

# Posudek práce

předložené na Matematicko-fyzikální fakultě  
Univerzity Karlovy

- posudek vedoucího     posudek oponenta  
 bakalářské práce     diplomové práce

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Název práce: Twistor equation on isolated horizons  
Studijní program a obor: Fyzika, Teoretická fyzika  
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## Odborná úroveň práce:

- vynikající     velmi dobrá     průměrná     podprůměrná     nevyhovující

## Věcné chyby:

- téměř žádné     vzhledem k rozsahu přiměřený počet     méně podstatné četné     závažné

## Výsledky:

- originální     původní i převzaté     netriviální kompilace     citované z literatury     opsané

## Rozsah práce:

- veliký     standardní     dostatečný     nedostatečný

## Grafická, jazyková a formální úroveň:

- vynikající     velmi dobrá     průměrná     podprůměrná     nevyhovující

## Tiskové chyby:

- téměř žádné     vzhledem k rozsahu a tématu přiměřený počet     četné

## Celková úroveň práce:

- vynikající     velmi dobrá     průměrná     podprůměrná     nevyhovující

## Slovní vyjádření, komentáře a připomínky vedoucího:

Isolated horizons represent a wide class of spacetimes containing black holes, but compared to more usual event horizons, these spacetimes allow for the presence of the matter or radiation outside the horizon. These properties make them an excellent tool to describe realistic astrophysical black holes. In asymptotically flat spacetimes there is a well-defined notion of Geroch-Hansen multipole moments at spatial infinity which can be used to reconstruct the spacetime geometry. For isolated horizons, however, there is a possibility to define (Ashtekar) multipole moments which are evaluated as quasi-local integrals on the horizon and, hence, they do not determine the full spacetime geometry but they encode all properties of a black hole. In particular, it is possible to define the mass and the angular momentum of a black hole.

On the other hand, there are many independent attempts to define quantities like energy or angular momentum quasi-locally in general spacetimes. It seems natural to compare the multipole moments in the sense of Ashtekar to corresponding quasi-local quantities. For this thesis, the so-called Penrose mass has been chosen as a reference point, i.e. the goal of the thesis was to investigate the relation between the Penrose mass and the mass of a black hole described by an isolated horizon.

The essential object necessary to construct the Penrose mass is a solution of the twistor equation. The twistor equation does not have a solution in general spacetime and the usual argument to show this is based on the Atiyah-Singer index theorem. Part of the thesis was to investigate the integrability conditions for twistor equation for isolated horizons explicitly in the Newman-Penrose formalism, in order to get a better understanding of the obstacles for solving the twistor equation. The author of the thesis analyzed these integrability conditions in detail for extremal and non-extremal horizons.

As a “by-product”, in the thesis it is shown that despite the generality of the notion of an isolated horizon, extremal horizons are pretty unique. In the earlier work by Lewandowski and Pawłowski it was shown that any regular extremal horizon is isometric to the Kerr-Newman horizon. We generalized this result by relaxing the assumption of regularity and shown that allowing for the conical singularities on the poles of axially symmetric horizon one obtains the Plebański-Demiański class of solutions (including the acceleration and NUT parameter). This is the most valuable result of the thesis and will be published in Physical Reviews D journal.

Finally, the author gives a detailed description of isolated horizons and their multipole moments, providing also some non-trivial details that are missing in the literature. He introduces the concept of the Penrose mass using the spinor formalism and gives the expression for the Penrose charge integral for isolated horizons. This part of the thesis is not finished: it turned out that the straightforward application of the Penrose procedure leads to inconsistent results and the origin of these difficulties is unclear so far; it will be a matter of further investigations.

To summarize, the thesis deals with highly non-trivial subject and the author shown a good understanding of advanced topics in differential geometry and spinor formalism, significant creativity and ability to work independently on given problem. The text of the thesis contains quite a few typos (like missing citations) but generally is on a very good scientific level. The thesis contains many auxiliary results and the proof of the uniqueness of extremal horizons with conical singularities, which is an important result. For these reasons, I **recommend** to accept this diploma thesis.

### Práci:

- doporučuji  
 nedoporučuji

uznat jako diplomovou.

### Navrhuj hodnocení stupněm:

- výborně    velmi dobře    dobře    neprospěl

Místo, datum a podpis vedoucího:

Martin Scholtz, Praha, 3. září 2018