FACULTY
OF MATHEMATICS
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Report on the thesis:

Multivariate stochastic dominance and its applications in portfolio optimization problems

by Barbora Petrová

The thesis is focused on multivariate utility functions and multivariate stochastic dominance, their generalizations and applications in dynamic portfolio selection problems.

The thesis is divided into 4 regular chapters, introductory and concluding parts. The introduction motivates the research and provides an extensive review of the related literature. Especially, the review of older papers published since 1940s really impressed me and at the same time confirmed that Barbora Petrová studied not only the recent papers but also the classical works.

Chapter 1 is devoted to the multivariate risk premiums which are used in multistage portfolio optimization problems as alternatives to risk measures to control the risk of the investments. Since the author considers non-separable utility functions, the traditional dynamic programming approach cannot be used. The underlying randomness is represented by a scenario tree. I must say that the notation used for the scenario trees is quite difficult, but after reading the following parts, it seems to be reasonable. The main results can be found in Sections 1.1.2 and 1.1.3 where two various multivariate risk premiums associated with the scenarios are defined and incorporated into the multistage portfolio optimization models. The next section introduces three multivariate utility functions which are then employed and compared in the empirical part. The numerical results are elaborated according to various criteria showing promising behaviour of the final wealth and its riskiness.

Chapter 2 is focused on the multivariate first-order stochastic dominance (FSD). It reviews various definitions available in the literature and introduces a new version called weak first-order stochastic dominance. The definition utilizes the survival functions of the compared random vectors. One of the fundamental results can be found in Theorem 4 that provides the description of the utility functions which induce the weak FSD. The proof employs the upper sets which are then used also in other proofs proposed in the thesis. Sufficient space is devoted to the relations between new

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weak FSD, the strong FSD by Levhari et al. (1975) and the linear FSD by Dentcheva and Ruszczýnski (2009). Section 2.3.1 provides the theorems and (simple) counter-examples which reveal these relations. It is obvious that the definitions of FSD are not equivalent. Although some of the proofs were already provided by Müller and Stoyan (2002), the author proposes new proof based on the "upper set approach". In Section 2.3.2, the relations between the univariate and multivariate stochastic dominance are investigated. New theorems 8 and 10 show that the weak FSD is equivalent to the univariate FSD (valid for all elements) only for the random vectors with independent marginals. Last section of this chapter provides an equivalent characterization of the strong and weak multivariate FSD using the multivariate risk premiums and the related generating utility functions.

Chapter 3 derives necessary and sufficient conditions for the strong multivariate FSD between two random vectors with special types of probability distributions. In particular, it considers a general discrete distribution and its special case with uniform distributions and equal cardinalities of possible outcomes. The relations between two vectors can be verified by optimization problems which are inspired by Armbruster and Luedtke (2015). Alternatively, the verification can be restricted to finding a feasible solution of these problems. My questions concerns the models (3.1)-(3.3) and (3.5)-(3.7). I think that the later one which is valid for a general discrete distribution is more general and at the same time is formulated as an easier linear program, whereas the model restricted to uniform distribution is a binary program. Can also this program be relaxed or reformulated to LP? I think that the most interesting results in this chapter concern the continuous distributions. Theorems 18 and 19 provide the necessary and sufficient conditions for the multivariate uniform and multivariate normal distributions where it is sufficient to compare their parameters instead of whole distributions. These findings are supported by long technical rigorous proofs using the "upper set technique". The chapter is concluded by a simulation study where the optimization approach for the uniform discrete distribution and a statistical testing are compared on FSD relations of two-dimensional normal vectors. In particular, the statistical approach uses the Holm-Bonferroni method to the composite hypothesis testing. The author concluded that the optimization method is more suitable for random vectors with smaller variances.

Chapter 4 applies the introduced multivariate stochastic dominance in several portfolio selection problems and compares it with univariate dominance applied to the elements. In particular, the problems use stochastic dominance constraints where the investor maximizes the expected final wealth under the dominance constraints that the wealth dominates a benchmark investment strategy in each period. Real data from the Kenneth R. French library are used to construct scenarios in several ways. Multiperiod as well as multistage formulations are considered leading to similar conclusions. The multivariate stochastic dominance constraint better describes the dynamic preferences of the investors, it leads to more diversified portfolios and the problems can be solved in lower computational time compared with the individual SD constraints. Last part of the chapter shows that the considered portfolio selection models are sensitive to the randomness representations using the scenario trees. My only question to this chapter is related to the tables 4.8 and 4.13 which compare descriptive statistics of the terminal wealth. Does the investor prefer lower or higher values of the VaR and CVaR measures? I would prefer lower values, but it seems that the opposite case is considered in the thesis.

The concluding part summarizes the contributions of the thesis and discusses open questions which mainly focus on multivariate generalizations of the higher order stochastic dominances which are already well established for the univariate case.

The work is clearly written, its graphical and stylistic level is high. The references are properly cited in the text and included in the list which appears at the end of the thesis.

Summary

The thesis contains many new deep and interesting results connected with the multivariate stochastic dominance. Many of the main results are already accepted or published in international journals. Several papers are still in the process of refereeing. All parts heavily use advanced techniques from the mathematical optimization, stochastic programming, utility and probability theory. On the basis of the thesis, I conclude that Barbora Petrová is capable of performing independent research.

Several highly successful researchers are focused on the same topic, namely prof. Darinka Dentcheva and James Luedtke among others. In such a competitive area, it is not easy to derive new strong results, but Barbora Petrová has achieved interesting findings which, I believe, are competitive to the top-level works and contribute significantly to our knowledge in this area. Therefore, the thesis certainly meets the requirements of doctoral thesis in mathematics (Probability theory, statistics, econometrics and financial mathematics) and I strongly recommend accepting this work.

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