In the present thesis we provide compact extended formulations for a wide range of polytopes associated with the constraint satisfaction problem (CSP), monadic second order logic (MSO) on graphs, and extensions of MSO, when the given instances have bounded treewidth. We show that our extended formulations have additional useful properties, and we uncover connections between MSO and CSP. We conclude that a combination of the MSO logic, CSP and geometry provides an extensible framework for the design of compact extended formulations and parameterized algorithms for graphs of bounded treewidth.

Putting our framework to use, we settle the parameterized complexity landscape for various extensions of MSO when parameterized by two important graph width parameters, namely treewidth and neighborhood diversity. We discover that the (non)linearity of the MSO extension determines the difference between fixedparameter tractability and intractability when parameterized by neighborhood diversity.

Finally, we study shifted combinatorial optimization, a new nonlinear optimization framework generalizing standard combinatorial optimization, and provide initial findings from the perspective of parameterized complexity