
Report on the habilitation thesis:

Mathematical aspects of quantum mechanics with non-self-adjoint operators

submitted by Dr. Krejčířik

The thesis submitted is focused on various aspects of unbounded non-self-adjoint operators and their role in quantum mechanics. Because of the absence of the spectral theorem and variational principles, the non-self adjoint operator theory represents a mathematical challenge and leads to new phenomena. On the side of physics, Scholtz, Geyer and Hahne (1992) introduced representation of quantum mechanical observables by operators that are generalizations of self-adjoint operators. These operators, called quasi-self adjoint operators, are similar to self-adjoint ones. This approach was an important motivation for the results in the present thesis. The author (together with his collaborators) developed the theory of non-self adjoint operators during an extensive research over more than 10 years. I am going to mention only some of his main achievements.

- He introduced simple PJ-symmetric (space reversal-time reversal symmetry) operator and established, for the first time, a closed form for its metric. The results have been extended to more general context including the curved manifolds.
- The concept of a quasi-self-adjoint operator was employed to show the reality of the spectrum of a generator of a stochastic process modelling the Brownian motion with random jumps from boundary.
- He has highlighted the concept of pseudo spectrum and showed that it is the right tool for studying non-self-adjoint operators. Among others he demonstrated that the eigenfunctions of imaginary cubic operator form a complete set that is not a Riesz basis. The connection between Riesz bases consisting of eigenvectors and quasi-self-adjoint operators was clarified. He proved that pseudo-spectra of Schrödinger operators are highly non-trivial and developed their theory.

The thesis is very extensive (460 pages). (In my opinion, only a part of the thesis would suffice to meet the criteria for a high quality work.) Its fundamental part is formed by a collection of papers (21) that were published in leading journals of mathematics and mathematical physics (J.Phys. A, Phys. Lett. A, Math Z., J. Spectral theory, etc.). This impressive number of publications acknowledges the high quality and recognition of the work. Besides, the thesis has a helpful introduction that clearly details main contributions and motivations. It shows that the author is a good writer with pedagogical skills. (Besides, the Appendix A serves as a good guide through the theory.) Some of the papers are coauthored by the students of the applicant which confirms his ability to attract the young collaborators.

I believe that Dr.Krejčířik has obtained a number of deep results in a difficult area of the current research. He is taking part in an international collaboration and has gained a recognition for his contributions.

I believe that the thesis submitted meets all criteria for obtaining the degree "docent" at Charles University, Faculty of Mathematics and Physics. I strongly recommend to award the title "docent" to Dr. David Krejčířik.

Jan Hamhalter
10 -04- 2018

Prof. RNDr. Jan Hamhalter, CSc.
Department of Mathematics
Faculty of Electrical Engineering
Technická 2, 166 27 Prague 6
Czech Technical University