Report on the MS thesis titled "Numerical simulation of viscoelastic fluid-like materials, as asphalt in particular" by Mr. Jan Kratochvil

At the outset let me mention that the thesis meets and exceeds what is demanded of Master's thesis in the United States in the best of universities. It is a solid piece of work and deserves to be accepted towards the partial fulfillment of a Master's degree. However, the work can be improved and several minor errors and misconceptions corrected and I shall discuss them in detail later in the review.

Mr. Kratochvil considers a wide variety of flows of a isotropic, incompressible, non-linear viscoelastic fluid, namely those involving plane flows, flows in a contraction, and steady and unsteady axial flows in a circular cylinder. In addition to solving the problem numerically for an Oldroyd-B fluid, he also considers the flows of a fluid, that he refers to as model 1, which is more general than the Oldroyd-B model but which reduces to the Oldroyd-B model for small deviatoric stresses.

The numerical study of the governing equations is far from trivial. In fact, there has been considerable time and effort devoted to just getting a hand on the contraction problem at high Weissenberg numbers. The study in this thesis concerns a much more complicated model than the Oldroyd-B model and thus Mr. Kratochvil should be commended on the work that he has done. It is unfortunate that neither of the models that were used was able to describe the experimental results but this should form the basis for future work.

I will now come to improvements that can be made to the thesis:

1) **D** is not the rate of deformation; it is merely the symmetric part of the velocity gradient.

2) It is incorrect to refer to the equation (2.5) as the balance of internal energy; it is in fact the balance of energy. Incidentally, internal energy is not conserved.

3) The procedure that is used eliminates the radiation from consideration. The reduced dissipation equation does not contain the radiation and this is a serious shortcoming in general. Luckily for the problem under consideration this does not matter. The issues concerning this ought to be discussed carefully. There are several important problems where it is important to be able to constitutively specify the radiant heating.

4) It makes no sense to call **A** the elastic extra stress (see comment 6).

5) The Cauchy-Green stretch tensor from the preferred natural configuration **C** is not frame indifferent but is merely invariant, i.e. it is the same in two different frames. It does not matter in the formulation of the problem as the material under consideration is isotropic. This ought to be mentioned.

6) One reaches the conclusion that the model 1 reduces to the Oldroyd-B model when the tensor **A** has a small Frobenius norm. However, it is not correct to call **A** a stress. In fact,

it has no dimension. It definitely does not have the dimension of stress. In any event what is being assumed is that the appropriate **B** tensor is close to the identity.

7) There are several grammatical and typographical errors. Even the title is incorrect. It ought to read "Numerical simulation of viscoelastic fluid-like materials; asphalt in particular".

The above mistakes are minor but nonetheless need to be corrected as they are conceptual errors.

I recommend the thesis to be accepted towards the granting of a Master's degree.