Circular matter rings are a natural zero approximation of stationary and axially symmetric structures which appear in astrophysics. If the rings are infinitesimally thin (line sources), they are singular, which in the general relativistic description typically implies weird deformation of space in their vicinity. In particular, and contrary to the Newtonian picture, such rings even tend to behave in a strongly directional manner. One solution is to consider non-singular, extended sources (toroids), which may however be difficult to treat exactly and/or be unsatisfactory in other respects. In this thesis we check another option, namely to abandon the "real matter" completely and consider a non-singular source represented by mere curvature arranged, at least at some instant, in a pattern possessing the above symmetries. One such solution of Einstein's equations is known as the Brill waves; we study its properties at the moment of time symmetry (when it is momentarily static), in order to compare it with the space-times of matter rings.