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Structure and Simplicity in Leibniz
Struktura a jednoduchost u Leibnize

Supervisor
(Vedoucí práce)

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ABSTRACT

The thesis aims to present Leibniz's monadic system as the simplest conceivable structure. To this end, the thesis employs both contemporary literature concerned with formal ontology and its logic, semantics and the nature of reference; and Leibniz's own writings coupled with correspondent commentaries, including articles transcending selected interpretive issues into present-day discussions on actualised versions of one of the essential Leibnizian principles: the identity of indiscernibles. The exposition proceeds in three steps: (i) developing concepts of *structure*, *determination* and *reference* in critical confrontation with contemporary approaches, then (ii) applying those concepts to the interpretation of Leibniz's principle of identity of indiscenibles and illuminating therewith links between ontology and semantics, and eventually (iii) utilizing those findings in clarifying the mirroring relation constitutive of the monadic structure as represented a) in Leibniz's own spatiotemporal illustrations and b) in formal models in secondary literature. Acknowledging that the simplest conceivable structure, which is the monadic structure, can neither be satisfactorily expressed by spatiotemporal illustrations, nor can it be represented in formal system, itself forming the limit of extrinsic determination, we can better appreciate a central role given to the mirroring structure within Leibniz's philosophical enterprise, as well as structural limitations inherent to our own expressive means.

KEY WORDS

Leibniz; structure; singularity, plurality, multiplicity; representation; extrinsic/intrinsic, real/semantic determination; homogeneity, heterogeneity

ABSTRAKT

Předkládaná práce má za cíl vykázat Leibnizův monadický systém jako nejjednodušší pojatelnou strukturu. K tomuto účelu využívá jak současnou literaturu zaobírající se formální ontologií a její logikou, sémantikou a povahou reference, tak Leibnizovy vlastní texty s odpovídající sekundární literaturou včetně článků přesahující textově interpretační otázky směrem k diskuzím o současných formulacích jednoho ze stěžejních Leibnizovských principů: principu identity nerozlišených. Výklad postupuje ve třech krocích: (i) kritickým vyrovnáváním se se současnými přístupy rozvíjí pojmy *struktury*, *determinace* a *reference*, (ii) užívaje těchto pojmů k interpretaci Leibnizova principu identity nerozlišených, osvětluje vztahy ontologie a sémantiky, (iii) s pomocí osvojených vhledů pojednává vztah zrcadlení konstituující monadickou strukturu v návaznosti na jeho reprezentaci a) v Leibnizových vlastních časoprostorových ilustracích a b) ve formálních modelech Leibnizových interpretů. Nahlížejíce, že nejjednodušší pojatelná struktura, jež je strukturou monadickou, není bezezbytku vyjádřitelná ani v časoprostorových zobrazeních, ani ve formálních systémech, jsouc sama mezí vnější determinace, můžeme lépe ocenit jak ústřední úlohu zrcadlové struktury v kontextu Leibnizova filosofického projektu, tak strukturní limity našich vlastních prostředků vyjádření.

KLÍČOVÁ SLOVA

Leibniz; struktura; singularita, pluralita, mnohost; reprezentace; vnější/vnitřní, reálná/sémantická determinace; homogenita, heterogenita

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TABLE OF CONTENTS

INTRODUCTION

Leibniz's notion of the monad is notoriously perplexing. In a concise definition boasting palette of hardly reconcilable designations – such as that of being ultimately simple, windowless and yet a living mirror of the entire universe – is encapsulated a concept of far-reaching philosophical import affecting areas as diverse as the structure of continuum, reality of time and space, the body-mind problem, the nature of sense perception, physical force, ontogeny or valuation of beings. However, the notion of the monad is above all an a priori ontological concept, establishing the most fundamental, further unanalysable, realm of genuinely real entities, or, which amounts to the same as will become clear later in the text, the ultimate structure of reality – thereby claiming to mark out a certain limit of conceivability. In the following I will try to take the claim seriously.

Philosophical ideas definitely draw their significance from the links they entwine into the map of thinking, but notwithstanding this, there is a sense of autarchy they keep – which justifies our pinpointing them from the text and treating them separately – the sense of an immediate persuasiveness coupled with their surprisingness: they are suggestive in virtue of being enlightening, not evident. In the course of the investigation I am to pursue, I will try to defend a thesis that the structure of Leibniz's monad is neither arbitrary, nor merely relatively requisite in respect of some metaphysical or logical desiderata, but indispensable as such in virtue of its testifying the limits of conceivability arousing philosophers' curiosity up to our days. Let the success of my enterprise be measured by the extent to which I manage to present the concept of monad as genuinely elucidating those constraints of our reasoning, on the one hand, and present those constraints so as to elucidate Leibniz's idea, on the other.

This twofold aspiration reflects two ways in which one can encounter the aforementioned constraints: (i) in recurrent failures of various attempts to cash out the monadic structure in terms of either logical formalisation or spatiotemporal illustrations, (ii) in a regress of determination inherent to the notion of structure itself – and therewith to every formal system as such. We will proceed by first investigating the indispensable regress involved in determination of every structure considered on its own – so far without recourse to Leibniz – in the first chapter; then, in the second chapter, binding those findings in semantics with the realm of ontology by means of interpreting Leibniz's original principle of identity of indiscernibles as perceived through the conceptual tools developed earlier; and eventually addressing the issues raised in both semantics and ontology by introducing a mirroring structure, which is the simplest conceivable structure – the monadic structure – and showing the systematic insufficiency of any of its representations – be they formal or illustrative.

I will make use of many words: some of them quite innocent sounding, some of them bearing a heavy load of philosophical import, some of them even bearing a load of logical or mathematical import. I cannot dispense with them; and I cannot relinquish inflecting their meanings – because playing the same "language game" is no philosophical interpretation. My reading of Leibniz, as well as my systematic approach to ontology, will be crucially dependent on the introduction of the notion of structure, which made its appearance neither in Leibniz's own texts nor in any other early modern conception of ontology, but significantly shaped the thinking of the twentieth century.

On the one hand, there are proponents of mathematical structuralism who claim that, since all and only properties of mathematical objects which really concern mathematicians are the structural ones, they should be in principle reducible to structures; and some physicalistic reductionists go even further in claiming that, since all and only properties of physical objects which really concern physicists are the measurable ones, the comparable ones, i. e. the structural ones, they should be reducible to structures as well – and with them all the ontology of scientific or philosophical theories. Yet, precise philosophical grounding and interpretation of such conspicuous reduction is far from clear. On the other hand, there are divergent approaches in interpreting what might be called Leibniz's "anti-structuralism" promoted under diverse headings of "nominalism" (Mates, 1986), "phenomenalism" (Adams, 1999), "reduction of relations to properties" (Russell, 1900), "reduction of extrinsic denominations to the intrinsic ones", etc. I believe that the notion of structure, and in particular the notion of the simplest conceivable structure, provides a much needed key to understanding the underlying idea behind all the purported "reductions" to be found in Leibniz, as well as our present perplexities regarding ontology and semantics of formal systems.

In the following chapter I will try to gradually develop an understanding of structure loaded with both philosophical and logical import enabling us to account for all the puzzling structural features of the monad. Along the way we will take into consideration some of the mathematical and logical concepts playing their part in contemporary formalized interpretations of Leibniz as well as old fashioned philosophical ones in order to delineate their position in our grand concept of the simplest structure.

BIBLIOGRAPHICAL NOTE

As a matter of convenience, since the thesis is written in English, rather systematic then historic in its scope and related predominantly to debates in "analytical philosophy" (though itself perhaps blending "analytical" and "continental" methods in its approach), all the sources I reference herein are either English written original texts, or standard English translations – with the sole exception of few Latin phrases taken from Gerhardt's edition of

Leibniz's philosophical works. My main source on English translations of Leibniz is the Loemker edition, which is highly esteemed and widely used in the English-speaking world, making it easier for the reader to consult other commentaries while reading my thesis. In addition I use less authoritative, nevertheless quite decent, English translations of Theodicy and New Essays. The same logic applies to my referencing other non-English primary literature (such as Kant's, Hegel's, etc.) via its standard English translations. All the abbreviations can be found in the bibliography. I use the Harvard referencing throughout.

1. THE NOTION OF STRUCTURE

1.1 Structure and elements. Speaking of a structure we find ourselves committed to a kind of manifoldness[1]. This we may perceive as contrasted with something completely simple, unstructured. We might say: complexity of a structure stands for a complexity of relations binding some elements. But, regarding this scheme, where then is to be found that "simple thing"?

The straightforward answer "in the element" is rather precarious, taking in consideration that the structure in our conception draws its determinateness from determinateness of its elements and therefore requires discrimination of the elements from one another (otherwise its relations would collide or multiply at will): either in virtue of discriminating signs inherent to each individual element, or in virtue of their position as delineated on a different independent structure. Both approaches seems to lead to an infinite regress: either multiplying ad infinitum the elements (discriminating signs of the elements, discriminating signs of the discriminating signs, discriminating signs of the discriminating signs of the discriminating signs, etc.), or multiplying ad infinitum the structures (structure underlying elements of the original structure, structure underlying elements of the structure underlying elements of the original structure, etc.). Contrary strategy may claim the elements to stand for extremities of a structure – turning the grounding relation upside down. Thus every element bears in its determination the structure, which precedes it. But then again, where is the simplicity to be found? Perhaps, the default dichotomy of structure and element is not that well founded as we originally thought.

If an element is to maintain its identity against *n* other elements in virtue of its differentiating sings – and the identity of all the elements is to be constituted by some combination of differentiating signs – the situation can be conceptualized thus: first we need an independent structure (S) within which the entire class of signs is differentiated to ground identity and difference relations between the signs themselves (otherwise we would identify some sign in an element and once again some sign in an element, but there would be nothing to determine whether they are the same or different sign – while the sameness or difference of elements should be determined by those signs in turn); but in addition we need a structure (E) inherent to every element, whose positions may be occupied by those signs[2]. In the case that individual signs might repeat within a definition of a single element, or in the case that their order matters, it is manifestly clear that we need some independent ground

[1] I do not mean structure or manifold in any technical, mathematical sense in this context.

[2] Given the terminology developed in 1.3.1 we can identify (S) structure with (D) semantics and (E) structure with (I) semantics.

for discriminating the positions – aside from the one determining the identity of the signs themselves (structure S); however even if no sign can appear twice in a definition of a single element, there is always something needed to secure, inter alia, precisely this – to determine how are the signs taken together in a form of a definition of an element. For example a set – a set may not be ordered, but nevertheless has definite "slots" for its members and so is indeed somehow structured (the point will be elaborated later in the text – *1.2.*). With an increasing complexity of (E) structure, a number of signs (or complexity of (S) structure) needed to discriminate a definite number of elements of the default structure proportionally decreases – but this means only to relegate the task of discriminating to another level. One limit case would be to employ only two signs (0, 1) in an infinite structure (E) to differentiate any number of elements[3]; the contrary strategy could cope with just as many signs as there are elements in the default structure.

There is no determinate difference between completely heterogeneous differentiating signs – and neither a structure[4]. Determinate difference, which needs to hold between elements of any structure, should itself be somehow structured: if it is to account for the various differential relations binding the element in question with the other elements of the default structure. Therefore, the structure of a differentiating sign must somehow correspond to the entire default structure. We can, of course, simplify the internal structure of every element (E), but only at the expenses of complicating the independent structure of differentiating signs (S) superadded to ground their distinctness – and vice versa. If no ultimate resolution of complexity of a structure into simplicity of immediate identities of its elements is ever feasible, then every addition of external structure intended to take over part of a differentiative task originally assigned to internal structuring of an element only multiplies regresses of determination. The most parsimonious way to determine the elements would then be to let them determine directly by one another without any mediation of a superadded structure. This yields an infinite regress as well, but the regress is just one, self-similar and isomorphic to the default structure on every level. The internal structure of an element is constituted by its differential relations to other elements, constituting in turn its position within a default structure. If there is no independent structure determining differential relations among the elements, each of the differential relations holding between one of the elements and the rest of them is unique and differentiate therefore not only the one element under consideration on the background of the entire multiplicity, but the remaining elements among themselves as well – being thereby a proper representation of the default structure. And so an element mirrors in itself the entire structure which extrinsically determines it.

[3] Compare Mates' (1986) model in 3.1.

[4] For further clarifications see 1.1.1. and 2.5.

This yields a basic depiction of the structure of monad, which mirrors the entire universe – and the universe which comprises nothing over and above the individual monads (no superadded structure of space, time, etc.).

What is the repeating pattern of such self-similar structure? – Or does it allow for any independently representable pattern to be repeated on various levels of the structure? Clearly not: if the pattern was representable independently of the contents (i. e. intrinsic structures) of the particular elements (monads) of the default structure (universe), there would hardly be any need to end up in an infinite regress while trying to determine it. If there was a semantically determinate interpretation of the pattern outside of the structure itself, the tools employed in determining this interpretation would themselves be in a need of determination – much like in the case of the aforementioned superadded structure of signs. Thus the pattern cannot be properly represented in a formal logical fashion. But then what is the status of the depiction of monadic structure, how does it relate to ontology and semantics?

1.1.1 Determination. A hidden assumption is involved in this reasoning: that the elements can be rendered distinct, exclusively either in virtue of having discriminating signs, or in virtue of occupying distinct positions on a structure, both of which therewith take over the priority regarding determination (without deciding yet, whether the priority is ontological, conceptual, or whichever). At least at the outset of our investigation we must acknowledge the dichotomy as exclusive: all determinations are considered to be either intrinsic (signs), or extrinsic (positions on some structure). This corresponds to Leibniz's distinction between intrinsic and extrinsic denominations, by means of which we refer to those determinations – and, in my view, succeeds in expressing his nominalistic and phenomenalistic intuitions better than commonplace distinction between properties and relations.

There is nothing like (in itself) indeterminate structure. There can be a rigid, or non-rigid structure (i. e. having non-trivial automorphisms), there can be a structure with vast areas of intrinsic indeterminacy – however, this indeterminacy is already precisely delimited within the structure – since structure is nothing but extrinsic determination of some content. Whichever status we ascribe to determination, however subjective, or objective, relative, or absolute, ontological, or epistemic, intensional, or extensional, the interrelatedness of the notion of structure and the notion of determination remains. If there is something like heterogeneous multiplicity, whose elements are manifold without having any determinate difference between them, then it simply lacks structure[5].

[5] We will return to this kind of multiplicity in *1.3.1*. and *2.5*.

Later on we will elaborate on the account of indiscernible, yet not identical objects[6]; for our present purpose it suffices to realize that the notion of determination, thus specified, may be more general than the notion of identity – and definitely is more general at least than the notion of numerical identity (since this provides countability of identified entities and is therefore merely an instance of extrinsic determination[7]) – and that it should not be plagued by any epistemic commitments (the question pertaining to "discernibility" – who discerns and how – is only subsidiary in the case of "determination").

1.2 Individual and gathering: immediate plurality. According to a common intuition we are inclined to say: the simple thing is one, independent individual, the simplest structure, thereafter, are two of these individuals taken together. But what do we really mean by "taken together" and what do we mean by "individual"?

There is more than one plausible account of such plurality. We may ask, according to which principles are they gathered – or, supposing we are proceeding from their plurality in turn, according to which principles are they to be discriminated. They have got something in common – they build the very same plurality (there are many ways to express this – in the language of predicate logic, for example: satisfy the same predicate) – and yet something differentiating them from one another.

Put in logico-mathematical terms, one may take the two together in an ordered pair <A, B> [standard notation] and discriminate therewith which one goes first and which one second. But this looks hardly as an ideal candidate for a minimal account of taking things together, if one can conceive of taking things together in a set {A, B} [standard notation] irrespectively of their order[8]. Set-gathering, nevertheless, prescribes a particular manner of taking things together specified by a set of axioms[9]. Some of the remarkable peculiarities of this axiomatisation display themselves manifestly exclusively on sets of higher cardinality, but at least one feature will be straightforwardly evident even in our elementary case of taking two things together: set-gathering entails existence of an additional abstract entity (i.e. a set) over and above the existence of the individual things gathered that way – and therewith a structure of coexistence absent for example in the case of gathering in a whole AB [my notation]. Such account of gathering in a whole is formalized in extensional

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[6] See *2.1*.

[7] For further details see *2.5*.

[8] Yet, there remains some intuitive hook in the idea of respecting the order: taking one thing after another: after all, we wanted to ground our concept of a structure on a simple thing, so it may sound quite reasonable to index some simple thing as first and proceed from it. In the subsequent we will return to the idea and further scrutinize it.

[9] I consider here set theory defined by Zermelo-Fraenkel axiomatisation plus axiom of choice (ZFC). This consists of eight finite formulas and two axiom schemas formulated in first-order logic with equality and only one binary relation symbol ∈ for membership.

mereology[10]. While members of a set are all and only the individual things taken that way together, parts of a whole are all parts of the things taken that way together and all wholes compoundable from them[11]. In our case: {A, B} has only two members (namely A and B), while AB at least three parts (namely A, B, AB), in case A and B are partless and indivisible, and many more, if A and/or B are compounds – e.g. if A = CD, then AB increases its parts up to seven: C, D, B, A, CB, DB, AB. Yet, AB = CDB, provided A = CD, while {{C, D}, B} and {C, D, B} differ. This points to yet another consequence: while both accounts of plurality are extensional[12] – in the sense that identity of a set (or a whole) is granted by the identity of its members (or parts) – only mereology prevents[13] parts from being counted twice over – e.g. AA equals to A, but {A, {A}} does not equal to {A}. Taken as a set, the members indiscernible per se can be discriminated in virtue of their different position on a structure prescribed them by this "taking them together" – which is not the case with parts of a whole.

Mereological whole looks in comparison indeed unstructured, but not at all simple – at least not in a sense Leibniz would put it, i.e. without parts, indivisible. Rather the opposite: by taking things together in a mereological sum we multiply the amount of the entities granting the being to all kinds of their mutual combinations and divisions[14]. Being

[10] Standard mereology would be Classical Extensional Mereology (CEM) as formalized in Goodman (1977) with the use of first-order logic with equality and only one primitive binary relation (Goodman used disjointness, but contemporary texts more conveniently use parthood or proper parthood, though overlap or sum might be used as well – for a discussion see (Varzi, 2003), or (Koslicki, 2004)).

[11] Quite illuminating depiction of the basic differences and possible reconciliation is found in (Lewis, 1993).

[12] While in ZFC is extensionality one of its axioms, in CEM is standardly deduced as a theorem. There are three variants of formulation of extensionality in mereology – extensionality of proper parts, extensionality of sums, extensionality of overlap – whose difference might in some circumstances matter. Deduction of the theorem is based on parthood being partial order (i.e. reflexive, transitive, and anti-symmetric) and the principle of Strong Supplementation (SSP) stating: If y is not part of x, then there's some z that's part of y but disjoint from x. Note: Different set of axioms might be chosen to define mereology and those standardly involve some version of the so called supplementation principle. We may adopt weaker supplementation amended with a requisite fusion axiom to achieve the same effect, but without considering the amendment, we need a full strength of a Strong Supplementation Principle (SSP) to secure extensionality of the theory. The general idea of a weak supplementation is that if something is a proper part of something else, then there is to be something left, when this proper part is subtracted; general idea of a strong one is that when something is not part of something else, then there is to be something which makes for the difference. For a discussion see (Cotnoir, 2013), for an overview (Varzi, 2003).

[13] This is given by parthood being partial ordering – i.e. reflexive, transitive, and anti-symmetric.

[14] The principle of unrestricted composition is standardly one of the axioms of CEM; decomposition is subject to supplementation principle and otherwise limitable by positing the existence of a partless entity, a mereological atom.

„structureless“ in this sense, after all, does not mean refraining from prescribing compositional and decompositional principles: we can easily think of different axiomatisations allowing different models.

If we turn to the second question mentioned above („according to which principles are they to be discriminated“) from this viewpoint, we may perceive the principles in question as unequivocally determined by the structure of a plurality in the case of set-gathering and left at (if restricted) will by it in the case of gathering in a whole; if we turn to the first one, on the other hand, („according to which principles are they gathered“), we may perceive the principles in question as unequivocally determined by the type of a whole (i.e. axioms of the mereological theory), but left at (if restricted) will by axioms of set theory. “Left at will” means delineable only on an external structure.

It became clear that it suffices to assume only a two-element plurality to face an abundant array of possible ways for it to be structured (in a broader sense of the notion promoted in this chapter) immediately, without any good reason to consider one way to be straightforwardly more simple or basal than the other[15]. Any such notion of plurality carries with itself implicit laws of its composition and/or decomposition – and circumscribed space of indeterminacy.

1.3 Proceeding from the simple to the complex: plurality mediated by representation. One might argue that those are only irrelevant peculiarities of certain formal theories incapable of corrupting our original idea of taking something simple – the simplest thing

[15] Of course, set theory is considered to be the standard foundation for mathematics in the sense that all mathematical objects can be construed as sets – those therefore being fundamental for other types of mathematical pluralities as well. Mereology was formerly conceived of as an alternative to set theory standing on purely nominalistic grounds (i.e. not entailing the existence of abstract entities like sets), but due to its exhibiting too small expressive power (that of a Boolean algebra without zero element) never have taken the role (even though Lewis (1993) tried to revive the ambition with the help of plural quantification once again recently). Yet set theory itself is in a need of a model (which in other cases is usually construed in terms of sets) and its structure is thus not self-evident from the axiomatisation, but rather presupposed in its naïve interpretation (see also 1.3.1). Moreover, paraphrasing everything in mathematics into one mode of expressing or another may be understood as a matter of practical convenience rather than a commitment to ontological or conceptual fundamentality of described pluralities in a philosophically relevant sense – even if such perspectives might get employed in discussions on foundations of mathematics – and the very possibility of such accounts justifies our discriminating between fundamentality in a sense of axiomatic formalization of mathematical expressive means, presupposed ontological fundamentality of what is being expressed and conceptual fundamentality conceived as mediating a grasp of those. The task of determining a concept of simplicity accountable for both structural features of Leibniz’s monad and our own conceptual resources for making it explicit is hardly to be satisfactorily solved by pointing to the features of contemporary mathematical formalism.

possible – together with something else quite as simple to obtain something complex – yet not that much complex: the second simplest thing, one would say.

What strikes me in this insight is the inclination to use row numbers: "second simplest thing". As if the ordering after all conceptually preceded all kinds of pluralities – in a manner comparable to Kant's construction in pure formal intuition of time (KRV, A142, A720). Behind this intellectual move lurks highly contentious claim that the complexity is to be built gradually from simplicity by some mathematical operation of multiplication (of a self-identical entity). Something is to be added to a simple unity to make it complex – so far, so good – but how is the addition to be interpreted? If we understand it as yielding ω–sequence structure does it not commit us to much more of a structure, than we originally intended? Standard model of arithmetic after all possesses a structure of considerable complexity – and that indeed is what we yield to when constructing a structure by adding simple units to one another in the manner suggested above. But how to add without forming a multiplicity of at least two: the former and the latter? If already together in a unity of their own, the "duality" may well be doubted – for example from a mereological perspective: this unity lets itself be divided however you like (especially liberally in an atomless universe, which is the case with the Leibniz's one). If separated, we take "the one" and "the one", but how do we know, they are two? What are we implying by naming the entity "the one"?

1.3.1 Semantic interpretation. The interpretation of addition in terms of ordering presupposes a determined ground for recognizing the entities added this way as equivalent. Two distinct semantics are to be at hand here: the one (D) assigning different position on a structure to counted entities and the one (I) enabling such assignment by assigning one and the same position on a structure to the entities. The scruples to accomplish blithely a transition from "the former and the latter" to "the first and the second" are analogous to those making us reluctant to concede that by taking something together with something else we obtain TWO things. The move may be blocked by indeterminacies in either D-semantics (by taking some water together with some water I cannot acquire anything like "two waters")[16] or in I-semantics (by taking some stone together with some pain I cannot acquire any plurality of two)[17]. Correspondingly in the case of "the former and the latter": the

[16] The problematic of non-count nouns is developed at length in Laycock (2004, 2006). In the following I draw substantially from his observation and I will return to it in more detail later in the text.

[17] Bergson (1910) once coined the term "heterogeneous multiplicity" as contrasted with homogenous multiplicity. Deficiencies in I-semantics leads to forming of a multiplicity which at least lacks some homogeneity, but there is no mention of other basic features of Bergson's heterogeneous multiplicity at this point. However, in the course of further development of our notion of simple

identification (I-semantics) of the entities in question as both being events of delimited duration ground the possibility of their differentiation in the structure of time (D-semantics). Perceived that way, our initial insight was not at all that innocuous as presented entailing commitments to universal arithmetized structure of "time intuition" (from Kantian viewpoint) – which straightforwardly corresponds neither to our subjective experience (we cannot discriminate "the first moment" from "the second one", since there is no natural demarcation line in the continuous passage of time and our consciousness is supposed to be directly acquainted only with present – articulated in identifications based on recollections of the past, whose memory in turn makes its appearance merely within the scheme of present identifications[18]), nor to its presentation in contemporary physics. In such construction of complexity by addition of simple elements we either way presuppose, what is to be constructed – and does not proceed any further behind our supposition.

The assumption having been made here is the one concerning structure as the interpretation of a talk about some multiplicity. This is what the notion of structure amounts to in model theory[19]. However, such structures are standardly construed set-theoretically (though there are attempts at developing a naïve model theory as well), which makes the interpretation of the axioms of set theory itself a bit problematic and after all commits us to presupposing a structure (standardly a very peculiar one – built from sets) instead of constructing it from something unstructured in the first place. Such is a price of discriminating between language and its interpretation and introducing therewith a principle of representation into our conceptual scheme. The move is enforced by a need to establish identity of distinct elements taken together in a single reference act. From this point on the initial question of what are the principles gathering entities into a plurality under consideration, or respectively what are the principles discriminating them therein – provided we are proceeding from their plurality in turn – mentioned at the outset of *1.2* lets itself rephrase into a question of how do we determine an interpretation identifying them with one another ((I) semantics), or respectively how do we determine an interpretation identifying every one of them individually ((D) semantics). "Principles" in question might have been operations of a subject as well as inner articulations of an object, but determination of an interpretation is subject to (and constituted by) criteria of meaningful reference to it.

Either way we were unable to fully determine a structure by means of something supposedly unstructured, simple. Even the simplest structure. Which, on top of it, we are unable to find – unsurprisingly, since we have no clue how to (in an unequivocal manner)

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structure other essential qualities of Bergson's heterogeneous multiplicity will come to the fore such as interpenetration of all its elements analogous to Leibniz's mirroring.

[18] This observation was notably pointed out by James, Bergson and Whitehead.

[19] For an introduction to model theory see Hodges (2013)

accomplish the transition from simplicity to complexity. On the other hand, do we have any definite grasp of simplicity, in the first place? And do we need any? We can posit a simple, unstructured thing by definition, indeed. By this we could mean an extreme of analysis, for example – but then again, there should be a structure of what is being analysed, or at least a structure of our analytical proceedings posited beforehand, which ultimately defines the elementary. Although – how far should we mingle *what* we mean to be there with *how* we mean it? Subsequent sub-chapter will give us some hints.

1.3.2 The identity in reference. There are some striking entailments involved in the suggested shift in conceptualisation. Gathering of entities is specified as a gathering in an act of reference, which gives certain criteria for what counts as an entity, or what counts as a multiplicity: clearly, we are excluding from our consideration whatever cannot be meaningfully referred to. Before determining the scope of this restriction – what can and what cannot be meaningfully referred to – we should acknowledge that the restriction is hardly a necessary one: there might be a plentiful of multifarious stuff which cannot be referred to out there (actually, it would not be inconceivable to posit a fundamental ontological level consisting exclusively of such stuff). Moreover, if concerns are raised, that therewith we engage in the talk of what cannot be properly talked about, the objection might be turned upside down in response, asking what grants the existence to entities whose identity is constituted solely in a reference act. It is not only possible, but manifestly inevitable, that existence transcends its determination in reference: what else would it mean to refer, than to represent something which itself is not present – if it was already right at hand with all its qualifications and potential manifestation further unaccountable, then it would conflate with this very reference act itself, which no longer would refer to anything external to it. If reference makes sense, then only at the proviso that what is being referred to transcends it. This alone does not grant the existence of something incapable of being referred to (since it is precisely in a reference act where has been this transcendence of existence established and since this transcending existence is precisely what is being referred to), but justifies scruples to grant existence to entities determined by the identity constituted in reference act – scruples symmetrically analogous to those precluding acknowledgement of entities outside of the realm of what can be meaningfully referred to.

Furthermore, it is not only for the concerns regarding ontology, which submit to a conceptualisation via means of reference only at the cost of transcending it at the same time, it is for the expressive means of natural language itself, which seems to *do* or *express* things[20] in various different ways having very little to do with straightforward mode of

[20] Austin's famous tenet (Austin, 1962) that by words we often actually *do* things in the world instead of just representing them is followed by a conviction that efficacy of linguistic acts is carried by their institutionalised regularity and therefore they bring about new things in the world only by representing the existing ones; on the other hand every linguistic representation is without a doubt

referring and enables forming of meaningful and interpersonally efficient expressions without determined referent, that we should be cautious about introducing restrictions regarding *what there is* based on what we can meaningfully refer to.

However, those objections notwithstanding, some prominent strands in philosophy have always sided with the view. One of the most notable among them epitomized in Quine's approach is based on singular reference as a fundamental mode of referring, to which all other forms of relating to extra-linguistic reality are to be ultimately reducible – which will be subject to our further enquiry in the subsequent sections – but the driving force behind this, a preoccupation with singularity of being, is motivated by reasons independent of search for identity criteria constitution in reference and we must scrutinize those as well.

Before doing so, let me summarize the preliminary outcome of this chapter. At the outset of the investigation we constructed infinite regresses as a result of an attempt to determine a structure by its elements, or elements by a structure. Proceeding this way, we relied on a very vague intuition of what determination and determinateness might mean. From the perspective developed in *1.3* we can specify "determinate" as "identifiable in reference" and conceptualize the suggested regresses as semantic regresses (in the more technical sense of the word "semantic" confining language to referring). As an alternative to this, the account of a "more realistic" regress might be taken into consideration – and in this sense "determination" would mean real determination of one entity (or event, or phenomenon, or whatever you judge to be fundamentally real) by another[21]. However, even if the criteria of identifiability in reference were inapt to determine *what there is*, they might nevertheless suffice to frame our search for the simplest conceivable structure – after all, their precise nature with respect to their congruence with the *limits of conceivability* of structural complexity and simplicity is yet to be determined. To this end we will investigate the notion of identity, identification and meaningful reference in the following section. Notwithstanding the possible criteria's ultimate insufficiency, the enterprise is worth

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already *a thing in the world*; and so the two accounts might converge to each other ultimately. Nevertheless, significant deviation from the straightforward mode of referring attributed to language is introduced alongside: what a linguistic act reproduces is a certain praxis, which not only defies standing for a proper object of the linguistic expression in question, but might defy identification in any possible act of reference performed in that language as such. That there must be some fundamental level of practices or „life forms" incapable to be made explicit is a well-established interpretation of later Wittgenstein.

[21] Typical example would be, provided extension is ultimately real (and therewith possibly transcending its geometrical representation), determination of extended entities in space by one another. The significance of the dichotomy will become clear when confronting our account with that to be found in Leibniz's own writings later in the text.

undertaking bearing in mind that vast majority of twentieth century secondary literature dedicated to uncovering hidden peculiarities a structure of past thinkers' ontologies does not even reflect on them, not to say question them.

1.4 Reference and structure. We are to investigate a determination construed as a constitution of identity in reference. We already hinted at the possibility that this does not exhaust the entire class of all determinations (we pondered an eventuality of real determination as well, concluding that the incompleteness of the overlap between those two might not be something merely accidental, but rather an indispensable requirement of a reference act), however, there are strong indications that without the identifiability in reference we cannot make sense of any structure. In fact, the concept of structure presupposes identifiability in reference, or: every reference is enabled by an underlying structure, or: every structure is constituted by multiple references.

The latter point is of the utmost importance: if structure is a principle of extrinsic determination, it presupposes, or constitutes borderlines between distinct interiors and exteriors (which is not the case with the intrinsic determination), so as to at least ground a possible abstraction from particular contents – if it is not straightforwardly identified with this abstraction. In this sense, a structure is a ground for rendering some contents equivalent. But equivalency of contents is exactly what identity as constituted in reference claims: this identity should secure that by referring to the same thing, we really refer to the same thing, or: that two references can be equivalent as to their contents. If a reference were a singular event, not grounding any subsequent repetition, we could hardly claim that it refers to something: to make this sole claim, we would need to use another, distinct reference to the thing – and assert the equivalence of the two references. Not only there is no way of claiming that a reference refers to something without the use of another reference, but there is no way of making sense of such unrepeatable reference, there is no difference between such referring and not referring at all, if the thing referred to is nowhere to be found outside of the solitary reference. So a possibility of repetition and equivalence of contents is indispensable for the notion of reference. And this merely potential multiple application of a reference already constitutes a structure binding the possible equivalent instances of the reference. In fact, it is hard to conceive of a structure (as contrasted with content, which inevitably transcends every reference) as transcending the identity criteria based on reference and so it can always be viewed as consisting of only representations – or as constituted by multiple references[22].

[22] This observation should be kept in mind while investigating Leibniz's ground for denying reality to structures. If the structures consist of mere representations, it is natural to think of their reality as dependent on the reality of what is represented – unless we do not acknowledge a representation to constitute the very essence of what is ultimately represented as real (as we eventually indeed will), but in that case a fundamental structure is to be found and all the other structures (of space, time,

1.4.1. Criteria for an entity. With this proviso in mind – that as an entity counts whatever is capable of being determined within a structure, or whatever is capable of determining a structure – and so, strictly speaking, not whatever there is – we can investigate the criteria of what could count as an entity based on its identity in reference.

Canonical for the issue is Quine's "On What There Is", where his famous dictum "to be is to be a value of a bound variable" is paradigmatically expressed. Delimiting minimal conditions of an account of entity at the beginning of his article he tosses a rhetorical question: "But what sense can be found in talking of entities which cannot meaningfully be said to be identical with themselves and distinct from one another?" (Quine, 1948) Needless to say that he finds no sense in such talk and considers the answer evident, but giving supposedly evident minimal condition for any entity he posits latently condition for meaningfulness as well. His later claim from the midst of the article that the "ways of talk about meaning boil down to two: the *having* of meanings, which is significance, and *sameness* of meaning, or synonymy" can be found already wrapped up in the former one. Having meaning for Quine means being substitutable. Bearing in mind, what does "ways of talk about meaning boil down to" we can infer what does ways of talk about entities boil down to: reference.

If we consider the qualification "to be identical with themselves and distinct from one another" on its own, it exhibits a bit nonsensical tinge: how can anything be identical to itself without being somehow doubled, and therefore distinct from itself at the same time?[23] And accordingly, how can they be "distinct from one another", if they were to be identical in the first place? However, this makes perfect sense if we acknowledge an entity only insofar as it casts its virtual existence in reference. Acknowledgement that an entity "x" exists, may be put in Quine's terms as follows: $\exists y(x=y)$. An entity cannot be taken separately on its own, because it is already taken together with the implication, that there is something else capable of standing for it – without this condition it cannot be qualified as an entity. This justifies the plural form of the qualification: it wouldn't be unconceivable to think of its singular version – "to be identical with itself and distinct from the other" – or half-singular one – "to be identical with itself and distinct from others" – but those versions strike us as being unsaturated: Which one do we mean by "the other" of the entity in question? Which ones do we mean by "others" of the entity in question? Do we need to talk about something

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etc.) will be representations of it as conjoined with its ultimate content and so not strictly isomorphic, i. e. less real.

[23] This resembles tractarian critique of the notion of identity (Roughly speaking: to say of two things that they are identical is nonsense, and to say of one thing that it is identical with itself is to say nothing. (Wittgenstein, Tractatus, 5.5303)). We will return to the issue of identity in the next chapter; for now it is worth keeping in mind that we already pondered an interpretation of an explicit plurality based on parallel reference to (I) semantic and (D) semantic.

else at the same time to meaningfully talk about something? And how many of them – perhaps all *the others*? This looks implausibly circular, yet there is something to it. The plural version keeps this[24], but dissolves strict dichotomy between *this* and *the other*[25]: we don't need a determinate other, only a substitutable one[26]. We don't need to posit and determine something external to the things being talked about – only a trace of their reference.

1.4.2 Singularity: identity and reality. However, Quine does not stick with the general formulation "identical with themselves and distinct from one another", which not only eschew commitments to determinate "others", or a determinate environment – a determinate content of what is posited alongside with the thing we want to "meaningfully talk about" – a determinate content of an accompanying structure of reference – but leaves unspecified number and countability of this thing being talked about as well; Quine goes on to specify identity of an entity to be constituted prominently in singular reference – and only derivatively in other ways of referring. He could have perfectly well retained the criterion "to be identical with themselves and distinct from one another" without specifying number or countability of what is being talked about; he could have equally well retain even the dictum "no entity without identity" (Quine 1969, p. 23) in its full generality without implying whether the identity in question is to be somehow numerical (and how exactly), or whether we should rather consider it as qualitative; and eventually he could have retained the idea of synonymy, and substitutivity wrapped up in it, as well – this latter case is a bit contentious, however, since we could do something like this in an informal manner and we could at least use plural quantification in his formal definition of existence (i. e. $\exists Y(X=Y)$, where X, Y are plural variables), but the interpretation of plural quantification itself regarding ontological commitments is spurious[27] with some claiming its ontological innocence and irreducibility to

[24] In Hegel's words from the chapter "Something and an other" in The Science of Logic: "It preserves itself in its non-being and is being; not, however, being in general but being with reference to itself in contrast to its reference to the other, as self-equality in contrast to its inequality. Such a being is *being-in-itself* " (Hegel, 2010, p. 92)

[25] Following Hegel: "It therefore stands in reference to an otherness without being just this otherness. The otherness is at once contained in it and yet separated from it; it is *being-for-other*." (Hegel, 2010, p. 92)

[26] Regarding conceivability and existence of parts of matter Leibniz opposes de Volder's contention that "that one part of an extended thing cannot exist or be conceived without the others" by conceding that from his arguments "it follows indeed that one part of matter cannot exist without some other part, but it hardly follows that it cannot exist without these particular others" (L, 519). We will return to the argument while discussing mutual determination of parts within a real structure.

[27] See: Linnebo (2017).

singular reference[28], while others its ontological significance (e. g. entailing the existence of concepts) conjoined with irreducibility to singular reference and yet others its ontological significance (entailing the existence of set-like entities) conjoined with reducibility to singular reference[29]. But those complications regarding formalisation (which, after all, in its conceptual resources is subject to some particular historically instituted structural preconceptions) should not plague the observation: we need identity in reference and for this we require substitutivity – but it is far less clear that we need singularity as well.

In fact, reasons for this might be rooted in a certain ontological preoccupation, rather than in contemplations of semantic requirements. Let us expound the idea in more detail. We already gave up the ambition that the criteria for something to be identifiable in reference, which we are trying to delimit, give us any general criteria for something to be. Moreover, we harbour a contention, that "what there is" must systematically deviate from what we can identify in reference – and that what marks a real being is precisely that it cannot be substitutable in any sense. An yet, despite this contention, we push the analysis one step further to insist that the reference must be singular and the identity therewith numerical, based on a historically well-established preoccupation with singularity of being: if we refer to more things (countably many, or uncountably many, constituting a continuous multiplicity), we are convinced that if not the *identity* in reference (e. g. of a continuous stuff being referred to), then certainly the *reality* of the multiplicity in question is based in the reality of its elements in themselves. We might be inclined to think that singular reference is really the most fundamental type of reference, to which all other types of reference are either reducible, or at least, on which they are heavily dependent – while the converse does not hold – and correspondingly that an identity of a singular object is a basic type of identity constituted in reference. This being the ground for the "singularity of being thesis". But our intuitions employed in evaluation of everyday experience teach us the contrary: we make reference to empirical objects with a contention that they should be identifiable in reference, that we should be capable to refer to the same object – in a sense of this identity – repeatedly, and yet, at the same time, on the level of ontology (regarding their real determination, or real identity) we deem them to be in permanent flux, not at all singular in itself, but rather multiple, divisible and ever dividing, loosing severed parts and gaining new ones: we indeed deem *the reality* of an object to be derived from the reality of its supposedly singular constituents, not its *identity* in reference. Moreover, genuine singularity of being far from being required for identity in reference is strictly opposed to it. As we have already seen, identity requires contrariwise non-singularity, multiplication and substitutivity.

[28] See: Boolos (1984).

[29] See: Catwright (1993).

Let us first elucidate our conviction that the criterion of identifiability in reference is not strong enough to secure singularity of the object referred to – and so that singular reference imposes additional structure to the one necessitated by identifiability alone; and then, in the next section, explore the independent root of the preoccupation with singularity.

1.4.3 Singular reference. Quine and with him most of the philosophers and logicians of the twentieth century insisted that all the other modes of referring available in natural languages should be in principle reducible, in a rigorous philosophical discourse, to singular reference, but from the 80's on the stance seems to be not that unanimously sided[30]. Laycock persuasively shows that some of the most recognised thinkers of the twentieth century[31] considered "object" to be the most basal, further unanalysable, philosophical category and calls up Tugendhat's words encouraging even more far reaching claim: "Tugendhat tells us that 'The fundamental question of ontology is: what is being as being?', and he reformulates this question as the question of 'what it means to speak of an object'." Whatever there is must satisfy the criteria of objecthood, because we were supposed to be unable to "meaningfully speak about" something failing them. A unique object of a reference was required to render the reference fully determined (however, this is, according to our preliminary analyses, neither necessary, nor sufficient). If this were the case, equality of references yielding an identity in reference would render the object of the reference singular; and a failure of a singular reference to refer uniquely would mark the boundary of our basic ontological category.

The serious challenge to this view – especially because of its coming from the camp of logicians, from which this metaphysical preoccupation to a considerable extent drew its legitimacy – was provided by Boolos' deliberations concerning plural reference. He has not only pointed out to the fact that in natural languages we use plural reference and quantification as surely and flawlessly as the singular one and that there are plural locutions in natural languages, which are either completely irreducible to first order formulas using only singular variables, or merely so at the cost of being highly counterintuitive and hard to entangle, which had been observed even before, but more importantly advocated the use of

[30] As canonical turning point is perceived Boolos' article (Boolos, 1984). For an introduction on Plural Quantification see Linnebo (2014), for a discussion of its overlap in metaphysics see Laycock (2014) and Laycock (2004), Laycock (2006).

[31] Laycock (2014) takes into consideration mainly thinkers from the so called analytical tradition listing Russel, Wittgenstein, Lowe, Strawson, Quine, Tugendhat among those embodying such a view. This is rather intuitive given the extent to which the topic is intertwined with logical issues, but perhaps analogical reflexion may very well be carried out with respect to thinkers of phenomenological tradition as well – we will not however delve into the issue any further.

plural logic as "ontologically uncommitted"[32], "pure logic"[33] variant provided with extra expressive power capable of solving some of the problems in formal logic[34].

Laycock (2014) takes the discussion even further pointing out the issue of genuinely non-count nouns. While approving plural reference alongside the singular one as two equally basic, mutually irreducible modes of referring made the up to then necessary conditions for something to be something look more arbitrary, it generally did not justified transgressing the constrains: "many" are only "many ones", "some" are "some ones". Plural logic codifies different form of referring to the old entities: that is necessitated by the claim to ontological innocence. As Laycock puts it: "Since, as Boolos in effect observes, there is no such thing as *the* (one, single) object of a plural reference, there is no such thing as *the* (one, single) value of a plural variable; such a variable has *some* values, *several* values, not just one." (Laycock, 2004). And these values are subject to the very same constrains as that of a singular variable. But if we attempted to introduce variable for non-count nouns as well, the situation would be remarkably different: "…given the non-singularity of non-count reference, there can be no such object as *the value* of a non-count variable. Much as a plural variable has *some values* (e.g. some clothes) and not one value, a non-count variable may be said to have **some value** (e.g. some clothing) and not one value. There is then a range of distinct substituents in cases of this kind; but there is no corresponding range of discrete values." (Laycock, 2004).

The introduction of something like non-count variable into logical formalisation is in every respect problematic, as hinted at earlier, but the reasons for the apparent incompatibility are likely to be sought not in the nature of reference, but rather in the nature of formal logical discourse, which is closely bind (in its origin as well as in its contemporary development) to mathematical discourse, where numerical identity plays a crucial role and

[32] Standardly a plural reference used to be transliterated as singular reference referring to a set. But this is seen as transgressing constrains of „pure logic"(see footnote 33), because of committing to the existence of the abstract entity referred to in the expression. Using plural variables, as Boolos suggested (Boolos, 1984), is ontologically innocent and does not refer to some set, class or collection of things taken together but only to the things themselves – though not singularly but plurally. As Boolos somehow emphatically puts it: "It is haywire to think that when you have some Cheerios, you are eating a *set* – what you're doing is: eating THE CHEERIOS."(Boolos, 1984). However matters are not that conclusively receipted and some argue against the ontological innocence thesis – for a discussion see (Linnebo, 2014).

[33] Linnebo (2014) concerning "pure logicality" of plural first-order language lists these three criteria: (i) absolute generality / universal applicability, (ii) formality / ontological innocence, (iii) cognitive primacy / perfectly well understood.

[34] These include showing how two of the axioms of Zermelo-Fraenkel set theory, Replacement Schema and Separation Schema, can be replaced by a finite formula in plural first-order logic and showing how monadic second-order logic can be interpreted in plural first-order logic, proving therewith its "pure logicality".

so having an extension not numerically structured at the bottom level seems hardly an option (this ultimate reason is only enforced by a well-established praxis of quantifying over sets, construing properties in terms of sets of objects satisfying them, etc.). If we, on the other hand, turn our attention to examples from natural languages, we see all kinds of references, which are no less basic, no less irreducible, and equally naturally understood as references to some purportedly singular objects: besides Laycock's notorious mass nouns "water", "gold" and "clothing" we can think of references to other quite concrete things like "grass", "rain", "sunshine" and "dawn" – or we can argue that concreteness of its object has nothing to do with simplicity of a reference and ponder possible singularity of such nouns as "love", or "traffic".

Let me summarise our account of the criteria imposed by identifiability in reference on all entities (capable of determining, or being determined by, a structure). Those entities, far from being singular in itself subsisting objects, are always already multiplied in their reference traces, in which they can be "identical with themselves and distinct from one another". For convenience we spoke of "repeated referring" as if the reference trace should be discretized by individual instances of possible reference, but this is clearly not the case: the reference trace must secure continuity (needless to add that this continuity should not be conceived of merely as a time continuity, because: firstly, what is being referred to does not need to be extended in time, secondly, repeated references themselves do not need to be linearly ordered in time – they might for example all be written on one sheet of paper). In a limit case we can think of the object of reference as nearly singular and its indeterminate neighbourhood of reference, on the background of which it differentiates itself, as nearly without content, nearly the sole borderline of exterior and interior (extrinsically differentiating structure and intrinsic self-differentiation). Then you refer to something extremely volatile, which disappears right after the referring, but nevertheless, the reference casts as if a shadow in virtuality, tracking the disappearance of its object, enliven either by an immediate reflection, or a shared presence. But without this specious possibility of re-reference there is no reference either.

1.4.4 Singularity of being. If a preoccupation with the singularity of being prevails nevertheless in the course of western thought (making Leibniz himself avow Aristotelian creed "being and one are convertible terms"[35]), it is not necessitated by a need to secure its identity in reference, but rather by a requirement of its transcending this identity. An object of representation taken as real must transcend its representation, as already indicated, and if its identity in representation renders it repeatable (representable) – identifiable in a homogenous repetition – then its real identity must render it escaping this identification,

[35] Aristotle (1984), 1003b20; Leibniz: L, 603.

being irreducibly singular, unique, one of its kind[36], heterogeneous, incomparable with others in any common extension. As Leibniz puts it:

"For the rest, so far removed am I from holding that a single individual is a plurality that I am even deeply convinced of the teaching of St. Thomas about intelligences but hold that it is valid generally, namely, that there cannot be two individuals entirely similar or differing only in number." (L, 337-336)

The unity demanded of every real being is not a mere numerical unity – which is accidental to it, delimited externally in an extension of reference as an extrinsic denomination – but a true unity, unity per se – by which a real being differentiates itself by itself – and in itself. The identity in reference is imposed from outside upon per se non-identical entities, while the intrinsic identity of those entities is procured by their unified self-differentiation – which precludes them to be "self-identical". The union of those two identities is to be established in the foreshadowed principle of identity of indiscernibles – which will be subject to a subsequent enquiry.

[36] In this respect all of Leibniz's substances are like Aquinas' separate substances: "…what St. Thomas says on this point about angels or intelligences (quod ibi omne individuum sit species infima) is true of all substances…" (L, 308). See also: L, 268, 308, 337

2. THE IDENTITY OF INDISCERNIBLES

In the construction of the default regresses we required distinct elements to be differentiated by discriminating signs or different positions on a structure – signs being inherent to each element, while positions being external to them. A suspicion might have been raised that any contradistinction necessarily entails perspective of a possible perceiver – be it a finite mind, or an infinite one, as Leibniz would put it, or generally whichever conceivable subject (e. g. animal, plant, infinitesimal consciousness of unanimated chunk of matter in panpsychist scheme, or an artificial sensor of a machine). One might adopt radically deflationary view and claim that there is no sense in talking about discriminating anything, if relevant perspective of an observer with its own relative criteria of difference and identity were not given; or take a moderate stance, claiming that things might not actually conflate (might not be actually identical) even if they were distinguishable neither in itself, nor in relation to other by any possible observer – and therefore that alongside all kinds of relative notions of differing, there is an ultimately inapproachable real difference as well. This latter view seems to straightforwardly violate the principle of identity of indiscernibles (PII), which, as a core principle of Leibniz's metaphysics, has already received extensive treatment in secondary literature and on the top of it over the course of past hundred years has remained in its modified version subject to lively debates in logic, but despite all this, or perhaps exactly because of it, has rarely been presented in an unprejudiced manner capable of illuminating our present riddle. The reason for this is to be sought in a preoccupation with formal logic and therewith a preoccupation with identity as constituted in singular reference, although roots of this superstition reach far back to the ontological load being bestowed on numerical identity as compared to the qualitative one early in the history of philosophy. There has always been good reasons to consider qualitative identity somehow more subjective than the numerical one – that being therefore somehow more objective in turn, but perhaps we should have been more cautious in translating this into ascription of ontological priority.

2.1 Semantic and ontological reading. If we take the case at its face value from the viewpoint of a dichotomy developed in the previous chapter, we can think of a structure either as a real organization of things, or as an interpretation of a talk about them (an organization of possible reference to them); in a semantic reading by claiming things to be non-identical, yet indistinguishable, we ascribe to them one and the same position in one (I) structure, while ascribing them two different positions in another (D) structure; in an ontological reading we mirror the move, but acknowledge those structures to be real organizations. Both of the views allow themselves to be internally hierarchized: either we are convinced that one of the semantics is actually meta-semantic (typically, though not necessarily, (D) structure being meta-semantics of (I) structure), or we posit one of the real

orders of organization to be more fundamental, bearing therewith ontological priority (again, typically this would be (D) structure). Introducing hierarchy means introducing ordering and this in turn means nothing less than projecting on yet another structure (whose particular form may differ depending on whether we want to speak only of a partial ordering or of a total order). And so the very construction of the infinite regress depends on presupposing an algebraic structure: either as an order of semantic levels or as an order of ontological levels.

However, combination of those is conceivable as well: since we inevitably face infinite semantic regresses, we should presuppose a fundamental order of organization of things grounding basic identity- and difference-relations to be ultimately real – and perhaps inapproachable in reference. Naturally, (D) structure would then be postulated as an inapproachable level of the ultimate, further unanalyzable differences, while (I) structure would yield only relative identities synthetized in reference. But it is not inconceivable to turn the scheme upside down to claim (I) structure to be the ultimate ontological level of further unsynthetizable identities and (D) structure to yield differences between perspectives of distinct references to it. Someone who rejects any notion of absolute (observer-independent) difference may side with this view – but then he is bound to explain why should there be absolute identity if absolute difference is banished. Analogical objection may be raised against the former view, if it means to establish the ontological level as ultimately transcendent to reference: why should there be absolute difference if absolute identity is not admitted? A deflationist may proceed even further in his critique insisting that both identity and difference (and with them, of course, every notion of structure or organization) should be considered solely as dependent on a perspective of an observer. But would not this amount to forbidding virtually all ontology? It is hard to conceive of this move being genuinely accomplished – especially when we think of all those great pronouncements of refraining from ontological judgements which ended up in the most flagrant ontological superstition unable to even make sense of realizing it[37]. However, could not it be the case

[37] Positivists has always been notorious for this; but contemporary debates over analytical metaphysics are no less representative: proponents (e. g. Fine, 1999; Koslicki, 2008; Sattig, 2015; Sider, 2008, 2011) are accused of arbitrarily reifying selected abstract (mathematical) structures (led by simplifying folk physics rather than complexity of a structure disclosed in up to date special sciences), while their critics (Ladyman, Ross, 2007) dream of ontology being reducible to mathematical structuralism as such. Needless to say that mathematical structuralism itself has hard time accomplishing proposed reduction of objects to structures and deciding on how to understand it and how to make those "grounding" these, if only in the realm of mathematical entities, without reifying them one way or another (we will return to the issue later) – and so the step looks rather desperate.

that reasonable constrains imposed on observer-dependency might nevertheless facilitate meaningful ontological talk?

2.2 Formulation of PII. Let us investigate the account given of the PII in more recent literature. It seems that Russell, in cooperation with Whitehead, was among the first to employ a variant of the principle[38] to define identity by means of logical formalization in *Principia Mathematica* (1910); his own curious comments in the *Critical Exposition of the Philosophy of Leibniz* (1900), however, reveal an understanding much more nuanced with respect to Leibniz's philosophy, than the one enforced by the phrasing in terms of modern logic. It is a commonplace in systematic philosophical discourses, as well as the logical ones, to cash out "indiscernibility" in terms of sharing all properties (satisfying the same predicative functions[39]): $\forall F(Fx \leftrightarrow Fy) \rightarrow x{=}y$ [40]
Using such paraphrasing, we clearly restrict ourselves solely to a variant of the first understanding sketched above, i. e. to the semantic one.

Mates hints at a possible source of this inaccuracy in adoption of Leibniz's original principle. He points out that: "Some scholars have considered the Identity of Indiscernibles to be just half, as it were, of the *salva veritate* criterion. They interpret the latter as a generalized biconditional: 'x is the same as y if and only if whatever is true of x is true of y, and vice versa', or, perhaps equivalently, 'x is the same as y if and only if every attribute of x is an attribute of y, and vice versa', and then they call the generalized conditional from left to right "the Indiscernibility of Identicals" and that from right to left "the Identity of Indiscernibles"." (Mates, 1986, 134)

The difference between the *salva veritate* criterion[41] and Leibniz's original PII seems to me striking: the *salva veritate* criterion speaks about substitutivity of terms (regarding possible truth value of linguistic expression), whereas PII speaks about discernibility of beings (eventually reducible to substances[42]) – in both previously developed conceptualisations, either semantic, or ontological, as well as in their combination.

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[38] He indeed explicitly relates himself to Leibniz's identity of indiscernibles in the Introduction, but one might argue that his proceedings should better be perceived as an application of another of Leibniz's principles, namely the *salva veritate* criterion.

[39] Russell could rephrase in virtue of the axiom of reducibility: $x{=}y \;\&\; \psi x \rightarrow \psi y$ taking into account both predicative and non-predicative functions. (Russell, 1910, 168)

[40] The formulation is given in Forrest (2016); surprisingly even Brandon C. (2017) does not hesitate to misrepresent Leibniz's original account in such a way.

[41] The *salva veritate* criterion is often called "Leibniz's Law": $\forall F(Fx \leftrightarrow Fy) \leftrightarrow x{=}y$
Even a plural version can be encountered in literature (e. g. Calosi (2016)), which must utilize Russell's rephrasing (see the footnote above): $x{=}y \;\&\; \psi x \rightarrow \psi y$ for x, y ranging over both singular and plural terms.

[42] This is a standard view promoted by inter alia Mates (1986), Russell, (1900), Nichols (2001).

Thoroughgoing analysis of eligible interpretations of the *salva veritate* criterion can be found in Mates (1986) – but however we choose to read the criterion, on every possible reading we speak of substitutivity[43], hence a representation in terms of variables might work. But this is not the case with the PII. It should be noted that there is still a considerable gap between dealing with something as by itself substitutable and dealing with something as having its identity constituted in reference (through substitutivity of references) – not to mention dealing with something as having its identity given in a real determination. But for now let us put aside the idea of substitutivity and focus on the complementary part of the criterion: unchanged truth value. This presents the thing whose identity we scrutinize in the context of a proposition – and that in turn in the context of its possible truth values, or rather as ultimately referring to truth values[44]. On the other hand, when we investigate discernibility, things are evaluated solely in the context of themselves. But those are only preliminary observations – we must go through the evidence, in order to really appreciate the import of both tenets.

Leibniz usually formulates PII in the following manner:

"there cannot be two individuals entirely similar or differing only in number" (L, 336)

Keywords in this definition are: (i) individuals, (ii) entirely similar and (iii) differing only in number. As for (i), most of the wordings read "individuals" (*individus*)[45], some of them "substances" (*substances*)[46], some "things in nature"[47] but once Leibniz choose the expression "real absolute beings" (*etres reels absolus*)[48] only to change for "sensible things" (choses sensibles)[49] a few lines later and eventually in the same writing (his fifth letter to Clarke) he even settled simply on "things" (*choses*)[50]. As concerns (ii), alternative phrases are

[43] L, 241, 245, 371

[44] A distinctively Fregean move (Beaney, The Frege Reader: The foundations of Arithmetics), indeed, but still helpful, I believe, to pinpoint certain indispensable conceptual shifts involved in the phrasing.

[45] Leibniz alternates between "individuals" (L, 336, 687), or "individual things" (L, 268), or "real or complete individuals" (NE, 331-2)

[46] L, 308 (G, IV, 433); L, 534-5 (G, II, 264)

[47] "there are never two things in nature which are perfectly alike and in which it is impossible to find a difference that is internal or founded on an intrinsic denomination" (L, 645)

[48] L, 699: "there are not in nature two real, absolute beings, indiscernible from each other" (qu'il n'y a point dans la nature deux etres reels absolus indiscernables (G, VII, 393))

[49] L, 699: "I said that in sensible things two that are indiscernible can never be found; that, for instance, two leaves in a garden or two drops of water perfectly alike are not to be found." (J'avois allegué que dans les choses sensibles on n'en trouve jamais deux indiscernables (G, VII, 393))

[50] L, 700 (G, VII, 395)

"resemble each other completely" (*ressemblent entierement*)[51], "perfectly alike"[52], "[not] different in nature" (diversae naturae)[53] and "indiscernible from each other" (*indiscernables*)[54]. The last keyword remains unaltered in its wording throughout most of the statements.

2.3 The first keyword: the real being. The first keyword pertains predominantly to the scope of the PII. Most of the texts dealing with the issue of the range of PII's application – either more or less independently of Leibniz's original account[55], or directly interpreting Leibniz[56] – are impaired by the preoccupation with a talk about properties, seeking the adequate restriction of the scope of PII by way of restricting properties allowed to be used in discriminating the things submitted to the principle: routinely relations are excluded (though, a version known as the Third Grade Principle excludes for example only symmetric relations[57]). However, such understanding requires knowledge of all the properties, before applying PII to establish the identity of the subject bearing them – but this is absurd. In the real world, where the principle is used by Leibniz, all the properties of something real can never be known – and we generally never ask whether this drop of water is identical with that drop of water, because the answer is already implied: obviously not. So the appropriate range of PII's application should not be determined in terms of delimiting permissible properties, which can never be known in advance, but rather in terms of the stipulated reality of a subject submitted to PII.

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[51] L, 308: "it is not true that two substances can resemble each other completely and differ only in number" (qu'il n'est pas vray que deux substances se ressemblent entierement, et soyent differentes solo numero (G, IV, 433))

[52] NE, 331-332: "In a word, we shall never find the final logical species, as I have already remarked above, and two real or complete individuals of one and the same species are never perfectly alike."

[53] L, 534-5: "all substances are different in nature, and there are no two things in nature which differ in number alone." (omnes substantias esse diversae naturae nec duo solo numero differentia in natura dari (G, II, 264))

[54] As far as I know, Leibniz actually started using the word no earlier than in his correspondence with Clarke: L, 687, 699, 700

[55] Consider the standard debates beginning with Black (1952), Rescher (1955), O'Connor (1954); or its more contemporary logicist alternates: Bar-Elli (1982), Ketland (2006), Forrest (2016). Though, it is true that a significant part of it steers to a question of determining identity in physical space, which is somewhat independent of the properties based talk about differences.

[56] Notoriously expositions like Brandon C. (2017); but even Mates (1986), who approaches the problem from Leibnizian subject-predicate logical scheme, eventually ends up in excluding "extrinsic denominations" with an addition that those are what "some philosophers now call relational properties" (Mates, 1986, 135)

[57] See Forrest (2016).

Those who transliterate the principle in a formal logical fashion sketched above (and therefore conflate it virtually with the *salva veritate* criterion, or at least with a half of it) might fancy its use in the realm of abstract, mathematical, logical, or generally structurally determined entities (we will expand on the account later on); however, that is exactly the application which we can without a doubt rule out straight from the beginning. All of Leibniz's efforts to arrive at sound definitions of *similarity*, *equality* and *homogeneity* (a pinnacle of those being without a doubt *The Metaphysical Foundations of Mathematics* (L, 666-674)) would otherwise go in vain[58]. Furthermore, Leibniz himself makes it pretty clear in his first systematic exposition of the idea in the *First Truths*:

"It follows also that *there cannot be two individual things in nature which differ only numerically*. […] Never are two eggs, two leaves, or two blades of grass in a garden to be found exactly similar to each other. So perfect similarity occurs only in incomplete and abstract concepts, where matters are conceived, not in their totality but according to a certain single viewpoint, as when we consider only figures and neglect the figured matter. So geometry is right in studying similar triangles, even though two perfectly similar material triangles are never found. And although gold or some other metal, or salt, and many liquids, may be taken for homogeneous bodies, this can be admitted only as concerns the senses and not as if it were true in an exact sense." (L, 268)

A symptomatic use of triangle example makes the comparison with a typical presentation of the *salva veritate* criterion somehow striking:

"Same or coincident terms are those which can be substituted for each other anywhere without affecting truth. For example, 'triangle' and 'trilateral', for in all the propositions demonstrated by Euclid about a triangle, trilateral can be substituted, and the converse, without affecting their truth." (L, 371)

The question whether Leibniz meant two concepts to be the same, or rather things denoted by them, which Mates tries to resolve, seems to me completely artificial[59]; what is important is that it clearly cannot be the case that PII would be as if a half[60] of the *salva veritate* criterion, because those principles are mutually exclusive, incapable of application to the same subject. Mates indicates that PII should be restricted to complete concepts and I generally espouse such a view, even though the qualification might turn out to be quite void, "nominal" to put it in Leibniz words, since we do not have full access to any complete concept and this therefore cannot be properly represented by any part of our language: boldly stated in a more exegetic manner, one can say that no fully determined singular

[58] An observation made explicit in Cook (2000).

[59] As will be vindicated further: see especially footnote 61.

[60] Actually I think that Leibniz meant by both principles something like biconditional, or simply did not care about the difference between a conditional and biconditional – in any case it does not make the difference.

reference to a complete individual concept is ever feasible for a finite mind. This being so, I find it more convenient to delimit the scope of PII negatively first: PII does not hold where the *salva veritate* criterion holds – i. e. in the cases of reference by means of incomplete concepts. These cases comprise not only reference to geometrical figures (such as triangles[61]), their homogeneous medium (such as space), mathematical objects in general (all of which deliberately violate PII by their very definition: consisting of either discrete or continuous repetition or multiplication of the same[62]), but all universals as well and ultimately all the concepts, whose account can be given in full, or, which amounts to the same, all the things which can be fully determined by their concept in a finite mind. The question of whether we speak more about the concepts themselves or rather about what is being conceptualized therewith is of a subsidiary nature (see footnote 61) and in fact Leibniz switches between a talk about concepts and a talk about what is being conceptualized quite freely, which perplexed many commentators and ultimately led them to accusations of use-mention confusion, despite acknowledging that Leibniz on other occasions showed to be well aware of the distinction.

A subject of the aforementioned kind seems well suited for the *salva veritate* criterion, since in its application we proceed from knowledge of truth values of certain propositions to knowledge of identity of its terms: and so we unequivocally determine the identity of the terms in question by their relations to other concepts. A full account of the incomplete concept can be given, but this, however fully determining within certain conceptual scheme, is nevertheless indeterminate, because its identity is merely relative

[61] The difference between the first and the second usage of the word "triangle" in the two quoted passages is worth noting: whereas the first speaks about a triangle more as a geometrical figure, the second speaks more about its concept – though what is being identified in the end cannot be this concept (because in this respect "triangle" and "trilateral" should differ and moreover, as Mates points out, the propositions would be the same, if they comprised the same concepts, and no substitution of termini would ever take place). It can be taken as evidence, that no strict distinction between concepts and geometrical figures or other abstract entities denoted by those concepts should be invoked at this point: the borderline separates not the concept and the conceptualized, but rather the real and its representation by a finite mind.

[62] Russell (1900) based his critique of Leibniz's purportedly self-defeating use of PII precisely on the fact that for Leibniz extension "is nothing but a certain [continuous] repetition of things in so far as they are similar or indiscernible" (Russell, 1900, 102; my emphasis) and correlatively number would be nothing but a certain discrete repetition of things in so far as they are indiscernible – which eventually, according to Russell, rules out the possibility that there is more than one substance. We shall examine the claim later on, but a qualification is to be made right now: Leibniz repeatedly stresses that "it is wrong to confuse extension with what is extended" (L, 667) and strictly discriminates between number and what is numbered (L, 516) as well as extension and what is extended.

with respect to other incomplete concepts. Such a thing cannot be conceived in itself, but only in relation, which Leibniz makes clear to de Volder with respect to extension:
"I do not think that substance is constituted by extension alone, since the concept of extension is incomplete. Nor do I think that extension can be conceived in itself, but I consider it an analyzable and relative concept, for it can be resolved into plurality, continuity, and coexistence or the existence of parts at one and the same time." (L, 516)
By "analysable" Leibniz means subject to finite analysis – as compared to concrete things, which can, of course, and routinely are analysed as well, but a finite mind can never reach a bottom: "the more we analyse things, the more they satisfy our intellect" (L, 516). By "relative" he means dependent on those defining concepts (i. e. in this case: plurality, continuity and coexistence), which in turn involve in their respective definitions yet another concepts, many of those involvements being reversible (as e. g. "substance and accident equally involve and are involved in each other" (L, 519)), without there being any unanalysable predicates ("Nor do I find any entirely absolute predicates in concepts, that is, any which do not involve connections with other predicates." (L, 528)) and eventually: "it is neither necessary nor easy to arrive at concepts of ultimate simplicity" (L, 526)[63]. The quoted

[63] This might indicate a remarkable departure from his former views apparently entailing a possibility of arriving at certain simple terms, from which all the other concepts (and therewith truths: true propositions comprising subject-predicate structure, where the predicate is involved in the subject, i. e. in the end complex concepts again) would arise by certain arithmetic operation. In fact even as late as in Monadology Leibniz mentions that: "When a truth is necessary, the reason for it can be found by analysis, resolving it into more simple ideas and truths until we reach the primitive" (L, 646). But "the primitive" in this case must be the merely relative constituent notion of a complex incomplete concept, which is being analysed (as for the extension those components were plurality, continuity and coexistence). The evidence is at least inconclusive and thoroughgoing investigation into it would lead us too far from our task, but I am inclined to believe that Leibniz shifted the emphasis from logic and category studies to somehow more naturalistic (and at the same time more "structural") grounding of metaphysics in the second half of nineties (as exemplified e. g. in *On the Correction of Metaphysics and the Concept of Substance* (1694)) and therewith, if not entirely abandoned, then at least largely suppressed the idea of hierarchy of concepts for the last time at length expounded in *On Universal Synthesis and Analysis, or the Art of Discovery* (1679(?)) in favour of a more horizontal organization of interrelated terms, none of which is truly primitive, which suited better his "democratized" ontology, where principles of unity and spontaneous action are granted not only to human souls, but to inhuman and seemingly unanimated matter as well. Moreover, Mates (1986, 59) wonders how an infinity of things can be achieved by combination of finitely many primitive concepts and I think that Leibniz must have realized this and exactly for that reason switched for an infinity of complete individual concept as an ultimate grounding of semantic regress instead. I deem the idea of a horizontal mirroring structure to be in strict opposition to the hierarchized traditional one and Leibniz to really invent, develop and multiply apply on various

passage from the *First Truths* already claimed this relativity of incomplete concepts: "where matters are conceived, not in their totality but according to a certain single viewpoint" (L, 268). Their relativization with respect to a certain conceptual scheme manifests itself as a relativization with respect to a particular viewpoint: a perspective abstracting from what makes things truly concrete, truly singular, differentiated from all others independently of any particular viewpoint, differentiated in itself, differentiating itself by its own activity, by its own expressing itself in the others: in a word, real. However, the perspectival relativization entails yet another significant feature: "perfect similarity [which] occurs only in incomplete and abstract concepts". If we apprehend incomplete concepts universally not only as negatively determined by one another, instituting differential relations between themselves, but positively determining as well, instituting relations of perfect similarity, we can finally make sense of them being substitutable: incomplete concepts positively determine themselves by multiplication in equivalent representations.

By contrast, in the application of PII we proceed from represented identity to a real difference – in the reverse order compared to the *salva veritate* criterion. We submitted two different terms, e. g. triangle and trilateral, to the *salva veritate* criterion, realizing that "in all the propositions demonstrated by Euclid about a triangle, trilateral can be substituted, and the converse, without affecting their truth" (L, 371), to institute an identity of a transcendent object[64] – relative to evaluation of geometrical propositions and achievable in singular reference by the two terms. It is somehow superfluous, or even misleading and harmful, to speculate whether this transcendent object is a mere concept or something else as well: its reality is fully determined by the possible references to it by means of those substitutable terms – and nothing more is needed to be added. But the point is that this determination of reality, even in its total, is still not complete, does not entail every eventuality, does not entail everything – and therefore does not render the thing ultimately

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problems the former one, while, pace Palkoska (2010), discarding the latter one on the way at best as an once convenient, now superfluous scaffolding.

[64] Bar-Elli (1982) argues to the effect that it is precisely PII (not the *salva veritate* criterion, explicitly considered by Bar-Elli as well), which is responsible for the formation of the notion of object in a particular language, or its "ontology", rendering these objects transcendent to the language. But the very opening line of his article clearly indicates the nature of a conceptual shift involved in his representation of the principle: whereas Leibniz writes "there cannot be two individuals entirely similar or differing only in number" (L, 336), Bar-Elli states that: "The principle of the identity of indiscernibles says that two things that are indiscernible – that share all their properties – are one and the same thing." Furthermore, he relies on the identity as constituted in singular reference only throughout the reasoning, cashing out indiscernibility in terms of equality in one semantics (I) and identity in terms of equality in meta-semantics (D) – a strategy formally expounded at the outset of this chapter.

real. On the other hand, by submitting a drop of water (which, being a "drop of water", is by definition similar to other drops of water) to PII, we establish a supposition of its intrinsic uniqueness as a guideline for an infinite analysis[65]. In this respect PII looks more like a heuristic criterion: we can hardly begin with a perfect grasp of two complete individual concepts only to realize a moment later that they are actually the same and therefore the things they express must be the same as well; rather we gradually complete our account of something supposedly real in the course of forthgoing analysis and the principle tells us that whenever we settle on a definite conceptualisation, which would fully determine the identity of the thing solely by means of conceptual determination, we already withdraw our initial supposition of the reality of the analysed object, which is therewith inevitably superseded by its idealization; it teaches us that behind every identification of representations, we should search for a real difference.

Leibniz famously invokes the principle in his discussion with Clarke (L, 675-721) to refute the *reality* of space: he does not argue to the effect that to comply with PII a "real space" should not be homogenous, but rather that space cannot be conceived of otherwise and therefore cannot be anything over and above the perfectly homogeneous geometrical space of mathematicians, cannot introduce any change, any difference, any heterogeneity[66] into the real world, cannot manifest itself (does nothing, is nothing[67]) – and therefore cannot be real. We could mean by "space" various things, but once we defined it as homogenous, we straightforwardly rule out the possibility for it to be something real. This touches upon a

[65] Leibniz distinguishes two types of analysis, both of them proceeding „from the posterior by nature to the prior by nature": an analysis of necessities, which reaches the ground level of primitive (though merely relative) elements in finitely many steps, and an analysis of contingents, which "proceeds to infinity without ever being reduced to primitive elements" (L, 664).

[66] "I have in fact demonstrated elsewhere, in my reply to Sturm which was published in the Leipzig Acts, that if matter were not heterogeneous (which it becomes through the entelechies), there could arise no variety of phenomena, and as a result equivalents would always replace each other." (L, 525) See also: L, 529 and 505.

[67] "It can be shown adequately from the essential principles of metaphysics that what does not act does not exist" (G, VII, 326-327) (translation taken from: L, 271, n.10) Moreover, what is ultimately real is a substance (everything else has its being dependent on some substance) and Leibniz occasionally even defines substance by activity: the opening line of *The Principles of Nature and of Grace, based on reason* for example reads: "1. Substance is a being capable of action." (L, 636) "So far as I have made the concept of action clear to myself, I believe that there follows from it and is established by it that most widely accepted principle of philosophy — that actions belong to substances [actiones esse suppositarum]. And hence I hold it also to be true that this is a reciprocal proposition, so that not only is everything that acts an individual substance but also every individual substance acts without interruption, not excepting body itself, in which no absolute rest is ever to be found." (L, 502)

common root of both Leibniz's phenomenalism, as highlighted by Adams (1999), and nominalism, as stressed by Mates (1986), and inevitably steers our attention to the second keyword.

2.4 The second keyword: discernibility. The second keyword pertains to the nature of the stipulated real difference. On the one hand, as "discernible" it seems to be dependent on some perceiving subject (this is the "phenomenalistic" side), on the other hand, as intrinsic it seems not to result from any particular relation holding between differentiated objects, but rather to be some strange difference independent of any differentiating, difference in itself[68], or a difference for itself (this is the "nominalistic" side). Separate study of either of those sides leads to tantalizing difficulties: the phenomenalistic accent opens questions of whether monads are to be differentiated from their own perspectives, or whether they are ultimately discernible only by God's eye, and whether the principle is contingent, or necessary; whereas the nominalistic accent poses a challenge of making sense of such radical idealisation of relations to the effect that even a difference between subjects of denominations (be it intrinsic or extrinsic), their very identity, cannot be instituted by any purely extrinsic denomination[69], any underlying structure (which taken as ultimately real would individuate the subjects by locating them in different positions[70]). I believe that the solution to those perplexities is to be sought in an acknowledgement that the two sides are not substantially different: they are only two incomplete conceptualizations of one and the same tendency in Leibniz's thinking, and in the end of the day, Leibniz's purported nominalism is nothing more and nothing less than his phenomenalism.

Let us have a look at those perplexities separately first. Commentators routinely could not help expressing dissatisfaction with the way Leibniz himself presents PII: it is usually considered that "Leibniz should have regarded his principle as necessary" (Russell, 1900, 55) and that: "Were the gentleman to whom Leibniz refers in an anecdote[71] from the

[68] Leibniz makes it clear that differeing things "must have within themselves some diversity that can be noted" (L, 529) and speaks in such cases of an "intrinsic difference" (L, 532), or a "difference that is internal or founded on an intrinsic denomination" (L, 645).

[69] If we stick to the distinciton between properties and relations – and even if we enrich the scheme by relational properties – the matter become contentios and it is unlclear if we are really to charge Leibniz with an attempt to reduce all relations to properties – after all, he acknowledged as genuine relations of representation, which only enables any such reduction, but he clearly states that there are no „purely extrinsic denominations" (L, 268, 269, 526) and I hope that my rendering in terms of structures rather than relations will prove to be better suited to Leibniz original intentions.

[70] Such as in an absolute space – but generally for example even a total order would suffice: rendering one monad to be the first, another one to be the second, etc.

[71] „An ingenious gentleman of my acquaintance, discoursing with me in the presence of Her Electoral Highness, the Princess Sophia, in the garden of Herrenhausen, thought he could find two leaves

Clarke correspondence to have found two leaves in a garden 'perfectly alike', that discovery would not imply the falsity of PII." (Nichols, 2001, 192) But if what Leibniz wrote to Clarke was to be just an irrelevant "anecdote", one wonders, why he concludes it with the claim: "Two drops of water or milk, viewed with a microscope, will appear distinguishable from each other. This is an argument against atoms, which are confuted, as well as a vacuum, by the principles of true metaphysics." (L, 687) Moreover, in his next letter, he expressly states: "I said that in sensible things two that are indiscernible can never be found; that, for instance, two leaves in a garden or two drops of water perfectly alike are not to be found." (L, 699; [my underline]) and as early as in the *First Truths* he formulates the principle in a similar fashion: "Never are two eggs, two leaves, or two blades of grass in a garden to be found exactly similar to each other. So perfect similarity occurs only in incomplete and abstract concepts…" (L, 268; [my underline]) Though it is generally true that PII pertains to substances, i. e. complete individual concepts, as Mates put it, we can never have a full intellectual (conceptual) grasp of those and what we eventually apply PII to (as a heuristic criterion) are really "sensible things" only.

Various attempts have been made to weaken the necessity of the principle. So Bar-Elli (1982) claims it to be *a priori*, but not analytic, Russell (1900) wants it to be necessary, however not of a logical necessity, but a metaphysical one: he points to the passage where Leibniz ponders a scenario contradicting the principle and founds it indeed "possible", but "contrary to the divine wisdom by which nothing is admitted without a reason" (L, 699, 700) and deems this, as well as the relevant article of the *Discourse on Metaphysics* (L, 308), to stand for a deduction of PII from the Law of Sufficient Reason, which is metaphysically necessary, rendering PII metaphysically necessary as well. The very same textual evidence, however, serves Mates (1986) to prove the principle to be a "contingent truth"[72] and Strawson (1959) to consider the principle "theological". A very compelling alternative is given in Hacking (1975): "Like the Principle of Sufficient Reason, I/I [=PII] is not true *in* each possible world. It is true *about* all possible worlds. It is a metaprinciple about possible descriptions." (Hacking, 1975, 255) The comparison with logical necessity is striking:

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perfectly alike. The princess defied him to do it, and he ran all over the garden a long time to look for some; but it was to no purpose." (L, 687)

[72] Though the precise nature of contingency in Leibniz is a contentious matter – see Adams (1999) for an exhaustive discussion of Leibniz views on contingency. Clearly, Russell wants the principle to be a priori – i. e. not empirical, not derived, or in principle derivable from any observation, nor refutable by it – whereas, by contrast, Mates acknowledges that "Leibniz seems to accept the relevance of empirical procedures like peering through the microscope at drops of water or hunting in the garden for pairs of exactly similar leaves." (Mates, 1986, 134) But how far would he venture into insisting that this "grand principle of reason" (L, 700) "changing the state of metaphysics" (L, 687) holds of the same contingency as factual truths?

"*Tractatus* 3.031 reminds us: 'It used to be said that God could create anything except what would be contrary to the laws of logic. The reason being that we could not *say* what an 'illogical' world would look like.' Leibniz replaces 'could not say' by 'would not say'. That is, Leibniz *could not* describe a world in which a law of logic was false and *would not* describe any world in such a way that it contravenes I/I. Whatever God might create, we are clever enough to describe it in such a way that the identity of indiscernibles is preserved." (Hacking, 1975, 255-6) The emphasis on language is of course anachronistic, but the point behind it remains: it is completely upon us what we consider to be the same: the possibility to think of it as really different is always open to us. But to render the things the same means making them into substitutable representations as determined within some merely relative representational scheme and therewith losing the ultimately real subject. The imperative, embodied in PII, viz. to search for an intrinsic difference, is the imperative to search for a real determination, as contrasted with the merely relative one – it is an imperative of a true metaphysics: to investigate what are the things "in themselves". If the principle is not logically necessary, but is nevertheless an a priori fundamental principle of reason, despite not pertaining to a priori objects, one cannot come up with any other way of demonstrating it than by its application. And I can hardly conceive of a more convincing demonstration of its force and utility than this leaves-on-a-tree example. Relevant estimates ascribe up to 2 million leaves to a fully grown oak tree. When I look at the tree, those leaves look pretty much alike, but I suppose that there are some differences between them, in fact I imagine those differences to result from some principle of variation: I can think of this part being smaller, that part bigger, this part lighter, that part darker, but I can never conceptualize a principle of differentiation rendering 2 million leaves completely unique. And then I look at the entire forest; and think of all the myriads of leaves growing anew every year. I am perfectly sure, the experience has testified this conviction countless times (and what other means of testifying it are there?), that any two random leaves I pick will be easily discernible even by a naked eye, nevertheless there is no chance for me to come up with a principle of identification, which would yield such richness of forms. I am left with only one option: to simply posit the individuality, or uniqueness of every single leaf prior to any attempt at its determination. I find this to be a pretty forceful argument to the effect that there is to be genuine individuality everywhere in nature, just as there is genuine individuality among human souls.

This once again shows how misguided it is to cash out indiscernibility in terms of having the same properties. The reality of our object must be presupposed in the first place – this entails individuality, which in turn entails an intrinsic difference, which may only eventually be determined by distinct properties (though in God's mind everything is present immediately at once). But even if discernibility is not relative to a particular abstracting or idealizing point of view of a particular conceptual scheme, it still seems to be dependent on

a perspective of an individual monad (a discerning human mind in particular). Nichols (2001) claims the "discernibility" in PII to be dependent exclusively on a God's eye perspective, but this repeats the flaw of those cashing it out in terms of properties and those restricting PII to complete individual concepts taken in themselves (i. e. as solely God manage – as compared to taking them in relation to us, from our point of view, in our perception of them): this would make the principle completely void. For of what use would it be for us to know that God might discriminate between things between which we cannot, and even in principle could not ever discriminate, if we had no clue as to which those for us indiscernible non-identities are? We would perceive some ultimately indiscernible phenomena, harbouring presumption that between some of them God would discriminate, while between others not, because they are in fact representations of one and the same substance – but for us all the cases would be ultimately indiscernible. This is a completely arbitrary and superfluous hypothesis. There cannot be any ultimately indiscernible perceptions. What makes us think otherwise and gives credibility to a fictitious scenario of having indiscernible yet different perceptions is a naïve identification of numerical difference with a difference of spatial locations ascribed to contents of our representations, but it is haywire to think so, as will be shown in a next section – and besides, space and time are only abstractions derived from differences of perceptions, which therefore must precede, not the other way round. Moreover, we rendered the principle at the outset of our enquiry as claiming equality of two structures (be they semantic or real), and in whichever reading I can conceive of, PII must at least initially presuppose some conceptual difference between numerical identity and discernibility (even if the difference between numerical identity and indiscernibility is eventually denied); but how could we conceptualize the difference which we cannot approach? What else would then the principle state over and above the God's omniscience? All this seems to me ridiculous. PII obviously speaks about discernibility, which is completely open to our view: "…all substances are different in nature, and there are no two things in nature which differ in number alone. When such a thing is conceived, it is by a fiction of the mind which is ignorant of the difference or is concealing it, or abstracting from it." (L, 534-5) When we institute a similarity between two distinct leaves, it is not due to us being completely unaware of the difference, which is manifest, but rather due to (i) a fiction of our mind determining similarity and (ii) our incapability to render the difference, which is real, determinate.

Nichols' interpretation is an attempt to address a difficulty posed by Strawson (1959) to the effect that monads cannot be uniquely individuated by their points of view, because the universe can be symmetric – but this idea does not need any refutation, as it straightforwardly twists Leibniz's scheme upside down. Strawson first presupposes a structure of the world and then shows that positions on this structure does not uniquely determine individuality of the elements occupying them. However, this is what structures

generally do, this is what they are good for: rendering their elements to be substitutable. To be sure, there are many structures which are rigid, i. e. devoid of any nontrivial automorphisms (but for some strange reason Strawson chooses exactly those nonrigid ones, such as chessboard), nevertheless even those form isomorphism classes and cannot therefore uniquely determine identity of the elements occupying the positions within them (hence the problem of a massive reduplication posed by Strawson). And nobody was more consistent in maintaining that structure is not sufficient for complete determination or individuation than Leibniz himself who holds that "there is no denomination so extrinsic that it does not have an intrinsic denomination as its basis" (L, 526-7). So in Leibniz's universe the order is reverse, there is no underlying structure posited, only intrinsic differences, which by themselves already individuate the monads, i. e. immediately for themselves structure their plurality on the first level, and upon those all the other structures supervene. But how should it work? To answer this we must steer our attention to the third keyword.

2.5 The third keyword: numerical identity. There is a significant strand of thinking about PII which focuses on devising spatial counterexamples and subsequently resisting them by offering equivalent redescriptions, which would not violate PII[73]. We already encountered Hacking (1975) providing a critique of them all in general, concluding that they are question-begging and suggesting that the possibility to give a PII-conform description is always open and, given certain metaphysical presuppositions, might be preferable. In the course of his reasoning, however, two remarkable observations are executed: 1. "The examples are 'underdetermined'" (Hacking, 1975, 249) as "there can be no determination of spatial relations without a study of the laws of nature attributed to objects in space." (Hacking, 1975, 250), which he illustrates by Mach's relativistic answer to Newton's rotating bucket argument. 2. "To avoid question-begging we must first show that there are two drops, or first show, on independent grounds, that absolute space exists." (Hacking, 1975, 251-2)

The first observation can be still expanded: a description of a fictitious scenario is by definition given merely in abstract terms, incomplete concepts, hence it cannot boast a real difference between depicted objects – PII is violated even if under an alternative description it seems to hold. The second observation is analogous to the conclusion rendered by a person A in Black's original dialogue: "Of course, if you began by assuming that the spheres were numerically different though qualitatively alike, you could end by 'providing' what you first assumed." (Black, 1952, 163) We must represent the scenario in a way which not only opens the possibility of describing the depicted objects as numerically distinct, but explicitly determines the numerical difference as well. The determination is achieved by representing

[73] A pioneering article was Black's *The Identity of Indiscernibles* (1952). Forrest (2016) gives a nice overview of various approaches to PII based on those spatial counterexamples.

the objects as occupying different positions in a postulated structure. And the structure is rendered therewith as real. Routinely an absolute space is used: two[74] in itself indiscernible objects are placed in some distance from one another in an absolute space to determine their numerical distinctness; then a re-description is offered conforming to the notion of numerical distinctness defined in terms of occupying different positions on a spatial structure, which aims to redefine a topology of the presupposed space so as to render the corresponding positions to be one and the same[75]. Numerical identity is therewith bound with a position in a spatial structure – and if the positions collapse into one, numerical identities collapse as well. This looks somehow arbitrary: it seems that we could have chosen other structures to render the objects non-identical – and besides, it may be suggested that space itself presupposes numerical difference, not the other way round.

Nevertheless, however we describe a scenario comprising two objects, an abstract structure responsible for the plurality of the objects – and therewith responsible for the singularity of every one of them taken separately – will already be given in this description. By putting forward a re-description we posit an alternative structure of the same kind or quality in which we ascribe one and the same position to both of the objects within the original description. What do we mean by the "same kind or quality"? The structures should be rendered homogeneous, in a broader sense, perhaps even a continuous transformation is to be defined between them, yet the specification is even stronger: if we presupposed a real absolute space, its re-description should represent a real absolute space as well – with not only homogenous structural properties, but with genuinely qualitative properties evincing reality of the spatial structure as well – taking into account a possible observer, who could judge the objects to be *really* two, or *really* one. However, even if the mutual determination of the elements in either of the structures is to be rendered ultimately real, the determination of the structures by one another is semantic[76]: the identity of the single object in one position of the new structure is determined in reference to the two objects in the original structure. We cannot easily discard the previous description, because it

[74] Sophisticated counterexamples may use more indiscernible objects instead – even an infinity of them. Nevertheless, everything I say in the subsequent is without loss of generality applicable to those cases as well.

[75] Although some of the redescriptions of the more complex scenarios comprising more indiscernible objects are pretty elaborate to be intrinsically "indiscernible" from the original one, it is worth noting that there are no determinate criteria of this intrinsic indiscernibility. Since there are no other fixed points in the structure than those multiple positions which eventually become one, it is hard to tell which additional features do we expect the structure to boast except discriminating the objects (plus possibly the shortest (and/or straight) path between them) and by which operations we are allowed to establish that it retains them even under the new description.

[76] A distinction developed at the outset of this chapter.

constituted not only the numerical difference between the indiscernible objects, but also the singular identity of each of them, which is transmitted by means of reference to the single object under the new description.

Absolute space proved to be quite convenient for grounding numerical identity, yet a suspicion has been raised that the identity it renders is more than numerical. What exactly is this "more" involved in spatially defined numerical identity? And how does the minimal structure of numerical identity and difference relations look like? A straightforward answer might read: a structure of natural numbers. However, this could, in a sense, be too much (it is clear, that we wanted two distinct things to be really *two*, but it is less clear that we insisted that many distinct things be, for example, exactly 1098, or that we insisted that these many things be countable and necessarily more, equal, or less than those many things), in another sense, too little (we definitely wanted those two things to be *really* two – not just theoretically, abstractly speaking, or in a certain respect, from a certain point of view, two). Indeed, it is hard to conceive that all the objects would instead be located in an absolute three-dimensional space in an absolute natural number line. Be it as it may, it makes hardly any difference, since for Leibniz the former is no more real than the latter. The only acceptable, or ultimately real, structure is the intrinsic structure of a simple unity. Substance as a unity per se needs to be determined by some minimal structure responsible for its numerical identity and yet not external to it.

How to diminish the specific impact of the structure, chosen arbitrarily only to render the things two? We pondered the notion of intrinsic difference in the previous section – but this does not seem to be a viable option for the present task: to "render the things two" is expressly to ascribe a number to them and the ascription of a number is an extrinsic denomination. No intrinsic denomination can do the job: by this we take something as something and once again something as something – and indeed there is an intrinsic difference rendering every one of our acts of referring unique, but lest we take something as one, we cannot establish its numerical identity. However, once we take something as one, we no longer take it as something specific, we no longer refer to it by means of intrinsic denomination and determine its identity in this unique act of reference, but rather let it be determined in an external structure. Even if intrinsic denominations differentiate the things, they are not sufficient to render them countable, or generally structured. For the things to "differ only in number" (L, 268, 308, 336, 535, 700)[77] means being identified merely in an external structure grounding their countability – and so we will need to presuppose one, in order to establish their numerical identity.

[77] It is worth noting that this is precisely the formulation Leibniz usually chooses. Leibniz was well aware of a peculiar nature of numerical difference.

Things differentiated "in themselves" do not form a single whole[78], nor a determined multiplicity (because their numerical identity is not given). If we were to speak about multiplicity nevertheless, we should call it a heterogeneous multiplicity[79]. What is peculiar about such heterogeneous multiplicity is the absence of any common measure, any principle of identity external to the elements themselves – enabling the elements to be carved out from the background of the whole of the multiplicity: you cannot single out and extract an individual element from heterogeneous multiplicity, because the elements are individuated intrinsically and so there is no common structure in which they could border with one another, or delimit one another: by taking a single element, you take the entire multiplicity. And correspondingly, even the whole of the multiplicity lacks any extrinsic delimitation of its numerical identity and so it can hardly be said to be self-identical – rather it is self-differentiating with respect to every one of its uncountable aspects represented by its elements.

Leibniz himself provides us with various pretty Bergsonian sounding descriptions of such multiplicity, pertaining to two crucial types of pluralities in Leibniz, which are regularly at the charge of being ill defined: (i) a plurality of monadic states[80] – criticised for leading to a bundle theory of substance, or a substratum theory of substance, or a presupposition of absolute time declaratively denied by Leibniz (see Whipple (2010)), (ii) a plurality of monads[81] – criticised for being inconsistent with a conception of numerical denominations

[78] For Leibniz a whole consists of parts, which are its homogeneous ingredients (L, 668).

[79] A term coined by Bergson (1888, 1939). Deleuze (2006) famously derived its origin from Riemann (2000), but I do not consider the evidence to be entirely convincing. Those are the basic characteristics of Bergson's heterogeneous multiplicity: it is heterogeneous, continuous (resisting arithmetization), its elements flow into one another, each of them containing all the rest. Additional characteristics connected with it being a "structure" of duration include being in permanent advance, where a novelty protracts what already is by including, yet qualitatively transforming it.

[80] The passage is a bit longer, but definitely worth quoting in full length, because in its entirety it fully reveals the striking similarity with Bergson's descriptions of consciousness: "An idea is, so to speak, something dead and unchangeable in itself, as is a figure; soul is rather something living and full of activity; and in this sense I do not say that it is any one idea which tends to change out of itself, but only various ideas succeeding each other, one of which can, however, be derived from another. But in another sense of the word, I could say that in some way the soul is a living or substantial idea or more correctly, that it is an 'ideating' substance. Nor do I think that you intend anything else when you say that ideas act upon each other in representing each other, for I do not believe that you regard ideas as substances colliding with each other as do bodies." (L, 520)

[81] "In my opinion there is nothing in the whole created universe which does not need, for its perfect concept, the concept of everything else in the universality of things, since everything flows into every other thing in such a way that if anything is removed or changed, everything in the world will be different from what it now is." (L, 524; my emphasis)

being external and therefore not entirely real, abstractions of mind, relative, dependent on a point of view (see Russell (1900)). Those descriptions tend to dissolve rigid limitation, or separation of the elements of the multiplicity, which prima facie looks more plausible in the case of monadic states (i), than individual monads (ii), that are to form "true unities", but one should bear in mind that this account expresses as if only a half of the truth about them. The second half is procured by their numerical identity. Bringing those two together – the union of the intrinsic and the extrinsic, the internalisation of structure – that is precisely what PII is entitled to establish.

Russell (1900) observed that PII presupposes both "difference of content" as well as numerical difference[82] and for that reason deemed it to be self-contradictory: since the ascription of contents presupposes numerical differentiation of subjects of those ascriptions. His argument depends at least in part on a substratum theory of substance he attributes to Leibniz: before ascribing differentiating intrinsic denominations (i. e. properties), a plurality of subjects of predication (i. e. substances) must already be there – and a plurality inevitably entails a structure, or an extrinsic difference. Furthermore he came up with two interconnected arguments partially deduced thereof, to the effect that the objective plurality of substances is untenable: concluding that Leibniz should have ended up with a single substance[83].

On the one hand, it would be way too hasty to deduce the singularity of the substance from the absence of a determination of plurality (as Russell himself partly admits): after all the very same principles (extrinsic denomination, or delimitation in a structure) are needed to render the substance singular, as already shown: in that case the substance would simply lack any numerical identity (though, it may still retain intrinsic, or qualitative identity[84]). On the other hand, Russell's argument cuts even deeper, when he puts his finger on what is crucial for Leibniz: the world is divisible – and therefore is actually divided – it is

[82] "The principle, so far from maintaining diversity of content alone, presupposes material or numerical diversity as well as diversity of content proper. To both these it is logically subsequent. […] This doctrine evidently presupposes both kinds of diversity, and asserts a relation between them." (Russell, 1900, 55)

[83] With this upshot he closes the discussion of PII as well as his treatment of Leibniz's solution to the labyrinth of the continuum. Though, in the end Russell seems to draw a conclusion to the effect that one substance is equally problematic as many substances – but this is a direct consequence of substratum theory of substance, which Russell ascribes to Leibniz and which, according to Russell, makes the term destitute of meaning.

[84] Compare it with the problem of non-count, or mass nouns expounded in the first chapter and the corresponding mode of reference.

definitely plural in its nature and this plurality is to be somehow built upon simple unities[85]; but if every determination of plurality is to be external denomination and as such merely relative (dependent on a determination of the entire extension, which is "an ideal thing", delimited within a particular conceptual scheme[86]), or subjective (dependent on monadic perspectives), there is no room for a real (fully determined) plurality in the world as it is in itself: all the plurality is either an idealisation, or a phenomenon, but nothing in itself, nothing real. Russell is as penetrating as no one else here[87] – and his argument loses nothing of its persuasiveness even if we transpose his talk of subjects, properties and relations into our notions of intrinsic and extrinsic (structural) determination. There cannot really be a plurality of monads – there cannot be even a single monad, as it seems after all. This looks indeed devastating. And yet completely consequent: every determinate plurality (be it three dimensional manifold of space, the real line, a set of two members, or a single self-identical unit) is just an incomplete concept – nothing real in itself. "Properties pertaining to extension are not to be assigned to souls, and their unity and multitude are not to be derived from the category of quantity but from the category of substance, that is, not from points but from the primitive force of action. But the action proper to the soul is perception, and the nexus of perceptions, according to which subsequent ones are derived from preceding ones, makes up the unity of the percipient." (L, 599)[88]

Our difficulties are similar to those exposed at the outset of the first chapter (1.1). If there is a way out of this riddle, it lies in the direction already hinted at: the conjunction of the intrinsic and the extrinsic (the elements and the structure) demanded by PII is to be accomplished by a mirroring relation binding the intrinsic ever self-differentiating heterogeneous multiplicity of a monad taken in itself with the extrinsic self-identical homogenous plurality of monads taken in their relations as the world. There clearly cannot be any immediately determinate number of monads (not even a number "one"): their plurality must be mediated by a reflection into their intrinsic self-differentiation in such a way that every attempt to determine it is caught up in an infinite regress of reflections. The monad mirroring the entire universe is the fulfilment of PII. And so the upshot addresses

[85] "Now I infer that where there is not even one thing that is a true being, there cannot be many beings and that every multitude presupposes a unity." (L, 342) "There must be simple substances, since there are compounds, for the compounded is but a collection or, an aggregate of simples." (L, 643) "Now a multitude can derive its reality only from the true unities, which have some other origin and are entirely different from points, for it is certain that the continuum cannot be compounded of points." (L, 454) See also: L, 456, 457.

[86] See 2.3.

[87] Russell, 1900, pp. 115-117.

[88] This confirms the definition of "true unities" in (L, 454) as that "which have some other origin and are entirely different from points" foreshadowed already in 1.4.4.

doubts raised at the beginning of this chapter (2.1): all relations of identity and difference are indeed dependent on a particular point of view and cannot be conceived as abstracted thereof and taken as universally objective (because, after all, every determination of entities as objective is necessarily connected with an external delimitation of their numerical identity, which is merely relative), but since the world consist solely of those particular points of view, this differentiation and identification of monads for themselves constitutes the ultimate reality, amounting to their differentiation and identification in themselves.

3. THE MIRRORING RELATION

For Leibniz, the fundamental ontological level comprises simple substances – monads: "true unities"[89], which are the very "principles of diversity"[90] by "expressing exactly each other"[91] and therewith "expressing the universe from their own points of view"[92]. This I deem to be a paradigmatic formulation of Leibniz's mature account of the monad with respect to its structural features. Moreover, I deem it to be a focal point of very much of his philosophising, at least in the later years, neither a logical consequent of ordinary superstitions disguised as irrefutable principles, nor any such purported principle itself, but rather a striking idea gradually dominating everything which in the virtual space of reasons surrounds it, eventually making Leibniz write to de Volder:
"You seem to have rightly grasped my doctrine of how every body whatever expresses all other things, and how every soul or entelechy whatever expresses its own body and through it all other things. But when you have uncovered the full force of this doctrine, you will find that I have said nothing else which does not follow from it." (June 20, 1703; L, 531)

3.1 Mirroring as semantic and real mutual determination. In the first chapter we established that the criteria for an entity to be identifiable in reference determine it insofar as it may stand for a constitutive element of a structure; mutual determination of such elements, therefore, mirrors mutual determination of their representations – and the numerical identity of an element is delimited within an underlying structure just as its identity in reference is delimited in its reference trace. In this way we conceived of a structure as an interpretation of a talk of certain multiplicity – rendering the determination in question as a semantic determination. However, we hinted at the possibility of a real determination among the elements, as well, yielding a real, ontological structure, which we further elaborated on in the second chapter. In order for the entities to exercise any influence upon one another, they cannot be completely heterogeneous, they must share some common ground on which to meet, there must be some underlying structure in which

[89] The term "true unity" is regularly used at least from 1687 (see: L, 339, 454, 456, 529) and I choose it for being less loaded with scholastic connotations than its alternative "unity *per se*". This latter term is a distinctive feature of a substance – as compared to being merely a "unity per accident", which for Leibniz ultimately amounts to being a unity per aggregation, i. e. a whole, or a fictional whole (the distinction between "wholes" and "fictional wholes" is comprehensively elaborated in Harmer (2014).

[90] "Entelechies must necessarily differ or not be completely similar to each other; in fact, they are principles of diversity, for they each express the universe from their own point of view."(L, 530)

[91] See: Monadology, 59. (L, 648)

[92] See: L, 337, 347, 360, 457, 473, 530, 579, 637, 640, 659, 711, 712; Th, 286, 322.

their mutual influence can be delimited. Yet, structure is an abstraction from a specific content, nothing but extrinsic principle of determination – which in itself cannot be real. The reality lies in the vanishing point of an infinite analysis: perfect, fully determined singularity, completely heterogeneous to anything representable – intrinsically heterogeneous – differentiating itself in itself[93]. The only structure admissible as real is the one, which is both extrinsic and intrinsic at the same time – monadic structure, which is to account for the plurality of monads as well as singularity of every one of them separately. This is the minimal structure, or the simplest conceivable structure, because in order for its determining elements or principles to be determined, no external independent structure is needed – nor allowed for. By the same token, on the other hand, the structure is inscrutable, incapable of being made explicit, or fully represented – precisely because it is inseparable from its content. And so the mutual *real* determination of the ultimate entities cannot be anything like mutual interaction of bodies in a physical space, causal dependence linearly ordered in time, or mutual mixing of hues in a space of colours – or any such interaction delimited in an independent structure – but rather mere mutual representation without any further universally applicable qualification (all its qualifications are subject to a particular point of view of a particular monad). In order for something to exist, it must manifest itself somehow (2.3.), it must act: "Substance is a being capable of action." (L, 636) – but its action is nothing but perception (L, 599): there is no objective, or independent reality of this action – if after all, the reality itself, whose essence is action, comprises nothing but points of view of its representations. Insofar as monads comprise the ultimate reality, they remain "windowless" (L, 643), because of being completely heterogeneous, in the sense that there is no external ground for their actions (they are fully spontaneous[94]), there is no independent structure, which could be taken as a measure of their interaction. Every kind of interaction and in general every kind of relation can be construed as a morphism, or mutual representation – mirroring – if an appropriate point of view is adopted: equality, or inequality pertains to representations and is always subject to a particular point of view, as suggested in 2.1. But there is no independent abstract space of points of view in Leibniz's ontology: all and only points of view are the monads themselves.

3.1.1 Real mutual determination. On several occasions Leibniz considers real mutual determination of beings construed as spatiotemporal interaction of extended objects. He employs correspondent examples in two kinds of arguments: either (i) as an illustration of

[93] Intrinsic heterogeneity of a "true unity": "Everywhere there are simple substances actually separated from each other by their own actions, which continually change their relations." (L, 636-637)

[94] Compare: L, 325, 457, 493, 577.

influence exercised by anything whatsoever on everything there is[95] (L, 269, 339, 637, 649), or (ii) as a basis of an argument to the effect that extension (and duration) is not sufficient to provide identity for extended objects within vacuum-free infinitely divisible mass (L, 505).

The crux of the first argument can be summed up thus: if there is something happening in one part of matter, then for it to really happen, to really be, it must be something – for something; i. e. it must manifest itself somehow (see 2.3): it must influence something, there must be something else happening in some other part of matter mirroring the change taking place here; but for there to be something happening, yet another manifestation is needed – else the two events would remain closed within an isolated system composed of the two correspondent parts of matter, meaning nothing to the rest of the world, meaning nothing to any conceivable observer (whose observation would not remain closed within the system as well). And so the chain of events spreads gradually to the farthest parts of the universe: one thing affects another, which affects yet another, etc. – and through all the intermediaries eventually everything mirrors everything.

But once we transpose this line of thought in a spatiotemporal scenario, several problems arise. For any change to occur in the first place some duration is already needed; and in the second place there is no such a thing as an immediate causal interaction.

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[95] "Every created individual substance exerts physical action and passion on all others. For if a change occurs in one, some corresponding change results in all others, because their denomination is changed. This is confirmed by our experience of nature, for we observe that in a vessel full of liquid (the whole universe is such a vessel) a motion made in the middle is propagated to the edges, though it may become more and more insensible as it recedes farther from its origin." (L, 269)
"… because of the continuity and divisibility of all matter, the slightest movement exerts its effect upon nearby bodies, and so from body to body to infinity, but in diminishing proportion. So our body must be affected in some way by the changes of all the rest." (L, 339)
"And since everything is connected because of the plenitude of the world, and each body acts on every other one more or less, depending on the distance, and is affected by its reaction, it follows that each monad is a living mirror, or a mirror endowed with an internal action, and that it represents the universe according to its point of view and is regulated as completely as is the universe itself." (L, 637)
"In this respect compound beings are in symbolic agreement with the simple. For everything is a plenum, so that all matter is bound together, and every motion in this plenum has some effect upon distant bodies in proportion to their distance, in such a way that every body not only is affected by those which touch it and somehow feels whatever happens to them but is also, by means of them, sensitive to others which adjoin those by which it is immediately touched. It follows that this communication extends to any distance whatever. As a result, every body responds to everything which happens in the universe, so that he who sees all could read in each everything that happens everywhere, and, indeed, even what has happened and will happen, observing in the present all that is removed from it, whether in space or in time." (L, 649)

Therefore the mirroring in question cannot take place among mere spatial parts of an instantaneous extended mass: there is no congruence between unmoved instantaneous chunks of matter, there is only congruence of their motions. It seems simply misguided to conceive of the ultimate subjects, the ultimate agents as if carved out of space in one simultaneous slice of space-time. Moreover, if the only admissible change was locomotion of infinitely divisible vacuum-free homogeneous mass, the identity of what is being moved could be established neither in itself (since each part of the mass would be homogeneous with every other part), nor by the effects of the motion (since motion is nothing but perpetual substitution of indistinguishables) – and therefore the motion itself would be both locally and globally indiscernible from the rest. This is the crux of the second argument, by which Leibniz means to show that some intrinsic principle of differentiation among the ultimate subjects is needed, some heterogeneity, which in the context of such spatiotemporal scenario amounts to "active forces or impetuses" within moving chunks of matter. "For if there is no difference between any portion of matter and another portion equal and congruent to it (which the illustrious man must admit, since he has destroyed active forces or impetuses and all other qualities and modifications except for existence in this place and successively some future existence or other, all qualities and modifications having been removed), and furthermore, if the state of this matter at one moment does not differ from its state at another moment except through the transposition of equal and congruent portions of matter which agree in everything, it obviously follows that because of the perpetual substitution of indistinguishables, the state of the corporeal world can in no way be distinguished at different moments." (L, 505)

Clearly enough, in terms of modern physics, the sole extension of bodies is not sufficient, vectors of momenta are needed as well; but the very dichotomy between bodies and their momenta looks arbitrary once we face philosophical problem of the fundamental ontology and indeed unsatisfactory even in physics, when we move from mechanics of a middle-sized objects to the scale of micro-particles. Moreover, Leibniz's arguments should be read in their full generality as pertaining to any kind of mutual determination delimited in an independently determinable structure, yet conceived as real. In any such idealisation, we ultimately end up in more or less entangled regresses of determination, as expounded at the beginning of the first chapter. We need some homogeneity to account for joint effects of mutual determination of real entities, but at the same time, we need some heterogeneity to account for the real difference between the ultimate entities – be they carved out of a three-dimensional Euclidean space, or any multidimensional vector space of modern physics. The (i) argument shows that between any two real entities, there is to be a morphism, but the (ii) argument shows that all the morphisms will ultimately become automorphisms in an

independently representable homogeneous structure of mutual determination, if we do not allow for some intrinsic principle of differentiation[96].

3.1.2 Principles of self-differentiation. The riddle exposes clash between mirroring construed as mutual determination of elements within a closed system and mutual determination of real entities, which cannot be but conceived of as taking place in an open system, allowing the entities to transcend themselves, to shed their identity, to self-differentiate. It is natural to think of this intrinsic self-differentiation as enabled by a superadded temporal dimension – prioritizing therewith time over space – and Leibniz himself did very little to denounce this preoccupation. In fact, Leibniz's notion of monadic state perplexed many commentators, ultimately leading them to ascribe him either a bundle theory of substance (Mates, 1986), or a substratum theory of substance (Russell, 1900), or some middle ground struggling with an implicit asymmetry in his treatment of the ideality of space and the ideality of time – as comprehensively put forward by Whipple (2010). Though the evidence is less conclusive than one would wish, there are systematic reasons to discount any such asymmetry between space and time. Monadic change insofar as it is temporal, is merely a phenomenal change; and insofar as it is a-temporal, is only an intrinsic self-differentiation, which is no more connected with time, than it is with space, or any other ideal structure into which it may be projected. Monadic state is in this respect perfectly comparable to "situs", which according to Leibniz (L, 667) involves both quantity and quality, or "location":

"I had said that extension is the order of possible coexistents[97] and that time is the order of possible inconsistents[98]. If this is so, you say you wonder how time enters into all things, spiritual as well as corporeal, while extension enters only into corporeal things. I reply that the relations are the same in the one case as in the other, for every change, spiritual as

[96] The problem generally arises once we spread the net of mutual determination on the entire universe, insisting that for anything to be, it must influence (if indirectly) everything there is, and so a universal common ground of all the interactions in the world is needed, which consequently cannot be but delimited independently from outside irrespective of particular points of view within the world, which makes it merely formal. This is precisely the consequence Leibniz claims to avoid with his monadic structure, yet various commentators failed to see this, accusing him of exactly the same deficiencies he charges his opponents with: namely, that the structure is merely formal, that there is no content to all the mirroring, and so the monads are ultimately indiscernible. We will investigate their puzzlement in the next section.

[97] See: L, 531, 583, 604, 656, 666; NE, 229; „time and space (or the order of possible existence) (NE, 694);

[98] See: L, 531, 583; „order of successive phenomena" (L, 604); „order of existence which is not simultaneous" (L, 656)

well as material, has its own place [sedes], so to speak, in the order of time, as well as its own location in the order of coexistents, or in space." (L, 531)

If we take the structural symmetry seriously, monadic states are at least equally eligible for introducing self-differentiation as monadic locations. Just as a single monadic state mirrors the entire universe as if extended in space, a single monadic location mirrors the entire universe as if extended in time. Both are merely inverse extremes of an ideal projection of a continuously spatiotemporal phenomenal world: either the entire world can be spread in space while shrinking the dimension of time to an unextended point (a state), or it can be spread in time while shrinking the dimension of space to an unextended point (a location) – in both projections the world is represented in all its variety, nothing is left out, nothing is added. But neither monadic state, nor monadic location can be subject of any real determination, which is influence or action, because none of them intrinsically changes or diversifies itself – rather by being arbitrarily defined slices within permanent flux of reality, they are the very (relative) norms of stability, forming a coordinate system of identification of events (spatiotemporal interaction requires homogeneity of certain properties of interacting entities, namely extension and duration, and in the absence of other discriminating signs, those entities qua spatiotemporally interacting entities are extrinsically identified in reference to the identity of a location and intrinsically differentiated in reference to the identity of different states).

Monads are neither mirrors of the entire space as perceived from one of its points, nor mirrors of the entire time as perceived from one of its points – and not even mirrors of the entire space-time as perceived from one of its points. They extend to the outer limits of space, just as they extend to the outer limits of time, because their identity therein is defined by their manifestations, i. e. by the effects of their actions, which correspondingly spread to the outer limits of the universe. We need not thoroughly examine a single spatial point or a single moment of time in order to delimit the identity of a monad within permanent flux of reality, rather we need to track a continuous series of action exhibiting a single pattern – or a law[99]. A monad is recognised in a body currently in charge of executing

[99] "The succeeding substance will be considered the same as the preceding as long as the same law of the series or of simple continuous transition persists, which makes us believe in the same subject of change, or the monad. The fact that a certain law persists which involves all of the future states of that which we conceive to be the same — this is the very fact, I say, which constitutes the enduring substance." (L, 535)

"But that which persists, insofar as it involves all cases, contains primitive force, so that primitive force is the law of the series, as it were, while derivative force is the determinate value which distinguishes some term in the series." (L, 533)

See also: L, 360,

its actions: all of the extended parts of the body are in permanent flux[100] and the sole principle of its identity is found in the correspondent law of a series of actions. The body assigned to the monad has blurred outlines[101]: its limits are not limits of the monad – rather, the monad itself is a limit of the identity of the body: it carves out a region within the whole of all the happenings – or manifestations – which mirrors according to unique law of morphism prescribed by the monad the rest of what there is. Imagined thus, the world consist of an infinity of intersecting closed boundaries[102], each of which encircles parts of the other boundaries and defines therewith a morphism holding between the parts of all the boundaries situated on one side of the boundary in question and those situated on the other side. In this sense, a monad is a measure of correlation between the internal and the external within a plenum of happenings: which entails, for example, that this event, which is a concussion, mirrors that event, which is a fall of a bough – and by defining specific correlations between all the events, it defines a particular point of view among them, which amounts to defining an individuum – a spontaneously acting being. There is no continuous

[100] "It is true, of course, that a soul cannot pass over from one organic body into another but remains always in the same organic body, not even death violating this law. But it must be remembered that even this organic body remains the same in the way in which the ship of Theseus or a river does; that is, it is in perpetual flux. And perhaps no portion of matter can be designated which always remains the property of the same animal or soul." (L, 597)

[101] "There is no actual determinate figure in things, for none can satisfy the infinity of impressions." (L, 270)

[102] Leibniz accomplishes his revision of the concept of substance under the heading of restitution of Aristotelian substantial forms: and a form can be seen as a definite shape or figure (a "closed boundary") encircling certain region of in itself undifferentiated subsistent content – i. e. matter. Though it is arguable to what extent did his commitments from the middle period survived till up to the later years, and for example the five-tier hylomorphic ontology ("(1) the primitive entelechy or soul; (2) primary matter or primitive passive power; (3) the complete monad formed by these two; (4) mass or secondary matter, or the organic machine in which innumerable subordinate monads concur; and (5) the animal or corporeal substance which the dominating monad makes into one machine" (L, 530)) which Leibniz develops in his correspondence with De Volder looks a bit too bloated to be considered a standard view, it is safe to say that something remains retained from the idea of a form even till *Monadology* and beyond. I am inclined to think that while he gradually deprived of independent reality all of that which formerly constituted the material – or passive, undifferentiated, homogeneous, indifferent – parts of the scheme (i. e. primary and secondary matter), he nevertheless retained the defining principles of the formal – or active, self-differentiating, heterogeneous, teleological (entelechy): "Forms are for me nothing but activities or entelechies, and substantial forms are the primary entelechies." (L, 511) and "… all the bodies in the world arise from an interaction of internal forces, and I have no doubt that these forces are coeval with matter itself, for I believe that matter cannot subsist in itself without forces." (L, 511.)

manifold of events needed to be posited beforehand, because events are nothing but intersections of boundaries, or their mirror images. Thus, a monad defines relations of mirroring (mutual determination) among events (phenomena), while being a mirror image of all the other monads.

3.1.3 Meanings of the mirroring relation. Therefore, there seems to be two meanings to the mirroring: mirroring among spatiotemporally interacting phenomena, i. e. causally related events, which are points of intersection of various identity criteria for living bodies provided by monads, and mirroring among the monads themselves[103]. In both cases the mirroring takes place between extrinsic determinations and intrinsic determinations: either between *phenomena* occurring outside of a living body and those occurring inside it, or between multiple beings and their representations within a single one. The latter is therefore the very principle of diversity, or variety, within the world on the most fundamental level, just as it is the principle of identity among bodies. Passages like the following one clearly show that Leibniz did not consider the two meanings of the mirroring to be essentially divergent, forming two distinct levels of ontology[104], rather, he meant the

[103] Thus, a monad as a principle of self-differentiation is a continuous function, a law of the series, whose values are individual events – which are the discriminating signs introduced in 1.1 construed as real determinants. This is what Leibniz had in mind in *Monadology*: "But besides the principle of change there must be some *distinguishing detail in that which changes*, which constitutes the specific nature and the variety, so to speak, of simple substances." (L, 645)
However, in his correspondence with De Volder and Des Bosses, Leibniz regularly uses other terms for the delimited dichotomy: primary and secondary force. Whereas primary force is the self-differentiating principle of a monad, which produces all the variety from itself and for itself, secondary force is that in virtue of which a being manifest itself, exercising its causal influence everything there is. Their relation is again that of a function and its value: "But that which persists, insofar as it involves all cases, contains primitive force, so that primitive force is the law of the series, as it were, while derivative force is the determinate value which distinguishes some term in the series." (L, 533)

[104] Certain commentators hold that Leibniz's ontology can be divided into two levels (Mates, 1986), while others maintain that it be divided into three levels (Anapolitanos, 1999); yet others stress smooth transitions between the levels: Adams (1994) wants mere ideal things to be seen in line with real phenomena "as an infinite gradation of approximations to reality" (Adams, 1994, p. 254) to revoke the „metaphysical apartheid" introduced by Harzt and Cover. As for my part, I do not see a need, nor a sense in stratifying Leibniz's ontology: what we mean by real being, or simply being, is always the same, but once we try to pin it down to a fully determined position in an independent extrinsic structure of representations (be it formalized extension of a natural language, formal ontology, or abstracted realm of sense perceptions), we inevitably already miss what was for it to be real (see my treatment of subjects of PII in 2.4). This does not preclude various degrees of perfection to be construed as "quantity of positive reality" (L, 647) and utilized in explication of monadic

mirroring to be in a way univocal (hence the "univocity of being" promoted in the footnote 104):

"In natural perception and feeling it suffices that what is divisible and material and is found dispersed among several beings should be expressed or represented in a single indivisible being or in a substance which is endowed with a true unity. […] Now this expression takes place everywhere, because every substance sympathizes with all the others and receives a proportional change corresponding to the slightest change which occurs in the whole world, although this change will be more or less noticeable as other bodies or their actions have more or less relationship with ours." (L, 339)

Yet, alongside with the aforementioned mirroring of causes in their joint effect and the mirroring of beings in their joint representation, Leibniz regularly gives examples of mirroring of a spatially larger organism into smaller ones, construed as its organs[105]. This should be understood along the lines of the proposed univocity of mirroring.

A misunderstanding might be lurking unseen at this point: namely, (i) that the regions demarcated within a plenum of happenings strictly correlate with spatial regions occupied by extended objects, or even worse, (ii) that spatial parts strictly correlate with parts of identity criteria. Neither of the aforementioned holds, of course, as expounded earlier: ad (i) complete spreading of phenomena in a spatial dimension is an ideal projection which effectively relaxes their interconnectedness and rules out any possibility of mirroring among them construed as a real mutual determination, ad (ii) spatial parts are in permanent flux, whereas identity criteria transcend the flux. The arguments connected with (i) elaborated earlier in the text facilitate to think of a continuous transition between causes and effects (as

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domination – as Look (2002) did. What is ultimately real are only monads, taken in general, of course – but they are not real as generalities: rather, this or that monad is ultimately real – this dog is ultimately real (not as a general term „a dog", supplemented, maybe, with additional qualifications like „at this moment standing one meter before me", but as an individual, truly singular monad, whose full conceptual determination would require infinite analysis – and whose identity, therefore, remains forever inscrutable for a finite mind).

[105] Consider the "cheese with worms" analogy (L, 521; NE, 722), or "pond full of fish" analogy in Theodicy (Th, 59) and vividly depicted in Monadology (66.-70.) (L, 650) as well; however closest to complete self-similarity among physical phenomena are his portrayals of organisms as machines consisting of yet another machines etc. ad infinitum: "For since the mechanisms of nature are mechanisms down to their smallest parts, they are indestructible, since smaller machines are enfolded in greater machines into infinity." (L, 589)

"A natural machine remains a machine even in its smallest parts, and what is more, it always remains the same machine that it has been, being merely transformed through the different foldings which it undergoes, and being now extended, now compressed and, as it were, concentrated, when it is thought to have perished." (L, 456)

Further references can be found in: L, 499, 529, 637, 649 (Monadology, 64.).

events) and organisms (as living bodies[106]) among all phenomena. Consider a passage from a letter to Jacob Thomasius:

"I observe in advance that numerically the same change may be the generation of one being and the alteration of another; for example, since we know that putrefaction consists in little worms invisible to the naked eye, any putrid infection is an alteration of man, a generation of the worm." (L, 96)

If we bear in mind that "in a strict metaphysical sense" the worm "cannot begin except by creation or end except by annihilation" (L, 643) and that this everlasting worm is not to be equated with a sum of its spatial parts, which are in permanent flux, but rather with the course of its operations, there is nothing controversial in concluding that the illness of a man, which is an *event* in the course of human life, be equated with a worm, which is a *living body*.

Only if the notion of mirroring is univocal in this sense, it can effectively facilitate the conjunction demanded by PII, as envisioned in 2.5 – because the conjunction in question is not merely a conjunction of the intrinsic and the extrinsic, of the ever self-differentiating heterogeneous multiplicity of a monad taken in itself and the self-identical homogenous plurality of monads taken in their relations as the world, but a conjunction of the real, the efficient, and its representation as well.

3.2 Proposed models of the mirroring. In the last few pages of the thesis I want to draw the attention of the reader to some models of the mirroring structure given in secondary literature in order to shed light on certain perplexing aspects of the system. It follows from what has already been said that those approaches are inevitably doomed to failure right from the beginning, because the monadic structure cannot be delimited independently of its specific content, but even in their failure, something substantial about the nature of the problems they try to address is revealed.

Commentators generally struggled with two interrelated peculiarities of the monadic system: (i) just as there is no structure independent of its content in the monadic system, there is no content independent of the structure either – there is no independently determinable ultimate content of all the mirroring; (ii) the structure is self-similar, but there is no independently determinable pattern, which would be repeated in different scales on various levels of the fractal structure – as the pattern is dependent on a point of view of a

[106] It is woth stressing that only living bodies are real – only they maintain their unity by themselves in their continuous action in the course of the entire world history – and their identity, therefore, cannot be delimited in recourse to extension only; whereas dead bodies are merely unities by aggregation – and their identity is therefore genuine subject of extrinsic determination in an independent structure, such as that of spatial extension.

particular monad – and so the riddle of finding a ground level in a self-similar structure is rather void. We will address these perplexities respectively.

3.2.1 The first riddle: What is ultimately expressed in all those expressions? Mates (1986) gives a nice overview of concerns raised by various authors regarding the issue[107]; Lotze's doubts are somehow symptomatic:

"… Therefore, what each monad can reflect is only the way in which it itself is reflected in others and these are reflected in one another; there would be no *independent state of affairs or content in the universe* to serve as grist for this process of reflection." (Lotze (1868), 13-14, quoted from Mates (1986, 79))

What Lotze searches for is analogous to Aristotelian ὑποκειμένον – which served Aristotle on the one hand as a logical subject[108], on the other hand as a subject of a real change[109] among beings: in Aristotle's words "something must be underlying" (Aristotle (1984), Physics, 191a31) – except that we need something underlying all those reflections, not changes[110]. Worth noting are Lotze's specifications "independent state of affairs", as contrasted with only relative state of affairs (or organization), and "content", as contrasted with empty form (or abstract structure). The first points in the direction of an agelong philosophers' endeavour to disclose things as they are "in themselves" (*per se* – though there is some curious shift of meaning between in itself and for itself alluding to subjectivity or self-reflection famously utilized by Hegel) – and not merely in relation to others (*per alia*) – with the "in itself" alluding to independence, or even separation. The second one further qualifies a role of a subject (ὑποκειμένον): every form, every relational structure presupposes something to be structured that way – and this can hardly be a structure itself, else we fall into an infinite regress: we need some "content" to the form. What is not a structure, what is opposed to every structure is ultimately simple.

But all of this seems to be exactly what is searched for in the notion of substance – or at least what Leibniz himself must have been searching in that notion. How does it come

[107] He mentions Lotze, Windelband, Heimsoeth, Joseph and Russel. It is somehow curious that the more contemporary authors do not seem to be bothered by those difficulties, even though solutions offered in later literature, including Mates (1986), are far from satisfactory, as will be shown in the subsequent.

[108] See: Aristotle (1984), Categories, 2 (1a20)

[109] See: Aristotle (1984), Physics, I, 8 (191a31)

[110] In Leibniz's system all the real mutual determination of the ultimate beings boils down to universally unqualified mutual representation, because other modes of interaction can be conceived of as qualified representations relative to some independent external point of view, which is excluded as a mere abstraction, since all and only metaphysical points of view are the ultimate beings themselves. See: 3.1.2.

that Leibniz's account manifestly betrays precisely those criteria which it supposedly sprang from?

Mates understands the difficulty as a logical paradox and offers "a numerical model of the possible worlds"[111]. He proceeds in two steps: first he comes up with a formalisation which assigns a real number greater than 1 to every single state within a continuous series of monadic states of every one of the infinitude of monads in every one of the infinitude of possible worlds[112]; and then he associates a set of simple properties or their negations to each of those numbers by associating every position of a nonterminating binary expansion of its reciprocal one member of a denumerable set of the simple properties and deciding if the monadic state has the property or not, based on whether 1 or 0 occupies the associated position. Correspondingly, every complete individual concept consists of a continuous series of ordered sets of simple properties – each of those sets standing for one monadic state. Moreover, with the complete knowledge of one single monadic state of an arbitrary monad in an arbitrary world (i. e. with the knowledge which one of the infinite set of simple properties is to be ascribed and which one denied to the state, with the knowledge of the order of those properties and with the knowledge of all the relevant formulas) one is capable of computing a real number assigned to this particular state and therewith real numbers assigned to all the other monadic states of all the monads in all the worlds and determining their sets of properties – in this sense every monadic state mirrors not only all the past and future states of a particular monad, but past and future states of all the monads in this world as well as in all the other possible worlds[113].

The outcome of his effort should be the proof "that there is no inconsistency in the mirroring aspect of Leibniz's monadology". This addresses only a fraction of the problem. Although it is true that some commentators worried about a circularity of Leibniz's description indicating that, since all the monads do is expressing one another's expressing one another, alongside with an ultimate content of those expressions the real difference between them is missing as well, the issue is not purely logical. Mates shows that a model can be found for the description – proving therewith its consistency. Insofar as the model is a mere mathematical structure it is purely formal and lacks any content by definition. If he

[111] See: Mates (1986, pp. 80-83)

[112] The corresponding formula for an ascending series of real numbers *r* representing successive monadic states of the *p*th monad in the *n*th world is: $p + (1 - 1/2^{n-1}) \leq r < p + (1 - 1/2^{n})$.

[113] If this is considered too much, the remedy will be no big deal: it suffices, for example, to come up with a unique formula ascribing real line positions to monads for every possible world, so that you can deduce monadic states in other worlds from a single state in this one only at the proviso of knowing the particular "laws" governing that world. Mates mentions other features of monadology not represented in his model, such as the directedness of time and clarity of perceptions, but this deficiency is not essential at this point.

had confined his enterprise to the first step solely, all of it would be isomorphic: possible worlds, monads and their states, all of this would be differentiated only by their arbitrarily chosen position on the presupposed real line structure – there would be structural difference between monadic states, which are points, and monads or possible worlds, which are line segments, but there would be nothing "in the states themselves" to differentiate them from one another (else we reify the real line itself and make it into the ultimate substance, half actual, half potential, of all worlds – a rather peculiar idea), since the assignment of a monadic state to a real number is arbitrary and could have been constructed equally well otherwise.

Leibniz had, of course, no idea of the arithmetization of the continuum pioneered by Dedekind and Cantor and so he could not have come up with a model like this, nevertheless he actually considered the case of differentiating line segments by their respective positions on a geometrical line and his remarks are quite illuminating: "Such concepts men can easily imagine to be diverse without diversity – for example, two equal parts of a straight line, since the straight line is something incomplete and abstract, which needs to be considered only in theory. But in nature every straight line is distinguishable by its contents from every other." (L, 529) The sole abstract structure, the form of geometrical line, does not grant individuality to the segments into which it can be cut, it is only in virtue of its varying content, once it is reified (realized, materialized), that those segments differ and do not conflate into one another.

However, Mates does not stop here and proceeds further to let the positions on the real line uniquely refer to infinite sets of properties. The real line does not have to be reified in order to render the monadic states different, because those are differentiated in virtue of having variant properties. The identity of monadic states is constituted in singular reference to sets of simple properties; by contrast, the simple properties are genuinely differentiated in themselves, thanks to their identity being qualitative. But this is not the end of the story, because the ascription of consecutive positions of the binary expansions of reciprocals of reals greater than 1 to simple properties cannot be unequivocally defined without those properties being already not only gathered in a set, but ordered as well – otherwise, there would be no chance to determine which property is to be associated with which position on a string of zeros and ones. The ultimate content of all the mirroring, the final non-relational differentiating signs genuinely distinct in themselves, the fundamental ontological level consist of qualitatively differentiated properties and an independent ω-sequence structure, granting them not the identity, which they already have in virtue of their qualitative difference, but the total order. It is no wonder that with those resources the arithmetized continuum may be reconstructed – and therewith the mirroring structure sketched above – but without the ordering, it would hardly be conceivable.

3.2.2 The fundamental level. It may be doubted that this must necessarily be the fundamental ontological level, despite being the ultimate content of mirroring and despite being the ground level of non-relational differentiating signs – after all being a differentiating sign as well as being content pertain to the realm of representation, which may be understood as merely conceptual, whereas the ontology may be understood as transcendent. Such a scheme would render only the determination of a monad being derived from its properties but not its being. This sounds utterly non-leibnizian, but even granted this, it would hardly offer any ground for localizing the ultimate source of reality within the system in monads: there are properties, their order, monadic states, monads, possible worlds, the structure of the entire continuum, on which everything is modelled – why would the reality enter into the scheme exactly at the level of monads?

If we exclude the realm of ordered simple properties, there still remain monadic states as the most plausible candidates for fundamental elements of reality[114]; or we can conclude that, since the fundamental elements are to be inherently structured in either case, there is no reason to presuppose as ultimately real a plurality (being an independent structure external to the in themselves individuated elements) of individual monads, rather than a single universe responsible for the reality of the monads gathered in it, yet structurally isomorphic to them – this being the most parsimonious ontology, jettisoning the superfluous structure of the plurality interrelating individual monads. The exclusion of monadic states might very well be justified on grounds that there is a structural difference between the way in which monadic states compose a monad and between the way in which individual monads compose the universe: because monadic states form a continuous series, the monad cannot be actually divided into them, in Leibniz's conception of continuity, and does not "result" from them, but rather the other way round – those states being merely potential results of a division which is conceived of as ideal, carried out merely in thought; whereas the universe, on the contrary, consist of a discrete manifold of monads and is therefore actually divided into them. However, it should be noted that (i) Mates' model does not explicitly favour this option, since the monadic composition of each possible world – as well as all of them taken together – is no less continuous than the composition of an individual monad, and (ii) the very continuity of monadic states is itself a controversial interpretive issue[115]. Furthermore, this does not yet give the plurality of monads any priority over the universe as a whole. Since every monad mirrors the entire universe, it retains in itself the structure of composition of all the monads (i. e. is isomorphic to the world) and if

[114] Which is the answer Mates in the end, contrary to Leibniz's explicit remarks, apparently opts for. However, we rejected any asymmetry between monadic states and monadic locations in 3.1.2. and Mates (1986) preoccupation with monadic states seems to us simply flawed. For a discussion see Whipple (2010).

[115] See: Whipple (2010).

the discreteness of the manifoldness of monads suffice for the universe to be actually divided into them, why would it be insufficient to render the monad's representation of the universe to be actually divided into representations of the individual monads (and those again into their representations of the individual monads etc. ad infinitum)? This undermines not only the thesis of simplicity of the monad, but the thesis of plurality of substances as well: if structural reasons led to the exclusion of a monadic state from the list of possible candidates for substance, they cannot settle a tie between the monad and the world – or rather, they decide it in favour of the world for the sake of ontological parsimony.

Other models have been devised, which treat monadic states on par with monadic locations, adding even more candidates to the list of equivalently eligible fundamental elements of the ontology. While Anapolitanos (1999) in his book-length effort develops only a topological description of four-dimensional manifold of space-time, Grene and Ravetz (1962), 37 years earlier, in their succinct article putted down a model of Leibnizian universe as a set of differential equations. Their approach is both challenging and yet pretty concordant with Leibniz's original intentions[116]. Nevertheless, they stick with a sole formal structure and consequently they cannot really address the issue of why should the reality enter into it exactly at the level of monads, if any set of values, which makes up a monad, can be computed given only one value of whichever equation you choose and the correspondent equations.

3.2.3 The second riddle: What could be a ground level in a self-similar structure? Self-similarity is a feature of a structure alone and ordinarily entails replication of the same pattern at different scales. But Leibnizian universe is not to be a pure structure: there is to be some content – delivering non-relational inherent diversity – if it was not for this content, monads would be only numerically distinct and the structure of the universe would be not only perfectly self-similar across the various levels of representation[117], but on the same level as well (and thus, monads would be indiscernible from one another). Where by levels of representation I mean this: an individual monad represents the entire universe as a multiplicity of a specific structure (relative to its particular point of view) – this is the first

[116] We have seen in 3.1.3. that Leibniz himself sometimes treated monad as a function with its individual discriminating signs as its values. But we have already seen as well, that those discriminating signs cannot be positions in a universally applicable coordinate system – as Grene and Ravetz (1962) would have it.

[117] It should be noted that Leibniz himself often in his multiple illustrations or examples of the mirroring structure cashes out those levels of self-similarity in terms of spatial scaling – as pointed out in 3.1.4. (e. g. "a machine inside a machine"). But this is misleading, since at least temporal dimension is needed as well – and even this would not suffice. All such illustrative scenarios are nothing but incomplete idealisations. For a detailed discussion see the entire section 3.1.

level of representation (a representation of "plurality in the simple"[118]); but the multiplicity in question is a multiplicity of monads, which are representations of the universe from their respective perspectives, i. e. multiplicities of specific structures relative to their particular points of view – this is the second level of representation (a representation of representations of "plurality in the simple"); and so on ad infinitum. In the case of self-similarity on a single level, a monad would represent multiplicity of multiplicities identical to it, while in the case of self-similarity across the levels, a monad would represent multiplicity of multiplicities distinct from it and distinct from one another, yet each of them consisting of all but one of the multiplicities represented by the original monad. All the monads are represented again and again on various levels, whereas their multiplicity gets multiplied on every next level once more. The structure of their multiplicity alternates, individuating the monads on different levels in manifold ways, but if the same monad was identified on a different level, it would exhibit the same intrinsic structure – hence self-similarity across the levels of representation. (However, needless to say, such identification would require infinite analysis and thus is not feasible for a finite mind.)

Though there is an infinite regress of determination inherent to the structure, the supposition of an intrinsically diversified content can be made to the effect of saving distinct monads from conflating into a single one. This has already been testified by our investigation of Mates' model. Moreover, Mates used the real line as a convenient tool to prove consistency of Leibniz's mirroring structure, without putting much emphasis on the peculiar coincidence that Leibniz himself considered his theory of monad as a way out of the labyrinth of the continuum[119]; now, we can clearly see why could he be actually more successful in this enterprise than usually admitted, foreshadowing the arithmetization of the continuum, which was to come nearly two centuries later: it suffices to posit countably many monads for their representations to boast a structure of an uncountable complexity – since on the ω-level of representation a representation of a single monad comprises $m^{\aleph_0}$ elements, where m indicates the number of monads (Leibniz seems to demand infinitely many of them, but generally any countable number $m \geq 2$ suffices: $2^{\aleph_0} = \aleph_1$). Furthermore, it is clear that we reach the cardinality of the continuum only at the farthest level of representation (i. e. phenomena – which are by definition representations of something ultimately real), whereas on the level of ontology, there are only countably many substances.

It is hard to evaluate the outcome from Leibniz's perspective, since Leibniz would most probably oppose the arithmetization of the continuum, and since he did not recognize the dichotomy of the countable and the uncountable, utilizing only the one between the

[118] L, 662.

[119] See: L, 343, 604.

finite and the infinite. However consider the case he made for denying reality to infinite number.

3.2.4 Actual infinity. Various commentators felt puzzled by Leibniz endorsing actual infinity of monads, while denying infinite number, or an infinite whole. It is not the case that Leibniz would consider all infinities to be merely potential (merely potential infinity is a false infinity: at every single step being only finite – if augmented with a prospect of further extension), rather he considers the infinity of monads to be complete and fully realized[120]. Any number whatsoever is an abstraction, nothing real, only an incomplete notion, which by the very fact of its incompleteness can be fully determined in language, or generally: have a fully determinate representation – if this is always merely relative, depending on a particular conceptual scheme and cannot account for all the difference relations to everything there is. But this is not be the case with the infinity, that is by definition non-terminating, encompassing either everything there is, or everything within a particular selection, which cannot be specified from within – by way of determining its individual constituents, which no finite mind can accomplish – but rather only from outside – by way of carving it out of its exterior: which is in turn once again non-terminating and cannot be gathered by a finite mind. Infinity lacks the "incompleteness" of abstract notions, which would enable it to be represented in full: it always requires a grasp of a certain realm in its entirety. Whereas a finite whole is merely a unity by aggregation, i. e. its unity is determined by extrinsic aggregation of intrinsic unities, an infinite whole is only a fictional whole, as Harmer (2014) put it, whose unity is determined neither in itself, nor by extrinsic aggregation of intrinsic unities. If the infinity lacks determinate unity, its constituents lack determinate numerical identity, and their plurality cannot be countable. There are, of course, countable infinities – such as the infinity of natural numbers – however, those are not actual infinities – as is the infinity of monads – but rather merely potential infinities: the infinity of natural numbers is indeed countable, composed by aggregation of determinate unities, yet a natural number is merely finite at every single step of the aggregating process, which only potentially

[120] This is how both Russell (1900) and Harmer (2014) understood Leibniz's position finding it consequent and justified, though Russell reportedly (Lison, 2006) denounced Leibniz's critique of infinite number later on, as did other commentators (see: Lison, 2006), based on his conflating two methods of measuring magnitudes later elaborated by Cantor (i. e. comparing distances between extreme members, or establishing one-to-one correspondence between all intermediary members). Lison (2006) does not address the issue of compatibility of Leibniz's rejection of infinite number and actual infinity of monads, but shows that his arguments against the infinite overlap with his arguments against the continuous (the diagonal paradox) and crucially depend on Euclidean part-whole axiom, which can be either accepted, or rejected, – and Leibniz actually considered the distinction sketched above, disposing with the axiom and foreshadowing Cantor's position, only to dismiss it few years later – and so his position is at least logically equivalent to Cantors.

approaches the infinity as its vanishing point. And so if the infinity is to be actual, it must be indeterminate in the aforementioned sense.

The claim that there is actual infinity of monads, thus, virtually amounts to the claim that their number is indeterminate[121], that there is no determinate number of them on the most fundamental (i. e. immediate) ontological level. Remember that we have stated in 2.5 that there is no independently determinable plurality of monads, there is no immediate, or objective, number of them: the structure of their multiplicity is always merely subjective, mediated by reflection into a particular monad. What is peculiar to the model sketched above is the fact that strictly speaking we do not need infinity of monads in order for them to yield a plurality of the cardinality of continuum: any countable number ($m \geq 2$) suffices. Regardless of the precise number of monads the cardinality of the multiplicity of their representations equals $\aleph_1$.

3.2.4 The model. This is the best we can do to make the monadic structure explicit. The immediate structure[122] of monads is completely indeterminate; however, the structure of monads as mediated by its reflection[123] into a particular monad finds its determination as intrinsic self-differentiation of this particular monad. Therefore, there is a multiplicity of relative structuring of the multiplicity of monads – which is this multiplicity itself. We have seen in 1.3 that there is not a single prominent structure (as for example that of natural numbers) and so the question of the particular structure of the multiplicity of monads does not boil down to the question of whether there are three or five monads (e. g. that from the point of view of a monad A there are three of them and from the point of view of a monad B there are five of them), rather a particular type of a structure carries with itself particular principles of identification and differentiation of its elements (i. e. the principles of how are the elements of the structure to be gathered together or how are they to be individuated within the structure – as discussed in 1.2) and so there is no external independent measure by which we could compare the particular representations of the multiplicity among themselves and tell: this one is bigger than this one, this one includes this one as its part, or this one is isomorphic to that one. The representations of the multiplicity are heterogeneous to one another, because every monad is completely singular, unique, establishing its own identity by itself – and yet, as "representations of multiplicity in the simple", they are the

[121] Bassler (1998) explores Leibniz's notion of syncategoremaic infinite as indefinite. Both Harmer (2014) and Bassler (1998) agree that Leibniz rejects the infinite in categorematic sense, while endorsing the infinite in syncategorematic sense (i. e. in a negative sense – in-finite: there is always a next one), and while limiting the infinite in „hypercategorematic" sense to God alone. Leibniz exposes his mature view at length in: NE, 161-164.

[122] The term immediate structure has been introduced in 1.2.

[123] The correspondent notion of a structure mediated by semantic reference has been introduced in 1.3.

very principles of homogeneity, they are the only determinate structures there are in the world. The very same monads could in principle be identified in all the monadic representations, but none of the monadic representations considered in itself can account for such identification, since the discerning capacities of monads are merely finite – only God could do this: as if to count the uncountable. From whichever point of view (point of view of a particular monad) you look at the multiplicity of monads, it suffices for their structure to be determinate and somehow at least dual ($m \geq 2$) – which should be construed as having at least some principle of diversification, or variety, within itself, aside from the intrinsic principle of unity – for its representation to boast the cardinality of continuum on the ω-th level. However, the assumption of well-ordered levels of representation should itself be viewed as problematic – after all taking it immediately for granted would commit us to acknowledging an extrinsically determinable ω-structure posited independently and prior to the monadic structure. Two concerns hinder the immediate commitment: (i) as shown in 1.3, the transition from "the former and the latter" to "the first and the second" is not at all straightforward, (ii) since there is no universal means of identification of monads across the levels, there is no universal mean of individuating the levels of representation either. On the other hand, if we reject the assumption of well-orderedness, could we still fancy the idea of approaching the cardinality of continuum in the analysis of the representation? Definitely not; Leibniz himself did not arithmetized the continuum – and the question lets itself be posed the other way round as well: if we could take for granted the assumption of well-orderedness of the analysis, we could in principle arithmetize the continuum, but since the identification of the levels of representation requires identification of its elements across the levels, which could not be established before the end of the infinite analysis, which is beyond our reach, the assumption will always remain unjustified for a finite mind – and correspondingly the continuum irreducible to the arithmetical structure.

AFTERWORD

So far we have explored all the essential features of the monadic structure as pertaining to semantics and ontology. I hope that I have satisfactorily shown that this is indeed the simplest conceivable structure; and, as I believe, every post-leibnizian attempt at fundamental ontology, as well as generally every philosophical enterprise concerned with delimiting some ultimate foundations (be they of mathematics, logic, semantic of natural languages or whatever) should face the challenge posed by the expounded deliberations outright.

I will not venture into any historical speculation, but one can at least vividly imagine that it was precisely Leibniz's lifetime struggle for devising something like "universal characteristic", which eventually led him to those very limits of conceivability, as embodied in the concept of monad, before which he stood astonished and stupefied, reluctant to fully acknowledge that his lifetime dream may well get completely shattered by the encounter. A different thesis with a vastly different approach to the issue would be needed in order to explore a source and nature of those limits of thinking: what does it mean that I share them with somebody who died more than 300 years ago, does it have something to do with the fact that my philosophical education consist of reading texts such as his and the like? I do not feel hard-pressed to address such perplexities in this paper, as I will content myself with the ascertainment that for one reason or another, after all, we indeed experience those limits: we encountered them in the course of the reasoning just undertaken – as various commentators of Leibniz's texts have encountered them already before.

In acknowledging this we do not need to make any far reaching commitments: all we need is to believe that philosophical interpretation is possible – that it is possible to "follow someone's thinking". I do not know if we share the same logical principles, the same inferential rules, I do not know if we share the same syntax, or the same meanings – and I am generally more or less sceptical about all of this – but if anything philosophers have ever done has to have any meaning at all, it must be possible to "follow" the ideas of others, there must be some principle of limitation to the freewheeling machine of thinking which guides its course from within so that we may eventually say: I know where is it going – and it must be so, I cannot think of it otherwise.

Regarding the issues of variety or plurality in a system, determinate difference and identity, multiplicities of all kind, the simplest conceivable structure is one such limit, or guideline of thinking. I do not think it must necessarily be some transcendental condition in a Kantian sense – rather, it is something hopefully each of us has in his peculiar way experienced in the course of the present reasoning, something which Leibniz's texts helped us disclose in our own thinking – and something which reflexively shed new light on Leibniz's texts.

BIBLIOGRAPHY

[L] Leibniz, Gottfried Wilhelm. *Philosophical Papers and Letters*. Edited and translated by Leroy E. Loemker. 2d ed., Dordrect: D. Reidel, 1969.

[NE] Leibniz, Gottfried Wilhelm. *New Essays Concerning Human Understanding*. Translated by Langley, Alfred G., Open Court, 1896.

[Th] Leibniz, Gottfried Wilhelm. *Theodicy: Essays on the Goodness of God, the Freedom on Man and the Origin of Evil*. Translated by E. M. Huggard. La Salle, IL: Open Court, 1985.

[G] Leibniz, Gottfried Wilhelm. *Die philosophischen Schriften*. 7 vols. Edited by C. I. Gerhardt. Berlin, 1875–90. Reprint, Hildesheim: Georg Olms, 1965.

Adams, Robert Merrihew (1999). *Leibniz: Determinist, Theist, Idealist*. Oup Usa.

Anapolitanos, Dionysios A. (1999). Leibniz: Representation, Continuity and the Spatiotemporal. *Studia Leibnitiana* 31 (2):215-217.

Aristotle, Jonathan ; Barnes, J. A. ; Smith, W. D. & Ross, (1984). *The Complete Works of Aristotle the Revised Oxford Translation /Edited by Jonathan Barnes.* --.

Bassler, O. Bradley (1998). Leibniz on the Indefinite as Infinite. *Review of Metaphysics* 51 (4):849 - 874.

Bar-Elli, Gilead (1982). Identity and the Formation of the Notion of Object. Or: The Identity of Indiscernibles: A Synthetic a Priori. *Erkenntnis* 17 (2):229 - 248.

Bell, John L., "Continuity and Infinitesimals", *The Stanford Encyclopedia of Philosophy* (Summer 2017 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/sum2017/entries/continuity/>.

Bergson, Henri (1913). *Time and Free Will: An Essay on the Immediate Data of Consciousness*. Dover Publications.

Black, Max (1952). The identity of indiscernibles. *Mind* 61 (242):153-164.

Boolos, George (1984). To be is to be a value of a variable (or to be some values of some variables). *Journal of Philosophy* 81 (8):430-449.

Cartwright, Helen Morris (1993). On plural reference and elementary set theory. *Synthese* 96 (2):201 - 254.

Cook, Roy T. (2000). Monads and Mathematics: The Logic of Leibniz's Mereology. *Studia Leibnitiana* 32 (1):1 - 20.

Cotnoir, Aaron (2013). Strange Parts: The Metaphysics of Non-classical Mereologies. *Philosophy Compass* 8 (9):834-845.

Forrest, Peter (2016). "The Identity of Indiscernibles", *The Stanford Encyclopedia of Philosophy* (Winter 2016 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/win2016/entries/identity-indiscernible/>.

Gasser, Marc (2015). Structuralism and Its Ontology. *Ergo: An Open Access Journal of Philosophy* 2:1-26.

Grene, M. & Ravetz, J. R. (1962). Leibniz's cosmic equation: A reconstruction. *Journal of Philosophy* 59 (6):141-146.

Gruszczyński, Rafał & Varzi, Achille C. (2015). Mereology then and now. *Logic and Logical Philosophy* 24 (4):409.

Hacking, Ian (1975). The identity of indiscernibles. *Journal of Philosophy* 72 (9):249-256.

Harmer, Adam (2014). Leibniz on Infinite Numbers, Infinite Wholes, and Composite Substances. *British Journal for the History of Philosophy* 22 (2):236-259.

Hawley, Katherine (2014). Ontological Innocence. In A. J. Cotnoir & Donald L. M. Baxter (eds.), *Composition as Identity*. Oxford University Press. pp. 70-89.

Hegel, Georg Wilhelm Friedrich (2010). The Science of Logic. Di Giovanni, George (ed.). Cambridge University Press.

Hellman, Geoffrey (2007). *Structuralism.* in *The Oxford Handbook of Philosophy of Mathematics and Logic*, Oxford: University Press.

Hodges, Wilfrid (2013). "Model Theory", *The Stanford Encyclopedia of Philosophy* (Fall 2013 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/fall2013/entries/model-theory/>.

Kant, Immanuel (1998). Critique of Pure Reason. Guyer, P., and Wood, A. (eds.). Cambridge: Cambridge University Press.

Ketland, Jeffrey (2006). Structuralism and the identity of indiscernibles. *Analysis* 66 (4):303–315.

Koslicki, Kathrin (2008). *The Structure of Objects.* Oxford: Oxford University Press.

Laycock, Henry (2010). "Object", *The Stanford Encyclopedia of Philosophy* (Winter 2014 Edition), Edward N. Zalta (ed.), URL =
<https://plato.stanford.edu/archives/win2014/entries/object/>.

Laycock, Henry (2006). *Words without Objects.* Oxford: Oxford University Press.

Laycock, Henry (2004). “Variables, Generality and Existence”, in Paolo Valore (ed.), *Topics on General and Formal Ontology*, Milano: Polimetrica, 27–52.

Lewis, D., 1993, ‘Mathematics Is Megethology’, *Philosophia Mathematica*, 3: 3–23.

Linnebo, Ø. (2017). "Plural Quantification", *The Stanford Encyclopedia of Philosophy* (Summer 2017 Edition), Edward N. Zalta (ed.), forthcoming URL =
<https://plato.stanford.edu/archives/sum2017/entries/plural-quant/>.

Lison, Elad (2006). The Philosophical Assumptions Underlying Leibniz's Use of the Diagonal Paradox in 1672. *Studia Leibnitiana* 38 (2):197 - 208.

Lodge, Paul & Puryear, Stephen (2006). Unconscious Conceiving and Leibniz's Argument for Primitive Concepts. *Studia Leibnitiana* 38 (2):177-196.

Look, Brandon C. (2017). "Gottfried Wilhelm Leibniz", *The Stanford Encyclopedia of Philosophy* (Summer 2017 Edition), Edward N. Zalta (ed.), URL =
<https://plato.stanford.edu/archives/sum2017/entries/leibniz/>.

Look, Brandon (2002). On monadic domination in Leibniz's metaphysics. *British Journal for the History of Philosophy* 10 (3):379 – 399.

Mates, Benson (1986). *The Philosophy of Leibniz: Metaphysics and Language*. Oxford: Oxford University Press.

Mugnai, Massimo (2001). Leibniz on Individuation: From the Early Years to the "Discourse" and Beyond. *Studia Leibnitiana* 33 (1):36 - 54.

Nichols, Ryan (1999). Space, Individuation and the Identity of Indiscernibles: Leibniz's Triumph Over Strawson. *Studia Leibnitiana* 31 (2):181-195.

O'Connor, D. J. (1953). The Identity of Indiscernibles. *Analysis* 14 (5):103 - 110.

Palkoska, Jan (2010). *Substance and Intelligibility in Leibniz's Metaphysics*. Franz Steiner.

Quine, Willard V. (1948). On What There Is. *Review of Metaphysics* 2 (1):21-38.

Quine, Willard V. (1969). Ontological Relativity and Other Essays. New York, Columbia University Press.

Rescher, Nicholas (1955). The identity of indiscernibles: A reinterpretation. *Journal of Philosophy* 52 (6):152-155.

Rodriguez-Pereyra, Gonzalo (1999). Leibniz's Argument for the Identity of Indiscernibles in his Correspondence with Clarke. *Australasian Journal of Philosophy* 77 (4):429 – 438.

Rodriguez-Pereyra, Gonzalo (2012). Leibniz's Argument for the Identity of Indiscernibles in his Letter to Casati. *The Leibniz Review* 22:137-150.

Russell, Bertrand (1900). *A Critical Exposition of the Philosophy of Leibniz*, Cambridge: Cambridge University Press.

Varzi, Achille (2003). "Mereology", *The Stanford Encyclopedia of Philosophy* (Winter 2016 Edition), Edward N. Zalta (ed.), URL =
<https://plato.stanford.edu/archives/win2016/entries/mereology/>.

Whipple, John (2010). The Structure of Leibnizian Simple Substances. *British Journal for the History of Philosophy* 18 (3):379-410.

Whitehead, Alfred North & Russell, Bertrand (1963). *Principia Mathematica by Alfred North Whitehead and Bertrand Russell*. University Press.

Zalta, Edward N. (2000). A (leibnizian) theory of concepts. *Logical Analysis and History of Philosophy* 3:137-183.