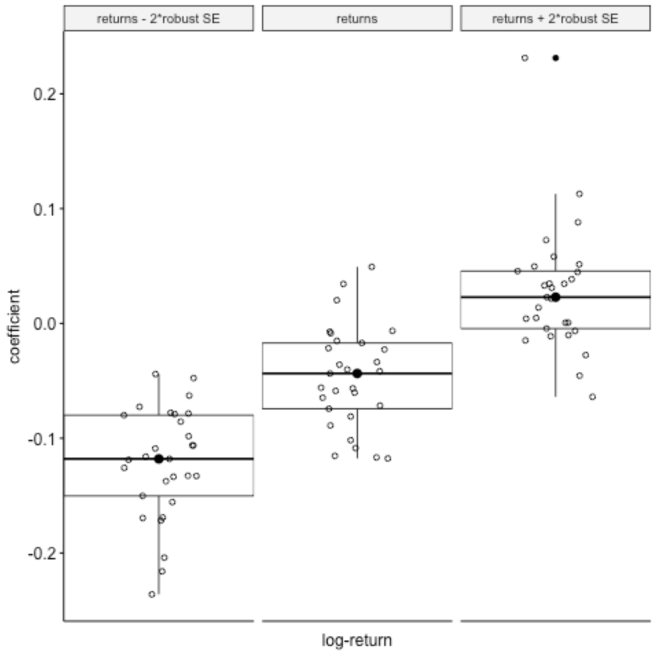

Figure 2: Boxplot of estimated coefficients for lagged returns in the AR1 model

While the AR1 model is relatively consistent across the individual stocks in terms of the estimated coefficient, more interesting is the evaluation of the model performance on the test set. The summary statistics for the model performance are presented in Table 1 below. As can be seen from the RMSE, as well as mean and median absolute deviations, in terms of prediction errors Amazon (ticker AMZN) is the company for which the model performs the worst. However, this does not mean that Amazon is also the company for which the predicted sign is most frequently wrong. It turns out, that the relatively naïve prediction for Amazon (positive return no matter what) is actually correct 50.5% of the time. Similarly, for Verizon (ticker VZ), where the model prediction is 7th and 8th best in terms of RMSE and mean absolute deviation respectively, the accuracy of the predicted sign is only 47.7%. Moreover, it can be seen that for Citigroup, the overall accuracy of predictions is 52.2%, while not a single positive return has been identified. This analysis suggest that we can’t simply look at statistical summary of the estimation when evaluating a model. As it has been shown, the summary can tell one story when we look at coefficient estimates on the training set, another story about model quality can be told by RMSE or accuracy in predicting the sign of the future returns. Equivalently, a Diebold-Mariano test focusing on comparison prediction errors of two non-nested models is not sufficient for a full analysis of model quality. Therefore, one needs to look at economic factors as well.

The economic part of the analysis should be tackled by the evaluation of the two trading strategies outlined in the *Methodology* chapter.

Table 1

Main Summary of AR1 individual stocks predictive power on test set

Company	RMSE	Mean absolute deviation	Median Absolute Deviation	Accuracy	True Positive	True Negative	False Negative	False Positive
AAPL	0.0123	0.0093	0.0075	0.514	103	212	199	99
AMZN	0.0140	0.0107	0.0085	0.505	310	0	0	304
BAC	0.0111	0.0084	0.0063	0.531	16	305	271	13
C	0.0107	0.0082	0.0065	0.522	0	318	291	0
CMCSA	0.0108	0.0080	0.0060	0.493	250	52	64	246
CSCO	0.0096	0.0073	0.0062	0.483	89	204	237	77
CVX	0.0112	0.0080	0.0060	0.476	176	116	122	199
DIS	0.0095	0.0070	0.0053	0.511	310	2	0	298
GE	0.0094	0.0069	0.0053	0.496	18	283	289	17
HD	0.0097	0.0072	0.0055	0.516	298	18	20	276
IBM	0.0090	0.0068	0.0054	0.500	306	0	1	305
INTC	0.0113	0.0083	0.0065	0.500	117	184	197	104
JNJ	0.0080	0.0060	0.0046	0.480	143	150	177	141
JPM	0.0098	0.0073	0.0055	0.480	169	125	156	163
KO	0.0072	0.0055	0.0042	0.507	184	124	135	164
MCD	0.0076	0.0055	0.0040	0.501	305	1	3	302
MRK	0.0103	0.0073	0.0053	0.492	167	132	122	187
MSFT	0.0113	0.0084	0.0065	0.481	123	170	198	118
ORCL	0.0100	0.0075	0.0056	0.515	262	51	56	239
PEP	0.0073	0.0056	0.0044	0.507	301	8	5	296
PFE	0.0092	0.0070	0.0057	0.526	33	286	256	31
PG	0.0073	0.0055	0.0043	0.493	270	30	33	275
QCOM	0.0111	0.0080	0.0060	0.485	135	162	205	111
SLB	0.0124	0.0093	0.0073	0.527	121	201	184	105
T	0.0079	0.0059	0.0046	0.509	29	280	272	26
VZ	0.0081	0.0062	0.0047	0.477	25	265	289	29
WFC	0.0084	0.0061	0.0048	0.497	169	132	133	172
WMT	0.0087	0.0058	0.0043	0.511	175	137	121	177
XOM	0.0093	0.0069	0.0053	0.516	217	99	85	211

Next, let's look at the overall performance of the model across the 29 companies. The results are summarized in *Table 2*. It turns out the AR1, while having relatively low RMSE and MAD, achieves just over 50% overall accuracy in predicting the sign of future returns. It turns out that one can achieve comparable or better accuracy by random guessing and sheer luck. The odds that given level of accuracy in forecasting the sign of future returns has been achieved by random guessing are summarized in *Table 25* provided in the *Appendix.* Moreover, for just 15 of the stocks the model achieves accuracy which is strictly greater than 50%, leaving the remaining 14 companies with accuracy of 50% or lower. This doesn't seem as much of an achievement regardless of the actual size of the prediction errors. More prediction accuracy statistics are reported in the *Table 27* in the *Appendix*. Table 27, among others indicate that when the model says the return will be positive, it is actually correct 51% of the time. This is better than always expecting a positive return, because the share of positive returns on all observations is 50.6%.

Table 2

Aggregate Summary of AR1 performance on test set

Model	RMSE	Mean Absolute Deviation	Median Absolute Deviation	Accuracy	Accuracy over 50
AR1	0.0097	0.0072	0.0056	0.5018	15

* For ratios, the metric average over all 29 stocks is used

As a next step of the analysis, the use of the predictions from AR1 in stock trading is presented. Instead of pinning down all future stock returns, investor might be interested in buying (selling) only stocks which the model identifies as the ones with highest (lowest) expected future return. As described in *The returns achievable by trading* chapter, two scenarios are considered: 1) for every day in the test set, make day ahead predictions for all of the stocks and select those with predicted return in the highest quartile. Subsequently, calculate the daily means of their returns; 2) again make day ahead predictions for every stock in the test set, but this time select those with positive predicted return. Then calculate daily means of their returns. The model that can identify stocks with better returns is deemed superior. The basic requirement for a quality model is that it can outperform the returns from holding all stocks for the entire time period covered by the test set. The results of this analysis are summarized in *Table 3*.

Let's start by reviewing the results for the top quartile trading strategy. Trading stocks within the highest quartile of predicted daily returns yields better results compared to investing in all 29 stocks present in the test set. Thus, the AR1 model should provide a decent benchmark for the pricing models leveraging realized variance, skewness and kurtosis when evaluating the economic criteria. The return achieved by this strategy is higher in 51.6% of cases, which translates to AR1 based strategy focusing on the top quartile achieving higher returns in 317 days out of 614 in the test set. Of course, the frequent trading comes with significant costs and thus the strategy is profitable only if the trading agent reaches a critical scale of operations. The returns from the top quartile strategy have variance higher by 32%, on the other hand, they are also less negatively skewed and have lower kurtosis. This translates to less negative returns and less extreme events. It is rather interesting that variance is higher for the AR1 strategy while kurtosis is much lower, as both of these measures can be interpreted as measures of risk.

However, if we wanted to buy stocks based on whether the predicted return is positive or not, the mean return over the entire period would have been comparable to buying all stocks at the beginning of the test period. On top of the lower returns, this strategy also yields variance of returns comparable to the quartile-based strategy, with negative skew and kurtosis going up. Additionally, the strategy based on positive predictions is beaten by 29 stocks strategy in 315 out of 615 days.

Table 3

Comparison of means of log returns from AR1 Top Quartile (TQ) and Positive Predictions (PP) strategies

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
AR1 TQ	-0.045774	-0.003744	0.00028	0.000202	0.004366	0.040566
$AR1$	-0.041756	-0.003279	0.00036	0.000146	0.003599	0.033092
29 Stocks	-0.045669	-0.003211	0.00046	0.000130	0.003537	0.033694

	Variance	Pearson skewness	Moors kurtosis
AR1 TQ	0.000051	-0.034566	6.992001
AR1 PP	0.000048	-0.112896	7.314694
29 Stocks	0.000039	-0.160277	8.700782

In this section, it has been shown that AR1 model provides a reasonable benchmark for the RMSE and MAD statistics, however it is not able to clearly outperform random guessing in identification of the sign of future returns. Moreover, when the AR1 model identify a stock as one with return in the top quartile, the stock usually ends up providing relatively good return compared to the overall market. However, related to its' inability to beat random guessing in identifying the sign of all returns is a fact that one isn't worse off passively holding the stocks rather than buying all stocks for which the model claims that the future return will be positive. The following chapter explores whether some of the deficiencies of the AR1 model can be fixed by using realized moments in the pricing equation.

4.1.2 Pricing model with realized variance, skewness and kurtosis

In this section, the results for $Model\ \theta^{RV}$ with traditional measures of realized variance, skewness and kurtosis are discussed. Comparison to the benchmark AR1 model and overall market is made.

While the statistical summary of the individual estimates can be found in *Table 29* presented in the appendix, the Figure 3 Figure 3: Estimated coefficients for $\mathbf{Model\ \theta^{RV}}$and Figure 4 provided bellow clearly indicate that the estimates are not 100% consistent across the 29 companies. As in case of Figure 1, Figure 3 shows the approximate 95% confidence interval of estimates for each of the companies. As before, triangle indicates values below 0, while solid dots indicate values above zero.

As Figure 3 indicates, the point estimate for coefficient of lagged return is still consistent with the estimates from AR1 model, although the 95% confidence interval now mostly straddles the 0 mark. However, for 8 of the companies we can be pretty sure that the relationship is negative. Putting this information together with the fact that in independent estimations, the point estimate for other 20 companies has been also negative, it seems reasonable to conclude that investors taking on the risk of investing in stocks that has been declining recently should be rewarded by higher risk premium. It is also worth pointing out, that not for a single one of the companies has the relationship been estimated to be significant and positive, and even for City and IBM we now have negative point estimates, as opposed to the results from AR1.

The results for realized variance are mixed. On one hand, we can see in second panel of Figure 3 and Figure 4 that the point estimate is positive for majority of the companies, which would be perfectly in line with our expectations as well as with previous literature on the topic. However, as Figure 3 also clearly shows, for 1 in 4 of the companies, the point estimate is negative (although significant only for City) and significantly positive for only 3 of them. Overall, there is perhaps some evidence of positive relationship between realized variance and return.

With skewness a first major surprise arises. Based on Amaya et al (2015) one would expect the relationship to be negative. Moreover, we can ask a question of why, when investors are rewarded for accepting higher variance by higher risk premium, we should not expect them to be rewarded for lower skewness with higher premium as well. First, it might be possible that historical skewness is not a good determinant of future skewness and thus does not really measure the risk investor is exposed to. Rather, the positive estimates could indicate that investors tend to react to skewed within-day returns and base their decisions on that. A better proxy for the long-term skewness is a combination of the lagged return and skewness. Indeed, this is in practice very similar to the proposition of Neuberger & Payne (2018). They argue that when calculating long-run skewness from high-frequency returns, the leverage effect (correlation of past low frequency return with variance over the previous period) is what determines the major part of the calculated measure. This means that as the horizon becomes longer, the skewness of high frequency returns becomes less important and the leverage effect takes over. The $Model\,\theta^{RV}$ contains all these components, only it separates the two effects. From a more technical perspective, the different results for coefficient on skewness compared to Amaya et al (2015) could be caused by inclusion of the lagged return in the $Model\,\theta^{RV}$. This theory is later revisited in chapter on robustness to model specification.

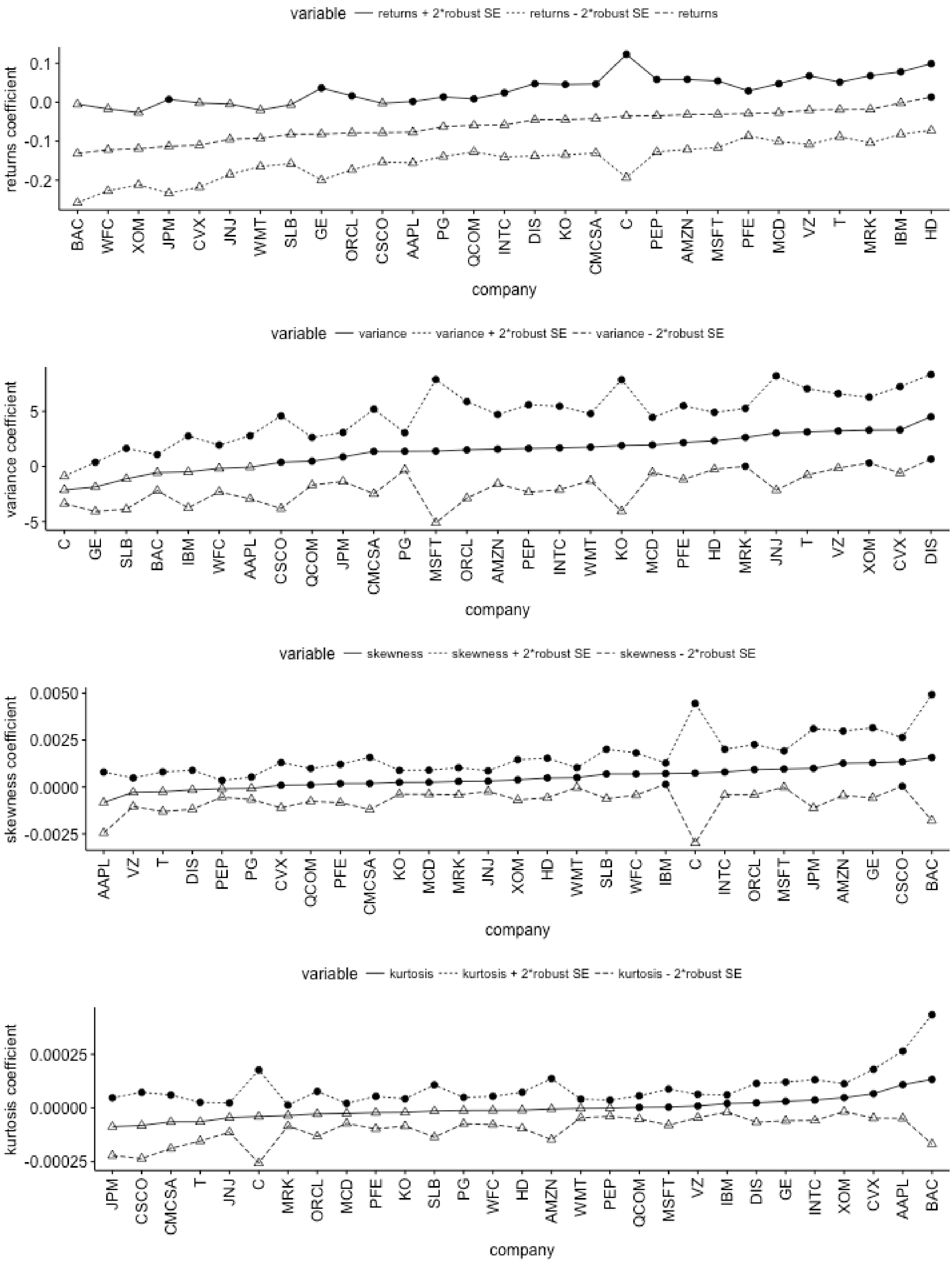

Figure 3: Estimated coefficients for Model θ^{RV}

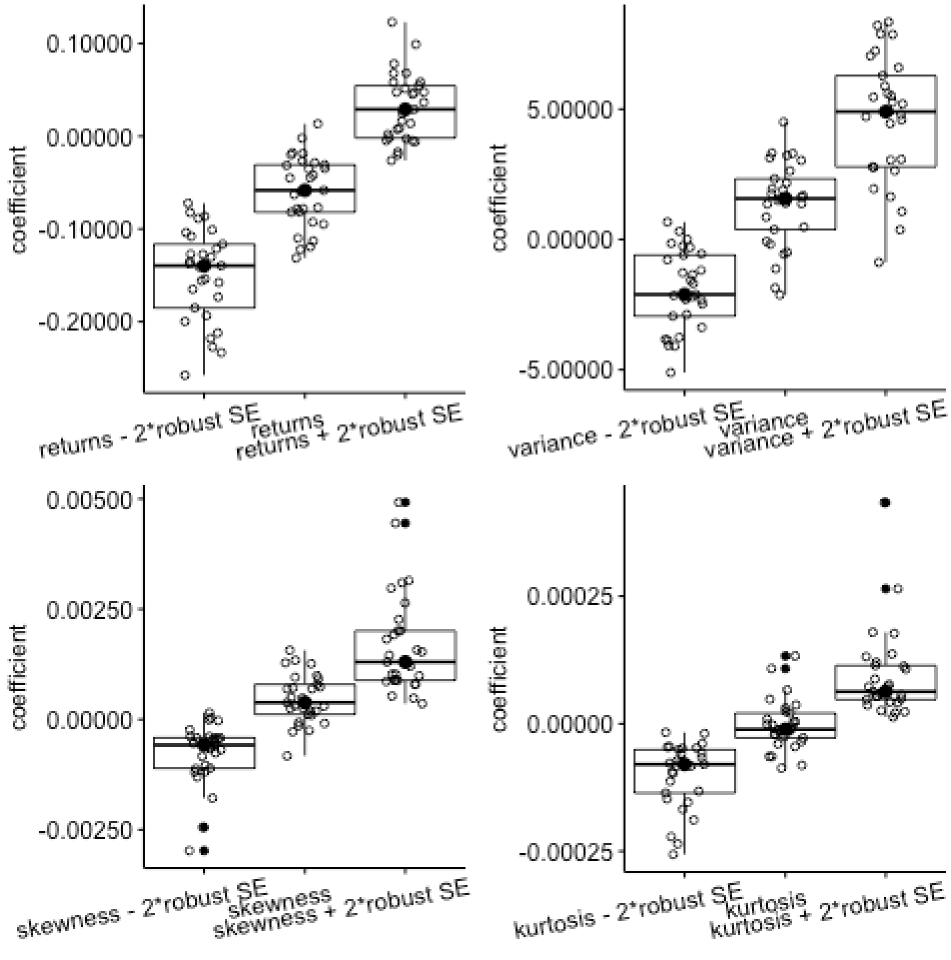

Figure 4: Boxplots of estimated coefficients for Model θ^{RV}

The estimates for kurtosis are not consistently different from zero in either of the directions. For 18 companies the point estimate is negative, for the remaining 11 it is positive. This is further underlined by the fact, that the approximate 95% confidence interval never stays clear of the 0 value. From the statistical perspective, as evaluated on the training set, it would seem that the conclusion here would be clear – we do not have enough evidence to suggest that there is really a relationship between kurtosis and risk premium. However, this is only part of the story, as will become clear throughout the rest of the thesis. First, let's turn to evaluation of the model performance on the test set.

The Figure 5 and Figure 6 presented below compare the AR1 model and Model θ^{RV} on RMSE and accuracy in terms of predicting the sign of future returns. Moreover, Figure 7 depicts the relationship between RMSE and accuracy of $Model\ \theta^{RV}$. The full summary of the performance on the test set is available in Table 30 presented in the appendix.

From Figure 5 we can see that $Model\ \theta^{RV}$ does not represent any improvement in terms of RMSE. As is clear from Table 30, the RMSE is actually much worse as it went up for 24 out of 29 stocks compared to AR1. While mean absolute deviation is also higher for $Model\ \theta^{RV}$, the median absolute deviation is actually better for 17 of the stocks. This would suggest, that the model has some relatively large prediction errors, while the median prediction is better compared to AR1.

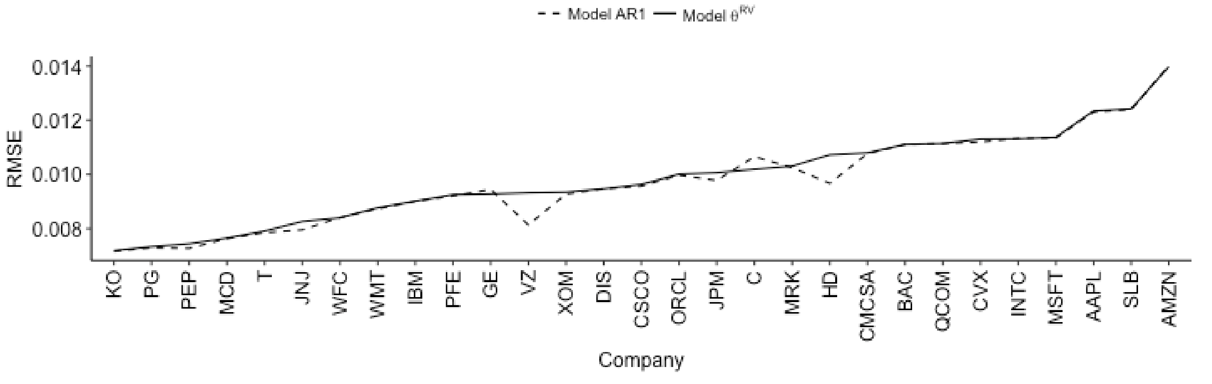

Figure 5: Comparison of test set RMSE for models AR1 and θ^{RV}

While the total error was not improved by $Model\ \theta^{RV}$, the accuracy was. As is clearly visible from Figure 6 below, the model achieved over 50% accuracy for 20 of the stocks, compared to just 14 in case of AR1. Moreover, compared to AR1, the model improved the accuracy for 19 of the stocks. Similarly, the precision is higher compared to AR1 on 18 of the stocks and negative predictive value is higher on 17 of the stocks. Still, as in case of AR1 the estimated model is rather useless for some of the stocks, such as Citigroup where it predicts a negative return over the entire time period in the test set and Procter & Gamble where the model insists on positive returns no matter what. For stocks like these, it is possible that the estimated model has been highjacked by some extreme observations. Thus, clearly a more robust method instead of OLS or a more robust estimators of the moments are needed.

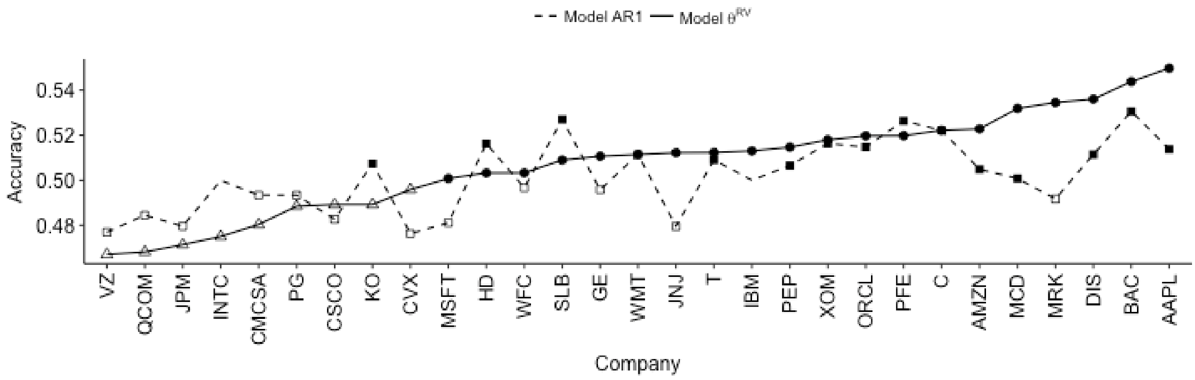

Figure 6: Comparison of test set accuracy for models AR1 and θ^{RV}

As is shown in Figure 7, there is no clear relationship between RMSE and accuracy in predicting the sign of future returns. This further highlights the notion that the models need to be evaluated on complex set of criteria, rather than focusing on statistic such as RMSE.

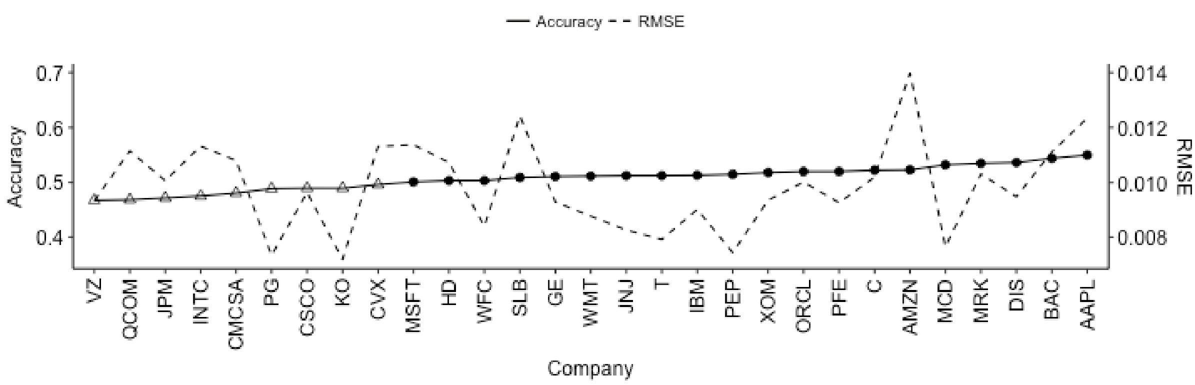

Figure 7: Relationship of test set accuracy and RMSE for Model θ^{RV}

The aggregate accuracy compared to AR1 is summarized in Table 4. While, as has been already mentioned, $Model\ \theta^{RV}$ is not superior to the AR1 in terms of the RMSE, there are other metrics which indicate variance, skewness and kurtosis are decent determinants of the returns and should be considered as part of the pricing equation. First, the aggregate accuracy of 50.7% is significantly better than random guessing (if we guessed the sign randomly, then given the test set size, we would observe accuracy over 50.5% in less than 10% of the attempts). We can say the same about the accuracy over individual time series – for our 29 stocks, we would get the accuracy over 50% for 20 of them in less than 4% of attempts if we were to guess the sign of the return randomly. Second, the precision is over 52.3% which means that when the model predicts a positive return it is right in 52% of the cases. That is over 1 percent more than AR1 which had only 51% precision. Moreover, the negative predictive value is almost 51% which means that also when predicting a negative return, the model is correct in slight majority of cases as opposed to AR1's 48.7% negative predictive value (see *Table 28* reported in the *Appendix*)

Table 4

Table: Aggregate Summary of $Model\ \theta^{RV}$ performance on test set

Model	RMSE	Mean Absolute Deviation	Median Absolute Deviation	Accuracy	Accuracy over 50
$Model\ \theta^{RV}$	0.0098	0.0072	0.0056	0.5074	20
	Precision	Negative Predictive Value	Improved RMSE	Improved Median absolute dev.	Improved Accuracy
$Model\ \theta^{RV}$	0.5234	0.5095	5	17	17

* For ratios, the metric average over all 29 stocks is used

While metrics such as RMSE, accuracy and precision provide useful insights, in order to fully evaluate the quality of the models it seems necessary to look at the properties of the returns achievable with strategies based on the model predictions. As in case of AR1 model, results for 2 strategies are evaluated: 1) every day choose stocks that have the predicted return in top quartile, 2) every day choose all stocks with predicted positive return. The properties of mean daily returns achieved in this way are summarized in Table 5. The following paragraph explains why the $Model\ \theta^{RV}$ should be considered superior to AR1 and why it works better compared to diversified portfolio of 29 stocks.

It's immediately obvious that both trading strategies based on the $Model\ \theta^{RV}$ beat the AR1 model. Looking at *Table 5* and starting from the left, the most negative return is only -0.0435 and -0.0437 for Top Quartile and Positive Predictions strategy respectively. Recall that for AR1 these were -0.0458 and -0.049 while the whole market had on its' worst day a mean return of -0.0457. While the median for Positive Predictions strategy doesn't beat that of whole market, both AR1 strategies are clearly beaten. Moreover, in terms of mean return, both of the strategies beat all of the AR1 strategies as well as the portfolio of all 29 stocks. Finally, while the risk (as measured by variance of mean returns over the test period) of the strategies is still higher compared to the portfolio of 29 stocks, the $Model\ \theta^{RV}$ Top Quartile strategy has 4% lower variance of returns while Positive prediction has 11% lower variance relative to their AR1 counterparts. On top of that, both strategies have less negatively skewed returns compared to the market return and the kurtosis (as measured by Moors Kurtosis) is significantly lower compared to market as well as AR1 based strategies. These results, together with the findings on improved accuracy in identifying positive and negative returns seem to provide sufficient evidence that realized variance, skewness and kurtosis bring significant value to understanding of the stock returns.

Table 5

Comparison of means of log returns from $\boldsymbol{Model\ \theta^{RV}}$ Top Quartile (TQ) and Positive Predictions (PP) strategies

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
$Model\ \theta^{RV}$ TQ	-0.043528	-0.003595	0.00048	0.000290	0.004431	0.034846
$Model\ \theta^{RV}$ PP	-0.043719	-0.003533	0.00037	0.000208	0.004170	0.034184

	Variance	Pearson skewness	Moors kurtosis
$Model\ \theta^{RV}$ TQ	0.000049	-0.080383	5.932262
$Model\ \theta^{RV}$ PP	0.000043	-0.072098	6.653542

* the reported statistics describe the distributions of daily means of log returns of chosen stocks

4.2 Robustness

Several robustness checks are performed in this section. First, results for jump-robust estimators of variance are analyzed. Second, these models, together with $Model\ \theta^{RV}$, are used in determining how well does an average model (based on coefficient mean or median) fit the data. Third, more robust estimators of skewness and kurtosis are explored. Finally, a sensitivity to model specification is explored. In the last section, several model restrictions and extensions are considered in order to determine whether each of the components of $Model\ \theta^{RV}$ bring additional value to the pricing formula.

4.2.1 Jump-robust estimators

In this section, the results of model given by (3.1) for $M \in \{BPV, MinRV, MedRV\}$ are explored. As a result of the estimation, we get the following set of models: $Model\ \theta^{BPV},\ Model\ \theta^{MinRV},\ and\ Model\ \theta^{MedRV}$. For more details on their exact definitions, please refer to the *Methodology* chapter, section with definitions of models. As these models follow the structure given by (3.1), they are very similar to $Model\ \theta^{RV}$, with the only difference being the use of jump robust estimators of realized variance. The statistical summaries are presented in the appendix in Table 33 - *Table 35* in order to keep main body of the text more readable. The 95% confidence intervals for the coefficient estimates are provided in Figure 8 for $Model\ \theta^{BPV}$, Figure 9 for $Model\ \theta^{MinRV}$ and finally Figure 10 for $Model\ \theta^{MedRV}$. All these figures are provided in the appendix.

When BPV is used the estimates on lagged return are still mostly negative and for 7 of the stocks the upper bound of the 95% confidence interval lies below 0. For Variance, there are again two stocks – GE and Citi – for which the estimate is negative and significant, while for 21 stocks we get the expected positive estimate. The skewness estimates still present the unexpected result of positive relationship between skewness and next period return. For kurtosis, we get 11 negative estimates and 18 positive estimates. However, the 95% CI crosses 0 for most of the stocks. In general, the results are comparable to the case of $Model\ \theta^{RV}$.

The individual estimates coming from models using *MinRV* and *MedRV* estimators of variance tell a very similar story. Again, in the tables reported in the appendix we can see that estimates on lagged return are mainly negative, estimates of variance are mostly positive. Skewness estimates are again mostly positive, as are estimates for kurtosis. In general, there are two emergent patterns: 1) the estimated coefficients for variance, skewness and kurtosis tend to be mostly positive, regardless of the definition of realized variance estimator we decide to use. 2) The estimates do not appear to be statistically significant anywhere.

While we do not have clear results on the estimates, we get at least get certain degree of consistency across the individual models. Table 6 summarizes how do the companies with positive and negative estimates of kurtosis coefficient change between the individual models. Included are all companies for which the estimated coefficient is larger than its' robust standard error. We can see that it never happens for one company to have such a positive estimate of the kurtosis coefficient under one model and then get a negative estimate under another model.

Table 6

Companies with differing relation between kurtosis and expected return

RV		BPV		MinRV		MedRv	
+	-	+	-	+	-	+	-
AAPL	CMCSA	AAPL	CSCO	DIS	AMZN	AAPL	C
IBM	CSCO	BAC	JPM	IBM	C	DIS	CMCSA
CVX	JNJ	DIS	MCD	PEP	CMCSA	IBM	CSCO
QCOM	JPM	IBM	ORCL	SLB	CSCO	INTC	JPM
XOM	MCD	PEP	T	WFC	JPM	PEP	ORCL
	MRK	SLB		XOM	ORCL	WFC	T
	T	WFC			T	XOM	
		XOM					

The *Table 7* summarizes the out of sample performances of models θ^{BPV}, θ^{MinRV}, $and\ \theta^{MedRV}$. As can be seen, in terms of magnitudes of prediction errors, they are relatively comparable as the RMSE is 0.0098 for models θ^{BPV} and θ^{MedRV}, and 0.0099 for the model θ^{MinRV}. Moreover, in terms of mean absolute deviation and median absolute deviation these models are very much alike. Accuracy-wise, all three models are performing worse compared to the *Model* θ^{RV}, however they are still able to outperform the AR1. On the other hand, if we were guessing the sign on random, we would be able to get higher accuracy in some 12% – 30%. Similarly, the models achieve accuracy over 50% for 14, 16 and 17 stocks for model θ^{BPV}, θ^{MinRV}, $and\ \theta^{MedRV}$ respectively. As a slight positive can be considered that the models achieve both precision and negative predictive value over 50% in which they clearly beat the AR1 model. Overall, based on fit of the data, the use of jump-robust realized variance estimators does not seem to offer any significant improvements in predictions compared to *Model* θ^{RV}. This could be caused by already mentioned fact that realized skewness and realized kurtosis measures are still impacted by any outliers mitigated in the realized variance and thus the outliers are likely to still hold a sway over the model.

Other possible explanation could be that omission of jump from the risk measures simply does not yield any significant benefit.

Table 7

Aggregate Summary of models $\boldsymbol{\theta^{BPV}}$, $\boldsymbol{\theta^{MinRV}}$, $\boldsymbol{\theta^{MedRV}}$ performance on test set

Model	RMSE	Mean Absolute Deviation	Median Absolute Deviation	Accuracy	Accuracy over 50
Model θ^{BPV}	0.0098	0.0072	0.0056	0.5033	14
Model θ^{MinRV}	0.0099	0.0072	0.0056	0.5021	15
Model θ^{MedRV}	0.0098	0.0072	0.0056	0.5046	17

	Precission	Negative Predictive Value	Improved RMSE	Improved Median absolute dev.	Improved Accuracy
Model θ^{BPV}	0.5093	0.5114	13	14	14
Model θ^{MinRV}	0.5060	0.5098	11	16	16
Model θ^{MedRV}	0.5090	0.5122	11	17	17

* For ratios, the metric average over all 29 stocks is used
* Improvements are relative to the Model θ^{RV}

As a next step of the analysis, let's take a look at returns achievable from trading based on predictions from the models. The Table 8 reported below shows that all of the jump-robust models provide some gains in terms of the mean return or they are at least comparable to the *Model* θ^{RV} based strategies. The median return for the Top Quartile strategy is higher for the Model θ^{BPV} and Model θ^{MedRV} compared to the baseline *Model* θ^{RV}. Moreover, the mean return from these two also beats the *Model* θ^{RV} Top Quartile strategy. On the other hand, the variance achieved by the Top Quartile strategy based on the jump-robust estimators is much higher than in case of *Model* θ^{RV} and is comparable to that of AR1. In terms of skewness and kurtosis these models also do not outperform the *Model* θ^{RV}, however in terms of kurtosis they clearly beat both AR1 and 29 Stocks portfolio. In case of the Positive Predictions strategy, the medians are quite comparable to the baseline *Model* θ^{RV}. However, the portfolio of 29 stocks still provide better median return. On the other hand, the mean return from this strategy beats *Model* θ^{RV} as well as AR1 and 29 Stocks portfolio. Moreover, these models yield a mean return higher than that achieved by AR1 Top

Quartile strategy, while offering much lower variance and kurtosis. In terms of variance, these models are at the same level as *Model* θ^{RV}, while kurtosis-wise, they offer less thick or less long tails compared to the *Model* θ^{RV}.

Table 8

Comparison of means of log returns from Top Quartile (TQ) and Positive Predictions (PP) strategies for models $\boldsymbol{\theta^{BPV}}$, $\boldsymbol{\theta^{MinRV}}$, and $\boldsymbol{\theta^{MedRV}}$

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
Model θ^{BPV} TQ	-0.048037	-0.003779	0.000505	0.000294	0.004323	0.034122
Model θ^{BPV} PP	-0.042947	-0.003652	0.000371	0.000214	0.004086	0.034184
Model θ^{MinRV} TQ	-0.048037	-0.003618	0.000430	0.000274	0.004200	0.034846
Model θ^{MinRV} PP	-0.042986	-0.003567	0.000265	0.000214	0.004103	0.034184
Model θ^{MedRV} TQ	-0.050305	-0.003603	0.000488	0.000294	0.004366	0.034846
Model θ^{MedRV} PP	-0.042529	-0.003609	0.000323	0.000237	0.004119	0.034184

	Variance	Pearson skewness	Moors kurtosis
Model θ^{BPV} TQ	0.000051	-0.088790	6.607812
Model θ^{BPV} PP	0.000043	-0.072240	6.533168
Model θ^{MinRV} TQ	0.000051	-0.065749	6.736116
Model θ^{MinRV} PP	0.000043	-0.023417	6.559622
Model θ^{MedRV} TQ	0.000051	-0.081381	7.328952
Model θ^{MedRV} PP	0.000043	-0.039226	6.407243

* the reported statistics describe the distributions of daily means of log returns of chosen stocks

Overall, the jump-robust estimators do not yield results too different from what was achieved with *Model* θ^{RV}. The coefficient estimates are relatively similar, the performance as measured by RMSE is comparable and the models θ^{BPV}, θ^{MinRV}, and θ^{MedRV} are slightly worse in terms of predicting the sign of future returns. This shortcoming is however compensated for by improved returns when the models are used in trading decisions. The most reasonable conclusion here seems to be that the models do not invalidate the findings for Model θ^{RV} and they do not bring significant additional value to asset pricing.

4.2.2 Do mean and median models fit the data?

It seems natural to ask how would the average model perform. If the model based on means or medians of the individual coefficients would perform relatively well, we could assume that the premiums for each company are determined by same or at least similar process. If, on the other hand, the average model would not fit the data at all, it would mean that the process of determining risk premium is different across companies. Another motivation for this exercise could be found in Tibshirani et al (2001) where model averaging is mentioned as strategy for improving the predictive capabilities of models. Hence, similar to Amaya et al (2015), average coefficients from the company specific estimates are calculated and used to make predictions on the test set. Same set of statistics as before is used to determine the performance of the resulting models. First, the average coefficients are reported, then the out of sample performance is analyzed and finally the achievable returns are discussed. The models evaluated here are $\overline{\theta^{(M)}}$ and $\widetilde{\theta^{(M)}}$, where $M \in \{RV, BPV, MinRV, MedRV\}$.

As *Table 9* reported below indicates, the models are consistent in terms of coefficients on intercept, return, variance and skewness. For these variables, both average and median indicate the same sign and relatively similar values of the estimates. We get the already expected result that past positive returns are penalized by lower return in the future and that higher risk as measured by variance is compensated for by risk premium. At the same time, both mean and median coefficient on skewness are positive, meaning that higher skewness means higher risk premium. This result is contradictory to that of Amaya et al (2015) but as is shown in the chapter on *robustness to model specification*, this is probably due to lagged returns being included as an explanatory variable in the models presented in the thesis. Slightly more problematic is the average and median coefficient for kurtosis. The coefficient is negative for models $\overline{\theta^{RV}}, \overline{\theta^{MinRV}}$ and $\overline{\theta^{MedRV}}$, which means that higher kurtosis of the returns in penalized by lower future returns. The coefficient is positive for $\overline{\theta^{BPV}}$. However, when we look at the median coefficient estimate, we can see that the relationship is positive for all models except $\widetilde{\theta^{RV}}$, which means that for majority of the companies, we get the expected positive relationship between tail risk and returns. Why the difference between the average and median estimates? It is a well-known behavior of the OLS method that it's not very robust to outliers in the data. As kurtosis is technically a measurement of outliers, we can expect that there might be some problems when extreme data points are observed and so some of our estimates might be influenced by the outliers. Overall, the consistency of coefficient estimates for the kurtosis variable remains questionable.

Table 9

Average and median coefficients of individual models

Model	Intercept	Return	Variance	Skewness	Kurtosis
Model $\overline{\theta^{RV}}$	-2.43E-04	-5.95E-02	1.35E+00	4.60E-04	-3.04E-06
Model $\overline{\theta^{BPV}}$	-2.79E-04	-5.63E-02	1.39E+00	2.86E-04	1.34E-06
Model $\overline{\theta^{MinRV}}$	-2.15E-04	-5.57E-02	1.36E+00	2.94E-04	-6.70E-06
Model $\overline{\theta^{MedRV}}$	-2.34E-04	-5.53E-02	1.42E+00	2.72E-04	-4.92E-06
Model $\widetilde{\theta^{RV}}$	-2.05E-04	-5.84E-02	1.57E+00	3.82E-04	-1.10E-05
Model $\widetilde{\theta^{BPV}}$	-2.16E-04	-5.53E-02	1.69E+00	2.04E-04	1.56E-06
Model $\widetilde{\theta^{MinRV}}$	-2.25E-04	-5.31E-02	1.67E+00	1.70E-04	1.62E-06
Model $\widetilde{\theta^{MedRV}}$	-2.19E-04	-5.44E-02	1.66E+00	1.46E-04	1.48E-06

The Table 10 summarizes the performance of models on the test set. All of the models based on coefficient means or medians outperform the Model θ^{RV} in terms of RMSE, as the worst performing of the aggregated models has RMSE of *0.00980*, while Model θ^{RV} achieved RMSE of *0.00984* over the entire test set. Moreover, the aggregated models perform better in terms of Mean Absolute Deviation. The results are mixed in terms of accuracy of the sign prediction, but we see some slight improvements from models $\overline{\theta^{RV}}, \overline{\theta^{MinRV}}$ and $\overline{\theta^{MedRV}}$, while the models based on the medians of the coefficient generally perform worse compared to Model θ^{RV}. Surprisingly, while the individual models using jump robust variance estimators do not outperform Model θ^{RV}, the accuracy is better when the coefficient averages are used. All of the models achieve accuracy significantly better than random guessing and they attain above 50% accuracy on majority of the time series. Both, precision and negative predictive value are above 50% indicating that the models are relatively good at identifying both negative and positive future returns. Compared to Model θ^{RV}, the models presented here improve the RMSE for 19-21 of the companies.

Table 10

Aggregate summary of the performance of models $\overline{\theta^{(M)}}$ and $\widetilde{\theta^{(M)}}$

Model	RMSE	Mean Absolute Deviation	Median Absolute Deviation	Accuracy	Accuracy over 50
Model $\overline{\theta^{RV}}$	0.00978	0.00721	0.00560	0.50589	21
Model $\overline{\theta^{BPV}}$	0.00978	0.00721	0.00561	0.50850	19
Model $\overline{\theta^{MinRV}}$	0.00977	0.00720	0.00560	0.50743	18
Model $\overline{\theta^{MedRV}}$	0.00978	0.00721	0.00560	0.50782	22
Model $\widetilde{\theta^{RV}}$	0.00979	0.00721	0.00560	0.50640	16
Model $\widetilde{\theta^{BPV}}$	0.00980	0.00721	0.00559	0.50724	15
Model $\widetilde{\theta^{MinRV}}$	0.00980	0.00721	0.00559	0.50775	16
Model $\widetilde{\theta^{MedRV}}$	0.00980	0.00721	0.00559	0.50646	16

	Precission	Negative Predictive Value	Improved RMSE	Improved Median absolute dev.	Improved Accuracy
Model $\overline{\theta^{RV}}$	0.51652	0.50027	21	15	14
Model $\overline{\theta^{BPV}}$	0.52070	0.50267	21	12	17
Model $\overline{\theta^{MinRV}}$	0.52022	0.50163	21	13	14
Model $\overline{\theta^{MedRV}}$	0.52079	0.50204	21	12	17
Model $\widetilde{\theta^{RV}}$	0.51794	0.50095	21	14	17
Model $\widetilde{\theta^{BPV}}$	0.51560	0.50203	20	15	13
Model $\widetilde{\theta^{MinRV}}$	0.51683	0.50249	20	16	14
Model $\widetilde{\theta^{MedRV}}$	0.51497	0.50151	19	17	14

* For ratios, the metric average over all 29 stocks is used
* Improvements are relative to the Model θ^{RV}

The results for returns are summarized in Table 11 presented below. Two patterns can be seen in the data. First, the Top Qurntile trading strategies based on predictions from models $\overline{\theta^{RV}}, \overline{\theta^{BPV}}, \overline{\theta^{MinRV}}, \overline{\theta^{MedRV}}, \widetilde{\theta^{RV}}, \widetilde{\theta^{BPV}}, \widetilde{\theta^{MinRV}}$ and $\widetilde{\theta^{MedRV}}$ achieve higher mean returns compared to their counterparts with separate coefficients for individual companies. The mean return is 2.8% to 31% higher compared to Model θ^{RV} with majority of the models offering at least 14% increase in the mean return. Moreover, another argument for resorting to model averages would be that the first quartile, 3rd quartile and maximum being higher compared to the Model θ^{RV}. In general, the right tail of the mean returns coming from Top Quartile strategy based on mean or median models is longer and thicker compared to the returns coming from its counterpart based on estimates for individual series.

Second, the Positive Predictions strategy appears to be useless across models when we think in terms of maximizing the return for the trader. It achieves lower mean return compared to both Model θ^{RV}, as well as all of the jump-robust variants. Moreover, it does not consistently beat the return on the entire portfolio of the 29 stocks. On the other hand, the median return is higher for 6 out of 8 models compared to the Model θ^{RV} and, specifically, both $\overline{\theta^{RV}}$ and $\widetilde{\theta^{RV}}$ model yield higher median return compared to the Model θ^{RV}. This means that the model is actually pretty good at identifying the stocks with positive return in general, however it sometimes misses larger negative returns (as demonstrated by lower minimum return and 1st quartile) while the broad strategy does not allow to capitalize on high returns as demonstrated by relatively low 3rd quartile and maximum of returns. Together with relatively higher variance and kurtosis, it would not make much sense for a trader to employ the strategy of buying all stocks with predicted positive returns based on the average or median models.

Table 11

Comparison of means of log returns from Top Quartile (TQ) and Positive Predictions (PP) strategies based on models $\overline{\boldsymbol{\theta}^{(M)}}$ *and* $\widetilde{\boldsymbol{\theta}^{(M)}}$

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
Model $\overline{\theta^{RV}}$ TQ	-0.046938	-0.003454	0.00044	0.000332	0.004510	0.039287
Model $\overline{\theta^{BPV}}$ TQ	-0.046938	-0.003489	0.00035	0.000317	0.004343	0.039287
Model $\overline{\theta^{MinRV}}$ TQ	-0.046938	-0.003397	0.00039	0.000351	0.004524	0.039287
Model $\overline{\theta^{MedRV}}$ TQ	-0.046938	-0.003408	0.00037	0.000316	0.004491	0.039287
Model $\widetilde{\theta^{RV}}$ TQ	-0.046938	-0.003408	0.00044	0.000353	0.004515	0.039287
Model $\widetilde{\theta^{BPV}}$ TQ	-0.046938	-0.003344	0.00049	0.000381	0.004721	0.039287
Model $\widetilde{\theta^{MinRV}}$ TQ	-0.046938	-0.003353	0.00023	0.000309	0.004736	0.039287
Model $\widetilde{\theta^{MedRV}}$ TQ	-0.046938	-0.003507	0.00026	0.000298	0.004463	0.039287
Model $\overline{\theta^{RV}}$ PP	-0.044670	-0.003513	0.00068	0.000179	0.004512	0.033725
Model $\overline{\theta^{BPV}}$ PP	-0.045669	-0.003959	0.00048	0.000098	0.004441	0.033725
Model $\overline{\theta^{MinRV}}$ PP	-0.045669	-0.003826	0.00031	0.000118	0.004330	0.033725
Model $\overline{\theta^{MedRV}}$ PP	-0.045669	-0.004039	0.00054	0.000051	0.004342	0.033725
Model $\widetilde{\theta^{RV}}$ PP	-0.045669	-0.003617	0.00063	0.000167	0.004486	0.033725
Model $\widetilde{\theta^{BPV}}$ PP	-0.045669	-0.003548	0.00051	0.000160	0.004062	0.033725
Model $\widetilde{\theta^{MinRV}}$ PP	-0.045669	-0.003548	0.00049	0.000161	0.004038	0.033725

Model $\widetilde{\theta^{MedRV}}$ PP	-0.045669	-0.003552	0.00050	0.000147	0.004203	0.033725

	Variance	Pearson skewness	Moors kurtosis
Model $\overline{\theta^{RV}}$ TQ	0.000054	-0.04406	6.449298
Model $\overline{\theta^{BPV}}$TQ	0.000055	-0.01450	6.165686
Model $\overline{\theta^{MinRV}}$ TQ	0.000054	-0.01679	6.309154
Model $\overline{\theta^{MedRV}}$ TQ	0.000055	-0.02287	6.238992
Model $\widetilde{\theta^{RV}}$ TQ	0.000057	-0.03513	6.204171
Model $\widetilde{\theta^{BPV}}$ TQ	0.000055	-0.04348	6.072788
Model $\widetilde{\theta^{MinRV}}$ TQ	0.000055	0.03281	6.073715
Model $\widetilde{\theta^{MedRV}}$ TQ	0.000055	0.01520	6.094608
Model $\overline{\theta^{RV}}$ PP	0.000051	-0.21018	6.237329
Model $\overline{\theta^{BPV}}$ PP	0.000055	-0.15375	5.985885
Model $\overline{\theta^{MinRV}}$ PP	0.000055	-0.07823	5.880643
Model $\overline{\theta^{MedRV}}$ PP	0.000057	-0.19363	5.716154
Model $\widetilde{\theta^{RV}}$ PP	0.000053	-0.19027	6.179175
Model $\widetilde{\theta^{BPV}}$ PP	0.000051	-0.14565	6.367972
Model $\widetilde{\theta^{MinRV}}$ PP	0.000051	-0.13748	6.184407
Model $\widetilde{\theta^{MedRV}}$ PP	0.000051	-0.14755	6.285976

* the reported statistics describe the distributions of daily means of log returns of chosen stocks

As has been shown in this section, the aggregated models fit the data relatively well, especially in case of $\overline{\theta^{(M)}}$. This is clearly visible in the improved RMSE and relatively good accuracy given that the models attempt to price all stocks using the same coefficient for all companies. Moreover, both $\overline{\theta^{(M)}}$ and $\widetilde{\theta^{(M)}}$ and the resulting Top Quartile strategies represent further improvement in the achievable returns. Overall, these results seem to provide further evidence of the usefulness of realized measures in asset pricing and suggest that the pricing process can be reasonably well fitted by one model. However, the models $\overline{\theta^{(M)}}$ and $\widetilde{\theta^{(M)}}$ did not bring huge improvements in the forecasting performance.

4.2.3 Robust estimators of skewness and kurtosis

Having established that the traditional definitions of realized variance, skewness and kurtosis, as well as their jump-robust equivalents, provide significant value in the analysis of stock market returns, it is time to explore how would the results change if robust estimators were used. As mentioned in the *Methodology* chapter, the considered estimators are: Bowley, Groeneveld-Meeden and Pearson measures of skewness, and

Moors and Crow & Siddiqui measures of kurtosis. The models considered here correspond to $Model\, \theta^{(M,P)}$, where $M \in \{Bowley, G\&M, Pearson\}$ and $P \in \{Moors, C\&S\}$.

The estimated coefficients and their robust standard errors are reported in the Appendix in Figure 11 - Figure 16 as well as in the Table 36 - Table 41 presented in *Appendix: Models with robust estimators of skewness and kurtosis.* As can be seen from the figures the point estimates for lagged return are still negative and even statistically significant for 6 to 8 stocks, depending on the robust measures used for estimation of skewness and kurtosis. While for majority of the stocks we couldn't say based on statistical criteria that they are significantly different from 0, the consistency of the sign is noteworthy. Similar results are obtained for variance and skewness, where we, for individual companies, obtain mostly positive point estimates. Clearly, the positive estimates on skewness are not limited to the RS^{RV} estimator. In case of kurtosis, however, the results do not appear to be consistent neither with the previous results nor across different robust estimators of skewness and kurtosis. It can be seen that when Moors estimator of kurtosis is used, we get positive the expected positive estimate on kurtosis coefficient for about 10 of the stocks. However, when we use the definition given by Crow & Siddiqui, we are suddenly down to about 5 stocks with positive estimates. That being said, the approximate 95% confidence intervals reported in Figure 11 - Figure 16 indicate that the impact of kurtosis variable on the risk premium should be non-significant. The fact that the approximate 95% confidence interval is spread around 0, together with the inconsistency of the estimated sign of the coefficients seems as an evidence, that we are not able to detect a link between returns and kurtosis when robust measures of kurtosis are used.

The Table 12 presented below summarizes the mean and median of the coefficients for the cross section of companies. The relative consistency of estimates for returns, variance and skewness across the different estimators is clear. At the same time the estimates for kurtosis differ based on the robust estimators being used.

Table 12

Mean and median coefficients of models with robust estimators of skewness and kurtosis

Model	Intercept	Return	Variance	Skewness	Kurtosis
$\overline{\theta^{Bowley,Moors}}$	-2.71E-04	-5.65E-02	1.42E+00	1.50E-03	1.01E-04
$\overline{\theta^{Bowley,C\&S}}$	-9.28E-05	-5.66E-02	1.43E+00	1.52E-03	-1.80E-04
$\overline{\theta^{G\&M,Moors}}$	-2.88E-04	-9.74E-02	1.47E+00	1.31E-02	1.01E-04
$\overline{\theta^{G\&M,C\&S}}$	-1.12E-04	-9.76E-02	1.48E+00	1.32E-02	-1.79E-04
$\overline{\theta^{Pearson,Moors}}$	-2.86E-04	-9.80E-02	1.47E+00	1.86E-02	1.00E-04
$\overline{\theta^{Pearson,C\&S}}$	-1.11E-04	-9.83E-02	1.48E+00	1.87E-02	-1.79E-04
$\theta^{\widetilde{Bowley,Moors}}$	-2.44E-04	-5.31E-02	1.65E+00	1.44E-03	-6.22E-05
$\theta^{\widetilde{Bowley,C\&S}}$	-8.39E-05	-5.35E-02	1.70E+00	1.49E-03	-3.45E-04
$\theta^{\widetilde{G\&M,Moors}}$	-2.41E-04	-1.03E-01	1.67E+00	9.65E-03	1.77E-06
$\theta^{\widetilde{G\&M,C\&S}}$	-9.09E-05	-1.03E-01	1.69E+00	9.62E-03	-3.12E-04
$\theta^{\widetilde{Pearson,Moors}}$	-2.36E-04	-1.01E-01	1.67E+00	1.42E-02	7.19E-07
$\theta^{\widetilde{Pearson,C\&S}}$	-9.32E-05	-1.02E-01	1.69E+00	1.42E-02	-3.13E-04

As a next step of the analysis, let's take a look at the models' predictive capabilities on the test set. The results are summarized in Table 13 reported below. All of the models with robust measures of skewness and kurtosis improve the RMSE, as well as mean and median absolute deviations, compared to the $Model\ \theta^{RV}$. At the same time, when Bowley skewness is used the accuracy in identifying the sign of future returns is worse and not too different from random guessing. The models using Bowley skewness further suffer from negative predictive value of 48.3% and 49.4% respectively, which means they are not very good at identifying negative returns properly. It can be seen that for 4 of the models, the precision is higher compared to $Model\ \theta^{RV}$, wile their negative predictive value is lower. Thus, they are better at identifying positive returns, however, they are also more likely to be wrong when predicting negative return. Some improvement coming from use of certain robust measures are the statistics on number of stocks for which RMSE, Median absolute deviation or accuracy are improved compared to $Model\ \theta^{RV}$. As the below table indicates, the models using Groeneveld-Meeden skewness or Pearson skewness improve the out of sample RMSE for 19-20 of the stocks, median absolute deviation for 17-18 stocks and overall accuracy is better for 17-18 of the individual series.

Table 13

Aggregate Summary of robust models' performance on the test set

Model	RMSE	Mean Absolute Deviation	Median Absolute Deviation	Accuracy	Accuracy over 50
$\theta^{Bowley,Moors}$	0.00979	0.00722	0.00559	0.50431	17
$\theta^{Bowley,C\&S}$	0.00979	0.00723	0.00560	0.50095	15
$\theta^{G\&M,Moors}$	0.00978	0.00722	0.00559	0.50679	21
$\theta^{G\&M,C\&S}$	0.00978	0.00722	0.00559	0.50944	21
$\theta^{Pearson,Moors}$	0.00977	0.00722	0.00560	0.50678	20
$\theta^{Pearson,C\&S}$	0.00977	0.00722	0.00560	0.50915	21

	Precision	Negative Predictive Value	Improved RMSE	Improved Median absolute dev.	Improved Accuracy
$\theta^{Bowley,Moors}$	0.53518	0.48286	18	14	14
$\theta^{Bowley,C\&S}$	0.51651	0.49425	18	15	15
$\theta^{G\&M,Moors}$	0.50812	0.50159	20	17	17
$\theta^{G\&M,C\&S}$	0.54733	0.48997	19	18	18
$\theta^{Pearson,Moors}$	0.52833	0.50130	20	17	17
$\theta^{Pearson,C\&S}$	0.53781	0.50606	20	18	18

* For ratios, the metric average over all 29 stocks is used
* Improvements are relative to the $Model\ \theta^{RV}$

As in previous scenarios, the returns achievable by the 2 strategies based on recommendations from the individual models are examined. These results are presented in the Table 14. Starting with the mean return from the Top Quartile strategy, we can see that 3 of the models result in much better return compared to $Model\ \theta^{RV}$. These models are $\theta^{Bowley,Moors}$, $\theta^{G\&M,Moors}$ and $\theta^{Pearson,Moors}$. All of these models have mean log return at least 20% above $Model\ \theta^{RV}$Top Quartile strategy. Interestingly, all of the well performing models use moors measure of kurtosis. On the other hand, we have two models which result in significantly lower returns from this strategy: model $\theta^{G\&M,C\&S}$ and model $\theta^{Pearson,C\&S}$. It appears that the models leveraging Siddiqui kurtosis perform worse in terms of predicting the extreme positive returns. In terms of the median return, there is also no consistent benefit over $Model\ \theta^{RV}$. Looking at IQR, minimum, maximum, variance and kurtosis of the resulting returns, one must conclude that for an optimizing trader, there doesn't seem to be any significant benefit from using the Top Quartile strategy based on the robust models.

Meanwhile, the robust models tend to yield better results when evaluated on the Positive Predictions strategy. Thus, it seems $Model\ \theta^{RV}$ is relatively good at spotting large future returns, whereas on average the robust models are better in identifying positive return, as when they do predict some positive returns, they tend to be mostly right and compared to $Model\ \theta^{RV}$ they are right more often. As a result, compared to $Model\ \theta^{RV}$, 5 of 6 robust models achieve higher returns from Positive Predictions strategy. The only model which achieved lower returns was $\theta^{Bowley,C\&S}$, but the difference compared to $Model\ \theta^{RV}$ was merely 1.4%. Moreover, for 4 of the 6 models, the returns from Positive Predictions strategy were higher compared to $Model\ \theta^{RV}$'s Top Quartile strategy. Not only do Positive Predictions strategies based on robust models achieve higher average return, they also provide lower variance, lower inter quartile range and, and that is important, much higher skewness, which in some cases is even positive, suggesting a preferable distribution of returns around the mean.

Overall, it seems that in terms of prediction errors, as well as achievable returns, the models based on robust estimators of skewness and kurtosis provide a non-negligible improvement over the $Model\ \theta^{RV}$

Table 14

Comparison of means of log returns from strategies based on models with robust measures

Model - Strategy	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
$\theta^{Bowley,Moors}$ TQ	-0.047129	-0.003608	0.000436	0.000380	0.004332	0.031807
$\theta^{Bowley,Moors}$ PP	-0.042947	-0.003415	0.000143	0.000270	0.004034	0.032476
$\theta^{Bowley,C\&S}$ TQ	-0.047129	-0.003757	0.000488	0.000284	0.004361	0.035267
$\theta^{Bowley,C\&S}$ PP	-0.042973	-0.003397	0.000305	0.000205	0.004051	0.032375
$\theta^{G\&M,Moors}$ TQ	-0.042635	-0.003584	0.000642	0.000382	0.004704	0.032583
$\theta^{G\&M,Moors}$ PP	-0.042920	-0.003060	0.000232	0.000338	0.004112	0.033596
$\theta^{G\&M,C\&S}$ TQ	-0.042635	-0.003615	0.000369	0.000203	0.004090	0.032583
$\theta^{G\&M,C\&S}$ PP	-0.042920	-0.003389	0.000427	0.000305	0.004163	0.033454
$\theta^{Pearson,Moors}$ TQ	-0.042635	-0.003454	0.000474	0.000353	0.004450	0.032583
$\theta^{Pearson,Moors}$ PP	-0.042920	-0.002940	0.000194	0.000359	0.004110	0.033596
$\theta^{Pearson,C\&S}$ TQ	-0.042635	-0.003596	0.000279	0.000184	0.004252	0.032583
$\theta^{Pearson,C\&S}$ PP	-0.042920	-0.003297	0.000381	0.000296	0.004073	0.033596

	Variance	Pearson skewness	Moors kurtosis
$\theta^{Bowley,Moors}$ TQ	0.000049	-0.023848	6.666691

$\theta^{Bowley,Moors}$ PP	0.000042	0.059371	6.550547
$\theta^{Bowley,C\&S}$ TQ	0.000050	-0.086396	6.829270
$\theta^{Bowley,C\&S}$ PP	0.000042	-0.046531	6.525773
$\theta^{G\&M,Moors}$ TQ	0.000047	-0.113260	5.544816
$\theta^{G\&M,Moors}$ PP	0.000041	0.049202	7.028134
$\theta^{G\&M,C\&S}$ TQ	0.000047	-0.072860	5.857397
$\theta^{G\&M,C\&S}$ PP	0.000041	-0.057024	7.040655
$\theta^{Pearson,Moors}$ TQ	0.000048	-0.052533	5.569126
$\theta^{Pearson,Moors}$ PP	0.000041	0.076935	7.156331
$\theta^{Pearson,C\&S}$ TQ	0.000047	-0.041460	5.816183
$\theta^{Pearson,C\&S}$ PP	0.000041	-0.039894	7.234082

* the reported statistics describe the distributions of daily means of log returns of chosen stocks

4.2.4 Robustness to model specification

In this final section of the results chapter, the impact of changes to model specification is analyzed. 1) Extending the model by interaction terms to at least partially account for the so-called leverage and GARCH effects; 2) Restricting the model to see how the estimates are influenced by, say, omission of daily log return from the explanatory variables. The impact of the changes to specification is evaluated in 3 steps: 1) mean and median coefficients are presented; 2) the performance on the test set is analyzed and results from Diebold-Mariano test are discussed; 3) return from the two trading strategies are compared.

Let’s start with the extended models. It can be observed in Table 15 and Table 16 that the inclusion of interaction terms does not change the results from model estimation too much. While the mean coefficient on realized kurtosis is now positive for all extended models except $\overline{\theta^{1,1,1,1,0,0,1}}$, Table 16 clearly shows that the estimates are still negative for majority of the stocks. Other than that, the estimates for the set of variables included in $Model\ \theta^{RV}$ are persistent across model extensions.

Slightly more interesting case presents itself when we look at the robustness to variable omission. As can be seen from the results for $\overline{\theta^{0,1,1,1,0,0,0}}$ and $\theta^{0,\overset{\frown}{1,1,1},0,0,0}$, both the mean and median coefficient on skewness become negative when lagged returns are excluded from the model. Thus, the difference between results of Amaya et al (2015) and this thesis seems to be explained purely by the decision to account for the lagged returns. Other than that, the estimated coefficients do not change much upon our decision to exclude either of the variables.

Table 15

Summary of mean coefficients across specifications

Model	Intercept	Return	Variance	Skewness	Kurtosis	Variance * Kurtosis	Skewness * Return	Variance * Return
$\overline{AR1}$	-4.23E-05	-4.70E-02	-	-	-	-	-	-
$\overline{\theta^{RV}}$	-2.43E-04	-5.95E-02	1.35E+00	4.60E-04	-3.04E-06	-	-	-
$\overline{\theta^{1,1,1,1,1,1,1}}$	-3.18E-04	-4.88E-02	1.53E+00	3.38E-04	2.29E-05	-1.95E-02	-2.40E-02	-1.07E+01
$\overline{\theta^{1,1,1,1,0,1,1}}$	-2.07E-04	-4.98E-02	1.31E+00	4.04E-04	9.76E-07	-	-1.88E-02	-8.98E+00
$\overline{\theta^{1,1,1,1,1,0,0}}$	-4.29E-04	-5.80E-02	1.65E+00	3.97E-04	2.32E-05	-2.21E-02	-	-
$\overline{\theta^{1,1,1,1,0,1,0}}$	-2.55E-04	-5.79E-02	1.45E+00	4.93E-04	3.80E-06	-	-2.08E-02	-
$\overline{\theta^{1,1,1,1,0,0,1}}$	-2.01E-04	-4.95E-02	1.18E+00	3.77E-04	-3.27E-06	-	-	-9.84E+00
$\overline{\theta^{1,1,1,0,0,0,0}}$	-2.75E-04	-5.91E-02	1.33E+00	4.24E-04	-	-	-	-
$\overline{\theta^{1,1,0,1,0,0,0}}$	-2.51E-04	-5.24E-02	1.35E+00	-	-1.04E-06	-	-	-
$\overline{\theta^{1,0,1,1,0,0,0}}$	-1.66E-05	-5.34E-02	-	4.20E-04	-3.68E-06	-	-	-
$\overline{\theta^{0,1,1,1,0,0,0}}$	-2.14E-04	-	1.34E+00	-1.59E-04	-7.26E-06	-	-	-
$\overline{\theta^{1,1,0,0,0,0,0}}$	-2.68E-04	-5.26E-02	1.33E+00	-	-	-	-	-

Table 16

Summary of median coefficients across specifications

Model	Intercept	Return	Variance	Skewness	Kurtosis	Variance * Kurtosis	Skewness * Return	Variance * Return
$\widetilde{AR1}$	5.92E-05	-4.36E-02	-	-	-	-	-	-
$\widetilde{\theta^{RV}}$	-2.05E-04	-5.84E-02	1.57E+00	3.82E-04	-1.10E-05	-	-	-
$\widetilde{\theta^{1,1,1,1,1,1,1}}$	-2.33E-04	-4.29E-02	1.71E+00	3.70E-04	-6.90E-06	-4.18E-03	-1.01E-02	-1.85E+00
$\widetilde{\theta^{1,1,1,1,0,1,1}}$	-2.22E-04	-5.04E-02	1.74E+00	3.75E-04	-5.81E-06	-	-8.51E-03	-3.00E+00
$\widetilde{\theta^{1,1,1,1,1,0,0}}$	-3.62E-04	-5.99E-02	2.08E+00	3.12E-04	1.56E-05	-3.34E-02	-	-
$\widetilde{\theta^{1,1,1,1,0,1,0}}$	-2.63E-04	-5.77E-02	1.81E+00	4.69E-04	-7.89E-07	-	-1.30E-02	-
$\widetilde{\theta^{1,1,1,1,0,0,1}}$	-1.59E-04	-4.26E-02	1.45E+00	3.22E-04	-9.42E-06	-	-	-4.47E+00
$\widetilde{\theta^{1,1,1,0,0,0,0}}$	-3.07E-04	-5.71E-02	1.57E+00	2.95E-04	-	-	-	-
$\widetilde{\theta^{1,1,0,1,0,0,0}}$	-2.38E-04	-4.82E-02	1.54E+00	-	-6.39E-06	-	-	-
$\widetilde{\theta^{1,0,1,1,0,0,0}}$	1.79E-04	-5.73E-02	-	3.22E-04	-1.01E-05	-	-	-
$\widetilde{\theta^{0,1,1,1,0,0,0}}$	-1.54E-04	-	1.49E+00	-4.64E-05	-1.10E-05	-	-	-
$\widetilde{\theta^{1,1,0,0,0,0,0}}$	-3.06E-04	-4.81E-02	1.54E+00	-	-	-	-	-

The out of sample predictive accuracy is discussed next. The accuracy of individual model specifications is presented in Table 17. In terms of RMSE, the AR1 model outperforms any of the models. Some of the extended models achieve comparatively poor results and their RMSE is up to 66% higher compare do AR1. Specifically, inclusion of the interaction term between variance and kurtosis seems to result in very steep increase of RMSE. As far as reduced models are concerned, omitting realized variance from the equation would lead to lower RMSE. However, it is difficult to imagine that AR1 would in pricing outperform models which make use of risk measures. And indeed, when we look at other indicators of prediction accuracy, such as Mean Absolute Deviation, we can see that the models are relatively comparable. This is again visible in Table 17, which shows that the difference between models with highest and lowest mean absolute deviation is only 5%. Furthermore, $Model\ \theta^{1,0,1,1,0,0,0}$ was actually second best when measured by RMSE, but there were only 2 other models doing worse in terms of Mean Absolute Deviation and none of them did worse in Median Absolute Deviation.

The model's ability to identify the sign of future returns correctly goes significantly down if either realized skewness or kurtosis are omitted. Without skewness and kurtosis, the model is no longer significantly better than random guessing. The least costly omission appears to be the realized variance, which results in accuracy decrease of only 0.2%. On the other hand, realized skewness is the costliest to omit as the omission causes 0.5% decrease in model's accuracy.

While restricting $Model\ \theta^{RV}$ appears to decrease the model's performance, inclusion of the interaction terms is mostly beneficial. When all of the proposed interaction terms are included, the model accuracy goes far beyond what could be achievable through random guessing in over 99% of cases. This means the models predictive power is very unlikely to be caused by coincidence or luck. On top of that, when the interaction term between skewness and returns is included the model achieves over 50% accuracy for 21 out of the 29 stocks. Meanwhile, any omission of variables decreases this value below 20/29, depending on model specification.

All of the extended models achieve precision and negative predictive value above 50% and precision-wise the extended models, as well as $Model\ \theta^{RV}$, perform better than their restricted counterparts. While $Model\ \theta^{RV}$ does a better job in predicting negative returns (as measured by Negative Predictive Value) than the extended models, the difference is not too large. On the other hand, omitting either realized skewness or kurtosis reduces the negative predictive value significantly. Meanwhile, omitting the return or variance allows us to achieve better performance in identifying the negative returns. However, this improvement fails to compensate for the decrease in precision.

Table 17

Comparison of aggregate accuracy across model specifications

Model	RMSE	Mean Absolute Deviation	Median Absolute Deviation	Accuracy	Accuracy over 50
$AR1$	0.00974	0.00723	0.00559	0.50177	15
θ^{RV}	0.00984	0.00724	0.00560	0.50741	20
$\theta^{1,1,1,1,1,1,1}$	0.01620	0.00757	0.00561	0.51227	21
$\theta^{1,1,1,1,0,1,1}$	0.00985	0.00725	0.00561	0.50324	18
$\theta^{1,1,1,1,1,0,0}$	0.01612	0.00755	0.00560	0.50904	20
$\theta^{1,1,1,1,0,1,0}$	0.00986	0.00725	0.00560	0.50873	21
$\theta^{1,1,1,1,0,0,1}$	0.00983	0.00723	0.00558	0.50601	20
$\theta^{1,1,1,0,0,0,0}$	0.00984	0.00723	0.00558	0.50463	18
$\theta^{1,1,0,1,0,0,0}$	0.00984	0.00723	0.00560	0.50221	16
$\theta^{1,0,1,1,0,0,0}$	0.00975	0.00725	0.00561	0.50571	18
$\theta^{0,1,1,1,0,0,0}$	0.00985	0.00723	0.00557	0.50417	19
$\theta^{1,1,0,0,0,0,0}$	0.00983	0.00722	0.00559	0.50187	16

	Precision	Negative Predictive Value	Improved RMSE	Improved Median absolute dev.	Improved Accuracy
$AR1$	0.51028	0.48726	24	12	10
θ^{RV}	0.52342	0.50947	0	0	0
$\theta^{1,1,1,1,1,1,1}$	0.52443	0.50692	7	14	15
$\theta^{1,1,1,1,0,1,1}$	0.52767	0.50266	14	15	8
$\theta^{1,1,1,1,1,0,0}$	0.52064	0.50711	11	14	14
$\theta^{1,1,1,1,0,1,0}$	0.53016	0.50816	14	13	15
$\theta^{1,1,1,1,0,0,1}$	0.51517	0.51015	19	19	13
$\theta^{1,1,1,0,0,0,0}$	0.51170	0.50414	15	17	10
$\theta^{1,1,0,1,0,0,0}$	0.51547	0.49209	19	14	9
$\theta^{1,0,1,1,0,0,0}$	0.51426	0.51243	20	13	13
$\theta^{0,1,1,1,0,0,0}$	0.50742	0.51247	9	17	14
$\theta^{1,1,0,0,0,0,0}$	0.51021	0.49132	16	16	8

* For ratios, the metric average over all 29 stocks is used
* Improvements are relative to the $Model\ \theta^{RV}$

The Table 18 shows the results of the Diebold-Mariano test of individual models against the $Model\ \theta^{RV}$. The test has been performed for two shapes of the cost function – linear (P1) and second power (P2). For the details of the Diebold-Mariano test, see Diebold & Mariano (1995) and Harvey et. al (1997). As the result for AR1 indicates, we find the use of realized measures to be of limited use when evaluating purely on the loss function in either linear of exponential. However, as the analysis of accuracy in predicting the sign of future returns and the analysis of returns achievable by trading show, the models relying on realized measures, and especially those that leverage skewness, kurtosis and potentially even the interaction terms, are extremely useful in analyzing and predicting future returns.

Table 18

Results of Diebold-Mariano test of individual Models against $\boldsymbol{Model\ \theta^{RV}}$. AR1 comes out as the most accurate.

Model	More Accurate (P1)	Less Accurate (P1)	More Accurate (P2)	Less Accurate (P2)
$AR1$	8	0	6	0
$\theta^{1,1,1,1,1,1,1}$	2	5	0	3
$\theta^{1,1,1,1,0,1,1}$	4	3	2	1
$\theta^{1,1,1,1,1,0,0}$	2	6	0	4
$\theta^{1,1,1,1,0,1,0}$	3	5	1	6
$\theta^{1,1,1,1,0,0,1}$	7	2	6	1
$\theta^{1,1,1,0,0,0,0}$	8	5	3	2
$\theta^{1,1,0,1,0,0,0}$	5	2	1	1
$\theta^{1,0,1,1,0,0,0}$	5	4	4	1
$\theta^{0,1,1,1,0,0,0}$	4	2	0	6
$\theta^{1,1,0,0,0,0,0}$	7	3	3	2

In the remainder of this chapter, the returns achievable by the two trading strategies are discussed. The Table 19 summarizes the returns achievable by the Top Quartile strategy. The extended models outperform the $Model\ \theta^{RV}$. While some of the models have a minimum return lower compared to the $Model\ \theta^{RV}$, they also tend to have a higher maximum return, with the possible exception of $Model\ \theta^{1,1,1,1,1,0,0}$ which achieves a spectacularly bad minimum return while not achieving the maximum return comparable to other extended models. Similarly, when we look at the interquartile range, the extended models tend to dominate the $Model\ \theta^{RV}$. While two of the extended models have lower value of 1st quartile of the returns, this is more than compensated by the increase in the 3rd quartile. For the $Model\ \theta^{1,1,1,1,1,1,1}$ we have 3rd Quartile higher by 7.6% compared to the $Model\ \theta^{RV}$, while 1st quartile is only 3% below that of $Model\ \theta^{RV}$. $Model\ \theta^{1,1,1,1,1,0,0}$ (which is the 2nd of the two with the lower 1st quartile) has slightly lower 3rd quartile than $Model\ \theta^{1,1,1,1,1,1,1}$, but it is still high enough to beat $Model\ \theta^{RV}$. In terms of the mean and median return, the benefits of including the interaction terms are clear.

Evaluated on the very same criteria, all the restricted models perform worse compared to $Model\ \theta^{RV}$. Not a single one of the restricted models achieve higher mean return than $Model\ \theta^{RV}$ and they perform much worse compared to the extended models. Similarly, their 1st quartiles tend to be much lower without the 3rd quartile compensating fully for this shortcoming.

Finally, the returns achieved by the Positive Predictions strategy are analyzed. The numbers are presented in the Table 20 and they generally confirm the results of the above analysis.

While two of the extended models – $Model\ \theta^{1,1,1,1,0,1,1}$ and $Model\ \theta^{1,1,1,1,0,1,0}$ - do not actually beat $Model\ \theta^{RV}$ in terms of mean return, the extended models all achieve higher median returns. At the same time, it's not the goal of this exercise to show that all of the model extensions perform better compared to $Model\ \theta^{RV}$. The important take away message is that $Model\ \theta^{1,1,1,1,1,1,1}$ tends to beat the other specifications across most of the performance measurement methods. And indeed, $Model\ \theta^{1,1,1,1,1,1,1}$ offers a mean return which is almost 30% higher than that of $Model\ \theta^{RV}$ when Positive Predictions strategy is used and 28% higher when Top Quartile strategy is used. Furthermore, as in case of the Top Quartile strategy, the extended models tend to offer a more lucrative distribution of the returns as measured by 1st and 3rd quartile. Moreover, the $Model\ \theta^{1,1,1,1,1,1,1}$ offers not only higher returns, but also lower volatility of the returns, as the variance is marginally lower in case of this model. The summary of the restricted models clearly shows that omitting skewness and kurtosis is very costly, resulting at drop of median return by over 60% and mean return by almost 40% relative to the $Model\ \theta^{RV}$. Omitting realized volatility is much less costly, but still results in decrease of mean return by 9%.

Table 19

Comparison of means of log returns from the Top Quartile strategy across model specifications

Model	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
$AR1$	-0.045774	-0.003744	0.000284	0.000202	0.004366	0.040566
θ^{RV}	-0.043528	-0.003595	0.000478	0.000290	0.004431	0.034846
$\theta^{1,1,1,1,1,1,1}$	-0.048090	-0.003702	0.000698	0.000371	0.004767	0.040306
$\theta^{1,1,1,1,0,1,1}$	-0.041083	-0.003459	0.000547	0.000360	0.004555	0.043187
$\theta^{1,1,1,1,1,0,0}$	-0.049642	-0.003704	0.000568	0.000366	0.004667	0.034122
$\theta^{1,1,1,1,0,1,0}$	-0.041083	-0.003494	0.000653	0.000427	0.004633	0.040156
$\theta^{1,1,1,1,0,0,1}$	-0.044427	-0.003479	0.000476	0.000337	0.004355	0.039196
$\theta^{1,1,1,0,0,0,0}$	-0.048037	-0.003913	0.000527	0.000272	0.004344	0.036811
$\theta^{1,1,0,1,0,0,0}$	-0.043528	-0.003789	0.000307	0.000247	0.004562	0.034846
$\theta^{1,0,1,1,0,0,0}$	-0.041360	-0.003634	0.000602	0.000241	0.004280	0.040566
$\theta^{0,1,1,1,0,0,0}$	-0.047129	-0.003515	0.000416	0.000276	0.004548	0.024706
$\theta^{1,1,0,0,0,0,0}$	-0.048037	-0.003894	0.000176	0.000205	0.004505	0.036811

	Variance	Pearson skewness	Moors kurtosis
$AR1$	0.000051	-0.034566	6.992001
θ^{RV}	0.000049	-0.080383	5.932262
$\theta^{1,1,1,1,1,1,1}$	0.000054	-0.133784	6.941249
$\theta^{1,1,1,1,0,1,1}$	0.000048	-0.080756	6.852278
$\theta^{1,1,1,1,1,0,0}$	0.000051	-0.084378	6.928928
$\theta^{1,1,1,1,0,1,0}$	0.000049	-0.096417	5.964541
$\theta^{1,1,1,1,0,0,1}$	0.000049	-0.059695	6.795978
$\theta^{1,1,1,0,0,0,0}$	0.000050	-0.108184	7.027449
$\theta^{1,1,0,1,0,0,0}$	0.000049	-0.025844	6.060263
$\theta^{1,0,1,1,0,0,0}$	0.000050	-0.152955	6.186843
$\theta^{0,1,1,1,0,0,0}$	0.000047	-0.061396	6.080137
$\theta^{1,1,0,0,0,0,0}$	0.000050	0.012254	7.162438

* the reported statistics describe the distributions of daily means of log returns of chosen stocks

Table 20

Comparison of means of log returns from the Positive Predictions strategy across model specifications

Model	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
$AR1$	-0.041756	-0.003279	0.000359	0.000146	0.003599	0.033092
θ^{RV}	-0.043719	-0.003533	0.000366	0.000208	0.004170	0.034184
$\theta^{1,1,1,1,1,1,1}$	-0.043530	-0.003237	0.000500	0.000269	0.003927	0.034144
$\theta^{1,1,1,1,0,1,1}$	-0.042965	-0.003399	0.000622	0.000192	0.003910	0.033354

$\theta^{1,1,1,1,1,0,0}$	-0.044224	-0.003381	0.000606	0.000325	0.004176	0.034184
$\theta^{1,1,1,1,0,1,0}$	-0.043788	-0.003400	0.000560	0.000190	0.004033	0.034576
$\theta^{1,1,1,1,0,0,1}$	-0.043326	-0.003301	0.000376	0.000240	0.004278	0.031477
$\theta^{1,1,1,0,0,0,0}$	-0.043193	-0.003700	0.000316	0.000230	0.004150	0.034184
$\theta^{1,1,0,1,0,0,0}$	-0.043788	-0.003644	0.000154	0.000134	0.004064	0.034184
$\theta^{1,0,1,1,0,0,0}$	-0.042955	-0.003213	0.000279	0.000189	0.003635	0.031537
$\theta^{0,1,1,1,0,0,0}$	-0.043780	-0.003377	0.000430	0.000206	0.004159	0.032636
$\theta^{1,1,0,0,0,0,0}$	-0.042973	-0.003736	0.000142	0.000126	0.004005	0.034184

	Variance	Pearson skewness	Moors kurtosis
$AR1$	0.000040	-0.100776	6.997802
θ^{RV}	0.000043	-0.072098	6.653542
$\theta^{1,1,1,1,1,1,1}$	0.000042	-0.106213	6.959884
$\theta^{1,1,1,1,0,1,1}$	0.000042	-0.200126	6.765637
$\theta^{1,1,1,1,1,0,0}$	0.000044	-0.126513	6.648649
$\theta^{1,1,1,1,0,1,0}$	0.000042	-0.170615	7.002441
$\theta^{1,1,1,1,0,0,1}$	0.000041	-0.064040	6.700909
$\theta^{1,1,1,0,0,0,0}$	0.000043	-0.038899	6.530684
$\theta^{1,1,0,1,0,0,0}$	0.000044	-0.009255	6.661986
$\theta^{1,0,1,1,0,0,0}$	0.000039	-0.043261	7.306972
$\theta^{0,1,1,1,0,0,0}$	0.000041	-0.105102	6.977001
$\theta^{1,1,0,0,0,0,0}$	0.000044	-0.007400	6.431554

* the reported statistics describe the distributions of daily means of log returns of chosen stocks

5 Conclusion

This thesis focuses on a highly relevant topic in the current financial literature, the use of realized moments in asset pricing. It extends the cross-sectional analysis done by Amaya et al (2015) to the time series and studies its properties. Moreover, it sets up a framework for the evaluation of the pricing model quality which allows for better understanding of the pros and cons of the method of realized moments. On top of a broad set of accuracy metrics for the model performance on the test set it is also examined what returns are achievable by a trader who would rely on recommendations from the model. Additionally, the use of robust estimators is explored. Furthermore, a simple extension to the pricing model is proposed based on lessons from Neuberger et al (2018). The considered extension accounts for interaction terms between variance and kurtosis, skewness and return, and variance and return.

First, it is shown that a time series model leveraging the realized moments ($Model\ \theta^{RV}$) has, as opposed to the benchmark AR1, ability to predict the sign of future returns with significantly better than random accuracy. Moreover, the $Model\ \theta^{RV}$ has been shown to allow an agent to achieve 60-120% higher return over the test period compared to passively holding the stocks over the same period. When looking at the *Top Quartile* strategy, the $Model\ \theta^{RV}$ resulted in return which was 40% higher compared to AR1. For *Positive Predictions* strategy, the achieved return was 330% higher compared to AR1, which is likely due to AR1's inability to consistently beat random guessing in predicting the sign of future returns. That being said, *the Positive Predictions* strategy based on $Model\ \theta^{RV}$ resulted in higher returns than the *Top Quartile* strategy for AR1 as well as passively holding entire portfolio over the time period covered by the test set. Finally, the $Model\ \theta^{RV}$ has been shown to provide solid predictions across the entire portfolio of 29 stocks. All of the above results were achieved despite of $Model\ \theta^{RV}$ underperforming simple AR1 as measured by RMSE or Diebold-Mariano test.

Another important finding is the robustness of the above-mentioned results to use of alternative estimators of variance, skewness and kurtosis, such as Moors or Siddiqui kurtosis and Bowley, Groeneveld-Meeden or Pearson measures of skewness. It has been shown that the outlier robust measures result in improvement of accuracy as measured by RMSE or MAD. Furthermore, the models based on the robust measures are still able to beat both AR1 and the passive investment strategy in terms of returns achieved over the period covered by the test set. Moreover, the use of Moors kurtosis in model specification and *Top Quartile* strategy for trading resulted in return which was 21-32% higher compared to $Model\ \theta^{RV}$. Same measure of kurtosis combined with the *Positive Predictions* strategy resulted in returns higher by 30-72% compared to the $Model\ \theta^{RV}$. At the same time, the models mostly kept their ability to beat the random guessing in terms of identifying the sign of future returns.

Finally, a robustness of the estimates to model specification has been tested. The difference between the coefficient on skewness in $Model\ \theta^{RV}$ and that reported by Amaya et al (2015) has been shown to be due to inclusion of the lagged return. Moreover, the omission of the lagged return has such a large negative impact on accuracy and achievable returns, that it clearly deserves to be included in the model – the omission of lagged return resulted in 1-5% lower expected return over the test period, higher RMSE and lower ability to correctly identify sign of future returns. Similarly, the omission of skewness and kurtosis variables from the model had significant impact on expected returns from trading, as well as on the model's ability to forecast the sign of future returns. On the other hand, it has been shown that accounting for the interaction terms between variance and return, variance and kurtosis, and skewness and return ($Model\ \theta^{1,1,1,1,1,1,1}$) is of great benefit to the model performance as measured by achievable returns or ability to predict sign of future returns. However, the RMSE and other standard accuracy measures do not reflect this, proving the point made in this thesis, that they shouldn't be solely relied on when evaluating pricing models. The extended model can yield expected returns which are 28-29% higher compared to $Model\ \theta^{RV}$ and median return higher by 37-46%, depending on the trading strategy employed by the agent. Furthermore, the $Model\ \theta^{1,1,1,1,1,1,1}$ has achieved over 51% accuracy in predicting the sign of future returns, which is significantly more than the other models based on realized moments. As far as identifying the sign of future return is concerned, the $Model\ \theta^{1,1,1,1,1,1,1}$ would be better than random guessing in over 99% of attempts.

It can be concluded that the realized moments are very useful in the analysis of stock market returns. Despite the literature on using realized moments in asset pricing is already quite extensive, and this thesis contributing to that literature, there remains a lot to be researched. Some of the future research might focus on longer time periods, or forecasting of variance instead of the returns. Similarly, it would be interesting to see the measure definitions of Neuberger & Payne (2018) applied to stock market returns of individual companies. Also, one might want to consider other estimation techniques – be it lasso regression which would allow for use of many more interesting determinants of returns; Poisson regression, which would allow for multiplicative structure of the model, although it couldn't be applied to log returns directly as it needs values of explained variables to be positive; or some other machine learning technique. If our goal was merely improving the forecasting accuracy, then online learning would be yet another reasonable step to take, as the data summary presented in this thesis shows that there might be a difference between the distribution of returns in the training set and the test set.

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Appendix A: Tables

Data summary and supporting tables

Table 21

Summary statistics of the training set 5-minute log returns

Company	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
AAPL	-0.002322	-0.000286	0.000000	0.000003	0.000286	0.002613
AMZN	-0.004222	-0.000274	0.000000	0.000004	0.000275	0.004467
BAC	-0.003497	-0.000291	0.000000	0.000030	0.000292	0.006452
C	-0.002660	-0.000222	0.000000	-0.000006	0.000222	0.001998
CMCSA	-0.001725	-0.000216	0.000000	-0.000004	0.000216	0.001929
CSCO	-0.004226	-0.000326	0.000000	0.000002	0.000326	0.003896
CVX	-0.002123	-0.000516	0.000000	0.000019	0.000517	0.002732
DIS	-0.003128	-0.000350	0.000000	0.000005	0.000349	0.002661
GE	-0.005166	-0.000399	0.000000	-0.000003	0.000399	0.003565
HD	-0.002787	-0.000254	0.000000	0.000020	0.000254	0.004303
IBM	-0.002106	-0.000253	0.000000	0.000022	0.000266	0.002697
INTC	-0.003013	-0.000371	0.000000	0.000019	0.000373	0.003755
JNJ	-0.001390	-0.000155	0.000000	0.000006	0.000155	0.001723
JPM	-0.002851	-0.000285	0.000000	0.000001	0.000285	0.002845
KO	-0.002140	-0.000236	0.000000	0.000009	0.000236	0.003273
MCD	-0.002866	-0.000350	0.000000	0.000011	0.000349	0.003592
MRK	-0.003644	-0.000323	0.000000	0.000013	0.000324	0.003653
MSFT	-0.002018	-0.000395	0.000000	0.000013	0.000396	0.002414
ORCL	-0.003028	-0.000729	0.000000	0.000020	0.000677	0.003037
PEP	-0.003287	-0.000188	0.000000	0.000003	0.000189	0.002460
PFE	-0.012951	-0.000370	0.000000	-0.000003	0.000370	0.009268
PG	-0.001901	-0.000190	0.000000	0.000014	0.000190	0.004198
QCOM	-0.004780	-0.000298	0.000000	0.000022	0.000300	0.004158
SLB	-0.004076	-0.000382	0.000000	0.000011	0.000386	0.002856
T	-0.003695	-0.000517	0.000000	0.000006	0.000518	0.003586
VZ	-0.002315	-0.000288	0.000000	0.000006	0.000289	0.002026
WFC	-0.001782	-0.000163	0.000000	0.000003	0.000164	0.003783
WMT	-0.002211	-0.000203	0.000000	0.000010	0.000206	0.002630
XOM	-0.002751	-0.000336	0.000000	0.000013	0.000337	0.002342

Table 22

Summary statistics of the training set daily log returns

Company	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
AAPL	-0.122318	-0.010627	0.000098	-0.000398	0.010136	0.112280
AMZN	-0.131336	-0.010545	0.000477	0.001454	0.012964	0.138765
BAC	-0.250927	-0.012125	-0.001522	-0.002016	0.008602	0.201400
C	-0.141597	-0.007890	0.000000	0.000307	0.008878	0.235337
CMCSA	-0.092217	-0.008604	0.000000	-0.000261	0.007871	0.076093
CSCO	-0.120896	-0.007354	0.000749	0.000187	0.007903	0.145960
CVX	-0.346825	-0.012736	-0.002008	-0.003470	0.008034	0.199238
DIS	-0.087877	-0.006736	0.000573	0.000822	0.008293	0.118534
GE	-0.118306	-0.007286	-0.000600	-0.000712	0.006511	0.113550
HD	-0.076310	-0.007545	0.000336	0.000500	0.008119	0.114312
IBM	-0.067651	-0.004848	0.000917	0.000821	0.006980	0.063860
INTC	-0.090707	-0.008672	0.000000	-0.000099	0.008718	0.087969
JNJ	-0.077473	-0.003952	0.000000	-0.000018	0.004166	0.072753
JPM	-0.180445	-0.009441	0.000000	0.000129	0.009274	0.156672
KO	-0.072791	-0.004763	0.000207	0.000060	0.004956	0.075928
MCD	-0.079908	-0.005783	0.000457	0.000238	0.006001	0.103508
MRK	-0.079823	-0.006643	0.000000	0.000007	0.007150	0.091907
MSFT	-0.075523	-0.006465	-0.000312	-0.000025	0.006694	0.110211
ORCL	-0.097522	-0.007759	0.000668	0.000500	0.009051	0.077413
PEP	-0.065697	-0.004505	0.000484	0.000492	0.005279	0.087823
PFE	-0.065597	-0.006704	0.000000	-0.000233	0.006207	0.069634
PG	-0.065549	-0.003933	0.000813	0.000549	0.004883	0.076587
QCOM	-0.111962	-0.008507	-0.000054	-0.000046	0.008627	0.104676
SLB	-0.155198	-0.010950	0.000117	-0.000243	0.011370	0.125305
T	-0.062872	-0.006402	0.000000	-0.000217	0.006148	0.124219
VZ	-0.076025	-0.006490	-0.000028	-0.000172	0.005859	0.109309
WFC	-0.192645	-0.008385	-0.000164	0.000099	0.008603	0.193286
WMT	-0.061708	-0.005364	0.000172	0.000092	0.005489	0.077426
XOM	-0.126113	-0.006010	0.000670	0.000401	0.007588	0.118856

Table 23

Summary statistics of the test set 5-minute log returns

Company	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
AAPL	-0.001985	-0.000284	0.000000	0.000007	0.000284	0.001701
AMZN	-0.002878	-0.000263	0.000000	0.000012	0.000521	0.002620
BAC	-0.001926	-0.000274	0.000000	0.000013	0.000274	0.001649
C	-0.002192	-0.000219	0.000000	0.000006	0.000220	0.001315
CMCSA	-0.001090	-0.000218	0.000000	-0.000010	0.000218	0.001531
CSCO	-0.002273	-0.000323	0.000000	0.000005	0.000323	0.001301
CVX	-0.002015	-0.000501	0.000000	0.000022	0.000502	0.002020

DIS	-0.002953	-0.000348	0.000000	-0.000009	0.000347	0.002780
GE	-0.001972	-0.000389	0.000000	0.000021	0.000391	0.001959
HD	-0.003641	-0.000243	0.000000	-0.000007	0.000243	0.002659
IBM	-0.001496	-0.000126	0.000000	0.000010	0.000232	0.002092
INTC	-0.002173	-0.000361	0.000000	-0.000005	0.000361	0.001449
JNJ	-0.001083	-0.000155	0.000000	-0.000008	0.000155	0.001241
JPM	-0.002830	-0.000282	0.000000	0.000013	0.000282	0.001590
KO	-0.001632	-0.000233	0.000000	0.000007	0.000233	0.001635
MCD	-0.001370	-0.000341	0.000000	0.000021	0.000341	0.002381
MRK	-0.002542	-0.000317	0.000000	0.000015	0.000317	0.002223
MSFT	-0.001569	-0.000321	0.000000	0.000012	0.000389	0.002577
ORCL	-0.002740	-0.000617	0.000000	0.000027	0.000717	0.002899
PEP	-0.001829	-0.000182	0.000000	-0.000008	0.000182	0.001824
PFE	-0.001114	-0.000369	0.000000	0.000019	0.000369	0.001853
PG	-0.001289	-0.000184	0.000000	0.000001	0.000184	0.001277
QCOM	-0.001999	-0.000288	0.000000	0.000018	0.000287	0.002851
SLB	-0.003315	-0.000379	0.000000	0.000001	0.000379	0.004341
T	-0.001564	-0.000520	0.000000	0.000014	0.000520	0.002092
VZ	-0.001451	-0.000290	0.000000	0.000006	0.000290	0.001450
WFC	-0.000813	-0.000162	0.000000	0.000004	0.000162	0.001780
WMT	-0.001597	-0.000200	0.000000	0.000011	0.000200	0.001797
XOM	-0.002346	-0.000334	0.000000	0.000000	0.000334	0.003684

Table 24

Summary statistics of the test set daily log returns

Company	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
AAPL	-0.054671	-0.007383	-0.000299	-0.000263	0.008110	0.080325
AMZN	-0.086145	-0.008167	0.000149	-0.000032	0.008588	0.052097
BAC	-0.055092	-0.006539	-0.000566	-0.000329	0.006191	0.048617
C	-0.041804	-0.006018	0.000416	0.000049	0.005919	0.077748
CMCSA	-0.048360	-0.005545	0.000866	0.000560	0.006555	0.055900
CSCO	-0.067277	-0.005927	-0.000367	0.000054	0.005810	0.047267
CVX	-0.049419	-0.006176	-0.000427	-0.000329	0.006242	0.034961
DIS	-0.040454	-0.005209	0.000144	-0.000222	0.005144	0.032825
GE	-0.059018	-0.004969	0.000188	0.000402	0.005768	0.073463
HD	-0.037934	-0.005075	0.000264	0.000192	0.006031	0.059355
IBM	-0.042421	-0.005442	0.000031	-0.000005	0.005357	0.030924
INTC	-0.043477	-0.005876	0.000408	0.001121	0.007131	0.064215
JNJ	-0.050814	-0.004361	0.000500	0.000078	0.004658	0.024809
JPM	-0.051400	-0.005102	0.000828	0.000142	0.005660	0.037576
KO	-0.024956	-0.003798	0.000468	0.000310	0.004509	0.025716
MCD	-0.044614	-0.003717	0.000104	0.000497	0.004383	0.040880
MRK	-0.073633	-0.005292	-0.000509	-0.000356	0.005220	0.055290
MSFT	-0.051059	-0.005982	0.000424	0.000744	0.007307	0.046109

ORCL	-0.033741	-0.004908	0.000500	0.000595	0.006646	0.049359
PEP	-0.044597	-0.004327	0.000000	0.000103	0.004654	0.022166
PFE	-0.042161	-0.005719	-0.000350	-0.000215	0.005576	0.036009
PG	-0.048345	-0.003950	0.000000	0.000258	0.004416	0.023447
QCOM	-0.075479	-0.005347	0.001128	0.000448	0.006674	0.048062
SLB	-0.049417	-0.007413	0.000000	0.000173	0.007345	0.060927
T	-0.035817	-0.004788	0.000000	-0.000253	0.004354	0.031091
VZ	-0.042732	-0.004553	0.000213	0.000200	0.005062	0.030033
WFC	-0.061536	-0.004506	0.000000	-0.000016	0.004926	0.056390
WMT	-0.107758	-0.004313	-0.000247	-0.000216	0.004326	0.025017
XOM	-0.032924	-0.004986	-0.000197	0.000108	0.005403	0.036231

Table 25

Empirical distribution function for an accuracy of randomly guessing the sign of the future returns

0%	**1%**	**2%**	**3%**	**4%**
48.82%	49.18%	49.27%	49.33%	49.37%
5%	**6%**	**7%**	**8%**	**9%**
49.39%	49.42%	49.44%	49.46%	49.49%
10%	**11%**	**12%**	**13%**	**14%**
49.52%	49.54%	49.58%	49.59%	49.61%
15%	**16%**	**17%**	**18%**	**19%**
49.63%	49.64%	49.66%	49.67%	49.69%
20%	**21%**	**22%**	**23%**	**24%**
49.70%	49.72%	49.73%	49.74%	49.76%
25%	**26%**	**27%**	**28%**	**29%**
49.77%	49.78%	49.79%	49.80%	49.81%
30%	**31%**	**32%**	**33%**	**34%**
49.82%	49.82%	49.83%	49.85%	49.85%
35%	**36%**	**37%**	**38%**	**39%**
49.86%	49.87%	49.88%	49.89%	49.90%
40%	**41%**	**42%**	**43%**	**44%**
49.91%	49.92%	49.93%	49.94%	49.94%
45%	**46%**	**47%**	**48%**	**49%**
49.95%	49.96%	49.97%	49.98%	49.99%
50%	**51%**	**52%**	**53%**	**54%**
50.00%	50.01%	50.01%	50.03%	50.04%
55%	**56%**	**57%**	**58%**	**59%**
50.05%	50.05%	50.06%	50.08%	50.09%
60%	**61%**	**62%**	**63%**	**64%**
50.10%	50.11%	50.12%	50.13%	50.15%
65%	**66%**	**67%**	**68%**	**69%**
50.15%	50.17%	50.18%	50.18%	50.19%

70%	**71%**	**72%**	**73%**	**74%**
50.20%	50.21%	50.23%	50.24%	50.24%
75%	**76%**	**77%**	**78%**	**79%**
50.26%	50.28%	50.29%	50.31%	50.32%
80%	**81%**	**82%**	**83%**	**84%**
50.34%	50.34%	50.36%	50.37%	50.39%
85%	**86%**	**87%**	**88%**	**89%**
50.40%	50.42%	50.44%	50.47%	50.49%
90%	**91%**	**92%**	**93%**	**94%**
50.51%	50.54%	50.57%	50.60%	50.63%
95%	**96%**	**97%**	**98%**	**99%**
50.69%	50.73%	50.76%	50.85%	50.92%
100%				
51.31%				

* based on 1000 trials

Models AR1 and $\theta^{(M)}$

Table 26

Summary of AR1 models estimated for each of the individual stocks

Company	Intercept	AR1	Intercept s.e.	AR s.e.	Intercept robust s.e.	AR1 robust s.e.
AAPL	-0.0004	-0.0889	0.0004	0.0223	0.0004	0.0307
AMZN	0.0015	-0.0065	0.0005	0.0224	0.0005	0.0396
BAC	-0.0022	-0.1176	0.0007	0.0222	0.0007	0.0592
C	-0.0033	0.0493	0.0007	0.0224	0.0007	0.0910
CMCSA	0.0003	-0.0417	0.0004	0.0224	0.0004	0.0323
CSCO	-0.0003	-0.0560	0.0003	0.0223	0.0003	0.0349
CVX	0.0002	-0.1086	0.0003	0.0222	0.0003	0.0316
DIS	0.0009	-0.0336	0.0003	0.0224	0.0003	0.0323
GE	-0.0008	-0.0588	0.0004	0.0223	0.0004	0.0551
HD	0.0005	0.0343	0.0004	0.0224	0.0004	0.0393
IBM	0.0008	0.0202	0.0003	0.0224	0.0003	0.0339
INTC	-0.0001	-0.0436	0.0004	0.0224	0.0004	0.0327
JNJ	0.0000	-0.0811	0.0002	0.0223	0.0002	0.0373
JPM	0.0001	-0.1017	0.0005	0.0223	0.0005	0.0512
KO	0.0001	-0.0401	0.0002	0.0224	0.0002	0.0393
MCD	0.0002	-0.0086	0.0003	0.0224	0.0002	0.0271
MRK	0.0000	-0.0072	0.0003	0.0224	0.0003	0.0327
MSFT	0.0000	-0.0152	0.0003	0.0224	0.0003	0.0324

ORCL	0.0005	-0.0603	0.0003	0.0223	0.0003	0.0279
PEP	0.0005	-0.0360	0.0002	0.0224	0.0002	0.0352
PFE	-0.0002	-0.0227	0.0003	0.0224	0.0003	0.0279
PG	0.0006	-0.0647	0.0002	0.0223	0.0002	0.0344
QCOM	0.0000	-0.0566	0.0004	0.0223	0.0004	0.0307
SLB	-0.0003	-0.0716	0.0005	0.0223	0.0005	0.0307
T	-0.0002	-0.0216	0.0003	0.0224	0.0003	0.0281
VZ	-0.0002	-0.0171	0.0003	0.0224	0.0003	0.0309
WFC	0.0001	-0.1154	0.0006	0.0222	0.0006	0.0503
WMT	0.0001	-0.0743	0.0002	0.0223	0.0002	0.0316
XOM	0.0004	-0.1167	0.0003	0.0222	0.0003	0.0264

* robust standard errors are heteroskedasticity and autocorrelation robust standard errors

Table 27

Extension of Summary of AR1 predictive power on test set

Company	Sensitivity	Specificity	Precision	Negative Predictive value	False Positive rate	False Negative Rate	False Discovery Rate
AAPL	0.341	0.682	0.510	0.516	0.318	0.659	0.490
AMZN	1.000	0.000	0.505		1.000	0.000	0.495
BAC	0.056	0.959	0.552	0.530	0.041	0.944	0.448
C	0.000	1.000		0.522	0.000	1.000	
CMCSA	0.796	0.174	0.504	0.448	0.826	0.204	0.496
CSCO	0.273	0.726	0.536	0.463	0.274	0.727	0.464
CVX	0.591	0.368	0.469	0.487	0.632	0.409	0.531
DIS	1.000	0.007	0.510	1.000	0.993	0.000	0.490
GE	0.059	0.943	0.514	0.495	0.057	0.941	0.486
HD	0.937	0.061	0.519	0.474	0.939	0.063	0.481
IBM	0.997	0.000	0.501	0.000	1.000	0.003	0.499
INTC	0.373	0.639	0.529	0.483	0.361	0.627	0.471
JNJ	0.447	0.515	0.504	0.459	0.485	0.553	0.496
JPM	0.520	0.434	0.509	0.445	0.566	0.480	0.491
KO	0.577	0.431	0.529	0.479	0.569	0.423	0.471
MCD	0.990	0.003	0.502	0.250	0.997	0.010	0.498
MRK	0.578	0.414	0.472	0.520	0.586	0.422	0.528
MSFT	0.383	0.590	0.510	0.462	0.410	0.617	0.490
ORCL	0.824	0.176	0.523	0.477	0.824	0.176	0.477
PEP	0.984	0.026	0.504	0.615	0.974	0.016	0.496
PFE	0.114	0.902	0.516	0.528	0.098	0.886	0.484
PG	0.891	0.098	0.495	0.476	0.902	0.109	0.505
QCOM	0.397	0.593	0.549	0.441	0.407	0.603	0.451
SLB	0.397	0.657	0.535	0.522	0.343	0.603	0.465
T	0.096	0.915	0.527	0.507	0.085	0.904	0.473
VZ	0.080	0.901	0.463	0.478	0.099	0.920	0.537

WFC	0.560	0.434	0.496	0.498	0.566	0.440	0.504
WMT	0.591	0.436	0.497	0.531	0.564	0.409	0.503
XOM	0.719	0.319	0.507	0.538	0.681	0.281	0.493

* table providing ratios calculated based on TP, TN, FN and FP statistics for AR(1) model
The source statistics are displayed in the AR(1) section of the main text

Table 28

Additional aggregate statistics for AR1 test set performance

Model	Precision	Sensitivity	Specificity	Negative Predictive Value	False Positive Rate
AR1	0.5103	0.5368	0.4623	0.4873	0.5377

* For ratios, the metric average over all 29 stocks is used

Table 29

Summary of Model θ^{RV} estimated for each of the individual stocks

Company		Intercept	Return	Variance	Skewness	Kurtosis
AAPL	Coef.	-1.12E-03	-7.70E-02	-7.82E-02	-8.28E-04	1.08E-04
	s.e	7.28E-04	2.58E-02	6.37E-01	8.06E-04	8.42E-05
	Robust s.e.	7.64E-04	3.94E-02	1.43E+00	8.10E-04	7.82E-05
AMZN	Coef.	6.04E-04	-3.14E-02	1.57E+00	1.26E-03	-5.32E-06
	s.e	8.82E-04	2.54E-02	6.08E-01	8.01E-04	9.01E-05
	Robust s.e.	9.57E-04	4.49E-02	1.57E+00	8.56E-04	7.11E-05
BAC	Coef.	-2.52E-03	-1.31E-01	-5.62E-01	1.57E-03	1.33E-04
	s.e	1.29E-03	2.43E-02	2.81E-01	1.40E-03	1.75E-04
	Robust s.e.	1.04E-03	6.32E-02	8.18E-01	1.68E-03	1.50E-04
C	Coef.	-5.66E-04	-3.52E-02	-2.13E+00	7.36E-04	-3.97E-05
	s.e	1.06E-03	2.42E-02	1.95E-01	1.37E-03	1.27E-04
	Robust s.e.	9.50E-04	7.91E-02	6.24E-01	1.86E-03	1.08E-04
CMCSA	Coef.	3.45E-04	-4.18E-02	1.36E+00	1.89E-04	-6.44E-05
	s.e	5.61E-04	2.50E-02	6.52E-01	5.60E-04	4.56E-05
	Robust s.e.	7.13E-04	4.42E-02	1.92E+00	6.93E-04	6.22E-05
CSCO	Coef.	6.01E-05	-7.83E-02	3.80E-01	1.34E-03	-8.16E-05
	s.e	5.74E-04	2.49E-02	6.63E-01	6.44E-04	6.73E-05
	Robust s.e.	7.61E-04	3.78E-02	2.10E+00	6.50E-04	7.71E-05
CVX	Coef.	-1.09E-03	-1.10E-01	3.32E+00	9.63E-05	6.61E-05
	s.e	5.04E-04	2.39E-02	4.56E-01	5.50E-04	5.85E-05

	Robust s.e.	6.44E-04	5.41E-02	1.96E+00	6.03E-04	5.66E-05
DIS	Coef.	-4.82E-04	-4.50E-02	4.51E+00	-1.50E-04	2.39E-05
	s.e	5.00E-04	2.48E-02	6.28E-01	5.28E-04	4.68E-05
	Robust s.e.	5.58E-04	4.63E-02	1.92E+00	5.17E-04	4.51E-05
GE	Coef.	-2.26E-04	-8.16E-02	-1.86E+00	1.28E-03	3.06E-05
	s.e	5.44E-04	2.43E-02	3.99E-01	7.37E-04	6.03E-05
	Robust s.e.	4.59E-04	5.91E-02	1.11E+00	9.32E-04	4.50E-05
HD	Coef.	-2.05E-04	1.34E-02	2.33E+00	4.81E-04	-1.10E-05
	s.e	5.54E-04	2.39E-02	5.85E-01	5.02E-04	4.74E-05
	Robust s.e.	5.21E-04	4.28E-02	1.29E+00	5.25E-04	4.19E-05
IBM	Coef.	7.15E-04	-2.06E-03	-4.96E-01	7.14E-04	2.07E-05
	s.e	3.37E-04	2.41E-02	5.77E-01	3.05E-04	2.42E-05
	Robust s.e.	3.53E-04	4.01E-02	1.63E+00	2.81E-04	2.00E-05
INTC	Coef.	-8.70E-04	-5.84E-02	1.68E+00	7.94E-04	3.68E-05
	s.e	5.96E-04	2.50E-02	7.67E-01	6.79E-04	6.61E-05
	Robust s.e.	6.37E-04	4.13E-02	1.89E+00	6.07E-04	4.74E-05
JNJ	Coef.	1.04E-05	-9.49E-02	3.03E+00	3.12E-04	-4.48E-05
	s.e	2.93E-04	2.37E-02	7.01E-01	2.59E-04	2.64E-05
	Robust s.e.	3.81E-04	4.51E-02	2.59E+00	2.74E-04	3.41E-05
JPM	Coef.	1.91E-04	-1.13E-01	8.60E-01	9.91E-04	-8.71E-05
	s.e	8.26E-04	2.42E-02	3.91E-01	9.40E-04	8.47E-05
	Robust s.e.	6.98E-04	6.02E-02	1.11E+00	1.05E-03	6.72E-05
KO	Coef.	-1.97E-05	-4.49E-02	1.90E+00	2.48E-04	-2.07E-05
	s.e	3.22E-04	2.41E-02	7.76E-01	2.97E-04	2.62E-05
	Robust s.e.	4.40E-04	4.52E-02	2.98E+00	3.17E-04	3.17E-05
MCD	Coef.	1.31E-04	-2.67E-02	1.95E+00	2.53E-04	-2.55E-05
	s.e	3.74E-04	2.45E-02	6.20E-01	3.37E-04	3.00E-05
	Robust s.e.	3.09E-04	3.71E-02	1.25E+00	3.21E-04	2.33E-05
MRK	Coef.	-3.68E-04	-1.80E-02	2.63E+00	3.00E-04	-3.51E-05
	s.e	4.06E-04	2.42E-02	5.44E-01	3.61E-04	2.61E-05
	Robust s.e.	4.03E-04	4.30E-02	1.31E+00	3.61E-04	2.41E-05
MSFT	Coef.	-3.80E-04	-3.09E-02	1.39E+00	9.55E-04	3.95E-06
	s.e	4.69E-04	2.43E-02	8.09E-01	5.22E-04	4.38E-05
	Robust s.e.	7.63E-04	4.27E-02	3.25E+00	4.83E-04	4.18E-05
ORCL	Coef.	2.21E-04	-7.85E-02	1.50E+00	9.24E-04	-2.77E-05
	s.e	4.87E-04	2.45E-02	7.00E-01	5.28E-04	4.10E-05
	Robust s.e.	6.99E-04	4.75E-02	2.19E+00	6.70E-04	5.23E-05
PEP	Coef.	2.86E-04	-3.46E-02	1.63E+00	-9.28E-05	-1.29E-06
	s.e	2.90E-04	2.37E-02	5.43E-01	2.23E-04	1.89E-05
	Robust s.e.	3.41E-04	4.64E-02	1.98E+00	2.24E-04	1.88E-05
PFE	Coef.	-6.09E-04	-2.87E-02	2.16E+00	1.85E-04	-2.17E-05
	s.e	4.00E-04	2.43E-02	7.96E-01	4.70E-04	3.79E-05
	Robust s.e.	4.46E-04	2.89E-02	1.67E+00	5.10E-04	3.78E-05

PG	Coef.	5.00E-04	-6.30E-02	1.37E+00	-6.93E-05	-1.24E-05
	s.e	3.09E-04	2.39E-02	6.34E-01	2.95E-04	2.74E-05
	Robust s.e.	3.05E-04	3.84E-02	8.38E-01	2.99E-04	3.06E-05
QCOM	Coef.	-2.51E-04	-5.92E-02	4.76E-01	1.13E-04	2.72E-06
	s.e	4.84E-04	2.44E-02	5.21E-01	4.64E-04	3.36E-05
	Robust s.e.	4.81E-04	3.40E-02	1.08E+00	4.37E-04	2.68E-05
SLB	Coef.	4.64E-04	-8.20E-02	-1.12E+00	6.92E-04	-1.44E-05
	s.e	6.93E-04	2.43E-02	5.15E-01	6.74E-04	6.08E-05
	Robust s.e.	8.09E-04	3.79E-02	1.38E+00	6.58E-04	6.08E-05
T	Coef.	-5.00E-04	-1.86E-02	3.13E+00	-2.53E-04	-6.42E-05
	s.e	3.96E-04	2.48E-02	5.09E-01	4.60E-04	3.60E-05
	Robust s.e.	4.02E-04	3.50E-02	1.96E+00	5.27E-04	4.50E-05
VZ	Coef.	-9.35E-04	-2.01E-02	3.22E+00	-2.81E-04	9.11E-06
	s.e	4.16E-04	2.39E-02	5.84E-01	3.84E-04	3.59E-05
	Robust s.e.	4.06E-04	4.40E-02	1.69E+00	3.80E-04	2.69E-05
WFC	Coef.	3.10E-04	-1.22E-01	-1.82E-01	6.93E-04	-1.17E-05
	s.e	7.63E-04	2.33E-02	3.36E-01	7.11E-04	5.71E-05
	Robust s.e.	6.32E-04	5.26E-02	1.06E+00	5.63E-04	3.28E-05
WMT	Coef.	-1.57E-04	-9.25E-02	1.75E+00	4.99E-04	-2.31E-06
	s.e	3.51E-04	2.40E-02	6.61E-01	3.26E-04	3.13E-05
	Robust s.e.	3.50E-04	3.62E-02	1.52E+00	2.63E-04	2.18E-05
XOM	Coef.	-5.94E-04	-1.19E-01	3.30E+00	3.82E-04	4.79E-05
	s.e	4.14E-04	2.40E-02	4.66E-01	5.41E-04	4.32E-05
	Robust s.e.	4.13E-04	4.65E-02	1.49E+00	5.36E-04	3.25E-05

Table 30

Main Summary of $Model\ \theta^{RV}$ predictive power on test set

Company	RMSE	Mean absolute deviation	Median Absolute Deviation	Accuracy	True Positive	True Negative	False Negative	False Positive
AAPL	0.0123	0.0093	0.0077	0.550	140	197	162	114
AMZN	0.0140	0.0107	0.0082	0.523	293	28	17	276
BAC	0.0111	0.0083	0.0062	0.544	35	294	252	24
C	0.0102	0.0078	0.0064	0.522	0	318	291	0
CMCSA	0.0108	0.0080	0.0061	0.480	133	161	181	137
CSCO	0.0096	0.0074	0.0062	0.489	91	206	235	75
CVX	0.0113	0.0080	0.0059	0.496	122	182	176	133
DIS	0.0095	0.0070	0.0053	0.536	192	135	118	165
GE	0.0093	0.0069	0.0054	0.511	117	193	190	107
HD	0.0107	0.0073	0.0056	0.503	148	160	170	134
IBM	0.0090	0.0068	0.0054	0.513	293	21	14	284
INTC	0.0113	0.0084	0.0065	0.475	75	211	239	77

JNJ	0.0083	0.0060	0.0046	0.512	118	195	202	96
JPM	0.0101	0.0074	0.0055	0.471	105	184	220	104
KO	0.0072	0.0055	0.0041	0.489	121	176	198	112
MCD	0.0077	0.0054	0.0040	0.532	204	121	104	182
MRK	0.0103	0.0074	0.0055	0.535	34	291	255	28
MSFT	0.0114	0.0084	0.0066	0.501	102	203	219	85
ORCL	0.0100	0.0075	0.0057	0.520	197	119	121	171
PEP	0.0074	0.0056	0.0043	0.515	277	37	29	267
PFE	0.0093	0.0070	0.0056	0.520	21	294	268	23
PG	0.0073	0.0054	0.0043	0.488	248	49	55	256
QCOM	0.0111	0.0080	0.0060	0.468	107	180	233	93
SLB	0.0124	0.0093	0.0075	0.509	178	133	127	173
T	0.0079	0.0059	0.0046	0.512	8	303	293	3
VZ	0.0093	0.0064	0.0048	0.467	21	263	293	31
WFC	0.0084	0.0061	0.0048	0.503	192	113	110	191
WMT	0.0088	0.0059	0.0043	0.511	139	173	157	141
XOM	0.0093	0.0069	0.0052	0.518	165	152	137	158

Table 31

Extension of Summary of $Model\ \theta^{RV}$ predictive power on test set

Company	Sensitivity	Specificity	Precision	Negative Predictive value	False Positive rate	False Negative Rate	False Discovery Rate
AAPL	0.464	0.633	0.551	0.549	0.367	0.536	0.449
AMZN	0.945	0.092	0.515	0.622	0.908	0.055	0.485
BAC	0.122	0.925	0.593	0.538	0.075	0.878	0.407
C	0.000	1.000		0.522	0.000	1.000	
CMCSA	0.424	0.540	0.493	0.471	0.460	0.576	0.507
CSCO	0.279	0.733	0.548	0.467	0.267	0.721	0.452
CVX	0.409	0.578	0.478	0.508	0.422	0.591	0.522
DIS	0.619	0.450	0.538	0.534	0.550	0.381	0.462
GE	0.381	0.643	0.522	0.504	0.357	0.619	0.478
HD	0.465	0.544	0.525	0.485	0.456	0.535	0.475
IBM	0.954	0.069	0.508	0.600	0.931	0.046	0.492
INTC	0.239	0.733	0.493	0.469	0.267	0.761	0.507
JNJ	0.369	0.670	0.551	0.491	0.330	0.631	0.449
JPM	0.323	0.639	0.502	0.455	0.361	0.677	0.498
KO	0.379	0.611	0.519	0.471	0.389	0.621	0.481
MCD	0.662	0.399	0.528	0.538	0.601	0.338	0.472
MRK	0.118	0.912	0.548	0.533	0.088	0.882	0.452
MSFT	0.318	0.705	0.545	0.481	0.295	0.682	0.455
ORCL	0.619	0.410	0.535	0.496	0.590	0.381	0.465
PEP	0.905	0.122	0.509	0.561	0.878	0.095	0.491

PFE	0.073	0.927	0.477	0.523	0.073	0.927	0.523
PG	0.818	0.161	0.492	0.471	0.839	0.182	0.508
QCOM	0.315	0.659	0.535	0.436	0.341	0.685	0.465
SLB	0.584	0.435	0.507	0.512	0.565	0.416	0.493
T	0.027	0.990	0.727	0.508	0.010	0.973	0.273
VZ	0.067	0.895	0.404	0.473	0.105	0.933	0.596
WFC	0.636	0.372	0.501	0.507	0.628	0.364	0.499
WMT	0.470	0.551	0.496	0.524	0.449	0.530	0.504
XOM	0.546	0.490	0.511	0.526	0.510	0.454	0.489

* table providing ratios calculated based on TP, TN, FN and FP statistics for Model θ^{RV}

The source statistics are displayed in the Variance, Skewness, Kurtosis section of the main text

Table 32

Extension of Summary of Averaged $\boldsymbol{Model\ \theta^{RV}}$ predictive power on test set

Company	Sensitivity	Specificity	Precision	Negative Predictive value	False Positive rate	False Negative Rate	False Discovery Rate
AAPL	0.397	0.569	0.472	0.493	0.431	0.603	0.528
AMZN	0.535	0.543	0.544	0.534	0.457	0.465	0.456
BAC	0.470	0.569	0.496	0.544	0.431	0.530	0.504
C	0.426	0.588	0.486	0.528	0.412	0.574	0.514
CMCSA	0.360	0.594	0.483	0.468	0.406	0.640	0.517
CSCO	0.399	0.598	0.535	0.462	0.402	0.601	0.465
CVX	0.409	0.603	0.494	0.519	0.397	0.591	0.506
DIS	0.416	0.680	0.573	0.530	0.320	0.584	0.427
GE	0.375	0.637	0.513	0.499	0.363	0.625	0.487
HD	0.403	0.636	0.545	0.496	0.364	0.597	0.455
IBM	0.375	0.659	0.525	0.511	0.341	0.625	0.475
INTC	0.417	0.625	0.548	0.496	0.375	0.583	0.452
JNJ	0.347	0.718	0.575	0.500	0.282	0.653	0.425
JPM	0.394	0.601	0.527	0.468	0.399	0.606	0.473
KO	0.310	0.740	0.569	0.492	0.260	0.690	0.431
MCD	0.364	0.670	0.528	0.509	0.330	0.636	0.472
MRK	0.391	0.608	0.475	0.524	0.392	0.609	0.525
MSFT	0.405	0.622	0.544	0.484	0.378	0.595	0.456
ORCL	0.377	0.638	0.533	0.483	0.362	0.623	0.467
PEP	0.314	0.701	0.513	0.504	0.299	0.686	0.487
PFE	0.367	0.634	0.477	0.523	0.366	0.633	0.523
PG	0.310	0.711	0.516	0.509	0.289	0.690	0.484
QCOM	0.362	0.593	0.526	0.427	0.407	0.638	0.474
SLB	0.459	0.549	0.504	0.505	0.451	0.541	0.496
T	0.332	0.725	0.543	0.525	0.275	0.668	0.457

VZ	0.315	0.673	0.508	0.479	0.327	0.685	0.492
WFC	0.341	0.615	0.468	0.484	0.385	0.659	0.532
WMT	0.301	0.659	0.454	0.500	0.341	0.699	0.546
XOM	0.377	0.635	0.502	0.512	0.365	0.623	0.498

* table providing ratios calculated based on TP, TN, FN and FP statistics for Averaged Model θ^{RV}

The source statistics are displayed in the Variance, Skewness, Kurtosis section of the main text, section on averaged model

Table 33

Summary of $\boldsymbol{Model\ \theta^{BPV}}$

Company		Intercept	Return	Variance	Skewness	Kurtosis
AAPL	Coef.	-8.57E-04	-7.75E-02	-5.18E-02	-7.34E-04	6.00E-05
	s.e	6.78E-04	2.54E-02	6.45E-01	6.87E-04	6.44E-05
	Robust s.e.	7.32E-04	3.98E-02	1.61E+00	6.70E-04	5.59E-05
AMZN	Coef.	9.74E-04	-2.71E-02	1.50E+00	9.34E-04	-3.69E-05
	s.e	7.35E-04	2.46E-02	6.38E-01	6.22E-04	5.28E-05
	Robust s.e.	9.06E-04	4.33E-02	1.64E+00	6.71E-04	5.26E-05
BAC	Coef.	-2.38E-03	-1.33E-01	-6.09E-01	1.50E-03	9.18E-05
	s.e	1.16E-03	2.39E-02	2.85E-01	1.12E-03	1.19E-04
	Robust s.e.	9.82E-04	6.53E-02	8.54E-01	1.21E-03	8.65E-05
C	Coef.	-6.17E-04	-3.63E-02	-2.20E+00	9.38E-04	-4.39E-05
	s.e	9.47E-04	2.40E-02	2.08E-01	1.06E-03	7.80E-05
	Robust s.e.	8.98E-04	7.83E-02	6.34E-01	1.50E-03	7.94E-05
CMCSA	Coef.	-1.11E-04	-2.59E-02	1.73E+00	-2.67E-04	-9.65E-06
	s.e	4.75E-04	2.45E-02	7.48E-01	2.30E-04	9.16E-06
	Robust s.e.	6.49E-04	5.05E-02	2.42E+00	2.72E-04	9.98E-06
CSCO	Coef.	5.53E-05	-7.57E-02	2.49E-01	9.85E-04	-5.54E-05
	s.e	4.54E-04	2.43E-02	6.83E-01	4.66E-04	2.83E-05
	Robust s.e.	6.59E-04	3.74E-02	2.23E+00	5.01E-04	2.80E-05
CVX	Coef.	-6.80E-04	-1.10E-01	3.16E+00	1.35E-04	8.65E-06
	s.e	3.77E-04	2.29E-02	4.64E-01	3.27E-04	2.03E-05
	Robust s.e.	5.50E-04	5.14E-02	2.14E+00	2.96E-04	1.56E-05
DIS	Coef.	-5.84E-04	-4.39E-02	4.46E+00	-1.19E-04	3.81E-05
	s.e	4.48E-04	2.42E-02	6.23E-01	3.92E-04	2.74E-05
	Robust s.e.	4.93E-04	5.24E-02	1.86E+00	3.89E-04	2.19E-05
GE	Coef.	-4.56E-05	-7.67E-02	-1.84E+00	8.23E-04	-5.60E-06
	s.e	4.96E-04	2.37E-02	4.18E-01	5.23E-04	3.65E-05

	Robust s.e.	4.29E-04	5.71E-02	1.16E+00	5.66E-04	2.42E-05
HD	Coef.	-1.33E-04	1.69E-02	2.22E+00	2.27E-04	-7.73E-06
	s.e	4.33E-04	2.33E-02	5.72E-01	3.11E-04	1.32E-05
	Robust s.e.	3.76E-04	4.33E-02	1.16E+00	3.16E-04	1.14E-05
IBM	Coef.	7.52E-04	5.42E-03	-3.61E-01	3.72E-04	9.29E-06
	s.e	2.82E-04	2.30E-02	5.84E-01	1.41E-04	3.50E-06
	Robust s.e.	3.11E-04	3.72E-02	1.63E+00	1.09E-04	2.57E-06
INTC	Coef.	-8.33E-04	-5.53E-02	1.75E+00	5.16E-04	2.64E-05
	s.e	5.36E-04	2.45E-02	7.93E-01	5.18E-04	4.02E-05
	Robust s.e.	6.14E-04	3.92E-02	1.91E+00	4.55E-04	2.81E-05
JNJ	Coef.	-1.90E-04	-8.95E-02	2.83E+00	1.49E-04	-1.12E-05
	s.e	2.51E-04	2.33E-02	8.15E-01	1.84E-04	1.34E-05
	Robust s.e.	3.56E-04	4.38E-02	3.31E+00	1.80E-04	1.27E-05
JPM	Coef.	-2.50E-05	-1.12E-01	9.92E-01	7.63E-04	-5.33E-05
	s.e	7.32E-04	2.37E-02	3.89E-01	7.30E-04	5.27E-05
	Robust s.e.	6.54E-04	5.78E-02	1.13E+00	7.18E-04	3.93E-05
KO	Coef.	-2.16E-04	-3.88E-02	1.98E+00	4.65E-05	2.55E-06
	s.e	2.62E-04	2.34E-02	7.87E-01	1.76E-04	7.69E-06
	Robust s.e.	4.12E-04	4.28E-02	3.24E+00	1.62E-04	6.36E-06
MCD	Coef.	2.93E-05	-2.68E-02	1.98E+00	1.78E-04	-8.91E-06
	s.e	2.97E-04	2.40E-02	6.06E-01	2.42E-04	1.07E-05
	Robust s.e.	2.70E-04	3.72E-02	1.24E+00	2.30E-04	8.66E-06
MRK	Coef.	-6.27E-04	-1.52E-02	2.73E+00	1.60E-04	-2.73E-06
	s.e	3.44E-04	2.32E-02	5.36E-01	2.05E-04	5.00E-06
	Robust s.e.	3.45E-04	4.73E-02	1.16E+00	1.60E-04	3.75E-06
MSFT	Coef.	-5.35E-04	-2.46E-02	1.67E+00	5.20E-04	1.57E-05
	s.e	4.00E-04	2.35E-02	8.04E-01	3.38E-04	1.89E-05
	Robust s.e.	7.09E-04	4.16E-02	3.23E+00	3.09E-04	1.73E-05
ORCL	Coef.	2.68E-04	-7.59E-02	1.48E+00	6.68E-04	-1.91E-05
	s.e	4.12E-04	2.37E-02	7.46E-01	3.39E-04	9.61E-06
	Robust s.e.	6.47E-04	4.41E-02	2.35E+00	4.43E-04	1.20E-05
PEP	Coef.	1.88E-04	-3.46E-02	1.69E+00	-6.91E-05	6.81E-06
	s.e	2.43E-04	2.29E-02	5.81E-01	1.06E-04	5.26E-06
	Robust s.e.	2.90E-04	4.79E-02	2.15E+00	8.16E-05	3.47E-06
PFE	Coef.	-7.48E-04	-2.37E-02	2.39E+00	-2.44E-05	-1.86E-07
	s.e	3.34E-04	2.31E-02	8.32E-01	2.19E-04	6.17E-06
	Robust s.e.	3.86E-04	2.73E-02	1.74E+00	2.03E-04	5.49E-06
PG	Coef.	3.76E-04	-6.31E-02	1.56E+00	-5.41E-05	1.56E-06
	s.e	2.59E-04	2.32E-02	6.48E-01	1.90E-04	1.25E-05
	Robust s.e.	2.19E-04	3.66E-02	8.45E-01	1.65E-04	9.56E-06
QCOM	Coef.	-2.24E-04	-5.61E-02	5.23E-01	5.39E-06	-2.52E-07
	s.e	4.00E-04	2.33E-02	4.95E-01	2.70E-04	5.90E-06
	Robust s.e.	3.65E-04	3.30E-02	5.52E-01	2.35E-04	5.03E-06

SLB	Coef.	1.78E-04	-7.85E-02	-1.10E+00	2.98E-04	1.49E-05
	s.e	5.55E-04	2.34E-02	5.26E-01	3.97E-04	1.53E-05
	Robust s.e.	7.02E-04	3.57E-02	1.41E+00	3.20E-04	1.11E-05
T	Coef.	-6.27E-04	-8.10E-03	3.42E+00	-2.56E-04	-3.44E-05
	s.e	3.42E-04	2.43E-02	5.33E-01	2.92E-04	1.65E-05
	Robust s.e.	4.39E-04	3.60E-02	2.17E+00	3.49E-04	1.90E-05
VZ	Coef.	-8.67E-04	-2.03E-02	3.12E+00	-2.11E-04	6.91E-06
	s.e	3.73E-04	2.35E-02	5.94E-01	2.74E-04	2.06E-05
	Robust s.e.	3.85E-04	4.25E-02	1.75E+00	2.58E-04	1.54E-05
WFC	Coef.	1.53E-04	-1.18E-01	-1.68E-01	2.51E-04	5.95E-06
	s.e	6.35E-04	2.24E-02	3.42E-01	2.37E-04	7.12E-06
	Robust s.e.	5.76E-04	5.26E-02	1.06E+00	8.55E-05	1.82E-06
WMT	Coef.	-2.56E-04	-9.15E-02	1.82E+00	3.63E-04	8.82E-06
	s.e	2.97E-04	2.35E-02	6.53E-01	2.30E-04	1.57E-05
	Robust s.e.	3.09E-04	3.58E-02	1.45E+00	1.82E-04	1.05E-05
XOM	Coef.	-5.07E-04	-1.16E-01	3.26E+00	2.04E-04	3.08E-05
	s.e	3.52E-04	2.32E-02	4.63E-01	3.72E-04	2.07E-05
	Robust s.e.	3.65E-04	4.98E-02	1.48E+00	3.35E-04	1.33E-05

Table 34

Summary of $Model\ \theta^{MinRV}$

Company		Intercept	Return	Variance	Skewness	Kurtosis
AAPL	Coef.	-7.03E-04	-7.72E-02	7.14E-02	-6.71E-04	3.14E-05
	s.e	6.92E-04	2.51E-02	6.49E-01	6.26E-04	6.31E-05
	Robust s.e.	7.17E-04	3.83E-02	1.44E+00	6.01E-04	5.32E-05
AMZN	Coef.	1.26E-03	-2.69E-02	1.37E+00	9.36E-04	-5.73E-05
	s.e	6.75E-04	2.45E-02	6.49E-01	5.77E-04	4.00E-05
	Robust s.e.	8.37E-04	4.29E-02	1.66E+00	6.11E-04	4.01E-05
BAC	Coef.	-2.04E-03	-1.33E-01	-6.14E-01	1.55E-03	5.19E-05
	s.e	1.06E-03	2.38E-02	2.79E-01	1.12E-03	1.07E-04
	Robust s.e.	9.21E-04	6.84E-02	8.62E-01	1.18E-03	7.57E-05
C	Coef.	-2.25E-04	-3.43E-02	-1.99E+00	1.41E-03	-1.03E-04
	s.e	9.07E-04	2.39E-02	1.98E-01	9.93E-04	7.21E-05
	Robust s.e.	9.02E-04	7.79E-02	5.66E-01	1.47E-03	8.41E-05
CMCSA	Coef.	-1.04E-04	-2.65E-02	1.60E+00	-2.11E-04	-7.56E-06
	s.e	4.79E-04	2.42E-02	7.66E-01	1.96E-04	7.35E-06
	Robust s.e.	6.86E-04	4.96E-02	2.50E+00	2.02E-04	6.76E-06
CSCO	Coef.	9.32E-05	-7.63E-02	6.99E-02	1.04E-03	-5.64E-05
	s.e	4.40E-04	2.41E-02	6.89E-01	4.56E-04	2.53E-05

	Robust s.e.	6.58E-04	3.83E-02	2.27E+00	4.30E-04	2.00E-05
CVX	Coef.	-6.48E-04	-1.10E-01	3.11E+00	1.34E-04	6.06E-06
	s.e	3.66E-04	2.28E-02	4.60E-01	2.81E-04	1.56E-05
	Robust s.e.	5.38E-04	4.98E-02	2.12E+00	2.36E-04	1.11E-05
DIS	Coef.	-4.91E-04	-4.42E-02	4.37E+00	-7.00E-05	2.87E-05
	s.e	4.20E-04	2.41E-02	6.12E-01	3.79E-04	2.30E-05
	Robust s.e.	4.67E-04	5.14E-02	1.80E+00	3.71E-04	1.71E-05
GE	Coef.	8.04E-06	-7.63E-02	-1.76E+00	8.04E-04	-1.55E-05
	s.e	4.87E-04	2.35E-02	4.19E-01	4.95E-04	3.70E-05
	Robust s.e.	4.42E-04	5.65E-02	1.19E+00	5.27E-04	2.82E-05
HD	Coef.	-1.18E-04	1.63E-02	2.18E+00	2.37E-04	-8.31E-06
	s.e	4.26E-04	2.33E-02	5.55E-01	2.94E-04	1.12E-05
	Robust s.e.	3.67E-04	4.38E-02	1.10E+00	3.04E-04	1.01E-05
IBM	Coef.	7.97E-04	1.05E-02	-3.41E-01	2.09E-04	4.21E-06
	s.e	2.80E-04	2.27E-02	5.98E-01	8.62E-05	1.70E-06
	Robust s.e.	3.01E-04	3.80E-02	1.59E+00	7.62E-05	1.46E-06
INTC	Coef.	-7.93E-04	-5.31E-02	1.72E+00	4.19E-04	2.14E-05
	s.e	5.02E-04	2.42E-02	7.86E-01	4.97E-04	3.40E-05
	Robust s.e.	5.93E-04	3.77E-02	1.79E+00	4.43E-04	2.36E-05
JNJ	Coef.	-2.31E-04	-8.72E-02	2.59E+00	9.07E-05	-4.64E-06
	s.e	2.44E-04	2.32E-02	8.44E-01	1.69E-04	1.14E-05
	Robust s.e.	3.58E-04	4.22E-02	3.21E+00	1.63E-04	9.90E-06
JPM	Coef.	2.67E-04	-1.13E-01	1.05E+00	9.29E-04	-9.51E-05
	s.e	7.80E-04	2.36E-02	3.83E-01	7.08E-04	6.30E-05
	Robust s.e.	6.88E-04	5.63E-02	1.16E+00	7.04E-04	4.61E-05
KO	Coef.	-2.28E-04	-3.90E-02	2.07E+00	4.32E-05	2.45E-06
	s.e	2.58E-04	2.31E-02	8.09E-01	1.46E-04	5.56E-06
	Robust s.e.	4.08E-04	4.29E-02	3.14E+00	1.21E-04	3.94E-06
MCD	Coef.	2.00E-05	-2.76E-02	1.97E+00	1.70E-04	-7.83E-06
	s.e	2.92E-04	2.39E-02	5.81E-01	2.25E-04	9.45E-06
	Robust s.e.	2.62E-04	3.72E-02	1.12E+00	2.15E-04	7.90E-06
MRK	Coef.	-6.32E-04	-1.35E-02	2.67E+00	1.16E-04	-1.72E-06
	s.e	3.41E-04	2.30E-02	5.10E-01	1.77E-04	4.12E-06
	Robust s.e.	3.35E-04	4.76E-02	1.06E+00	1.31E-04	2.94E-06
MSFT	Coef.	-4.74E-04	-2.24E-02	1.54E+00	4.18E-04	1.16E-05
	s.e	3.92E-04	2.33E-02	7.94E-01	2.99E-04	1.70E-05
	Robust s.e.	7.24E-04	4.03E-02	3.24E+00	2.83E-04	1.59E-05
ORCL	Coef.	2.52E-04	-7.40E-02	1.37E+00	6.16E-04	-1.65E-05
	s.e	4.18E-04	2.35E-02	7.71E-01	3.21E-04	8.42E-06
	Robust s.e.	6.77E-04	4.35E-02	2.39E+00	4.28E-04	1.08E-05
PEP	Coef.	2.04E-04	-3.45E-02	1.69E+00	-5.40E-05	5.18E-06
	s.e	2.39E-04	2.27E-02	5.93E-01	8.54E-05	3.71E-06
	Robust s.e.	2.97E-04	4.82E-02	2.27E+00	5.94E-05	2.11E-06

PFE	Coef.	-7.37E-04	-2.21E-02	2.28E+00	-7.77E-05	-1.64E-06
	s.e	3.38E-04	2.30E-02	8.29E-01	2.08E-04	6.33E-06
	Robust s.e.	4.02E-04	2.71E-02	1.75E+00	2.11E-04	6.08E-06
PG	Coef.	3.64E-04	-6.47E-02	1.67E+00	-5.19E-06	1.62E-06
	s.e	2.44E-04	2.30E-02	6.65E-01	1.57E-04	8.54E-06
	Robust s.e.	2.06E-04	3.60E-02	8.87E-01	1.25E-04	5.47E-06
QCOM	Coef.	-2.37E-04	-5.63E-02	5.64E-01	1.09E-05	-2.86E-07
	s.e	3.95E-04	2.31E-02	4.74E-01	2.05E-04	3.93E-06
	Robust s.e.	3.69E-04	3.26E-02	5.84E-01	1.65E-04	3.10E-06
SLB	Coef.	1.81E-04	-7.83E-02	-1.09E+00	2.51E-04	1.25E-05
	s.e	5.56E-04	2.33E-02	5.38E-01	3.57E-04	1.37E-05
	Robust s.e.	7.09E-04	3.52E-02	1.44E+00	2.95E-04	1.04E-05
T	Coef.	-6.81E-04	-7.97E-03	3.42E+00	-2.13E-04	-3.13E-05
	s.e	3.33E-04	2.41E-02	5.38E-01	2.64E-04	1.45E-05
	Robust s.e.	4.93E-04	3.65E-02	2.36E+00	3.41E-04	1.74E-05
VZ	Coef.	-8.32E-04	-2.11E-02	3.03E+00	-1.88E-04	4.39E-06
	s.e	3.52E-04	2.33E-02	5.80E-01	2.49E-04	1.73E-05
	Robust s.e.	3.53E-04	4.78E-02	1.57E+00	2.39E-04	1.35E-05
WFC	Coef.	1.59E-04	-1.18E-01	-1.51E-01	1.92E-04	4.16E-06
	s.e	6.33E-04	2.24E-02	3.45E-01	1.78E-04	4.56E-06
	Robust s.e.	5.73E-04	5.24E-02	1.07E+00	5.73E-05	1.14E-06
WMT	Coef.	-2.21E-04	-9.06E-02	1.85E+00	3.21E-04	4.49E-06
	s.e	2.88E-04	2.34E-02	6.47E-01	2.10E-04	1.33E-05
	Robust s.e.	3.02E-04	3.57E-02	1.44E+00	1.68E-04	9.03E-06
XOM	Coef.	-4.56E-04	-1.15E-01	3.25E+00	1.23E-04	2.30E-05
	s.e	3.46E-04	2.30E-02	4.59E-01	3.27E-04	1.82E-05
	Robust s.e.	3.64E-04	5.20E-02	1.50E+00	2.90E-04	1.33E-05

Table 35

Summary of $Model\ \theta^{MedRV}$

Company		Intercept	Return	Variance	Skewness	Kurtosis
AAPL	Coef.	-8.26E-04	-7.66E-02	-8.90E-02	-7.54E-04	5.47E-05
	s.e	6.97E-04	2.51E-02	6.62E-01	6.29E-04	6.42E-05

	Robust s.e.	7.62E-04	4.00E-02	1.78E+00	6.08E-04	5.33E-05
AMZN	Coef.	1.22E-03	-2.45E-02	1.20E+00	8.19E-04	-4.41E-05
	s.e	7.05E-04	2.45E-02	6.50E-01	5.75E-04	4.53E-05
	Robust s.e.	9.15E-04	4.30E-02	1.74E+00	6.27E-04	4.87E-05
BAC	Coef.	-2.12E-03	-1.32E-01	-6.80E-01	1.44E-03	6.97E-05
	s.e	1.05E-03	2.38E-02	2.87E-01	1.12E-03	1.02E-04
	Robust s.e.	9.05E-04	6.51E-02	8.76E-01	1.16E-03	7.04E-05
C	Coef.	-2.20E-04	-3.31E-02	-2.00E+00	1.44E-03	-1.06E-04
	s.e	9.01E-04	2.38E-02	2.00E-01	9.92E-04	7.00E-05
	Robust s.e.	8.83E-04	7.85E-02	5.81E-01	1.44E-03	7.84E-05
CMCSA	Coef.	-1.94E-04	-2.56E-02	1.87E+00	-2.18E-04	-6.98E-06
	s.e	4.71E-04	2.40E-02	7.16E-01	1.93E-04	7.21E-06
	Robust s.e.	6.32E-04	4.87E-02	2.25E+00	1.94E-04	6.54E-06
CSCO	Coef.	6.98E-05	-7.66E-02	1.90E-01	1.05E-03	-5.82E-05
	s.e	4.43E-04	2.42E-02	6.94E-01	4.66E-04	2.65E-05
	Robust s.e.	6.56E-04	3.79E-02	2.28E+00	4.40E-04	2.08E-05
CVX	Coef.	-6.73E-04	-1.10E-01	3.32E+00	1.32E-04	3.56E-06
	s.e	3.67E-04	2.28E-02	4.68E-01	2.84E-04	1.60E-05
	Robust s.e.	5.16E-04	4.91E-02	2.02E+00	2.30E-04	1.07E-05
DIS	Coef.	-5.62E-04	-4.36E-02	4.66E+00	-1.56E-04	3.02E-05
	s.e	4.48E-04	2.42E-02	6.43E-01	3.81E-04	2.81E-05
	Robust s.e.	5.09E-04	5.18E-02	1.92E+00	3.70E-04	2.47E-05
GE	Coef.	-1.09E-04	-7.67E-02	-1.86E+00	8.07E-04	5.03E-06
	s.e	4.92E-04	2.35E-02	4.11E-01	4.87E-04	3.79E-05
	Robust s.e.	4.51E-04	5.65E-02	1.14E+00	5.54E-04	3.47E-05
HD	Coef.	-1.70E-04	1.69E-02	2.35E+00	2.12E-04	-7.20E-06
	s.e	4.27E-04	2.32E-02	5.95E-01	2.84E-04	1.03E-05
	Robust s.e.	3.73E-04	4.39E-02	1.21E+00	3.04E-04	9.76E-06
IBM	Coef.	8.25E-04	1.15E-02	-4.51E-01	1.83E-04	3.59E-06
	s.e	2.79E-04	2.27E-02	5.91E-01	7.99E-05	1.53E-06
	Robust s.e.	3.07E-04	3.77E-02	1.62E+00	6.77E-05	1.26E-06
INTC	Coef.	-7.71E-04	-5.44E-02	1.66E+00	4.79E-04	2.14E-05
	s.e	4.89E-04	2.42E-02	7.87E-01	4.86E-04	3.07E-05
	Robust s.e.	5.98E-04	3.91E-02	1.92E+00	4.16E-04	2.04E-05
JNJ	Coef.	-2.14E-04	-8.92E-02	3.16E+00	1.17E-04	-1.11E-05
	s.e	2.40E-04	2.31E-02	8.09E-01	1.64E-04	1.08E-05
	Robust s.e.	3.32E-04	4.74E-02	3.11E+00	1.64E-04	1.18E-05
JPM	Coef.	3.00E-04	-1.12E-01	1.02E+00	8.53E-04	-9.44E-05
	s.e	7.75E-04	2.36E-02	3.89E-01	6.95E-04	6.13E-05
	Robust s.e.	7.01E-04	5.61E-02	1.20E+00	6.62E-04	4.45E-05
KO	Coef.	-2.28E-04	-3.65E-02	2.33E+00	-2.14E-05	-3.73E-07
	s.e	2.56E-04	2.31E-02	8.18E-01	1.37E-04	5.02E-06
	Robust s.e.	4.07E-04	4.27E-02	3.27E+00	1.28E-04	4.41E-06

MCD	Coef.	2.65E-06	-2.64E-02	1.99E+00	1.46E-04	-6.35E-06
	s.e	2.88E-04	2.39E-02	5.89E-01	2.19E-04	8.36E-06
	Robust s.e.	2.62E-04	3.69E-02	1.13E+00	2.07E-04	7.01E-06
MRK	Coef.	-6.50E-04	-1.21E-02	2.73E+00	6.40E-05	-5.72E-07
	s.e	3.42E-04	2.30E-02	5.38E-01	1.56E-04	3.50E-06
	Robust s.e.	3.43E-04	4.90E-02	1.19E+00	1.24E-04	2.66E-06
MSFT	Coef.	-4.12E-04	-2.28E-02	1.28E+00	4.17E-04	1.20E-05
	s.e	3.95E-04	2.33E-02	8.29E-01	2.97E-04	1.70E-05
	Robust s.e.	7.32E-04	4.13E-02	3.36E+00	2.86E-04	1.61E-05
ORCL	Coef.	1.95E-04	-7.52E-02	1.58E+00	6.45E-04	-1.71E-05
	s.e	4.11E-04	2.35E-02	7.41E-01	3.18E-04	8.21E-06
	Robust s.e.	6.48E-04	4.39E-02	2.28E+00	4.19E-04	1.04E-05
PEP	Coef.	1.83E-04	-3.64E-02	1.87E+00	-4.52E-05	5.05E-06
	s.e	2.36E-04	2.27E-02	5.57E-01	8.53E-05	3.59E-06
	Robust s.e.	2.67E-04	4.32E-02	1.91E+00	5.64E-05	1.99E-06
PFE	Coef.	-7.48E-04	-2.26E-02	2.35E+00	-6.72E-05	-1.32E-06
	s.e	3.37E-04	2.31E-02	8.29E-01	2.16E-04	6.53E-06
	Robust s.e.	3.97E-04	2.72E-02	1.75E+00	2.12E-04	6.10E-06
PG	Coef.	3.82E-04	-6.53E-02	1.54E+00	9.78E-06	1.48E-06
	s.e	2.40E-04	2.30E-02	6.47E-01	1.57E-04	7.67E-06
	Robust s.e.	2.06E-04	3.59E-02	8.43E-01	1.25E-04	4.80E-06
QCOM	Coef.	-2.42E-04	-5.60E-02	5.80E-01	6.87E-06	-1.99E-07
	s.e	3.99E-04	2.30E-02	5.07E-01	2.02E-04	3.75E-06
	Robust s.e.	3.60E-04	3.27E-02	5.49E-01	1.51E-04	2.74E-06
SLB	Coef.	1.68E-04	-7.62E-02	-9.87E-01	1.64E-04	7.74E-06
	s.e	5.61E-04	2.33E-02	5.37E-01	3.44E-04	1.56E-05
	Robust s.e.	7.11E-04	3.54E-02	1.45E+00	2.73E-04	1.20E-05
T	Coef.	-6.39E-04	-5.60E-03	3.37E+00	-2.68E-04	-3.43E-05
	s.e	3.44E-04	2.43E-02	5.57E-01	2.87E-04	1.69E-05
	Robust s.e.	5.12E-04	4.16E-02	2.46E+00	3.42E-04	1.78E-05
VZ	Coef.	-8.39E-04	-2.16E-02	3.21E+00	-1.86E-04	2.59E-06
	s.e	3.53E-04	2.33E-02	6.01E-01	2.46E-04	1.72E-05
	Robust s.e.	3.61E-04	4.69E-02	1.69E+00	2.36E-04	1.36E-05
WFC	Coef.	1.51E-04	-1.18E-01	-1.35E-01	1.77E-04	3.80E-06
	s.e	6.33E-04	2.24E-02	3.49E-01	1.70E-04	4.54E-06
	Robust s.e.	5.79E-04	5.26E-02	1.08E+00	5.27E-05	1.04E-06
WMT	Coef.	-2.19E-04	-8.93E-02	1.87E+00	2.92E-04	4.15E-06
	s.e	2.83E-04	2.34E-02	6.39E-01	2.10E-04	1.25E-05
	Robust s.e.	2.93E-04	3.56E-02	1.41E+00	1.59E-04	7.81E-06
XOM	Coef.	-4.40E-04	-1.14E-01	3.29E+00	1.46E-04	2.09E-05
	s.e	3.52E-04	2.31E-02	4.48E-01	3.44E-04	2.04E-05
	Robust s.e.	3.57E-04	4.65E-02	1.33E+00	2.88E-04	1.64E-05

Models with robust estimators of skewness and kurtosis

Table 36

Summary of Model $\theta^{Bowley,Moors}$ estimated for each of the individual stocks

Company		Intercept	Return	Variance	Skewness	Kurtosis
AAPL	Coef.	-1.59E-03	-8.99E-02	3.85E-02	2.01E-03	7.57E-03
	s.e	6.64E-04	2.29E-02	6.63E-01	5.30E-03	2.83E-03
	Robust s.e.	8.44E-04	3.57E-02	1.73E+00	5.31E-03	3.02E-03
AMZN	Coef.	1.37E-03	-1.34E-02	1.15E+00	1.84E-03	-1.93E-03
	s.e	7.57E-04	2.34E-02	6.52E-01	5.04E-03	1.99E-03
	Robust s.e.	9.76E-04	4.07E-02	1.72E+00	4.77E-03	1.82E-03
BAC	Coef.	-2.01E-03	-1.20E-01	-6.58E-01	-1.44E-03	1.42E-03
	s.e	1.00E-03	2.40E-02	2.89E-01	8.49E-03	2.45E-03
	Robust s.e.	9.70E-04	5.92E-02	8.50E-01	1.10E-02	1.81E-03
C	Coef.	-9.99E-04	-3.25E-02	-2.02E+00	4.18E-03	1.72E-04
	s.e	7.72E-04	2.45E-02	2.01E-01	5.86E-03	1.07E-04
	Robust s.e.	8.79E-04	7.53E-02	7.37E-01	8.70E-03	1.29E-04
CMCSA	Coef.	-2.14E-04	-3.93E-02	1.71E+00	1.76E-04	-6.22E-05
	s.e	5.47E-04	2.45E-02	7.10E-01	4.25E-03	1.27E-03
	Robust s.e.	6.83E-04	3.94E-02	2.15E+00	4.22E-03	1.08E-03
CSCO	Coef.	-3.81E-04	-4.69E-02	2.36E-01	-3.04E-03	9.74E-05
	s.e	4.26E-04	2.45E-02	6.95E-01	3.26E-03	7.30E-04
	Robust s.e.	6.12E-04	3.75E-02	2.09E+00	3.86E-03	5.34E-04
CVX	Coef.	-5.82E-04	-1.07E-01	3.31E+00	-1.23E-05	-3.90E-04
	s.e	4.41E-04	2.31E-02	4.69E-01	3.16E-03	1.71E-03
	Robust s.e.	5.95E-04	4.57E-02	2.08E+00	3.44E-03	1.42E-03
DIS	Coef.	-3.77E-04	-7.21E-02	4.69E+00	8.58E-03	5.57E-04
	s.e	4.45E-04	2.49E-02	6.42E-01	4.00E-03	1.16E-03
	Robust s.e.	5.11E-04	4.22E-02	1.83E+00	4.76E-03	1.02E-03
GE	Coef.	-2.58E-04	-5.31E-02	-1.83E+00	-4.99E-03	1.35E-03
	s.e	4.45E-04	2.48E-02	4.11E-01	4.79E-03	1.30E-03
	Robust s.e.	4.24E-04	6.02E-02	1.14E+00	5.81E-03	1.01E-03
HD	Coef.	-2.09E-04	2.65E-02	2.32E+00	-1.67E-03	-1.14E-04
	s.e	5.47E-04	2.57E-02	5.99E-01	3.83E-03	1.43E-03
	Robust s.e.	5.32E-04	4.87E-02	1.26E+00	4.42E-03	1.40E-03
IBM	Coef.	1.27E-03	1.89E-02	-4.76E-01	4.93E-04	-1.98E-03
	s.e	3.85E-04	2.33E-02	5.92E-01	2.64E-03	1.37E-03
	Robust s.e.	3.81E-04	3.68E-02	1.69E+00	2.48E-03	1.19E-03
INTC	Coef.	-1.06E-03	-5.35E-02	1.65E+00	3.50E-03	2.83E-03
	s.e	4.70E-04	2.51E-02	7.88E-01	4.56E-03	1.31E-03
	Robust s.e.	5.45E-04	3.59E-02	1.67E+00	4.78E-03	1.23E-03

JNJ	Coef.	-6.43E-05	-1.17E-01	3.21E+00	5.70E-03	-1.03E-03
	s.e	2.81E-04	2.50E-02	8.08E-01	2.10E-03	6.96E-04
	Robust s.e.	3.87E-04	4.53E-02	2.60E+00	2.20E-03	6.90E-04
JPM	Coef.	-5.94E-04	-1.08E-01	1.03E+00	2.81E-03	6.70E-04
	s.e	7.44E-04	2.35E-02	3.90E-01	5.61E-03	2.28E-03
	Robust s.e.	6.18E-04	6.05E-02	1.02E+00	7.21E-03	1.71E-03
KO	Coef.	-2.44E-04	-4.45E-02	2.34E+00	1.44E-03	3.04E-05
	s.e	3.19E-04	2.54E-02	8.19E-01	2.46E-03	7.90E-04
	Robust s.e.	4.39E-04	4.76E-02	2.96E+00	2.80E-03	6.54E-04
MCD	Coef.	1.06E-04	-1.25E-02	1.92E+00	-2.12E-03	-6.44E-04
	s.e	3.56E-04	2.49E-02	5.91E-01	2.49E-03	8.92E-04
	Robust s.e.	3.26E-04	4.03E-02	1.17E+00	2.70E-03	7.65E-04
MRK	Coef.	-2.26E-04	-1.29E-02	2.75E+00	1.33E-03	-1.50E-03
	s.e	4.63E-04	2.47E-02	5.39E-01	3.11E-03	1.09E-03
	Robust s.e.	4.32E-04	4.28E-02	1.19E+00	3.52E-03	9.98E-04
MSFT	Coef.	-3.76E-04	-9.55E-03	1.25E+00	-1.30E-03	4.76E-04
	s.e	3.96E-04	2.56E-02	8.30E-01	3.57E-03	1.15E-03
	Robust s.e.	7.15E-04	3.99E-02	3.36E+00	4.02E-03	1.01E-03
ORCL	Coef.	-5.82E-05	-4.72E-02	1.62E+00	-4.25E-03	2.23E-04
	s.e	4.10E-04	2.37E-02	7.41E-01	2.62E-03	1.48E-04
	Robust s.e.	4.94E-04	3.12E-02	1.50E+00	2.48E-03	6.22E-05
PEP	Coef.	4.61E-04	-5.63E-02	1.90E+00	3.39E-03	-9.12E-04
	s.e	2.99E-04	2.48E-02	5.57E-01	2.05E-03	7.63E-04
	Robust s.e.	3.23E-04	4.32E-02	1.77E+00	2.40E-03	6.70E-04
PFE	Coef.	-7.35E-04	-1.38E-02	2.26E+00	-3.55E-03	-1.90E-05
	s.e	3.55E-04	2.46E-02	8.31E-01	3.42E-03	8.34E-04
	Robust s.e.	4.22E-04	3.19E-02	1.86E+00	3.23E-03	6.93E-04
PG	Coef.	2.85E-04	-7.43E-02	1.57E+00	1.91E-03	3.91E-04
	s.e	3.01E-04	2.48E-02	6.48E-01	2.22E-03	7.92E-04
	Robust s.e.	2.83E-04	4.47E-02	1.20E+00	2.50E-03	7.09E-04
QCOM	Coef.	-4.91E-05	-5.90E-02	5.75E-01	8.45E-04	-8.87E-04
	s.e	5.03E-04	2.49E-02	5.07E-01	3.18E-03	1.37E-03
	Robust s.e.	4.96E-04	3.62E-02	9.20E-01	3.45E-03	1.29E-03
SLB	Coef.	2.43E-04	-8.62E-02	-9.55E-01	1.23E-02	-3.70E-04
	s.e	7.36E-04	2.29E-02	5.37E-01	4.93E-03	2.80E-03
	Robust s.e.	8.68E-04	3.36E-02	1.43E+00	4.92E-03	2.68E-03
T	Coef.	-8.47E-04	-3.41E-02	3.38E+00	2.72E-03	-8.46E-04
	s.e	3.32E-04	2.45E-02	5.61E-01	2.88E-03	9.98E-04
	Robust s.e.	5.19E-04	3.44E-02	2.49E+00	2.73E-03	1.01E-03
VZ	Coef.	-5.72E-04	-3.41E-02	3.26E+00	2.35E-03	-1.27E-03
	s.e	3.57E-04	2.46E-02	6.01E-01	3.21E-03	9.85E-04
	Robust s.e.	3.30E-04	3.70E-02	1.40E+00	3.51E-03	8.05E-04
WFC	Coef.	2.73E-04	-1.13E-01	-1.40E-01	-2.09E-03	-3.59E-04

	s.e	7.59E-04	2.34E-02	3.49E-01	5.52E-03	2.22E-03
	Robust s.e.	6.57E-04	5.39E-02	1.10E+00	5.11E-03	1.59E-03
WMT	Coef.	9.75E-06	-1.05E-01	1.90E+00	5.67E-03	-8.60E-04
	s.e	3.49E-04	2.46E-02	6.40E-01	2.37E-03	9.03E-04
	Robust s.e.	3.61E-04	3.76E-02	1.14E+00	2.50E-03	8.68E-04
XOM	Coef.	-4.49E-04	-1.31E-01	3.32E+00	6.77E-03	3.40E-04
	s.e	3.91E-04	2.31E-02	4.48E-01	2.78E-03	1.59E-03
	Robust s.e.	3.96E-04	4.40E-02	1.33E+00	3.05E-03	1.35E-03

Table 37

Summary of Model $\theta^{Bowley,C\&S}$ estimated for each of the individual stocks

Company		Intercept	Return	Variance	Skewness	Kurtosis
AAPL	Coef.	-2.39E-03	-9.21E-02	3.95E-02	1.68E-03	2.37E-03
	s.e	7.95E-04	2.29E-02	6.62E-01	5.30E-03	7.44E-04
	Robust s.e.	8.80E-04	3.48E-02	1.62E+00	5.36E-03	6.94E-04
AMZN	Coef.	1.32E-03	-1.28E-02	1.16E+00	1.83E-03	-3.45E-04
	s.e	9.62E-04	2.34E-02	6.55E-01	5.05E-03	6.34E-04
	Robust s.e.	1.19E-03	4.18E-02	1.82E+00	4.74E-03	6.08E-04
BAC	Coef.	-1.34E-03	-1.20E-01	-6.69E-01	-1.26E-03	-4.18E-04
	s.e	1.06E-03	2.40E-02	2.88E-01	8.49E-03	1.06E-03
	Robust s.e.	8.91E-04	6.40E-02	8.71E-01	1.02E-02	8.23E-04
C	Coef.	-1.00E-03	-3.34E-02	-2.02E+00	4.80E-03	1.01E-04
	s.e	7.75E-04	2.45E-02	2.02E-01	5.86E-03	8.33E-05
	Robust s.e.	8.91E-04	7.51E-02	7.41E-01	8.66E-03	9.62E-05
CMCSA	Coef.	4.20E-05	-3.93E-02	1.74E+00	1.18E-04	-3.47E-04
	s.e	6.37E-04	2.45E-02	7.11E-01	4.25E-03	5.59E-04
	Robust s.e.	7.13E-04	4.37E-02	2.24E+00	4.34E-03	5.10E-04
CSCO	Coef.	-2.33E-04	-4.65E-02	2.54E-01	-3.24E-03	-1.92E-04
	s.e	4.73E-04	2.44E-02	6.96E-01	3.25E-03	3.87E-04
	Robust s.e.	5.89E-04	3.74E-02	2.11E+00	3.82E-03	3.37E-04
CVX	Coef.	-5.54E-04	-1.07E-01	3.31E+00	-2.13E-05	-1.17E-04
	s.e	5.74E-04	2.32E-02	4.71E-01	3.16E-03	5.88E-04
	Robust s.e.	7.63E-04	5.24E-02	2.10E+00	3.58E-03	5.58E-04
DIS	Coef.	8.02E-05	-7.30E-02	4.75E+00	8.70E-03	-4.32E-04
	s.e	4.97E-04	2.49E-02	6.45E-01	4.00E-03	4.30E-04
	Robust s.e.	4.85E-04	4.09E-02	1.72E+00	4.70E-03	4.32E-04
GE	Coef.	-4.62E-04	-5.35E-02	-1.90E+00	-4.96E-03	7.59E-04
	s.e	4.87E-04	2.48E-02	4.15E-01	4.78E-03	5.21E-04

	Robust s.e.	4.22E-04	5.97E-02	1.17E+00	5.77E-03	5.10E-04
HD	Coef.	4.00E-04	2.66E-02	2.29E+00	-1.58E-03	-5.77E-04
	s.e	7.05E-04	2.57E-02	5.98E-01	3.83E-03	5.17E-04
	Robust s.e.	7.29E-04	4.61E-02	1.25E+00	4.24E-03	5.83E-04
IBM	Coef.	1.69E-03	2.16E-02	-5.22E-01	4.41E-04	-8.26E-04
	s.e	5.03E-04	2.34E-02	5.92E-01	2.64E-03	4.29E-04
	Robust s.e.	5.11E-04	3.52E-02	1.65E+00	2.48E-03	4.08E-04
INTC	Coef.	-6.48E-04	-5.40E-02	1.70E+00	3.48E-03	1.64E-05
	s.e	5.16E-04	2.52E-02	7.93E-01	4.57E-03	5.19E-04
	Robust s.e.	5.70E-04	3.86E-02	1.93E+00	4.80E-03	5.18E-04
JNJ	Coef.	7.21E-05	-1.18E-01	3.22E+00	5.76E-03	-3.52E-04
	s.e	3.62E-04	2.50E-02	8.08E-01	2.10E-03	2.51E-04
	Robust s.e.	4.53E-04	4.51E-02	2.77E+00	2.21E-03	2.54E-04
JPM	Coef.	1.07E-04	-1.07E-01	1.00E+00	2.74E-03	-6.22E-04
	s.e	9.60E-04	2.35E-02	3.91E-01	5.61E-03	8.28E-04
	Robust s.e.	8.76E-04	5.94E-02	1.07E+00	7.02E-03	7.37E-04
KO	Coef.	-2.30E-04	-4.45E-02	2.34E+00	1.44E-03	-6.92E-06
	s.e	3.91E-04	2.54E-02	8.18E-01	2.46E-03	3.06E-04
	Robust s.e.	4.35E-04	4.77E-02	3.05E+00	2.79E-03	2.64E-04
MCD	Coef.	5.59E-04	-1.30E-02	1.92E+00	-2.17E-03	-5.56E-04
	s.e	4.48E-04	2.49E-02	5.90E-01	2.49E-03	3.20E-04
	Robust s.e.	4.07E-04	3.95E-02	1.15E+00	2.68E-03	3.04E-04
MRK	Coef.	-5.67E-04	-1.44E-02	2.75E+00	1.49E-03	-8.21E-05
	s.e	5.36E-04	2.46E-02	5.43E-01	3.11E-03	3.67E-04
	Robust s.e.	5.18E-04	3.86E-02	1.11E+00	3.49E-03	3.96E-04
MSFT	Coef.	-2.45E-05	-1.05E-02	1.35E+00	-1.03E-03	-4.72E-04
	s.e	4.46E-04	2.56E-02	8.33E-01	3.57E-03	4.25E-04
	Robust s.e.	6.60E-04	3.72E-02	3.21E+00	3.98E-03	4.00E-04
ORCL	Coef.	-8.39E-05	-4.69E-02	1.58E+00	-4.33E-03	1.42E-04
	s.e	4.15E-04	2.37E-02	7.41E-01	2.62E-03	1.14E-04
	Robust s.e.	6.53E-04	3.38E-02	2.24E+00	2.40E-03	6.87E-05
PEP	Coef.	5.86E-04	-5.64E-02	1.91E+00	3.35E-03	-2.97E-04
	s.e	4.03E-04	2.48E-02	5.57E-01	2.05E-03	2.79E-04
	Robust s.e.	3.93E-04	4.26E-02	1.39E+00	2.38E-03	2.52E-04
PFE	Coef.	-6.78E-04	-1.37E-02	2.33E+00	-3.55E-03	-1.69E-04
	s.e	3.59E-04	2.46E-02	8.44E-01	3.41E-03	3.80E-04
	Robust s.e.	4.06E-04	3.15E-02	1.86E+00	3.22E-03	3.52E-04
PG	Coef.	5.59E-04	-7.37E-02	1.54E+00	1.87E-03	-1.48E-04
	s.e	4.01E-04	2.48E-02	6.48E-01	2.22E-03	2.82E-04
	Robust s.e.	3.88E-04	3.87E-02	8.90E-01	2.30E-03	2.68E-04
QCOM	Coef.	-1.46E-04	-5.93E-02	5.75E-01	9.00E-04	-8.97E-05
	s.e	6.63E-04	2.49E-02	5.08E-01	3.18E-03	4.67E-04
	Robust s.e.	6.82E-04	3.66E-02	1.06E+00	3.58E-03	4.60E-04

SLB	Coef.	5.93E-04	-8.57E-02	-9.78E-01	1.23E-02	-5.06E-04
	s.e	9.26E-04	2.30E-02	5.38E-01	4.93E-03	9.14E-04
	Robust s.e.	1.11E-03	3.28E-02	1.42E+00	4.89E-03	9.17E-04
T	Coef.	-5.90E-04	-3.57E-02	3.48E+00	2.76E-03	-5.91E-04
	s.e	3.78E-04	2.45E-02	5.64E-01	2.88E-03	3.59E-04
	Robust s.e.	5.20E-04	3.46E-02	2.47E+00	2.75E-03	3.66E-04
VZ	Coef.	-3.02E-04	-3.44E-02	3.32E+00	2.25E-03	-6.36E-04
	s.e	4.26E-04	2.46E-02	6.03E-01	3.21E-03	3.70E-04
	Robust s.e.	4.16E-04	4.00E-02	1.79E+00	3.62E-03	3.57E-04
WFC	Coef.	9.40E-04	-1.12E-01	-1.55E-01	-1.97E-03	-8.09E-04
	s.e	9.56E-04	2.34E-02	3.49E-01	5.52E-03	7.91E-04
	Robust s.e.	8.00E-04	5.38E-02	1.08E+00	5.03E-03	6.54E-04
WMT	Coef.	3.68E-04	-1.04E-01	1.88E+00	5.59E-03	-5.10E-04
	s.e	4.61E-04	2.46E-02	6.40E-01	2.37E-03	3.34E-04
	Robust s.e.	4.89E-04	3.77E-02	1.21E+00	2.49E-03	3.21E-04
XOM	Coef.	-7.54E-04	-1.31E-01	3.35E+00	6.69E-03	4.86E-04
	s.e	5.12E-04	2.31E-02	4.49E-01	2.78E-03	5.49E-04
	Robust s.e.	5.06E-04	3.12E-02	1.09E+00	2.58E-03	4.80E-04

Table 38

Summary of Model $\theta^{G\&M,Moors}$ estimated for each of the individual stocks

Company		Intercept	Return	Variance	Skewness	Kurtosis
AAPL	Coef.	-1.61E-03	-7.71E-02	3.31E-02	-8.52E-03	7.61E-03
	s.e	6.64E-04	2.48E-02	6.62E-01	8.20E-03	2.83E-03
	Robust s.e.	8.46E-04	4.00E-02	1.71E+00	9.39E-03	3.03E-03
AMZN	Coef.	1.33E-03	-2.14E-02	1.17E+00	7.28E-03	-1.91E-03
	s.e	7.59E-04	2.59E-02	6.52E-01	9.15E-03	1.99E-03
	Robust s.e.	9.67E-04	4.77E-02	1.70E+00	1.05E-02	1.82E-03
BAC	Coef.	-1.87E-03	-1.55E-01	-6.79E-01	2.75E-02	1.19E-03
	s.e	1.00E-03	2.83E-02	2.89E-01	1.44E-02	2.45E-03
	Robust s.e.	9.56E-04	7.44E-02	8.60E-01	2.05E-02	1.81E-03
C	Coef.	-1.05E-03	-2.22E-02	-1.99E+00	-3.68E-03	1.93E-04
	s.e	7.72E-04	2.95E-02	2.05E-01	1.55E-02	1.03E-04
	Robust s.e.	8.62E-04	1.02E-01	7.35E-01	3.38E-02	1.15E-04
CMCSA	Coef.	-2.19E-04	-6.87E-02	1.67E+00	9.51E-03	-6.94E-05
	s.e	5.47E-04	4.28E-02	7.12E-01	1.17E-02	1.27E-03
	Robust s.e.	6.75E-04	8.66E-02	2.14E+00	1.77E-02	1.07E-03
CSCO	Coef.	-3.91E-04	-7.89E-02	2.40E-01	6.49E-03	1.92E-04

	s.e	4.26E-04	4.72E-02	6.95E-01	1.18E-02	7.25E-04
	Robust s.e.	6.60E-04	1.09E-01	2.29E+00	2.42E-02	5.05E-04
CVX	Coef.	-5.97E-04	-1.15E-01	3.30E+00	3.88E-03	-4.19E-04
	s.e	4.41E-04	2.56E-02	4.70E-01	5.97E-03	1.71E-03
	Robust s.e.	5.92E-04	5.28E-02	2.05E+00	8.55E-03	1.40E-03
DIS	Coef.	-4.84E-04	-1.13E-01	4.79E+00	1.88E-02	5.41E-04
	s.e	4.47E-04	3.71E-02	6.44E-01	8.51E-03	1.16E-03
	Robust s.e.	5.12E-04	1.08E-01	1.95E+00	1.90E-02	1.03E-03
GE	Coef.	-2.27E-04	-1.08E-01	-1.88E+00	1.67E-02	1.24E-03
	s.e	4.45E-04	3.51E-02	4.11E-01	1.04E-02	1.30E-03
	Robust s.e.	4.20E-04	9.83E-02	1.15E+00	1.92E-02	1.02E-03
HD	Coef.	-2.41E-04	-2.60E-03	2.41E+00	7.76E-03	-1.17E-04
	s.e	5.47E-04	3.64E-02	6.03E-01	9.35E-03	1.43E-03
	Robust s.e.	5.32E-04	7.43E-02	1.28E+00	1.33E-02	1.40E-03
IBM	Coef.	1.26E-03	8.22E-03	-4.64E-01	4.43E-03	-2.01E-03
	s.e	3.85E-04	2.62E-02	5.92E-01	5.06E-03	1.37E-03
	Robust s.e.	3.80E-04	4.28E-02	1.68E+00	5.86E-03	1.19E-03
INTC	Coef.	-1.12E-03	-2.32E-01	1.83E+00	5.50E-02	2.74E-03
	s.e	4.67E-04	4.98E-02	7.84E-01	1.30E-02	1.30E-03
	Robust s.e.	5.23E-04	8.33E-02	1.68E+00	1.77E-02	1.22E-03
JNJ	Coef.	-1.20E-04	-2.21E-01	3.51E+00	2.60E-02	-1.02E-03
	s.e	2.80E-04	3.51E-02	8.08E-01	5.28E-03	6.93E-04
	Robust s.e.	3.96E-04	6.12E-02	2.83E+00	6.67E-03	6.88E-04
JPM	Coef.	-5.83E-04	-1.10E-01	1.03E+00	4.73E-03	6.19E-04
	s.e	7.43E-04	2.62E-02	3.90E-01	1.02E-02	2.28E-03
	Robust s.e.	6.16E-04	7.60E-02	1.03E+00	1.90E-02	1.72E-03
KO	Coef.	-2.82E-04	-1.40E-01	2.35E+00	2.18E-02	1.30E-05
	s.e	3.19E-04	3.66E-02	8.16E-01	6.18E-03	7.87E-04
	Robust s.e.	4.66E-04	8.00E-02	3.40E+00	9.74E-03	6.53E-04
MCD	Coef.	7.20E-05	-4.28E-02	2.06E+00	4.82E-03	-6.29E-04
	s.e	3.57E-04	3.60E-02	6.01E-01	6.31E-03	8.92E-04
	Robust s.e.	3.39E-04	8.30E-02	1.31E+00	1.21E-02	7.64E-04
MRK	Coef.	-2.17E-04	-1.27E-02	2.74E+00	1.17E-03	-1.52E-03
	s.e	4.63E-04	3.91E-02	5.40E-01	8.65E-03	1.09E-03
	Robust s.e.	4.38E-04	9.03E-02	1.21E+00	1.61E-02	9.98E-04
MSFT	Coef.	-3.92E-04	-4.83E-02	1.25E+00	9.13E-03	4.41E-04
	s.e	3.96E-04	4.41E-02	8.30E-01	1.01E-02	1.15E-03
	Robust s.e.	7.19E-04	8.39E-02	3.38E+00	1.55E-02	1.01E-03
ORCL	Coef.	-6.08E-05	-8.96E-02	1.63E+00	8.31E-03	2.51E-04
	s.e	4.12E-04	4.93E-02	7.43E-01	1.25E-02	1.47E-04
	Robust s.e.	6.85E-04	1.04E-01	2.31E+00	2.30E-02	6.12E-05
PEP	Coef.	4.68E-04	-8.65E-02	1.92E+00	9.65E-03	-9.50E-04
	s.e	2.98E-04	3.63E-02	5.57E-01	5.74E-03	7.63E-04

	Robust s.e.	3.38E-04	7.85E-02	1.96E+00	9.84E-03	6.70E-04
PFE	Coef.	-8.13E-04	-1.36E-01	2.52E+00	2.71E-02	1.77E-06
	s.e	3.54E-04	5.39E-02	8.31E-01	1.19E-02	8.32E-04
	Robust s.e.	4.25E-04	8.09E-02	1.90E+00	1.48E-02	6.93E-04
PG	Coef.	2.38E-04	-1.71E-01	1.61E+00	2.21E-02	2.81E-04
	s.e	3.00E-04	3.54E-02	6.46E-01	5.78E-03	7.90E-04
	Robust s.e.	2.80E-04	7.31E-02	1.11E+00	9.62E-03	7.00E-04
QCOM	Coef.	-3.78E-05	-5.02E-02	5.77E-01	-1.96E-03	-9.05E-04
	s.e	5.03E-04	3.60E-02	5.07E-01	9.38E-03	1.37E-03
	Robust s.e.	5.00E-04	6.53E-02	9.29E-01	1.48E-02	1.29E-03
SLB	Coef.	2.34E-04	-1.03E-01	-9.91E-01	2.48E-02	5.81E-05
	s.e	7.36E-04	2.44E-02	5.37E-01	8.24E-03	2.80E-03
	Robust s.e.	8.66E-04	3.80E-02	1.43E+00	9.19E-03	2.67E-03
T	Coef.	-8.87E-04	-1.04E-01	3.44E+00	2.04E-02	-8.55E-04
	s.e	3.33E-04	4.10E-02	5.61E-01	8.80E-03	9.97E-04
	Robust s.e.	5.33E-04	6.89E-02	2.58E+00	1.17E-02	1.01E-03
VZ	Coef.	-5.92E-04	-6.98E-02	3.34E+00	1.09E-02	-1.27E-03
	s.e	3.57E-04	3.67E-02	6.04E-01	7.36E-03	9.85E-04
	Robust s.e.	3.31E-04	6.71E-02	1.44E+00	1.09E-02	8.02E-04
WFC	Coef.	2.71E-04	-1.32E-01	-1.31E-01	1.27E-02	-3.99E-04
	s.e	7.59E-04	2.65E-02	3.49E-01	1.10E-02	2.21E-03
	Robust s.e.	6.58E-04	6.69E-02	1.11E+00	1.65E-02	1.60E-03
WMT	Coef.	-1.90E-05	-1.84E-01	2.06E+00	2.28E-02	-8.37E-04
	s.e	3.49E-04	3.71E-02	6.42E-01	6.50E-03	9.02E-04
	Robust s.e.	3.67E-04	6.44E-02	1.28E+00	9.35E-03	8.64E-04
XOM	Coef.	-4.02E-04	-1.38E-01	3.29E+00	1.05E-02	4.78E-04
	s.e	3.90E-04	2.62E-02	4.48E-01	5.84E-03	1.59E-03
	Robust s.e.	3.95E-04	5.52E-02	1.33E+00	8.57E-03	1.36E-03

Table 39

Summary of Model $\theta^{G\&M,C\&S}$ estimated for each of the individual stocks

Company		Intercept	Return	Variance	Skewness	Kurtosis
AAPL	Coef.	-2.42E-03	-7.95E-02	3.50E-02	-8.56E-03	2.39E-03
	s.e	7.95E-04	2.48E-02	6.61E-01	8.20E-03	7.44E-04
	Robust s.e.	8.90E-04	4.01E-02	1.73E+00	9.35E-03	6.86E-04
AMZN	Coef.	1.29E-03	-2.10E-02	1.17E+00	7.45E-03	-3.49E-04
	s.e	9.63E-04	2.59E-02	6.56E-01	9.15E-03	6.34E-04
	Robust s.e.	1.18E-03	4.73E-02	1.76E+00	1.04E-02	6.08E-04

BAC	Coef.	-1.25E-03	-1.55E-01	-6.87E-01	2.79E-02	-4.44E-04
	s.e	1.06E-03	2.83E-02	2.87E-01	1.44E-02	1.06E-03
	Robust s.e.	8.74E-04	7.93E-02	8.75E-01	2.02E-02	8.22E-04
C	Coef.	-1.07E-03	-2.19E-02	-1.98E+00	-3.97E-03	1.20E-04
	s.e	7.75E-04	2.95E-02	2.05E-01	1.56E-02	8.05E-05
	Robust s.e.	8.72E-04	1.02E-01	7.39E-01	3.38E-02	8.59E-05
CMCSA	Coef.	4.07E-05	-6.91E-02	1.69E+00	9.62E-03	-3.54E-04
	s.e	6.37E-04	4.28E-02	7.13E-01	1.17E-02	5.59E-04
	Robust s.e.	7.01E-04	1.02E-01	2.21E+00	2.05E-02	5.09E-04
CSCO	Coef.	-2.49E-04	-7.83E-02	2.52E-01	6.25E-03	-1.50E-04
	s.e	4.74E-04	4.72E-02	6.96E-01	1.18E-02	3.86E-04
	Robust s.e.	6.03E-04	1.03E-01	2.18E+00	2.29E-02	3.31E-04
CVX	Coef.	-5.67E-04	-1.15E-01	3.30E+00	3.86E-03	-1.24E-04
	s.e	5.73E-04	2.56E-02	4.71E-01	5.97E-03	5.88E-04
	Robust s.e.	7.45E-04	5.85E-02	2.04E+00	8.85E-03	5.43E-04
DIS	Coef.	-6.40E-05	-1.14E-01	4.85E+00	1.87E-02	-3.88E-04
	s.e	4.98E-04	3.70E-02	6.47E-01	8.50E-03	4.30E-04
	Robust s.e.	5.05E-04	7.55E-02	1.87E+00	1.32E-02	4.32E-04
GE	Coef.	-4.39E-04	-1.09E-01	-1.95E+00	1.69E-02	7.43E-04
	s.e	4.87E-04	3.50E-02	4.15E-01	1.04E-02	5.21E-04
	Robust s.e.	4.17E-04	9.77E-02	1.17E+00	1.91E-02	5.06E-04
HD	Coef.	3.79E-04	-2.72E-03	2.39E+00	7.92E-03	-5.87E-04
	s.e	7.06E-04	3.64E-02	6.02E-01	9.34E-03	5.17E-04
	Robust s.e.	7.28E-04	6.89E-02	1.26E+00	1.23E-02	5.85E-04
IBM	Coef.	1.69E-03	1.04E-02	-5.10E-01	4.55E-03	-8.40E-04
	s.e	5.02E-04	2.62E-02	5.92E-01	5.06E-03	4.29E-04
	Robust s.e.	5.08E-04	4.15E-02	1.62E+00	5.83E-03	4.08E-04
INTC	Coef.	-6.84E-04	-2.35E-01	1.89E+00	5.55E-02	-4.89E-05
	s.e	5.13E-04	4.98E-02	7.89E-01	1.31E-02	5.16E-04
	Robust s.e.	5.59E-04	9.52E-02	1.92E+00	2.00E-02	5.13E-04
JNJ	Coef.	-2.65E-05	-2.20E-01	3.52E+00	2.59E-02	-3.12E-04
	s.e	3.61E-04	3.51E-02	8.08E-01	5.29E-03	2.50E-04
	Robust s.e.	4.66E-04	6.79E-02	3.13E+00	7.47E-03	2.56E-04
JPM	Coef.	1.20E-04	-1.10E-01	1.00E+00	5.02E-03	-6.37E-04
	s.e	9.60E-04	2.62E-02	3.91E-01	1.02E-02	8.28E-04
	Robust s.e.	8.76E-04	7.39E-02	1.07E+00	1.84E-02	7.41E-04
KO	Coef.	-2.44E-04	-1.40E-01	2.35E+00	2.18E-02	-3.52E-05
	s.e	3.90E-04	3.66E-02	8.16E-01	6.18E-03	3.05E-04
	Robust s.e.	4.70E-04	8.42E-02	3.46E+00	1.02E-02	2.70E-04
MCD	Coef.	5.20E-04	-4.29E-02	2.06E+00	4.71E-03	-5.48E-04
	s.e	4.49E-04	3.59E-02	6.00E-01	6.31E-03	3.20E-04
	Robust s.e.	4.20E-04	7.81E-02	1.31E+00	1.15E-02	3.05E-04
MRK	Coef.	-5.63E-04	-1.40E-02	2.75E+00	1.26E-03	-8.09E-05

	s.e	5.36E-04	3.91E-02	5.44E-01	8.66E-03	3.67E-04
	Robust s.e.	5.15E-04	7.81E-02	1.12E+00	1.42E-02	3.97E-04
MSFT	Coef.	-3.60E-05	-4.96E-02	1.36E+00	9.46E-03	-4.87E-04
	s.e	4.46E-04	4.41E-02	8.32E-01	1.01E-02	4.24E-04
	Robust s.e.	6.73E-04	7.74E-02	3.31E+00	1.45E-02	4.00E-04
ORCL	Coef.	-9.09E-05	-8.91E-02	1.59E+00	8.19E-03	1.62E-04
	s.e	4.17E-04	4.94E-02	7.43E-01	1.25E-02	1.13E-04
	Robust s.e.	6.79E-04	1.02E-01	2.29E+00	2.25E-02	6.40E-05
PEP	Coef.	5.92E-04	-8.57E-02	1.93E+00	9.42E-03	-3.05E-04
	s.e	4.02E-04	3.63E-02	5.57E-01	5.73E-03	2.79E-04
	Robust s.e.	4.04E-04	7.76E-02	1.53E+00	9.75E-03	2.51E-04
PFE	Coef.	-7.39E-04	-1.37E-01	2.60E+00	2.74E-02	-2.10E-04
	s.e	3.59E-04	5.39E-02	8.44E-01	1.19E-02	3.80E-04
	Robust s.e.	4.10E-04	8.35E-02	1.95E+00	1.53E-02	3.56E-04
PG	Coef.	5.17E-04	-1.71E-01	1.59E+00	2.23E-02	-1.78E-04
	s.e	4.00E-04	3.54E-02	6.45E-01	5.78E-03	2.81E-04
	Robust s.e.	3.89E-04	6.93E-02	1.00E+00	9.24E-03	2.72E-04
QCOM	Coef.	-1.33E-04	-5.05E-02	5.77E-01	-1.90E-03	-9.49E-05
	s.e	6.64E-04	3.60E-02	5.08E-01	9.39E-03	4.67E-04
	Robust s.e.	6.83E-04	6.79E-02	1.03E+00	1.55E-02	4.60E-04
SLB	Coef.	5.46E-04	-1.03E-01	-1.01E+00	2.48E-02	-3.66E-04
	s.e	9.26E-04	2.45E-02	5.38E-01	8.24E-03	9.13E-04
	Robust s.e.	1.10E-03	3.72E-02	1.45E+00	9.09E-03	9.12E-04
T	Coef.	-6.33E-04	-1.05E-01	3.54E+00	2.03E-02	-5.88E-04
	s.e	3.78E-04	4.10E-02	5.64E-01	8.79E-03	3.59E-04
	Robust s.e.	5.30E-04	6.88E-02	2.53E+00	1.17E-02	3.64E-04
VZ	Coef.	-3.13E-04	-7.11E-02	3.40E+00	1.11E-02	-6.48E-04
	s.e	4.25E-04	3.67E-02	6.06E-01	7.35E-03	3.69E-04
	Robust s.e.	4.15E-04	7.33E-02	1.84E+00	1.19E-02	3.55E-04
WFC	Coef.	9.64E-04	-1.32E-01	-1.46E-01	1.31E-02	-8.45E-04
	s.e	9.56E-04	2.65E-02	3.49E-01	1.10E-02	7.91E-04
	Robust s.e.	8.02E-04	6.64E-02	1.09E+00	1.62E-02	6.59E-04
WMT	Coef.	3.37E-04	-1.82E-01	2.03E+00	2.26E-02	-5.02E-04
	s.e	4.60E-04	3.71E-02	6.41E-01	6.50E-03	3.33E-04
	Robust s.e.	4.94E-04	6.68E-02	1.45E+00	9.65E-03	3.22E-04
XOM	Coef.	-7.17E-04	-1.39E-01	3.32E+00	1.04E-02	5.27E-04
	s.e	5.12E-04	2.62E-02	4.49E-01	5.83E-03	5.49E-04
	Robust s.e.	5.05E-04	3.79E-02	1.11E+00	6.62E-03	4.83E-04

Table 40

Summary of Model $\theta^{Pearson,Moors}$ estimated for each of the individual stocks

Company		Intercept	Return	Variance	Skewness	Kurtosis
AAPL	Coef.	-1.62E-03	-7.70E-02	3.58E-02	-1.18E-02	7.62E-03
	s.e	6.64E-04	2.48E-02	6.62E-01	1.13E-02	2.83E-03
	Robust s.e.	8.47E-04	4.01E-02	1.72E+00	1.30E-02	3.03E-03
AMZN	Coef.	1.34E-03	-2.04E-02	1.17E+00	9.52E-03	-1.91E-03
	s.e	7.59E-04	2.58E-02	6.52E-01	1.30E-02	1.99E-03
	Robust s.e.	9.69E-04	4.74E-02	1.71E+00	1.49E-02	1.82E-03
BAC	Coef.	-1.86E-03	-1.54E-01	-6.80E-01	3.74E-02	1.17E-03
	s.e	1.00E-03	2.83E-02	2.89E-01	2.01E-02	2.45E-03
	Robust s.e.	9.56E-04	7.42E-02	8.60E-01	2.86E-02	1.81E-03
C	Coef.	-1.06E-03	-1.99E-02	-1.98E+00	-7.96E-03	1.93E-04
	s.e	7.73E-04	2.94E-02	2.05E-01	2.17E-02	1.03E-04
	Robust s.e.	8.64E-04	1.02E-01	7.37E-01	4.75E-02	1.15E-04
CMCSA	Coef.	-2.20E-04	-7.24E-02	1.67E+00	1.55E-02	-6.86E-05
	s.e	5.47E-04	4.29E-02	7.12E-01	1.69E-02	1.27E-03
	Robust s.e.	6.74E-04	8.45E-02	2.13E+00	2.49E-02	1.07E-03
CSCO	Coef.	-3.88E-04	-7.55E-02	2.40E-01	7.71E-03	1.88E-04
	s.e	4.26E-04	4.81E-02	6.95E-01	1.70E-02	7.25E-04
	Robust s.e.	6.60E-04	1.11E-01	2.29E+00	3.51E-02	5.03E-04
CVX	Coef.	-5.96E-04	-1.15E-01	3.30E+00	5.33E-03	-4.17E-04
	s.e	4.41E-04	2.55E-02	4.70E-01	8.18E-03	1.71E-03
	Robust s.e.	5.92E-04	5.30E-02	2.06E+00	1.18E-02	1.40E-03
DIS	Coef.	-4.82E-04	-1.17E-01	4.79E+00	2.79E-02	5.36E-04
	s.e	4.47E-04	3.71E-02	6.43E-01	1.21E-02	1.16E-03
	Robust s.e.	5.11E-04	1.08E-01	1.96E+00	2.72E-02	1.03E-03
GE	Coef.	-2.26E-04	-1.07E-01	-1.88E+00	2.31E-02	1.24E-03
	s.e	4.45E-04	3.51E-02	4.11E-01	1.46E-02	1.30E-03
	Robust s.e.	4.20E-04	9.79E-02	1.15E+00	2.67E-02	1.01E-03
HD	Coef.	-2.36E-04	9.46E-04	2.40E+00	9.35E-03	-1.14E-04
	s.e	5.47E-04	3.65E-02	6.03E-01	1.33E-02	1.43E-03
	Robust s.e.	5.32E-04	7.39E-02	1.28E+00	1.89E-02	1.40E-03
IBM	Coef.	1.26E-03	9.00E-03	-4.64E-01	5.87E-03	-2.01E-03
	s.e	3.85E-04	2.61E-02	5.92E-01	7.08E-03	1.37E-03
	Robust s.e.	3.80E-04	4.29E-02	1.69E+00	8.21E-03	1.19E-03
INTC	Coef.	-1.12E-03	-2.41E-01	1.84E+00	8.01E-02	2.73E-03
	s.e	4.67E-04	5.06E-02	7.84E-01	1.85E-02	1.30E-03
	Robust s.e.	5.24E-04	8.46E-02	1.69E+00	2.52E-02	1.22E-03
JNJ	Coef.	-1.19E-04	-2.16E-01	3.48E+00	3.59E-02	-1.01E-03
	s.e	2.80E-04	3.50E-02	8.08E-01	7.51E-03	6.93E-04

	Robust s.e.	3.95E-04	6.10E-02	2.80E+00	9.50E-03	6.88E-04
JPM	Coef.	-5.79E-04	-1.09E-01	1.03E+00	5.22E-03	6.23E-04
	s.e	7.43E-04	2.62E-02	3.90E-01	1.42E-02	2.28E-03
	Robust s.e.	6.16E-04	7.61E-02	1.02E+00	2.67E-02	1.72E-03
KO	Coef.	-2.80E-04	-1.41E-01	2.35E+00	3.14E-02	9.33E-06
	s.e	3.19E-04	3.66E-02	8.16E-01	8.83E-03	7.87E-04
	Robust s.e.	4.65E-04	7.94E-02	3.39E+00	1.38E-02	6.52E-04
MCD	Coef.	7.46E-05	-4.10E-02	2.05E+00	6.38E-03	-6.29E-04
	s.e	3.57E-04	3.58E-02	6.01E-01	9.06E-03	8.92E-04
	Robust s.e.	3.39E-04	8.24E-02	1.31E+00	1.74E-02	7.65E-04
MRK	Coef.	-2.26E-04	-2.28E-02	2.75E+00	5.77E-03	-1.52E-03
	s.e	4.63E-04	3.85E-02	5.40E-01	1.25E-02	1.09E-03
	Robust s.e.	4.37E-04	8.72E-02	1.22E+00	2.25E-02	9.98E-04
MSFT	Coef.	-3.91E-04	-5.05E-02	1.25E+00	1.36E-02	4.38E-04
	s.e	3.96E-04	4.49E-02	8.30E-01	1.46E-02	1.15E-03
	Robust s.e.	7.18E-04	8.59E-02	3.38E+00	2.26E-02	1.01E-03
ORCL	Coef.	-6.29E-05	-9.57E-02	1.63E+00	1.42E-02	2.51E-04
	s.e	4.11E-04	4.99E-02	7.43E-01	1.79E-02	1.47E-04
	Robust s.e.	6.80E-04	1.04E-01	2.30E+00	3.28E-02	6.12E-05
PEP	Coef.	4.67E-04	-8.68E-02	1.92E+00	1.41E-02	-9.47E-04
	s.e	2.98E-04	3.64E-02	5.57E-01	8.34E-03	7.63E-04
	Robust s.e.	3.38E-04	7.77E-02	1.95E+00	1.41E-02	6.70E-04
PFE	Coef.	-8.13E-04	-1.43E-01	2.52E+00	4.06E-02	7.19E-07
	s.e	3.54E-04	5.42E-02	8.31E-01	1.70E-02	8.32E-04
	Robust s.e.	4.25E-04	8.12E-02	1.90E+00	2.10E-02	6.92E-04
PG	Coef.	2.42E-04	-1.71E-01	1.60E+00	3.14E-02	2.81E-04
	s.e	3.00E-04	3.55E-02	6.45E-01	8.21E-03	7.90E-04
	Robust s.e.	2.80E-04	7.45E-02	1.13E+00	1.40E-02	7.00E-04
QCOM	Coef.	-3.90E-05	-5.11E-02	5.77E-01	-2.41E-03	-9.04E-04
	s.e	5.03E-04	3.57E-02	5.07E-01	1.32E-02	1.37E-03
	Robust s.e.	5.00E-04	6.53E-02	9.30E-01	2.11E-02	1.29E-03
SLB	Coef.	2.41E-04	-1.03E-01	-9.91E-01	3.52E-02	3.75E-05
	s.e	7.36E-04	2.43E-02	5.36E-01	1.14E-02	2.80E-03
	Robust s.e.	8.66E-04	3.77E-02	1.43E+00	1.27E-02	2.67E-03
T	Coef.	-8.83E-04	-1.01E-01	3.43E+00	2.80E-02	-8.57E-04
	s.e	3.32E-04	4.10E-02	5.61E-01	1.26E-02	9.97E-04
	Robust s.e.	5.32E-04	6.78E-02	2.58E+00	1.64E-02	1.01E-03
VZ	Coef.	-5.91E-04	-7.25E-02	3.34E+00	1.67E-02	-1.28E-03
	s.e	3.57E-04	3.68E-02	6.04E-01	1.06E-02	9.85E-04
	Robust s.e.	3.30E-04	6.74E-02	1.44E+00	1.59E-02	8.02E-04
WFC	Coef.	2.74E-04	-1.30E-01	-1.31E-01	1.61E-02	-3.98E-04
	s.e	7.59E-04	2.66E-02	3.49E-01	1.56E-02	2.21E-03
	Robust s.e.	6.57E-04	6.71E-02	1.11E+00	2.38E-02	1.60E-03

WMT	Coef.	-1.27E-05	-1.81E-01	2.05E+00	3.14E-02	-8.43E-04
	s.e	3.49E-04	3.74E-02	6.42E-01	9.31E-03	9.02E-04
	Robust s.e.	3.66E-04	6.49E-02	1.27E+00	1.35E-02	8.64E-04
XOM	Coef.	-4.01E-04	-1.38E-01	3.29E+00	1.41E-02	4.84E-04
	s.e	3.90E-04	2.61E-02	4.48E-01	7.90E-03	1.59E-03
	Robust s.e.	3.95E-04	5.55E-02	1.33E+00	1.17E-02	1.36E-03

Table 41

Summary of Model $\theta^{Pearson,C\&S}$ estimated for each of the individual stocks

Company		Intercept	Return	Variance	Skewness	Kurtosis
AAPL	Coef.	-2.42E-03	-7.93E-02	3.79E-02	-1.21E-02	2.40E-03
	s.e	7.96E-04	2.48E-02	6.61E-01	1.13E-02	7.44E-04
	Robust s.e.	8.90E-04	4.02E-02	1.74E+00	1.30E-02	6.86E-04
AMZN	Coef.	1.29E-03	-2.00E-02	1.17E+00	9.71E-03	-3.48E-04
	s.e	9.63E-04	2.58E-02	6.55E-01	1.30E-02	6.34E-04
	Robust s.e.	1.18E-03	4.72E-02	1.77E+00	1.47E-02	6.08E-04
BAC	Coef.	-1.24E-03	-1.54E-01	-6.88E-01	3.81E-02	-4.58E-04
	s.e	1.06E-03	2.83E-02	2.87E-01	2.01E-02	1.06E-03
	Robust s.e.	8.73E-04	7.93E-02	8.75E-01	2.83E-02	8.24E-04
C	Coef.	-1.08E-03	-1.96E-02	-1.98E+00	-8.34E-03	1.20E-04
	s.e	7.75E-04	2.94E-02	2.05E-01	2.17E-02	8.05E-05
	Robust s.e.	8.74E-04	1.02E-01	7.41E-01	4.75E-02	8.58E-05
CMCSA	Coef.	4.14E-05	-7.29E-02	1.69E+00	1.57E-02	-3.56E-04
	s.e	6.37E-04	4.29E-02	7.12E-01	1.69E-02	5.59E-04
	Robust s.e.	7.00E-04	1.01E-01	2.20E+00	2.92E-02	5.09E-04
CSCO	Coef.	-2.45E-04	-7.50E-02	2.51E-01	7.42E-03	-1.52E-04
	s.e	4.74E-04	4.81E-02	6.96E-01	1.70E-02	3.86E-04
	Robust s.e.	6.01E-04	1.06E-01	2.17E+00	3.35E-02	3.30E-04
CVX	Coef.	-5.68E-04	-1.15E-01	3.30E+00	5.31E-03	-1.23E-04
	s.e	5.73E-04	2.56E-02	4.71E-01	8.18E-03	5.88E-04
	Robust s.e.	7.45E-04	5.86E-02	2.04E+00	1.22E-02	5.44E-04
DIS	Coef.	-6.23E-05	-1.17E-01	4.85E+00	2.78E-02	-3.89E-04
	s.e	4.98E-04	3.71E-02	6.47E-01	1.21E-02	4.30E-04
	Robust s.e.	5.03E-04	7.52E-02	1.86E+00	1.88E-02	4.30E-04
GE	Coef.	-4.38E-04	-1.08E-01	-1.95E+00	2.34E-02	7.44E-04
	s.e	4.87E-04	3.51E-02	4.15E-01	1.46E-02	5.21E-04
	Robust s.e.	4.16E-04	9.73E-02	1.17E+00	2.65E-02	5.07E-04
HD	Coef.	3.82E-04	9.51E-04	2.38E+00	9.51E-03	-5.85E-04
	s.e	7.06E-04	3.65E-02	6.02E-01	1.33E-02	5.17E-04
	Robust s.e.	7.28E-04	6.86E-02	1.26E+00	1.75E-02	5.85E-04

IBM	Coef.	1.69E-03	1.13E-02	-5.11E-01	6.02E-03	-8.38E-04
	s.e	5.02E-04	2.61E-02	5.92E-01	7.08E-03	4.29E-04
	Robust s.e.	5.09E-04	4.13E-02	1.63E+00	8.15E-03	4.09E-04
INTC	Coef.	-6.85E-04	-2.43E-01	1.90E+00	8.08E-02	-4.45E-05
	s.e	5.13E-04	5.07E-02	7.89E-01	1.86E-02	5.16E-04
	Robust s.e.	5.54E-04	9.37E-02	1.89E+00	2.77E-02	5.14E-04
JNJ	Coef.	-2.15E-05	-2.16E-01	3.49E+00	3.58E-02	-3.13E-04
	s.e	3.61E-04	3.50E-02	8.08E-01	7.52E-03	2.50E-04
	Robust s.e.	4.63E-04	6.59E-02	3.06E+00	1.03E-02	2.56E-04
JPM	Coef.	1.21E-04	-1.09E-01	1.00E+00	5.58E-03	-6.33E-04
	s.e	9.60E-04	2.62E-02	3.91E-01	1.42E-02	8.28E-04
	Robust s.e.	8.76E-04	7.40E-02	1.07E+00	2.59E-02	7.40E-04
KO	Coef.	-2.39E-04	-1.41E-01	2.35E+00	3.15E-02	-3.97E-05
	s.e	3.90E-04	3.66E-02	8.16E-01	8.84E-03	3.05E-04
	Robust s.e.	4.68E-04	8.13E-02	3.45E+00	1.41E-02	2.68E-04
MCD	Coef.	5.24E-04	-4.12E-02	2.05E+00	6.26E-03	-5.49E-04
	s.e	4.49E-04	3.58E-02	6.00E-01	9.06E-03	3.20E-04
	Robust s.e.	4.19E-04	7.79E-02	1.31E+00	1.66E-02	3.05E-04
MRK	Coef.	-5.71E-04	-2.42E-02	2.76E+00	5.90E-03	-8.24E-05
	s.e	5.36E-04	3.85E-02	5.44E-01	1.25E-02	3.67E-04
	Robust s.e.	5.15E-04	7.56E-02	1.12E+00	2.00E-02	3.97E-04
MSFT	Coef.	-3.48E-05	-5.19E-02	1.35E+00	1.42E-02	-4.88E-04
	s.e	4.46E-04	4.49E-02	8.32E-01	1.46E-02	4.24E-04
	Robust s.e.	6.71E-04	7.90E-02	3.30E+00	2.12E-02	4.00E-04
ORCL	Coef.	-9.32E-05	-9.53E-02	1.60E+00	1.41E-02	1.62E-04
	s.e	4.17E-04	4.99E-02	7.42E-01	1.79E-02	1.13E-04
	Robust s.e.	6.79E-04	1.04E-01	2.30E+00	3.27E-02	6.37E-05
PEP	Coef.	5.91E-04	-8.61E-02	1.93E+00	1.38E-02	-3.04E-04
	s.e	4.02E-04	3.64E-02	5.57E-01	8.34E-03	2.79E-04
	Robust s.e.	4.03E-04	7.70E-02	1.52E+00	1.40E-02	2.51E-04
PFE	Coef.	-7.39E-04	-1.44E-01	2.60E+00	4.10E-02	-2.12E-04
	s.e	3.59E-04	5.43E-02	8.43E-01	1.70E-02	3.80E-04
	Robust s.e.	4.10E-04	8.48E-02	1.95E+00	2.19E-02	3.57E-04
PG	Coef.	5.24E-04	-1.71E-01	1.58E+00	3.16E-02	-1.79E-04
	s.e	4.00E-04	3.54E-02	6.45E-01	8.21E-03	2.81E-04
	Robust s.e.	3.88E-04	6.97E-02	9.92E-01	1.33E-02	2.72E-04
QCOM	Coef.	-1.35E-04	-5.13E-02	5.77E-01	-2.30E-03	-9.43E-05
	s.e	6.63E-04	3.57E-02	5.08E-01	1.32E-02	4.67E-04
	Robust s.e.	6.82E-04	6.77E-02	1.03E+00	2.21E-02	4.60E-04
SLB	Coef.	5.53E-04	-1.03E-01	-1.01E+00	3.51E-02	-3.71E-04
	s.e	9.26E-04	2.44E-02	5.38E-01	1.14E-02	9.13E-04
	Robust s.e.	1.10E-03	3.68E-02	1.44E+00	1.25E-02	9.12E-04
T	Coef.	-6.26E-04	-1.02E-01	3.53E+00	2.80E-02	-5.93E-04

	s.e	3.78E-04	4.10E-02	5.64E-01	1.26E-02	3.59E-04
	Robust s.e.	5.28E-04	6.78E-02	2.53E+00	1.64E-02	3.64E-04
VZ	Coef.	-3.10E-04	-7.40E-02	3.41E+00	1.70E-02	-6.51E-04
	s.e	4.25E-04	3.68E-02	6.06E-01	1.06E-02	3.69E-04
	Robust s.e.	4.14E-04	7.34E-02	1.84E+00	1.72E-02	3.56E-04
WFC	Coef.	9.64E-04	-1.30E-01	-1.46E-01	1.66E-02	-8.42E-04
	s.e	9.56E-04	2.66E-02	3.49E-01	1.56E-02	7.91E-04
	Robust s.e.	8.02E-04	6.65E-02	1.09E+00	2.33E-02	6.59E-04
WMT	Coef.	3.42E-04	-1.79E-01	2.03E+00	3.12E-02	-5.03E-04
	s.e	4.60E-04	3.74E-02	6.42E-01	9.31E-03	3.34E-04
	Robust s.e.	4.93E-04	6.72E-02	1.42E+00	1.39E-02	3.22E-04
XOM	Coef.	-7.18E-04	-1.39E-01	3.32E+00	1.40E-02	5.30E-04
	s.e	5.12E-04	2.61E-02	4.49E-01	7.90E-03	5.49E-04
	Robust s.e.	5.05E-04	3.75E-02	1.10E+00	9.01E-03	4.83E-04

Appendix B: Figures

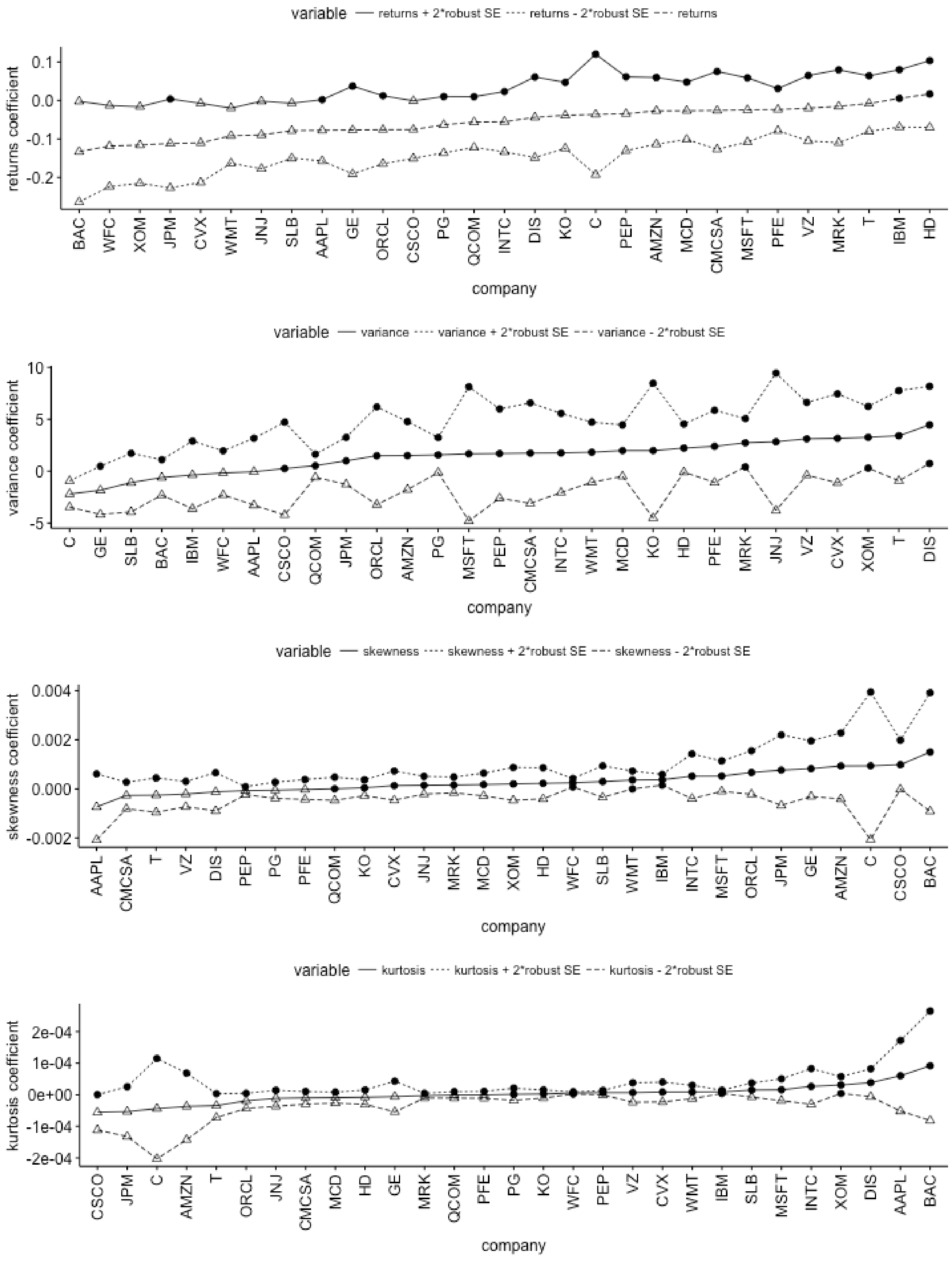

Figure 8: Estimated coefficients for Model θ^{BPV}

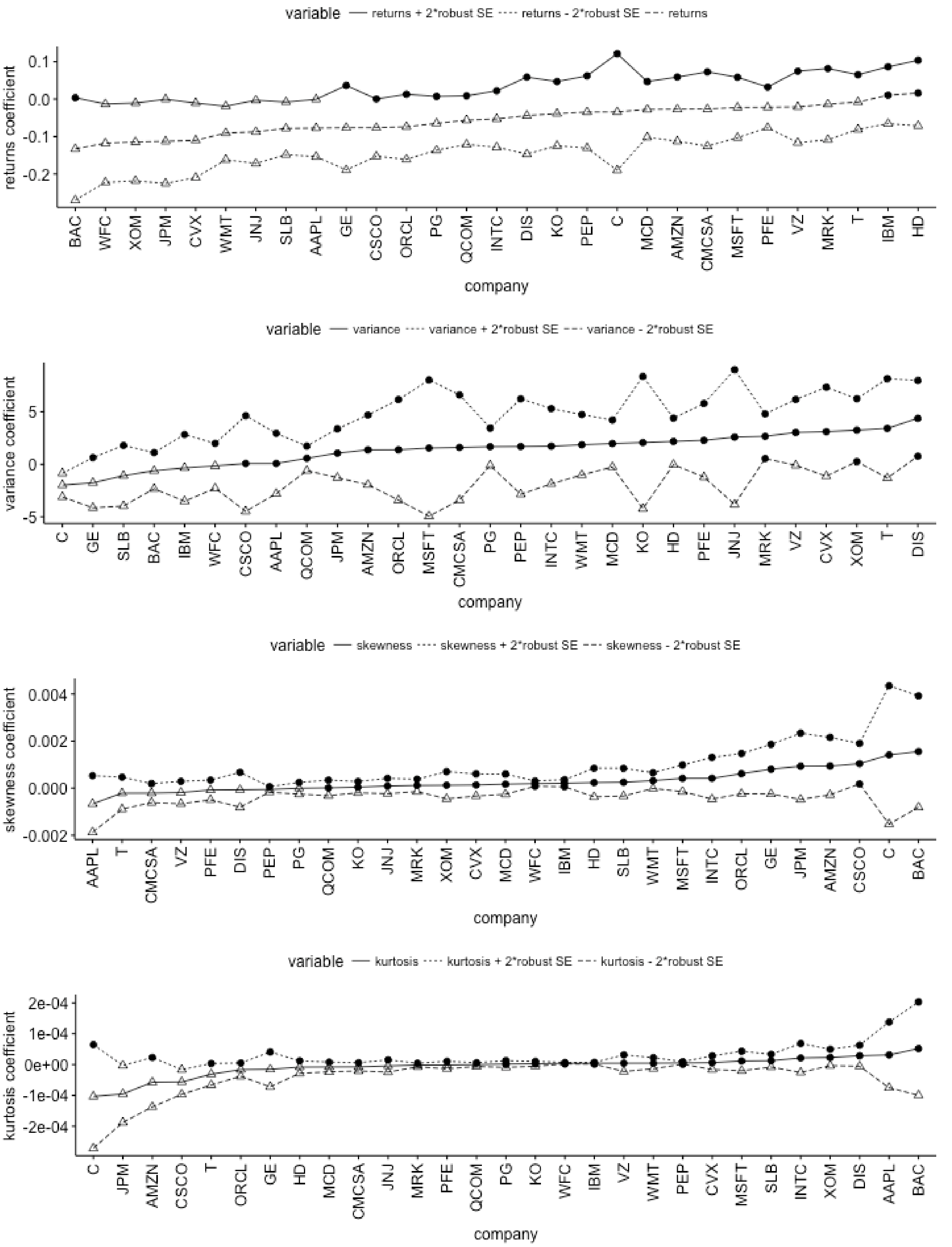

Figure 9: Estimated coefficients for Model θ^{MinRV}

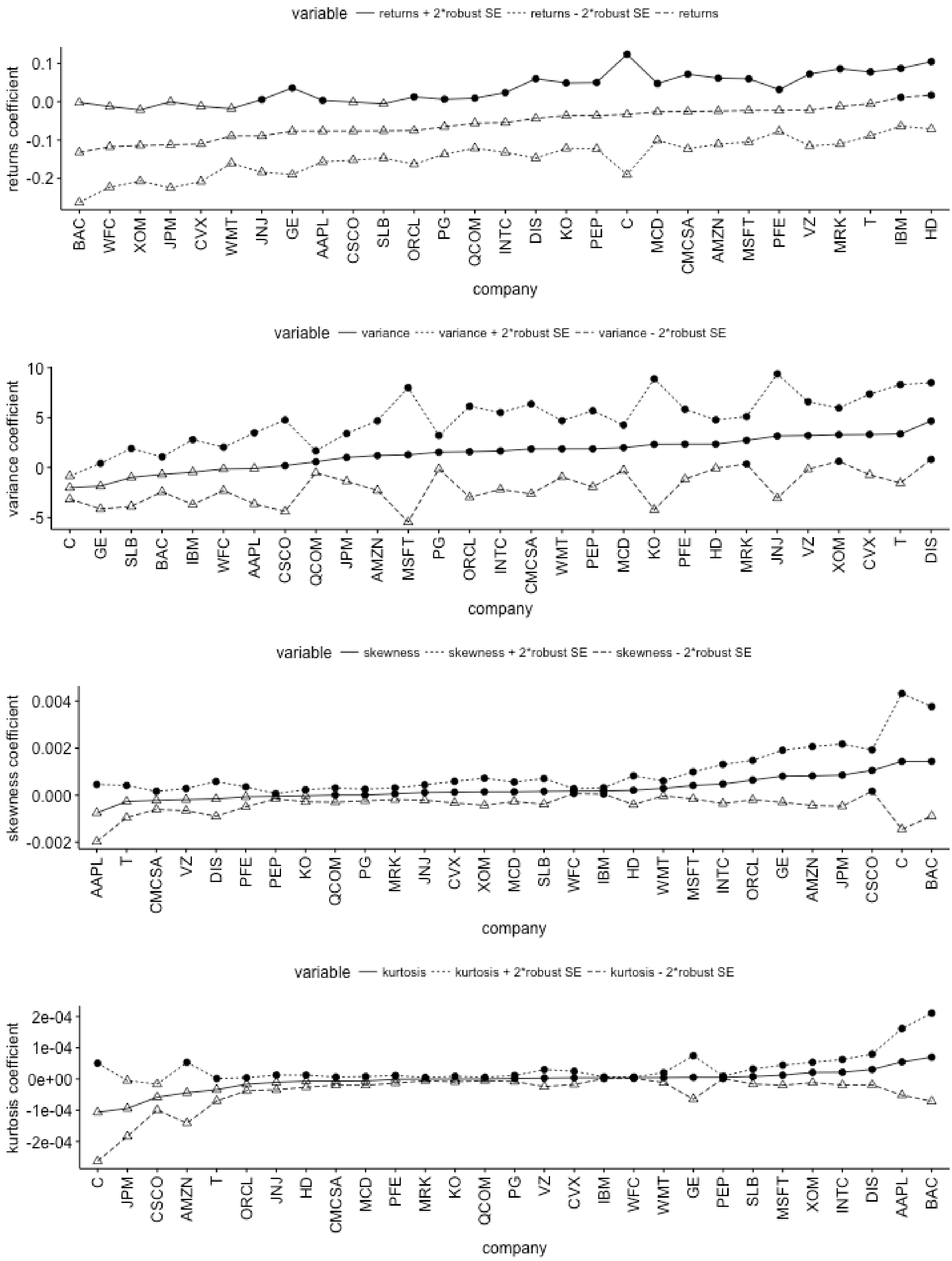

Figure 10: Estimated coefficients for Model θ^{MedBPV}

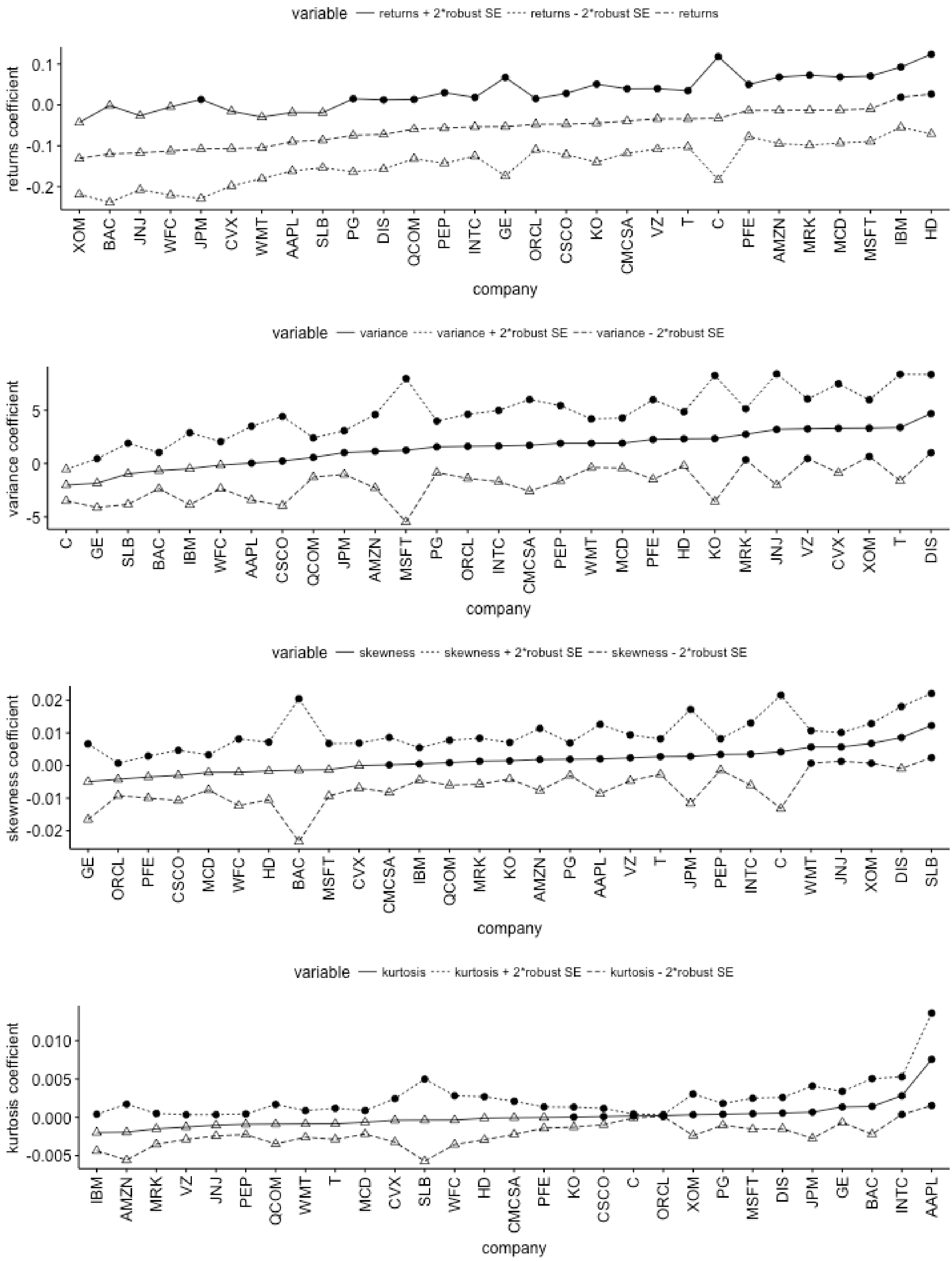

Figure 11: Estimated coefficients for model $\theta^{BS,MK}$

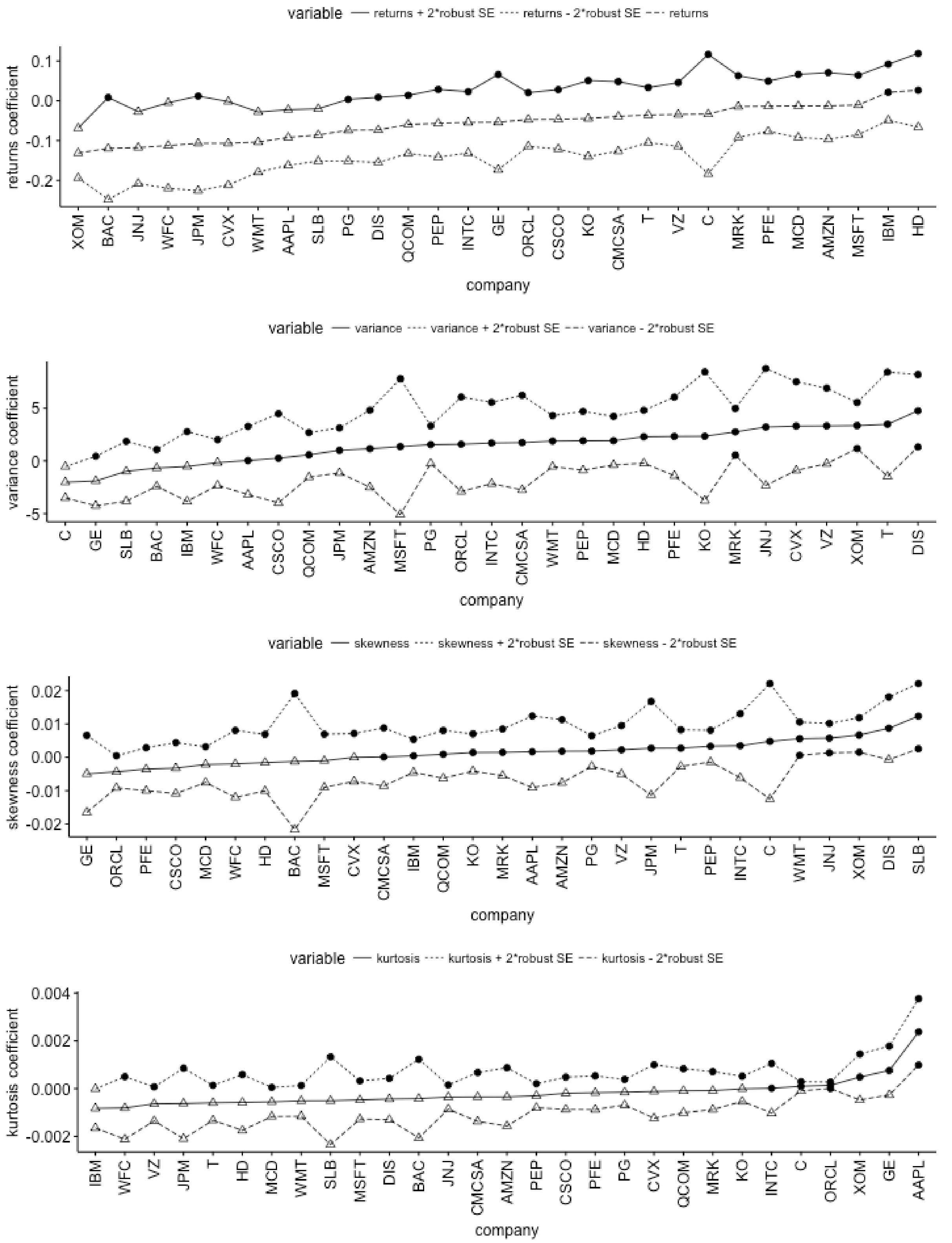

Figure 12: Estimated coefficients for model $\theta^{BS,SK}$

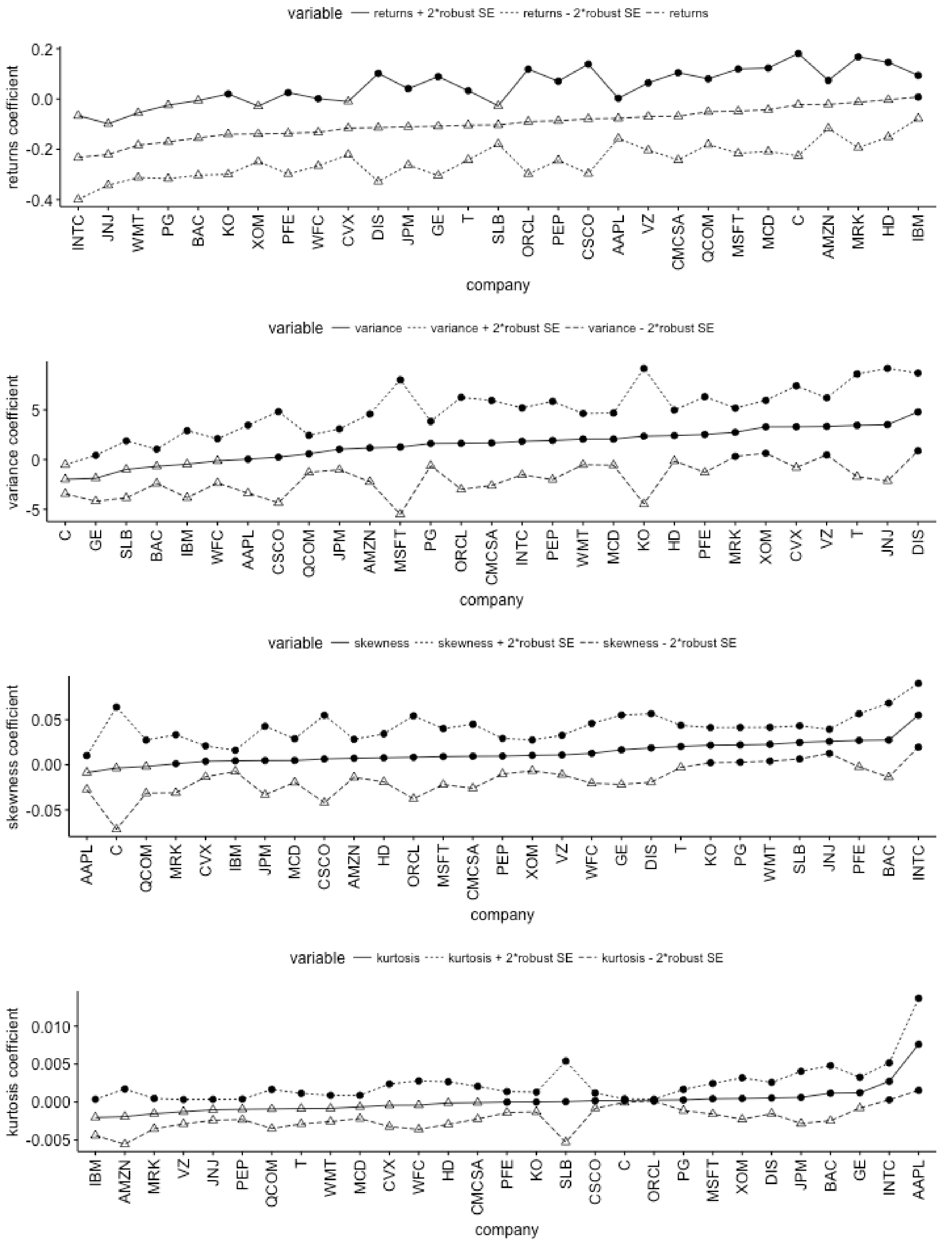

Figure 13: Estimated coefficients for model $\theta^{GMS,MK}$

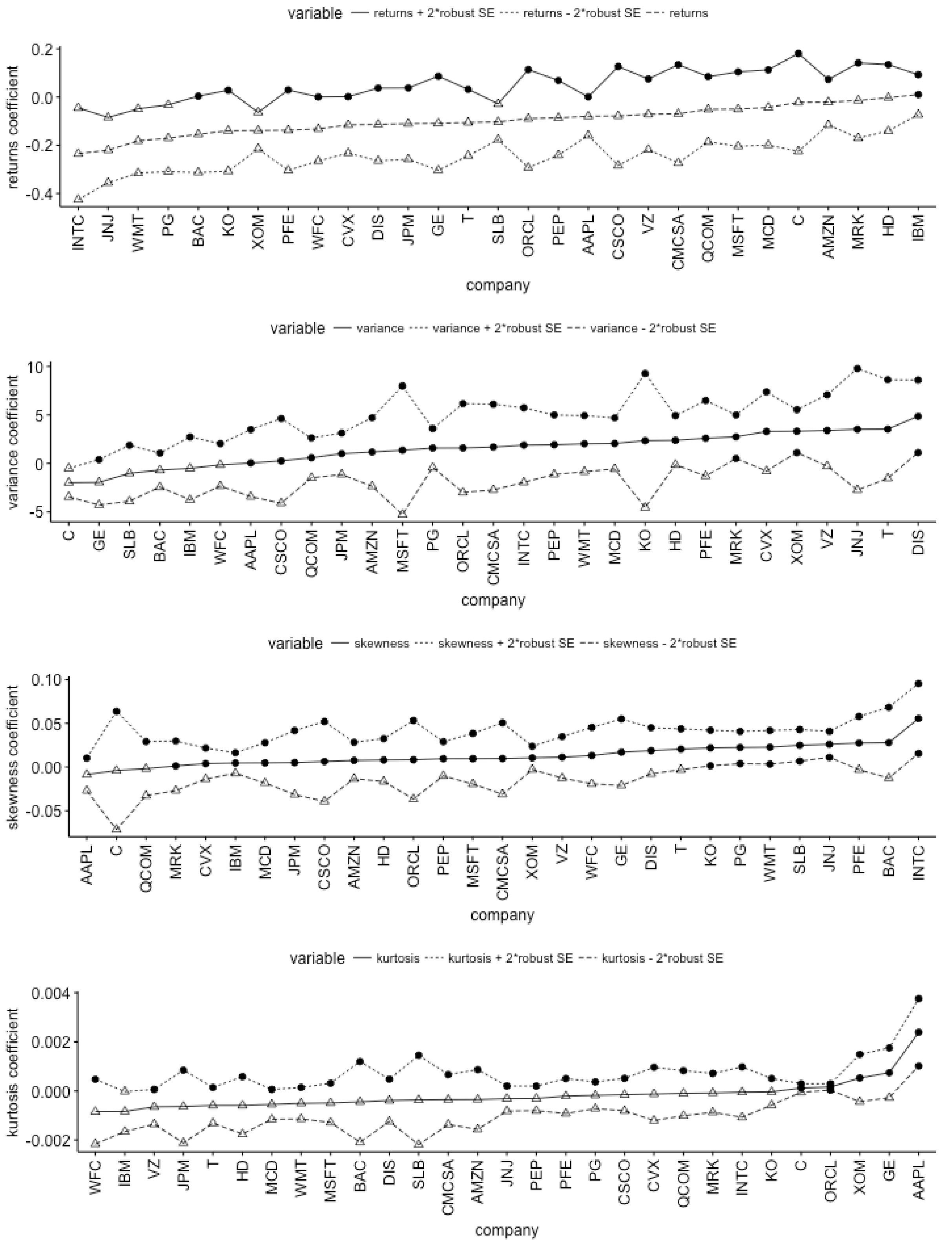

Figure 14: Estimated coefficients for model $\theta^{GMS,SK}$

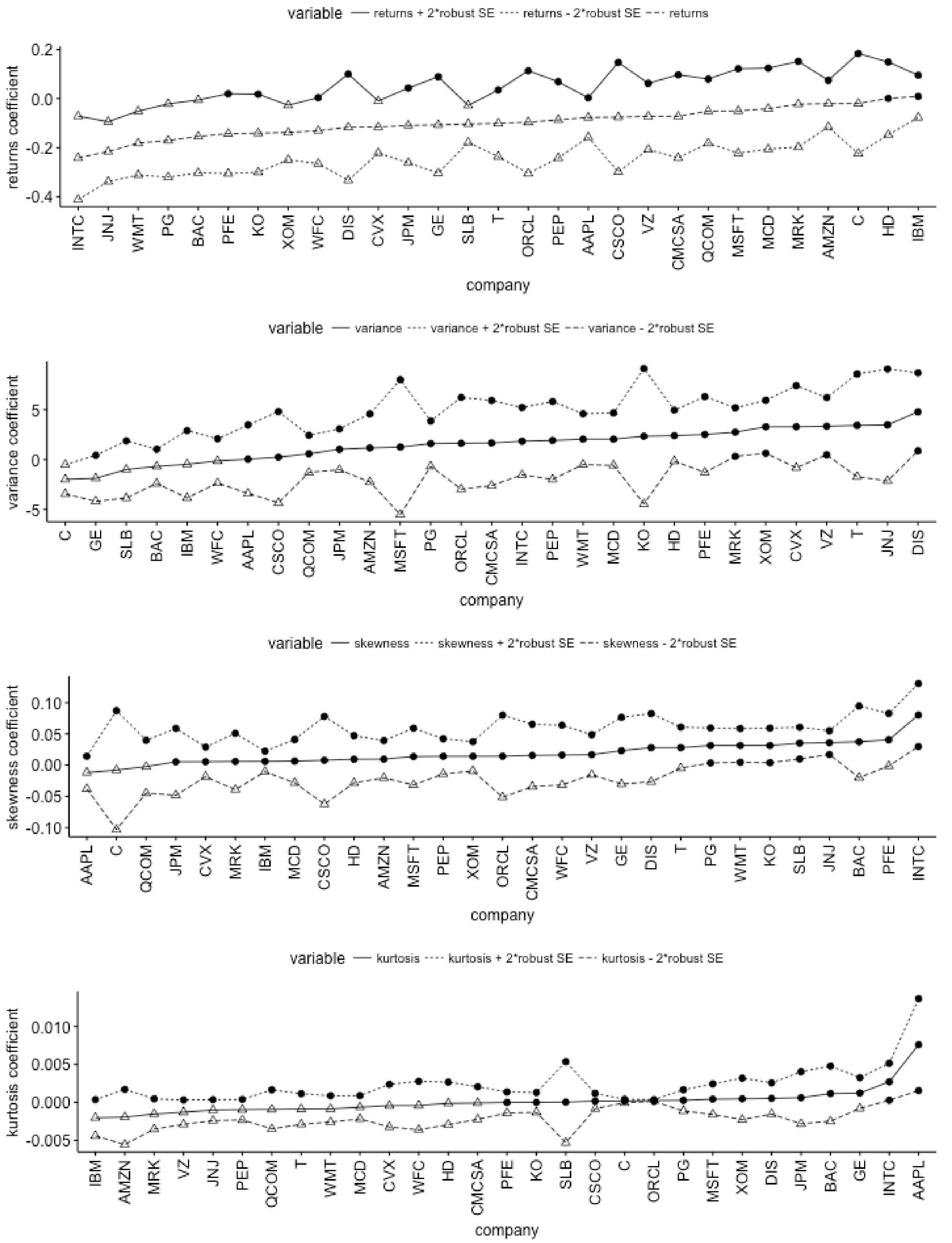

Figure 15: Estimated coefficients for model $\theta^{PS,MK}$

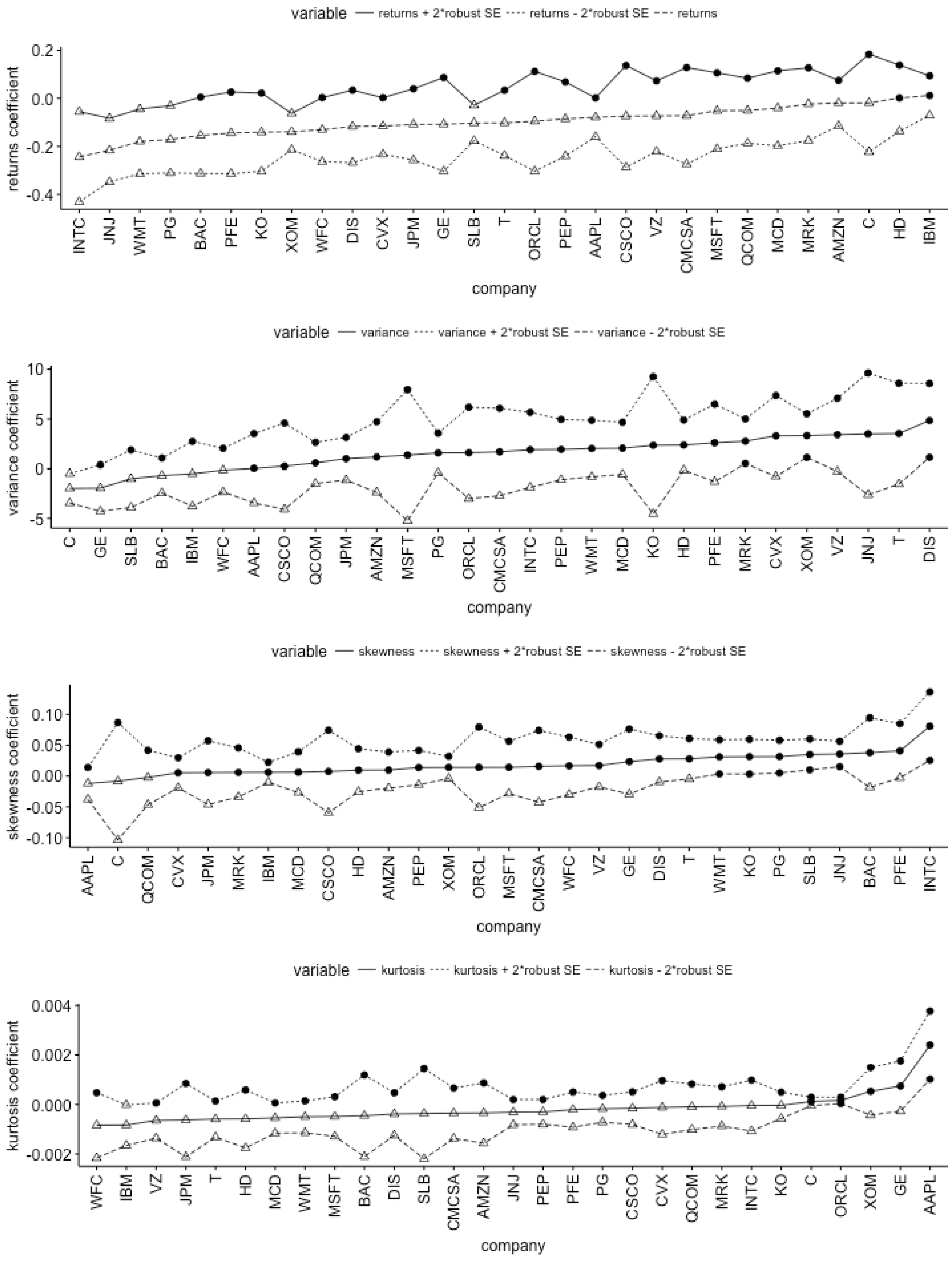

Figure 16: Estimated coefficients for model $\theta^{PS,SK}$