The thesis contains a survey of numerical tools for studying thermomechanical interactions of a two-phase system contained in a domain with an upper boundary that forms a free surface. The enthalpy diffused-interface formulation is used for an approximation of the phase change interface and the computing algorithm is benchmarked against an analytical solution of the Stefan problem. Arbitrary Lagrangian-Eulerian kinematical description of the continuum is applied to overcome the difficulty in the form of the free surface. The validity of the approach is examined on a thermal convection benchmark problem.