

Abstract: In the following thesis we will be mostly concerned with questions related to the regularity of solutions to non-linear elasticity models in the calculus of variations. An important step in this question is the approximation of Sobolev homeomorphisms by diffeomorphisms. We refine an approximation result of Hencl and Pratelli's which, for a given planar Sobolev (or Sobolev-Orlicz) homeomorphism, constructs a diffeomorphism arbitrarily close to the original map in uniform convergence and in terms of the Sobolev-Orlicz norm. Further we show, in dimension 4 or higher, that such an approximation result cannot hold in Sobolev spaces $W^{1,p}$ where p is too small by constructing a sense-preserving homeomorphism with Jacobian negative on a set of positive measure.

The class of mappings referred to as mappings of finite distortion have been proposed as possible models for deformations of bodies in non-linear elasticity. In this context a key property is their continuity. We show, by counter-example, the surprising sharpness of the modulus of continuity with respect to the integrability of the distortion function. Also we prove an optimal regularity result for the inverse of a bi-Lipschitz Sobolev map in $W^{k,p}$ and composition of Lipschitz maps in $W^{k,p}$ comparable with the classical inverse mapping theorem. As a consequence we retrieve a Sobolev equivalent of the implicit function theorem.