Report on the doctoral dissertation thesis

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TITLE: Multivariate stochastic dominance and its application in portfolio optimization problems
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Barbora Petrova's doctoral dissertation thesis deals the multivariate first stochastic dominance order and some possible applications in portfolio theory. In the introduction, the thesis recalls the status quo of the topic and the aim of the dissertation. In the first chapter, it is introduced a multistage portfolio problem where investors maximize the utility of the wealth sample path controlling the risk exposure by proper constraints on the multivariate risk premium. The second chapter discusses multivariate stochastic dominance for economic agents who prefer more to less (i.e. non-satiable agents). In particular, the author firstly examines the relationships between the first order multivariate stochastic dominance, the linear stochastic dominance and orthant stochastic dominance. Secondly, the author evaluates the relationships among stochastic orders and multivariate risk premiums. The third chapter discusses multivariate orderings for discrete and continuous uniform distributions and then it extends the ordering analysis to multivariate Gaussian distributions. The Chapter fourth proposes some empirical applications in portfolio multiperiod and multistage problems. Finally, in the conclusion Barbora Petrova summarizes the main ideas and discusses few possible developments of her work.
Since the thesis not only addresses some of the classical approaches but also introduces some innovative and original ideas, formulations and methods, I have no doubt that the dissertation thesis fulfils the requirements for a doctoral dissertation thesis. Therefore, I **recommend** Barbora Petrovà's **thesis for the final defence**.

Comments and questions:

1. I believe that the most innovative part of the thesis is the one linked to the multivariate risk premium and the proposed multivariate applications in portfolio problems. Unfortunately, some of them are not clear, maybe some additional assumptions are missing:
a) When it is defined the risk premium and its relationship with utility, it is not defined in a proper way the dependence of the risk premium with respect to the used utility function (see pages 41, 42 Theorems 13, 14 and Proposition 15). Thus, that definition can lead some misunderstanding problems for the reader.
b) When it is evaluated the multivariate risk premium for some useful utility functions, it is fundamental to point out the multivariate distribution used for the set of returns, because different return distributions imply different risk premiums. In the thesis this point is completely missing. Thus, I believe that you are using a multivariate Gaussian distribution to compute risk premium functions of pages 19, 20 (formulas 1.21, 1.22, 1.23). Is it true ? Furthermore, it could be interesting evaluate the impact of different distributional assumptions (with heavy tails and skewness according to joint asset returns behaviour) on the multivariate risk premium.
c) I believe that alternative portfolio problems could be proposed considering the minimization of risk premium functions with constraints on the investors' expected utility. Have you try some experiments with this alternative portfolio problem?

2. In the several proposed empirical comparisons the author often uses some scenarios without dealing the distributional aspect (only in few cases referring to some other works for this topic). I believe that dealing this aspect more deeply (at least once) in the thesis could help the reader, even considering that there are several fundamental works on the topic.

3. The alternative hypothesis H_1 proposed for the test at page 61 is not the complementary of hypothesis H_0 because you are working in a multivariate framework. Probably you should specify better your alternative hypothesis that I believe must be $H_1 = \cup_n {H_1}^n$.

4. Since univariate first order stochastic dominance implies arbitrage opportunities (see Jarrow), clearly multivariate first order stochastic dominance implies the multivariate linear stochastic dominance and thus it implies arbitrage opportunities among several portfolios. On the one hand, it is not easy to find first order univariate stochastic dominance between single portfolios of returns and it is much more difficult determine the multivariate first order stochastic dominance. In particular, I believe that the presence of this dominance justifies some momentum strategies and it appears more frequently in multiperiod (related to specific periods) and multistage framework, as also suggested by your work. Clearly your study could be also important to identify some momentum strategies. Have you think about this alternative aspect? On the other hand, the difficulty to determine this stochastic dominance relationship has moved several studies on the direction of multivariate concave and convex dominance (increasing concave dominance and increasing convex dominance). Did you find any presence of the other multivariate stochastic dominance orders when the first multivariate stochastic dominance is not fulfilled?

5. The empirical comparisons proposed presents two limits: a) you have used only few assets; b) you have used a very brief ex-post period in the empirical analysis (see chapter 1 and chapter 4) that could imply the lack of ex-post robustness. About the first problem, I understand that is difficult find first order dominance even for two assets I imagine that for large scale problems you need probably parallel programming. Have you test the computational complexity of the problems at least with some experiments? The second problem is intrinsic in the stochastic programming literature and I'm curious to know if you have investigated it in the specific literature.

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