Review of a Master Thesis
from Mathematical-Physical Faculty of Charles University

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Title: Balanced and almost balanced group presentations from algorithmic viewpoint
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The student has chosen an interesting and difficult topic of understanding balanced presentations of groups. Every group can be presented by giving a set of its generators and relations among them. Unfortunately, many presentations are - from the practical point of view - useless: even the triviality problem, i.e. deciding whether the given presentation describes the trivial group, is undecidable. Hence the endeavor to find some conditions under which this would be easy. Being balanced is one candidate of such a condition.

A presentation is called balanced if the number of generators equals the number of relations. It is an open (and difficult) question to decide the complexity of the triviality problem for balanced presentations. Although not solving this question, Chapter 2 of the thesis shows that some properties of the group are algorithmically undecidable even for balanced presentations. Such properties include "being free" or "having a finite presentation with 12 generators". It also shows that one cannot decide whether two presentations (from which one is balanced) define the same group. The proofs consist in an easy reduction from the original undecidability result for general presentations. In Chapter 4, the thesis continues by considering two algorithmic tasks. 1) Deciding whether a graph is a cycle, 2) Deciding whether a graph is 2-edge connected. It shows that these tasks can be reduced to the triviality problem for balanced presentations and hence provide a (very weak) lower bound for its complexity. The last chapter shows a standard construction of a simplicial complex whose fundamental group has a given presentation. This allows one to use topological methods to study groups. A part of the thesis is also a computer implementation of the construction.

There are, however, severe inaccuracies. First of all, the author calls the groups the same even if they are just isomorphic, even on places where it may lead to confusion. The second problem are the definitions in the Chapter 5. Practically none of them is correct. The most sever trouble of Chapter 5 is an abundance of typos, for example:

1. Definition 5.2: "Let *P(S)* denote a power set of *X*." (Should be *X*)
2. "Homotopy equivalence: A continuous function *H* ... is said to be homotopy between *f* and *g* if *H(x,0)=f(x)* and *H(x,1)=f(x)*. " (The second *f(x)* should be *g(x)*.)
3. "Definition 5.4: Topological spaces *X* and *Y* are said to be homotopy equivalent if there exist continuous functions $f: X \to Y$ and $f: Y \to X$ such that $f \circ g$ is homotopic to Id_X and $g \circ f$ is homotopic to Id_y." (The second or first *f* should be *g*, Id_y should be Id_Y, and likely a wrong order of function composition.)
4. "Construction a standard complex of presentation" (A preposition is missing)

And the list could go on. However, there are also factical mistakes. For example, Definition 5.5 defines elementary collapse only as a collapse of face that has codimension 1 in the whole complex. This is too restrictive, one cannot even collapse an edge glued to a boundary of a tetrahedra. Definition 5.6 defines link of a vertex *v* in a 2-dimensional simplicial complex *K* as all edges *ef* for which *{v,e,f}* is a triangle in *K*. This is a crude mistake, as the proper link consists of all simplices *S* in *K* that do not contain *v* and for which $v \cup S$ is a simplex of *K*. In particular it may contain isolated points, which is important in many situations.

The least severe mistakes are repeated non-English formulations, mostly wrong usage of English articles. (E.g. Theorem 4.5: Let *G=(V,E)* be the graph.) I would also suggest to rephrase "There exist" to "There is/are".

Questions:
What is your opinion about the actual complexity class of the triviality problem for balanced presentations?

Suggested Evaluation: 2
I do not suggest the work for any special prize.

In Klosterneuburg 3. 9. 2018 Pavel Paták