

In this thesis we study algorithmic aspects of balanced group presentations which are finite presentations with the same number of generators and relations. The main motivation is that the decidability of some problems, such as the triviality problem, is open for balanced presentations.

First, we summarize known results on decision problems for general finite presentations and we show two group properties which are undecidable even for balanced presentations - the property of “being a free group” and the property of “having a finite presentation with 12 generators”.

We also show reductions of some graph problems to the triviality problem for group presentations, such as determining whether a graph is connected, k -connected or connected including an odd cycle. Then we show a reduction of the determining whether a graph with the same number of vertices and edges is a cycle to the triviality problem for balanced presentations. On the other hand, there is also a limitation of reduction to balanced presentations. We prove that there is no balanced presentation with two generators $\langle a, b | a^{p(m)}b^{q(m)}, a^{r(m)}b^{s(m)} \rangle$ for $p(m), q(m), r(m), s(m) \in \mathbb{Z}[m]$ which describes the trivial group if and only if m is odd.

In the last part of this thesis, we describe a relation between group presentations and topology. In addition, this thesis contains a program which constructs a simplicial 2-complex from a group presentation.