Monetary Policy and Stock Market Returns: Does the ZLB make a difference?
(Case of United States, European Union and United Kingdom)

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Academic Year: 2017/2018
Declaration of Authorship

The author hereby declares that he compiled this thesis independently; using only the listed resources and literature, and the thesis has not been used to obtain a different or the same degree.

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Prague, May 11, 2018

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Abstract

This thesis provides analyses of the impact of monetary policy on stock market returns under the zero lower bound. Using a VAR model with time-varying parameters and stochastic volatility, it aims to verify and reconfirm the relevance of monetary policy for stock market returns. This investigation has been carried out on the cases of the U.S., the E.U., and the U.K. When the interest rate is being constrained by the zero lower bound, the interest rate is approximated by the shadow interest rate in the spirit of Krippner or Wu-Xia. The findings can be summarized in three main points. Firstly, it is shown that stock markets react positively to negative shock into shadow interest rate, so the central banks were able to affect asset prices even after the interest rates hit the zero lower bound. Secondly, the impulse response functions suggest that even though the monetary policy is able to affect asset prices, it does so by being less effective. Thirdly, the analyses revealed the cross-country differences in each of the cases, as the monetary policy impact changes across samples both in terms of efficacy and magnitude.

JEL Classification C32, E43, E44, E47, G12, G15,
Keywords ZLB, VAR model, time-varying parameter, stochastic volatility, stock market returns, shadow interest rate, monetary policy

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# Acronyms

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<th>Description</th>
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<tbody>
<tr>
<td>ZLB</td>
<td>Zero Level Bound</td>
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<tr>
<td>US</td>
<td>United States</td>
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<tr>
<td>EU</td>
<td>European Union</td>
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<tr>
<td>UK</td>
<td>United Kingdom</td>
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<tr>
<td>MCMC</td>
<td>Markov Chain Monte Carlo</td>
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<tr>
<td>CRSP</td>
<td>Center for Research in Security Prices</td>
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<tr>
<td>FIT</td>
<td>Flexible Inflation Targeting</td>
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<tr>
<td>OECD</td>
<td>Organization for Economic Co-operation and Development</td>
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<tr>
<td>S&amp;P</td>
<td>Standard &amp; Poor’s 500 Index</td>
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<tr>
<td>SRTSM</td>
<td>Shadow Rate Term Structure Model</td>
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<tr>
<td>GATSM</td>
<td>Gaussian Affine Term Structure Models</td>
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<tr>
<td>SSR</td>
<td>Shadow Short Rate</td>
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<tr>
<td>SML</td>
<td>Shadow Lower Bound</td>
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<tr>
<td>LSE</td>
<td>London Stock Exchange Index</td>
</tr>
<tr>
<td>N</td>
<td>Euronext 100 Index</td>
</tr>
<tr>
<td>ADF</td>
<td>Augmented Dickey-Fuller Test</td>
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<tr>
<td>KPSS</td>
<td>Kwiatkowski–Phillips–Schmidt–Shin</td>
</tr>
<tr>
<td>TVP</td>
<td>Time Varying Parameter</td>
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<tr>
<td>IRF</td>
<td>Impulse Response Function</td>
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Monetary Policy and Stock Market Returns: Does the ZLB make a difference? (Case of United States, European Union and United Kingdom)

Motivation:
The monetary policy during years was largely conducted by using conventional instruments, which were the main tool of policymakers. However, with the financial crises of 2008, the interest rates were decreased to boost spending and to support the stability of the financial system. At that time the usage of interest rates was the main tool through which the monetary policy was implemented and transmitted, but ever since the interest rates reached the so-called zero level bound they never recovered. That situation led to a new way of conducting the monetary policy, which was through the use of unconventional instruments. The set of unconventional instruments includes among others credit easing, quantitative easing, forward guidance, and signaling. Considering that many papers in the past have perceived the relationship between monetary policy and stock market returns by studying the conduct of the policy based on conventional instruments, it is judged that an assessment of the current situation by considering the implementation of unconventional policies could contribute to the literature. Moreover, the research can be further developed in form of a comparative analysis of the cases of US and EU.

Hypotheses:
1. The impact of monetary policy on stock prices after the financial crisis when interest rate hit the zero level bound became more relevant.
2. Significant movements in asset prices require the intervention of monetary policymakers, which pursue different strategies with respect to the movement’s direction.
3. Zero level bound does not make a difference in terms of monetary policy’s efficacy and efficiency.
4. Shadow interest rates represent a tool able to restore the classical interest rates impact on asset prices.

Methodology:
The methodology to be employed in this research is based on vector autoregressive models and is further extended by allowing for time-varying parameters. Former studies have found TVP-VAR very useful in modeling the monetary policy impact on stock market returns. In addition, inspired from a study conducted by Gali & Gambetti (2013) on assessing the impact of monetary policy in stock market bubbles,
it is intended to further extend their analyses till nowadays in order to assess the change in policy application due to the ZLB issue. This model further allows for analyses of shocks and assesses the severity of their impacts. It will be used to test the effectiveness of the policy and its speed of adjustment. In addition, further analyses through variance decomposition can determine the pure impact of the stimulated shock on stock returns. The econometric assessment will be conducted for the case of the United States and Europe, aiming to reveal two alternatives of monetary policy application by analyzing their differences and similarities. Moreover, it is intended to use shadow interest rate as a core variable representing the unconventional monetary policy. Its measure will be based firstly on the calculation by Wu & Xia (2015) and secondly on Krippner’s (2016) measure, which will serve as a robustness check. Apart from shadow rate, other control variables based on literature suggestions will be employed in the model, in order to provide a correct analysis.

Expected Contribution:

Considering the shift from the traditional conventional tools to unconventional tools of monetary policy, a study of this kind can contribute by revealing the uncertainties of this change. It is important to quantify to what extent the use of unconventional tools impacts stock returns. In addition, the way of pursuing the policies needs to be identified in order to push the variable of interest to the desired direction and exert the necessary impact on it. In addition, measuring the impact of unconventional policies through the usage of shadow interest rate, with the proposed methodology contributes to the accuracy of results. Moreover, the application of this study to US and EU cases would contribute to a broader understanding of the monetary policy interventions, by revealing the successes and drawbacks of each intervention. Also, the comparison on the basis of centralized and decentralized systems would further contribute to the identification of monetary policy strategy, which would be necessary to be applied in our sample's countries. All in all, the contributions of this study can be expected on the accuracy of the methodology to be applied, on the measures of unconventional monetary policy and on the chosen sample.

Outline:

The study will be structured into six chapters, starting with the introduction which among others includes a presentation of the topic, the main research questions, the limitations of the study, the structure of the paper, etc. It will be followed by literature review chapter which will provide a summary and a detailed analysis regarding the existing research on the topic. The third chapter will provide a detailed assessment of the data, variables and the methodology to be employed. Next chapter will treat the comparative analysis between US and EU in order to reveal two different perspectives of the situation. It will be followed by the empirical findings chapter, which is concerned with the estimation of the model and the interpretation. The final chapter will be about the conclusion on the topic, summarizing the findings.
and making some suggestions on further research on this topic.

**Core Bibliography:**

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1 Introduction

Monetary policy is one of the key determinants standing behind the behavior of many economically important variables. Its study has always been of a very high importance for numerous facts, where we can mention the potential impact on the macroeconomic environment and the fact that its main aim is financial stability. In our case, the research is specifically oriented into revealing the influence of monetary policy on stock market returns with a greater emphasis on recent years after the global financial crisis. This focus has been motivated by the fact that the approach of monetary policy in terms of its available instruments has experienced a significant shift towards unconventional tools once the zero lower bound (ZLB) was reached.

Having stated that, it is important to highlight that such change has created a lot of uncertainty and has fogged the relationship of monetary policy with stock market returns. In addition, uncertainty has been expanded towards modeling consistency in terms of estimation, interpretation and forecasting accuracy. Being in front of a situation where uncertainty and breach of consistency have misted the impact of monetary policy, it is important to identify a factor with a potential ability to restore consistency. A major part of the current understanding of the monetary policy developments after the ZLB has been largely built based on models with constant parameters and non-stochastic volatility, thus ignoring the flexible nature of the economy. Recall the studies by Beningo & Paciello (2010), Swanson (2015) and Borio & Hofmann (2017). Even the existing studies taking into consideration the time variation in parameters and employing them in the methodologies applied, like in the cases of Gali & Gambetti (2015), Jansen & Zervou (2017) and Pascal (2018), are incomplete as they disregard stochastic volatility, miss recent developments and their focus rests on the United States case only.

In this thesis, we focus on revealing the impact of monetary policy on stock market returns while focusing on whether the ZLB has shaped the direction and the persistence of the relationship. Due to the breach of consistency in terms of policy pursuance and modeling interpretation, we make an attempt at restoring the consistency by making use of the concept of shadow interest rate. This variable has been initially introduced by Black (1995) and calculated by Wu & Xia (2015) and Krippner (2016). Shadow interest rate simply serves as an approximation of the end of the yield curve, by providing a mimicking behavior of how the interest rate would look like in normal times. In order to provide robust results, this thesis employs both measures of shadow interest rate and this constitutes one of the main contributions as
well. In addition, the used methodology is based on a vector autoregressive model with time-varying parameters and stochastic volatility similarly to the one described by Primicieri (2005). This choice is motivated based on the belief that the modeling of constant parameters might be deficient, as suggested by Stock & Watson (2002). The rationale behind allowance for time variation rests on the belief that the economy is characterized by a time-varying nature. In addition, allowing for time variation in residuals as well is strongly believed to improve the results, as suggested by Sims & Zha (2006) and Nakajima, Kasuya, & Watanabe (2011).

Moreover, this thesis provides a detailed analysis of three main samples, which are United States (US), European Union (EU) and United Kingdom (UK). The choice of these samples has been motivated based on two main rationales. Firstly, it would be relevant to consider the developments across countries with significant distinctions in their financial systems. Secondly, there is a gap in the literature regarding the availability of comparative analyses in terms of monetary policy and stock market returns. In all the formerly mentioned samples this thesis tries to identify the change in dynamics after the reach of the ZLB and explains if the effects of this change represent any significant metamorphose. In addition, it tests the claim that the shadow interest rate is the factor which can restore the consistency breach caused by the ZLB. Lastly, the third claim to be tested is related to the relevance of monetary policy in explaining the developments in stock market returns.

The contribution to the current literature is twofold. Firstly, it consists on the extension of existing studies for the impact of stochastic volatility and secondly on the extension of the analysis on multiple countries. Focusing on recent dynamics is quite beneficial for policymakers as there is an extensive need to clarify and understand how the zero lower bound has affected monetary policy. Moreover, the investigation of multiple cases contributes by checking for cross-country heterogeneity and for increasing the understanding of how the monetary policy works under different systems. Also, another important contribution of this thesis is the fact that the analysis has been built by taking into account both measures of the shadow interest rate, thus providing rich and robust results.

Our results suggest that the impact of monetary policy on stock market returns is quite relevant even under the ZLB, thus reconfirming the claim of Bernanke & Reinhart (2004). Unfortunately, the effectiveness of monetary policy seems to have decreased as suggested by the output of the impulse response functions. In addition, the findings are consistent across the three samples suggesting that a tightening in monetary policy negatively affects asset prices. Such implication rests on the same line with the findings by Gali & Gambetti (2015), Jansen & Zervou (2017) and Pascal
(2018), while suggesting for the higher persistence of the impact. In addition, the results reveal the relevance of shadow interest rate in explaining the policy implications and measuring their ability to ease the financial conditions. This further implicates that this variable is one of the best candidates in restoring the breach of consistency in terms of unclear policy impact and blurred modeling interpretations caused by the ZLB.

Furthermore, this thesis is subject to several limitations. The formerly described variable of shadow interest rate is grouped together with stock market returns and inflation rate, thus forming a dataset of three variables. While the literature clearly suggests a wider range of variables, we have limited the number to three for two main reasons. Firstly, vector autoregressive models have always been of a significant concern when it comes to over-parameterization and this concern gets fueled even more when allowing for time variation, as indicated by Koop (2012). In addition, the application of time-varying models, in general, has been characterized by a very small number of variables and a group of three in our case has been motivated by the studies of Primicieri (2005) and Cogley & Sargent (2005). As Koop (2012) suggests, there is no need to concern for the over-parameterization issue with a small number of variables and in addition to that, it helps to avoid prior shrinkage or model averaging.

Apart from the formerly mentioned issue, another important limitation faced with the model is related to its computability. It takes more than twenty minutes to get 55000 Markov Chain Monte Carlo (MCMC) draws, thus making the computation with additional lags much more time-consuming. This issue is even more pronounced in this case, since the analysis involves three samples repeated over the two variables of shadow interest rates. Another important limitation is related to the sample's length, which is constrained by the data availability on shadow rates and in our case is restricted to 156 monthly observations. As pointed out by Koop (2012), the low number of observations constitutes an issue but can be overcome by the Bayesian methods through the use of informative priors. For this thesis, we have made use of the priors described by Primicieri (2005), which are believed to overcome the formerly mentioned issue.

The rest of the paper is organized as follows. Chapter 2 provides a review of literature while aiming to bestow different streams of literature and familiarize with the topic. In addition, the third chapter is focused on conveying the methodology used to analyze the available samples. The fourth chapter is focused on analyzing the data in accordance with their respective samples and to dispense the developments over time by making use of descriptive statistics. After elaborating the methodology and
data, the fifth chapter of empirical findings is focused on discussing the estimated results, explaining the rationale behind them and contrasting with the literature. Finally, the sixth chapter of conclusion summarizes the investigation by focusing on providing the implications from the results and revealing the satisfaction of the hypothesis.
2 Literature Review

Monetary policy, which is conducted by central banks of each country, can be performed either by using traditional policies - conventional tools - or through recent non-traditional policies - unconventional tools. The formerly mentioned conventional tools include open market operations, discount window and reserve requirements as described by Meulendyke (1998), and the latter group of unconventional tools includes quantitative easing (QE), comprehensive monetary easing, the zero interest rate policy, etc., as described by Kuroda (2016). These two groups of tools have drawn a lot of attention in the recent years as it has been experienced a shift towards unconventional policies.

For many years now, the interest rates have reached the ZLB leaving the policymakers without their most important conventional tool and pushing them towards alternative solutions. But as discussed by Bernanke & Reinhart (2004), the low level of short-term interest rates does not mean that monetary policy would be less efficient. Considering that many papers in the past have perceived the relationship between monetary policy and stock market returns by studying the conduct of the policy based on conventional instruments, it is judged that an assessment of the current situation by considering the implementation of unconventional policies could contribute to the literature.

Early, Thorbecke (1997), inspired by the debate of whether the monetary policy was neutrally aimed to contribute to the discussion by studying the response of stock returns to monetary policy shocks. The author used the innovation in the federal funds rate, the non-borrowed reserves and a dummy variable indicating the policy changes to measure monetary policy. Its findings suggested an ex-post positive impact on stock returns in case of an expansionary policy and a positive ex-ante return as well for the assets exposed to the monetary policy, thus confirming the existence of a significant relationship.

Another approach was used by Sellin (2001), which investigated the impact of monetary policy on stock market returns for the case of US. Aiming to provide both a theoretical and empirical approach the author used the variables of money growth and inflation, interacting with real stock market returns. Its findings suggested that money growth would be a good indicator for predicting future real stock returns, while inflation provides mixed evidence in terms of short-run where it exerts a negative impact on stock returns and in terms of long-run where its impact becomes positive.
In addition, the latter relation is further supported theoretically by the type of monetary policy, which may be counter-cyclical or pro-cyclical.

In contrary to the former study, Rigobon & Sack (2003) studied the relation between monetary policy and stock market returns in a different angle, by investigating the reaction of monetary policy towards changes in stock market returns. By making use of an identification technique based on the heteroskedasticity of stock returns, the authors concluded the existence of a response of monetary policy to changes in stock market returns. This response was found to be moderate, indicating that the monetary policymaker would respond to such changes only to the extent that it would want to impact the macro-economy. The heteroskedasticity identification technique developed by Rigobon & Sack (2003) was recently used to investigate the response of stock market returns to European Central Bank’s policies by Haitsma, Unalmis, & de Haan (2016). The results suggested that the main influence on stock market returns is exerted by the unexpected unconventional policies. In addition, the results indicate a higher influence on the returns of the stocks which have been over-performing and under-performing in the past.

Another research by Bernanke & Kuttner (2005), tries to explain the impact of unanticipated changes in federal funds rate target on equity prices in order to estimate the reaction and investigate the market’s behavior. By making use of CRSP (Center for Research in Security Prices) value-weighted index results, the author finds that a 0.25% unanticipated cut impacts the stock prices by 1% increase. The authors claim that the impact is in line with the capital asset pricing model predictions, even though the magnitude varies across sectors. More importantly, unlike the theory predicts the majority of the impact is explained by forecasted equity risk premiums and surprising monetary policies rather than by the expectations of real interest rates.

Moreover, in a more recent study, the linkage between inflation, monetary policy, and stock market returns has been reassessed by Bordo, Dueker, & Wheelock (2008). By applying a hybrid latent variable VAR model on a large number of data regarding the second half of 20th century, extended the findings of the former authors significantly. It was concluded that inflation shocks contributed to busts while disinflation shocks contributed to booms. In addition, the findings suggested a greater ability of inflation in explaining the variation of stock market returns when market conditions were taken into account.

Besides the consideration of inflation, Laeven & Tong (2012) investigated the impact of US monetary policy on global stock markets by taking into account the change in interest rates. The authors’ findings suggest a strong and significant effect of US policy, as stock markets returns increase in reaction to unexpected loosening
monetary policy and vice versa when an unexpected tightening policy is pursued. Also, according to the estimated results the impact varies across sectors and across firms as well, since the impact is expected to be more significant among sectors and firms dependent on external financing. These findings confirm the results of an earlier study by Ehrmann & Fratzscher (2009), which also concludes strong impact of US policy on global stock markets. Apart from the same conclusions, the former study highlights the important role of foreign exchange rates and financial integration of the countries in explaining this significant impact.

In contrary to the two former studies, Gali & Gambetti (2015) agree that a tightening of the monetary policy can decrease stock market returns, but only for a short time and after that, the response will be positive causing a persistent increase on the returns. Also, the authors do strongly claim that the observed relationship is unlikely to be caused by an endogenous response of the equity premium to the monetary policy shocks. In addition to the former study, Jansen & Zervou (2017) find consistent results in terms of monetary policy and stock market returns. They reveal that the impact of the monetary policy to have become on average 5 times stronger during 2000-2007 compared to the pre-2000 period. In contrary to the two former studies, Pascal (2018) indicates that the negative response of stock market returns with respect to a tightening monetary policy remains constant across all periods, thus removing the ambiguity presented in Gali & Gambetti (2015).

As many papers provide mixed evidence regarding the sign of interest rate's impact on stock market returns, Chen & Wu (2013) constructed a threshold regression model to define the real impact of interest rates. According to their suspicions, interest rates were exerting two types of impacts and one of them should be superior to the other, leading to the final results that we observe. Even though the economic theory assumes a negative relationship between interest rates and stock prices, the author's findings do not fully support this idea. When estimating the results interest rates positively impact the stock prices and at the same time they are strongly significant. However, the situation only differs after a certain threshold is crossed and the results go in the same line with the theoretical predictions. Implying that the interest rates are a very important tool for central bankers, it has been proved through co-integration that they would be very useful in forecasting the indexes.

Moreover, Hojat (2015) investigated the impact of monetary policy on stock market returns by considering the impact of a change in money supply (M2), change in federal funds rate and change of federal funds futures. The regression results suggest a positive moderation effect of M2 and a negative effect of federal funds rate and change of federal funds futures on expected returns. In a more recent study,
Neuhierl & Weber (2016) used the federal fund futures changes in different periods to build a slope rate. The authors find the slope suitable for making predictions for the expected returns. In addition, the slope can predict changes in future interest rates and according to the calculations can help investors revise their forecast and achieve a weekly sharp ratio improvement of 20%. These findings imply the importance of future interest rates in determining asset prices and the consideration of monetary policy impact not only when announced but during the entire period.

Although the former studies acknowledge the relevance of interest rates, the situation needs to be assessed for the case of ZLB as well, where interest rates are stuck at zero. A study by Bernanke, Reinhart, & Sack (2004), indicates the monetary policy alternatives when the interest rates have hit the zero level bound. This study empirically investigates alternative policies like, like shifting people's expectations, expanding central bank's balance sheet and changing its composition as well, with the intention of evaluating in terms of efficacy. In addition, the study highlights vulnerability to shocks originating from the level of inflation under ZLB, and further analyzes regarding this issue are conducted by Bean (2007). It tries to answer the question if inflation targeting is enough to keep the economy safe from inflation originating shocks. In addition, this study highlights the relevance of monetary policy response to imbalances in asset prices, aiming to decrease the chances of a bubble creation. While agreeing with the literature in terms of a tightening monetary policy, Bean (2007) also introduces a solution to the issue based on a so-called flexible inflation targeting (FIT) policy. The FIT framework does not describe a specially designed policy to calibrate asset prices but rather derives the trade-offs between inflation and output gap, which should be in the same line with the main objectives.

Following the discussion on the optimal level of inflation, Roger (2009) addresses the same analyzes as the former author by indicating the issues on the inflation targeting policy and increasing the awareness on a new framework based on FIT. In addition, Williams (2009) indicates that the traditional target of 2% may not be able to provide a good buffer against ZLB. But unlike the former studies suggestion in favor of flexible targeting policies, Coibion, Gorodnichenko, & Wieland (2012) argue on the contrary with intent to increase the target. By making use of different approaches they try to measure the right level of inflation, which would result to be more efficient under ZLB condition. The results indicated that the level of inflation would still be less than 2%, like in the current state, indicating the lack of FIT efficiency in reducing the severe costs of ZLB. In addition, Beningo & Paciello (2010) suggest the pursuance of an inflation targeting strategy in shaping the asset prices as well. By making use of the New Keynesian models, it has been
implied that strict inflation targeting decreases co-movement with assets prices and vice versa when it comes to less strict policies.

Apart from the FIT framework, one of the alternative policies discussed by Joyce, Miles, Scott, & Vayanos (2012) is the QE approach. It mainly consists in large-scale asset purchase by central banks and results with an expansion of their balance sheet, but on the other hand, accommodates the economy by providing liquidity. Going one step ahead, Chung, Laforte, Reišschneider, & Williams (2012) tested on the effectiveness of QE for the case of US. Its results suggested that the severity and consequences of hitting ZLB were underestimated and the implementation of QE was not able to prevent the impact on economic activity and inflation as well. But unlike the for study’s findings, Baumeister & Benati (2012) conclude that the relevance of QE was unquestionable. In addition, this study highlights that the consequences of ZLB, which were prevented due to QE, could have resulted in similar outcomes to those of 1930’s Great Depression. On the other hand, Gambacorta, Hofmann, & Peersman (2014) only moderately agree on the impact of QE, as in their analyses the boost in economic activity seems to have only temporary effects.

In a later study, Nakazono & Ikeda (2016) evaluated the effectiveness of monetary policy under ZLB for the case of Japan. Their findings are quite ambivalent as stock markets were not responding to monetary policy surprises as expected and they were negatively influenced by surprise monetary easing. The authors conclude that it is quite difficult to implement unconventional policies effectively, especially when there are unclear inflation dynamics for the future. Based on the authors’ conclusions can be easily stated that inflation uncertainties are a characteristic of Japan and do not necessarily apply to other cases. Such claim is further confirmed by Lima, Vasconcelos, Simão, & de Mendonça (2016), which studied the impact of QE on stock market returns after the global financial crisis. The findings reveal that the impact was positive by confirming the relevance of monetary policy in influencing the asset prices.

More recently, Swanson (2015) measures the impact of unconventional policies on stock market returns by making use of changes in the large-scale asset purchase and forward guidance. The results suggest that the influence of both variables was quite comparable and the author finds it ambivalent due to the irrelevant impact of forward guidance over other long duration assets. While trying to assess the effectiveness of monetary policy under the ZLB by considering its impact on aggregate demand and output, Borio & Hofmann (2017) claim loss of efficacy. Such conclusion was drawn based on the presence of headwinds originating from
balance sheet recessions and non-linearities resulting after the interest rates impact spending.

Even though U.S case remains in the center of the research, other cases provide good insights regarding the relationship monetary policy and stock market returns. The relationship has been early investigated for the European case by Cassola & Morana (2004), which have focused their research on determining the relevance of stock market as a transmission channel of monetary policy. Their findings confirm the suspicion that stock markets through assets prices were playing a key role in transmitting the policies. However, when considering inflation no significant evidence could be found to prove the impact of stock prices on inflation, but the results do not deny the existence of a relationship between monetary policy and the stock market. Evidence suggests that the policies focused on maintaining long-run stability contribute to the stock market stability.

Moreover, Ioannidis & Kontonikas (2006) investigated the relationship between monetary policy and stock markets for a group of OECD countries aiming to understand the relation. Their results confirmed that the monetary policy shifts significantly affected stock returns. In addition, they claim that their results support the hypothesis that the stock market serves as a policy transmission channel, thus confirming the findings of the former author. The investigation of the UK case by Bredin, Hyde, Nitzsche, & Reilly (2007) provides good grounds for information from an important EU country. The study was not focused only on the relationship between monetary policy and stock market returns but, also on the reasons behind the returns behavior and the response to shocks. By using a baseline regression and a variance decomposition model the authors concluded that monetary policy shocks were negatively influencing future excess returns in many sectors.

In addition, Gregoriou, Kontonikas, MacDonald, & Montagnoli (2009), investigated the relationship by using the variables of anticipated and unanticipated interest rate change for the case of UK. The findings from the empirical analyses helped to identify structural breaks in the relation between returns and monetary policy shifts. More concretely, it was estimated that before the crisis when credit is available and no issues are faced the stock market returns were responding negatively to both anticipated and unanticipated interest rate change. However, as soon as crisis strike and a credit crunch situation occurred, the relationship turned positive indicating the inability of policymakers to reverse the relationship to the pre-crisis situation. The link between monetary policy, inflation and aggregated stock returns for all sectors has also been assessed for the UK case by Li (2009). Author's results flow in the same line with the literature pointing out a negative impact of monetary
policy announcements on stock returns. Regarding inflation, it is claimed that UK's stock market fails to provide a good hedge in short or medium term, but in the long term, the situation reverses. Moreover, the author claims that the benefits or the drawbacks of high unexpected inflation will depend on the firms' operations as debtors or creditors and as a consequence, this will impact the aggregated returns.

In a latter research, Furlanetto (2011), conducts a study for a group of countries including, the US, UK, and Australia, in order to reveal the differences among them regarding the monetary policy response to stock market fluctuations. According to the estimated results it has been found that, while in the US the market fluctuations become significant only after acknowledging for the dependence between stock prices and interest rates, the situation is not the same for other countries. Moreover, no response of monetary policy to changes in asset prices could be observed in any of the countries besides Australia. Even in the case of US, the response declines over time and it is found to be significant only during the housing bubble period. The difference between Australia and other countries in the sample can be explained based on Wang & Mayes (2012) study, which explains the different patterns of Australian case with the fact that it hasn't reached the zero level bound. Also according to the estimation, there could not be found any evidence suggesting that the country was impacted by the latest financial crises.

Moreover, Shibamoto & Tachibana (2013) studied the relationship between monetary policy and individual stock returns for the case of Japan by making use of firm-level data. Aiming to identify the behavior of different firms stocks to monetary policy changes, the authors have also considered the firms' characteristics. The findings suggest a relation one to three to quantify the impact of one unit cut in the call rate target to stock returns. Additionally, it has been found that monetary policy has a greater impact on the stock markets during the recessions rather than during expansionary periods. Another study by Zeng (2010) investigates the impact of monetary policy announcements in stock market returns for a short period after the announcement for the case of China. The results of the author do not provide enough evidence to support the claim of a short-term impact after the announcement and he explains the lack of significance by assuming information leakage. The trend observed in the market supports the idea of information leakage as days before the announcement stock market returns are substantially changing.

Additionally, Guo, Hu, & Jiang (2013) considered the case of China for monetary shocks and asymmetric effects on the stock market. The author's findings suggest the presence of asymmetric effects on different periods and on different cycles. It is proved the relevance of interest rate shocks but the money supply and
exchange rate shocks were not found to be significant. All in all the research concludes that the presence of monetary policy shocks increases the volatility of stock markets, hence when implementing the policies central bankers should take into account the market condition and the impact on the stock market. In a more recent study, Liu & Sun (2016) investigates the impact of monetary policy actions and central bank communication on stock market bubbles by using a time-varying parameter SVAR. The author's findings suggest that a contractionary monetary policy positively impacts the stock prices to rise during periods of large bubbles. Also, the central banks' communication seems to have a more significant impact in the long term, suggesting that it should be used as an effective long-term stabilizing instrument.

Moreover, there is a large stream of literature, which discusses and protects the view that rather than dependent on current economic environment, the reaction of stock markets towards changes in monetary policy is a state-dependent issue. The suspicion for asymmetric effects of monetary policy on stock market returns, studied by Chen (2005), uses Markov-Switching models to test this hypothesis. The findings suggest that the impact of monetary policy varies among bull and bear markets with a larger impact on the latter one. Additionally, it concludes that a tighter monetary policy increases the possibility of shifting to bear market regime. Following the same logic and methodology, Davig & Gerlach (2006), was able to identify two types of regimes. Based on the S&P 500 index, the first regime revealed a negative significant impact on the returns in case there was an unexpected change in the federal funds rate. The second regime identified in this study is characterized by the insignificance of stock price response to monetary policy shocks, and at the same time, the volatility is much higher compared to the previous regime.

As the former studies are able to confirm the asymmetries and as a consequence validate the assumption of state-dependent reaction, Kurov (2012), emphasizes the importance of information transmitted through monetary policies. Its findings suggest that the state dependence can be explained by future corporate cash flows and expected equity premiums, whose information is somehow transmitted through the policy. Hence the author gives an explanation which confirms the existence of state dependency and furthermore, explains the reasons behind it.

Going one step forward, Guo, Hung, & Kontonikas (2016) consider the influence of investor's sentiment regimes on the relationship between monetary policy shocks and stock market returns. The authors have determined that when investor's sentiment is high and an unexpected expansionary monetary policy is pursued the impact on stock market returns is strongly positive. However, it is
determined that the effect's magnitude is higher on the large stocks with a bad performance in the past and a low book-to-market value. Also, an important finding of this paper is that it gives evidence that the response of stock prices to unconventional policies is significant.

Furthermore, Floro & van Roye (2017) considers the state dependent response of monetary policy in a highly financially stressed environment. Through the appliance of a factor-augmented dynamic panel threshold model, the authors were able to support their state dependency policy response hypothesis. It was confirmed the existence of a state dependency policy response in advanced economies as these country’s central banks aggressively pursue an expansionary policy when financial markets volatility is high.

All in all, we can agree that the monetary policy represents a very relevant tool in affecting the economy. The formerly presented literature stipulates the importance of the monetary policy role and the need for understanding its impacts and behavior perfectly. Recent years developments have significantly shaped the way of conducting the monetary policy, thus making it more and more difficult to accurately forecast and interpret the developments of monetary policy. In addition, its relevance has been largely questioned after the interest rate hit the ZLB, thus creating a lot of uncertainty. But even with all the uncertainties and different opinions dividing the researchers, monetary policy cannot be ignored as it still has a core role in shaping the economy and the financial system as well.
3 Methodology

Based on the literature, the transmission of monetary policy in various channels of the economy has been historically measured through VAR models. Taking into account the former research, the empirical part of this study will be carried out based on the Vector Autoregressive models methodology. It aims to improve the results by improving the basic model, in order to allow for time-varying dynamics on its parameters like in the studies of Franta (2011), Nakajima, Kasuya & Watanabe (2011) and Gali & Gambetti (2015). Such modification is believed to provide better results, as accounting for parameters dynamics and flexibility addresses the time patterns.

Moreover, as the main aim of the study is to test the relevance of unconventional monetary policies under ZLB, the main variable to be used will be the shadow interest rate. Based on the calculations of shadow interest rate by Wu & Xia (2015), it is intended to carry the research for the case of US, EU, and the UK. In addition, aside from the shadow interest rate generated based on former author's calculation, this study will also use as a robustness check the shadow interest rate calculated by Krippner (2016).

3.1 Shadow Interest Rate

3.1.1 Shadow Interest Rate by Wu & Xia (2015)

Based on an early analysis by Black (1995), the nominal short-term interest rate is the shadow real interest rate plus inflation in case it is positive or plus zero in case it is negative. From such argumentation and based the idea that the nominal short rate cannot be negative since people can still hold currency at ZLB and, Wu & Xia (2015) built their measure of shadow interest rate.

By denoting shadow interest rate by $s_t$ and the lower bound by $r_t$, the calculation of short-term interest rate would require maximizing the following function:

$$r_t = \max(r_t, s_t).$$ (3.1)

Like it was formerly assessed in Black’s (1995) analyses, the shadow rate would be considered as the nominal short-term rate in case the lower bound would be bind to zero level. Being expressed as a maximization of two factors, the absence of
one due to ZLB would automatically transfer the economic information to the shadow interest rate. Thus to some extent, we could expect that the inference that might be taken from the former variable might be quite relevant once we hit the ZLB.

When explaining the shadow interest rate, the authors based their further calculation on the factor dynamics and stochastic discount factor. By assuming that \( s_t \) could be represented as a function of a state variable \( x_t \) the equation turns out to be as follows:

\[
 s_t = \delta_0 + \delta_1' x_t. 
\]  
(3.2)

And the state variable is expected to follow a first-order autoregressive process (VAR(1)) under the physical measure (\( \mathbb{P} \)):

\[
 x_{t+1} = \mu + \rho x_t + \sum \epsilon_{t+1} \epsilon_{t+1} \sim N(0, I) 
\]  
(3.3)

While dynamics were presented in the former equation, the stochastic discount factor is constructed based on Duffé (2002):

\[
 LogM_{t+1} = -r_t - \frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \epsilon_{t+1}, 
\]  
(3.4)

Where lambda is considered to be the price of risk and is considered to be linear in factors as well:

\[
 \lambda_t = \lambda_0 + \lambda_1 x_t. 
\]

Consideration of lambda to be linear in factors implies that the dynamics for the factors under the risk-neutral measure (\( \mathbb{Q} \)) should be as well VAR(1):

\[
 x_{t+1} = \mu^Q + \rho^Q x_t + \sum \epsilon^Q_{t+1}, \mathbb{Q} N(0, I). 
\]  
(3.5)

While \( \mathbb{P} \) and \( \mathbb{Q} \) measures relate to the parameters as follows:

\[
 \mu - \mu^Q = \sum \lambda_0, 
\]

\[
 \rho - \rho^Q = \sum \lambda_1. 
\]

The pricing formula for the Shadow Rate Term Structure Model (SRTSM) has a closed form and as described through the former equations cannot be extended beyond one factor. For this reason, Wu & Xia (2015) proposed an approximation of the forward rate in SRTSM, which according to their calculation would result in a very low error margin, quantified to vary only a few basis points. In order to define
the formula for the forward rate, firstly denote $f_{n,n+1,t}$ to be the forward rate at time $t$ of the loan starting in time $t+1$ and maturing in $t+n+1$,

$$f_{n,n+1,t} = (n + 1)y_{n+1,t} - ny_{nt}. \quad (3.6)$$

With the former equation being a linear function of yields on risk-free $n$ and $n+1$ period pure discounting bonds, the SRTSM developed through equations (3.1) to (3.5) approximates as follows,

$$f^{SRTSM}_{n,n+1,t} = \Gamma + \sigma_n^Q g \left( \frac{a_n + b_n'X_t - \Gamma}{\sigma_n^Q} \right). \quad (3.7)$$

In the former equation, we defined $(\sigma_n^Q)^2 = \text{Var}_t^Q(s_{t+n})$. while $g(z) = z\Phi(z) + \phi(z)$, consists of a normal cumulative distribution function $\Phi()$ and a normal probability density function $\phi()$.

Moreover, when explaining the relationship to Gaussian Affine Term Structure Models (GATSM) Wu & Xia (2015), replace equation (1) by $r_t = s_t$ in order to transform SRTSM into GATSM, and the forward rate in the latter one is defined as,

$$f^{GATSM}_{n,n+1,t} = a_n + b_n'X_t. \quad (3.8)$$

While $a$ and $b$ in equation (3.8) are the same as those in (3.7), the only difference among them is the function $g(.)$, which is found to be nonlinear and increasing in the same time. The study indicated a limited behavior of the former function for $z=2$ and $z=-2$ when the function is plotted against a 45-degree line, indicating that GATSM is an approximation of SRTSM when ZLB is not reached yet.

The estimation process continues with the state space representation of SRTSM and GATSM. Starting firstly with SRTSM, the state transition equation is (3.3) and from equation (3.7) the SRTSM is written as a nonlinear space model with measurement equation relating the observed forward rate $f^{0}_{n,n+1,t}$ as follows,

$$f^{0}_{n,n+1,t} = \Gamma + \sigma_n^Q g \left( \frac{a_n + b_n'X_t - \Gamma}{\sigma_n^Q} \right) + \eta_{nt}, \quad (3.9)$$

where $\eta_{nt}$ is the measurement error, normally distributed as $\eta_{nt}\sim N(0, \omega)$. Like formerly stated equation (3.9) is not linear and for this reason, the authors applied the extended Kalman filter to linearize function $g(.)$. 
The GATSM as well has the same state transition equation as SRTSM, equation (3.3), and according to equation (3.8) the implied measurement equation is as follows,

$$f_{n,n+1,t}^0 = a_n + b_n'X_t + \eta_{nt},$$

(3.10)

where $\eta_{nt} \sim N(0, \omega)$. Unlike in the SRTSM case here the Kalman filter is applied directly as the measurement equation is linear.

In the normalization process, the authors based on the literature suggested that three factors would be enough to account for all cross-sectional variation in yields and as a result proceeds with a three-factor model. The group of parameters to be estimated includes $(\mu, \mu^Q, \rho, \rho^Q, \Sigma \delta_0, \delta_1)$, and for identification purposes, the following restrictions on $Q$ parameters are imposed: $\delta_1 = [1, 1, 0]'$, $\mu^Q = 0$, $\rho^Q 1$ is in real Jordan normal form with eigenvalues in descending order, and $\Sigma$ is lower triangular. The imposition of these restrictions does not change the implications of the model.

### 3.1.2 Shadow Interest Rate by Krippner (2016)

In his re-estimation of the shadow interest rate Krippner (2016) followed the same logic as Wu & Xia (2015) by making use of Black’s (1995) concept on the lower bound mechanism:

$$r(t) = \max[r(t), r_{LB}],$$

(3.11)

where $r(t)$ is the shadow short rate (SSR), $r(t)$ is the actual short rate which is constrained to the minimum value of $r_{LB}$ parameter standing for the lower bound. Through the derivation of a GATSM process for the SSR, Krippner (2016) was able to approximate Black’s (1995) framework by making it more tractable for any number of factors. By adding to the former derivation the discrete-time equivalent developed by with Wu & Xia (2015), becomes possible to get a closed form expression for the lower bounded forward rates $f(x_t, \tau)$ as follows,

$$f(x_t, \tau) = r_{LB} + [f(x_t, \tau) - r_{LB}] \times \Phi[z(x_t, \tau)] + \omega(\tau) \times \phi[z(\tau, \tau)],$$

(3.12)

with:

$$z(x_t, \tau) = \frac{f(x_t, \tau) - r_{LB}}{\omega(\tau)},$$

(3.13)

---

1 Considering the repetition of eigenvalues the real Jordan normal form transforms as follows, $\rho^Q = \begin{bmatrix} \rho^Q_1 & 0 & 0 \\ 0 & \rho^Q_2 & 1 \\ 0 & 0 & \rho^Q_3 \end{bmatrix}$.
where $\tau$ is time to maturity, $\Phi[.]$ and $\phi[.]$ are the unit normal cumulative density and density function. As it is displayed in the former equations, the lower bounded forward rate and volatility function $\omega(\tau)$ are dependent on the specification in terms of state variables $x_t$ and their associates.

Equation (3.12) is very relevant to the analyses as it compares the shadow lower bound term structure model (SML) results with the yield curve. Krippner defines the state variable by making use of vector Ornstein-Uhlenbeck process followed by the variable under the physical $\mathbb{P}$ measure:

$$x_t = \theta + \kappa[\theta - x_{t-1}] + \sigma,$$

(3.14)

where $x_t$ is the vector of state variables (N x 1) with a long-run value of $\theta$, a mean reversion matrix $\kappa$ and a volatility matrix $\sigma$.

On the other hand, the risk-adjusted $\mathbb{Q}$ measure process for state variables is obtained by making use of the market linear risk specification $\Pi(t) = \gamma + \Gamma x_t$, which is analogous to equation 3.14 with $\tilde{\kappa} = \kappa + \Gamma$ and $\tilde{\theta} = \tilde{\kappa}^{-1}(\kappa \theta - \gamma)$. The formerly defined time variables and the other parameters explaining it, together define the closed-form analytic expressions for $f(x_t, \tau)$, and $\omega(\tau)$, which together with $r_{LB}$ form the closed-form analytic expression for $f(x_t, \tau)$ in equation (3.12).

Through the former derivations the SML can be compounded and by applying a non-linear Kalman filter the SSR point estimate would be as follows:

$$r(t) = a_0 + b_0' x_t,$$

(3.15)

which is the zero maturity rate on the estimated shadow forward rate or interest rate curve. Krippner shows in his paper that SSR can freely take negative values, which is interpreted as a combination of a close to zero policy rate and unconventional policy tools, which is considered to be more accommodative than a near-zero policy rate alone.

3.2 Empirical Model

Early introduced in the paper by Primiceri (2005), the vector autoregressive model would allow for both time-varying parameters and time-varying variance-covariance matrix. Through such implementation, it was aimed to capture the time variations and nonlinearities of the model. In addition, the model's multivariate stochastic volatility is meant to capture existing heteroskedasticity of the shocks and nonlinearities fueled through the relations within the variables. The idea behind the allowance for time variation is to attribute to the data to determine if the time variation in the linear...
structure comes as a result of changes in the severity and size of shocks or due to changes in the response strategy.

Such strategy, which is also closely followed by Gali & Gambetti (2015), initially considers an autoregressive model as follows,

\[ y_t = c_t + \beta_1 y_{t-1} + \cdots + \beta_{k,t} y_{t-k} + u_t, \quad t = 1, \ldots, T. \]  

(3.16)

In the former equation, \( y_t \) stands for an \( n \times 1 \) vector of dependent variables, \( c_t \), and \( \beta_{k,t} \) respectively stand for an \( n \times 1 \) and \( n \times n \) matrices of time-varying parameters, while \( u_t \) represent the heteroskedastic shocks with variance-covariance matrix \( \Omega_t \). Such matrix can be defined as,

\[ A_t \Omega_t A_t' = \Sigma_t \Sigma_t', \]  

(3.17)

where \( A_t \) can be defined as the lower triangular matrix,

\[
A_t = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
\sigma_{21,t} & 1 & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
\sigma_{n1,t} & \cdots & \sigma_{n-1,n,t} & 1
\end{bmatrix}
\]

And the diagonal matrix \( \Sigma_t \) can be displayed as follows,

\[
\Sigma_t = \begin{bmatrix}
\sigma_{1,t} & 0 & \cdots & 0 \\
0 & \sigma_{2,t} & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & \sigma_{n,t}
\end{bmatrix}
\]

The next step requires to substitute the \( u_t \) component in equation (3.16) as follows,

\[ y_t = c_t + \beta_1 y_{t-1} + \cdots + \beta_{k,t} y_{t-k} + A_t^{-1} \Sigma_t \varepsilon_t, \]

\[ V(\varepsilon_t) = I_n. \]  

(3.18)

By loading into a vector \( B_t \) all the R.H.S coefficients, we can redefine (3.18) as follows,

\[ y_t = X_t' B_t + A_t^{-1} \Sigma_t \varepsilon_t, \]

\[ X_t' = I_n \otimes [1, y_{t-1}', \ldots, y_{t-k}']. \]  

(3.19)
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where $\otimes$ stands for the Kronecker product.

After defining the former equations it is important to emphasize the relevance of having a time-varying variance-covariance matrix. According to Primicieri (2005), maintaining a constant $A_t$ would indicate that a change in the $i$-th variable would have a time-invariant effect of the $j$-th variable. Such development is unappealing as the goal is to study the time-variation in a simultaneous equation model, with a core importance on the interactions between variables.

So the strategy requires modeling the coefficient processes in equation (3.19). In order to follow the strategy, we define $\Sigma_t$ as the vector of non-zero and non-one elements in matrix $A_t$, while $\sigma_t$ is defined as the vector of diagonal elements in matrix $\Sigma_t$. Having made the former specifications the dynamics of the model's time-varying parameters are specified as,

$$B_t = B_{t-1} + v_t,$$

$$a_t = a_{t-1} + \zeta_t,$$

$$\log \sigma_t = \log \sigma_{t-1} + \eta_t.\tag{3.22}$$

In the equation (3.20), we have the components of vector $B_t$, which are modeled as random walk due to the fact that we earlier defined them as free elements of matrix $A_t$. In addition, the standard deviations are supposed to be developed as geometric random walks, and as a result, the variances produced in (3.22) are unobservable components.

Due to the fact that a random walk process has a probability of one to hit the lower or upper bound and due to the fact that this is not wanted in the model, the set of assumptions (3.20), (3.21) and (3.22) will ensure the model to be protected from such issue. In addition, such assumptions come with the advantage of having a reduced number of estimates.

Furthermore, the model's alterations are expected to be jointly normally distributed with the following assumptions in the variance-covariance matrix,

$$V = Var \begin{pmatrix} \varepsilon_t \\ \nu_t \\ \zeta_t \\ \eta_t \end{pmatrix} = \begin{bmatrix} I_t & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & W \end{bmatrix}.\tag{3.23}$$

With $I_t$ being the identity matrix and $Q$, $S$ and $W$ being positive definite matrices. In addition, in matrix $V$ it is worth stating that the restrictions imposed are
not absolutely necessary as they can be simply replaced by non-zero elements by editing the estimation procedure.

3.3 Bayesian Inference

Following the description of the Bayesian inference as in the paper of Primicieri (2005), a generic vector of variables $\omega^\tau$ for a generic time $\tau$ is denoted as follows,

$$\omega^\tau = [\omega_1^\tau, ..., \omega_T^\tau]' .$$

While a generic matrix of variables and constant terms $M_t$ is denoted as follows,

$$M^\tau = [m_1^\tau, ..., m_T^\tau]' ,$$

where $m_t$ stands for the column vector built with time-varying components of $M_t$.

The Bayesian inference introduced in this section aims to estimate the formerly described model by providing the necessary econometric techniques. In addition, the goal of such estimation is the evaluation of the posterior distribution of the parameters of interest.

Moreover, there have been identified four major reasons that make such estimation technique more preferable to others. As stated in the paper, firstly the fact that the parameters cannot be observed makes it very difficult to tell the difference between parameters and shocks, and as a result, the Bayesian technique is more suitable. Secondly, while comparing with the maximum likelihood estimation there are two major issues identified. The first issue is related to the size of the variance of the time-varying coefficient, which if it is small would cause the maximum likelihood estimator to have a point mass at zero. In addition, the second issue is connected with the high dimensions and nonlinearity. Such problems would produce likelihood with numerous peaks and increase the probability that some of them are in uninteresting areas of the parameter space. Such issue is corrected under the Bayesian approach through the usage of uninformative priors by ruling out undesired behavior patterns. Furthermore, another drawback of maximum likelihood estimation is related to the difficulty of maximizing the likelihood function while having large dimensional space. Again the Bayesian technique is able to overcome such problem since it operates by splitting the original problem into easier and smaller ones. For the valuation of the posterior numerical evaluation of the parameters of interest is used Gibbs sampling, which is a variant of MCMC.
3.3.1 Priors and Ordering

The priors in this paper assume independence at the initial state among coefficients, covariances, volatilities, and hyperparameters. In addition, $Q$, $W$, and $S$ priors of hyperparameters are built based on the assumption of having independent inverse Wishart distribution. On the other hand, priors for the rest are implied to be normally distributed. With the assumptions on hyperparameters and the hold of former specification together with the assumptions regarding the dynamics of time-varying parameters, (3.5), (3.6), (3.7), the normality of priors is ensured.

3.3.2 MCMC Algorithm

Through the simulation of the distribution of the parameters, it is achieved the estimation of the model for a given dataset. In Primiceri (2005) the MCMC algorithm was designed in three steps. In the first step $\Sigma^T$ is drawn from $p(\Sigma^T|y^T, \theta, s^T)$, secondly $s^T$ is drawn from $p(s^T|y^T, \Sigma^T, \theta)$ and thirdly $\theta$ is drawn from $p(s^T|y^T, \Sigma^T)$. The highlights in the first two steps stand for the conditional posteriors of the parameters, corresponding to the product of their priors. The last step indicates the estimation of the conditional posterior of $\theta$ by using the likelihood.

Such algorithm sketch is proven to be incorrect and the issue has been revised in the paper by Del Negro & Primiceri (2015). As they point out there are two main reasons why such algorithm is wrong. Firstly, it is identified that the first two steps, the algorithm alternates by using two different types of likelihood functions. Secondly, in the wrong version, the Gibbs sampling has been performed incorrectly as according to the theory one has to draw on each block conditional on others. Instead, it can be easily observed in step three that $\theta$ is not conditional on $s^T$. So the initial algorithm would produce incorrect posterior draws.

Moreover, Del Negro & Primiceri (2015), propose to continue using the Gibbs sampling but with different blocking. Unlike using three blocks like above, the authors designed the new algorithm in two blocks. In the first step $\Sigma^T$ can be drawn from $p(\Sigma^T|y^T, \theta, s^T)$ and in the second step, both $s^T$ and $\theta$ are drawn from $p(\theta, s^T|y^T, \Sigma^T)$. The second step is achieved through two additional sub-steps, where $\theta$ is drawn from $p(\theta|y^T, \Sigma^T)$ and $s^T$ is drawn from $p(s^T|y^T, \Sigma^T, \theta)$.

Such alternative is very similar to the initial algorithm and what changed are the steps. In the correct algorithm $s^T$ are sampled after $\theta$, but prior to $\Sigma^T$. This means that ordering matters and the estimation can be carried similarly by respecting the order.
4 Data

This study makes use of a set of economic and financial data for the case of US, EU, and the UK. The choice of such sample contributes to the aim of developing a comparative analysis for countries where both financial and economic developments are different. Regarding the dataset, it has been built by making use of multiple sources in order to provide complete and trustful information. Such sources include among others the Federal Reserve Bank of St. Louis, European Central Banks, Bank of England, Quandl and Reserve Bank of New Zealand. In order to compare the cases with each other, the stock market returns data that this study employed are from S&P 500, LSE and N100 indexes. The motivation behind this choice rests on the fact that these indexes represent the benchmark for equity prices in their respective countries and as a result represent their economies as well.

In addition, this study employs the variable of shadow interest rate, through which the impact of monetary policy will be measured on stock market returns. Such variable represents a recent development in literature and it would contribute to a better understanding of the impact that this unconventional tool may exert on stock market returns. Moreover, in order to motivate different views and extend the findings of this study, the analysis will be repeated for both estimations of Wu & Xia (2015), and Krippner (2016).

Furthermore, another variable of interest is inflation rate, which according to the literature is considered to be quite relevant. There are 156 observations for each variable for the period from 2005 to 2017, except for the case of US under Wu & Xia rate, which is limited to December 2015. Such adjustment comes due to the fact that after 2015 the interest rates in the US have been greater than 0.25%, thus exceeding the necessary benchmark in calculating the shadow rate. The frequency of the data is monthly.

4.1 Descriptive analysis

This section will be organized in three parts by providing a description for each of the cases. General analysis through graphs and descriptive statistics will be carried on for each case. It will reveal the main developments during the study period and will highlight the main findings.
4.1.1 United States

S&P 500

S&P 500\(^2\) the index represents a market capitalization of USD 23.9 trillion as of December 2017. It is built based on best 500 US companies and it is a decent measure for the market. The sample that is being analyzed contains interesting developments within it, as it includes the pre-crisis, crisis and post-crisis periods.

During the sample period the closing price has averaged to USD 1550.162, and its median is just a bit lower compared to the mean. Regarding maximum and minimum value it is worth pointing out that the difference is quite substantial. That is also for the fact that the sample includes the period of global financial crisis as well. The maximum value of USD 2673.61 is reached just very recently at the end of 2017, while the minimum value of USD 735.09 is reached during February 2009. Regarding the dates associated with such values, it makes sense as 2017 represents the recovery period, while 2009 represents the "peak" of the global financial crisis. Such indication can also be observed in figure 4.1, which shows the increasing trend up to the end of 2007, followed by an immediate drop during 2008 with the lowest point in April 2009. After that, as a result of Federal Reserve's interventions through liquidity provisions, bank's bailouts, etc., the market started to recover by maintaining an increasing trend afterward.

The returns as well could be interpreted similarly to the closing prices. On average during the period the returns are found to be less than one percentage point, with a maximum of 10.77% and a minimum of -16.94%. Again like explained above the significant drop is registered during the global financial crises and the maximum during the post-crisis period. Also, an interesting development here is the huge volatility that is observed during 2008 and 2009, and the fact that it is almost halved in the post-crisis period.

The descriptive statistics summary of the historical closing price and returns for S&P 500 is provided in table 4.1. In addition, the plot of prices and returns can be found respectively in figure 4.1 and 4.2.

\(^2\) Data are available at: http://siblisresearch.com/data/total-market-cap-sp-500/
Table 4.1: Standard & Poor’s 500 Historical Prices and Returns Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Max (MM/YY)</th>
<th>Min (MM/YY)</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>1550.162</td>
<td>1406.580</td>
<td>2673.610</td>
<td>735.0900</td>
<td>446.4465</td>
</tr>
<tr>
<td>Price</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>0.5881%</td>
<td>1.0583%</td>
<td>10.7723%</td>
<td>-16.9425%</td>
<td>3.9638%</td>
</tr>
<tr>
<td>Returns</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Author’s computations, E-views.

Figure 4.1: Standard & Poor’s 500 Closing Price 01/05 - 12/17 (USD)

Source: Author’s computations, E-views.

Figure 4.2: S&P 500 Returns

Source: Author’s computations, E-views.
Shadow Rate

While being a relatively new concept, shadow rate, has proven to be very useful in measuring the effects of monetary policy after the interest rate hit the ZLB. Traditionally central banks have used the federal funds rate to measure the impact of monetary policy due to its correlation with various economic and financial variables, but such practice is no longer possible. In addition, even the introduction of shadow rate, which could be simply defined as an approximation of the end of the yield curve by reflecting the federal funds rate in normal times, is quite challenging. Challenging for the fact that it is very hard to impose some sort of coherence between conventional and unconventional modeling of monetary policy. But regardless of such issue, investigation through shadow rate seems to be a very promising path.

There are currently two measures of shadow rate, which are relevant to be analyzed and explained. The shadow rate by Wu & Xia has an average of 0.59% and a mean of -0.54%, while Krippner’s shadow rate seems to be more normally distributed with mean and median closer to each other, respectively estimated to be 0.041% and -0.242%. In addition, the maximum and minimum values for the former one are found to be 5.26% and -2.98% for the periods of July 2007 and May 2014 respectively. The latter one, on the other hand, has a maximum value of 5.33% reached during July 2006 and a minimum value of -5.36% reached during April 2013. Such development is similar in terms of maximum value, but much more negative in terms of minimum values for the Krippner’s rate. Moreover, the latter one is more volatile as well. Such volatility can be also observed on the distribution on the graph. So the Krippner’s rate is more dispersed and puts more weight on its estimations compared to Wu & Xia rate. But apart from it, the general picture is very similar. As it may be understood by a simple graph, in normal times the shadow rate should be positive and similar to the federal funds rate. Once, the federal funds rate hits the ZLB, the shadow rate will work as an approximated for the upper end of the yield curve and it will be negative, thus reflecting the way how federal funds rate would be in normal times.

In table 4.2 can be found the descriptive statistics summary. In addition, figures A.1 and A.2 in Appendix A display the plot for each of the shadow rates, while figure 4.3 below plots the shadow rates against the federal funds rate.
Table 4.2: US Shadow Rates by Wu & Xia and by Krippner Descriptive Statistics

<table>
<thead>
<tr>
<th>Shadow Rate (Wu &amp; Xia)</th>
<th>Mean</th>
<th>Median</th>
<th>Max (MM/YY)</th>
<th>Min (MM/YY)</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shadow Rate (Wu &amp; Xia)</td>
<td>0.596098%</td>
<td>-0.542088%</td>
<td>5.26% (07/07)</td>
<td>-2.985644% (05/14)</td>
<td>2.675514%</td>
</tr>
<tr>
<td>Shadow Rate (Krippner)</td>
<td>0.041053%</td>
<td>-0.242690%</td>
<td>5.331815% (07/06)</td>
<td>-5.3693% (04/13)</td>
<td>3.0041%</td>
</tr>
</tbody>
</table>

Source: Author’s computations, E-views.

Figure 4.3: US Shadow Rates Vs. Federal Funds Rate

Source: Author’s computations, E-views.

Inflation Rate

For the sample period, the inflation rate has been quite stable and at the target, averaging at 1.9%. In addition, its mean is very close to the median thus suggesting a normal distribution of this time series. Maximum of 2.9% has been observed during September 2006, while the minimum of 0.6% has been observed during October 2010. The minimum value observed during the post-crisis period fueled the fear of a possible deflation. This outcome came as a result of a dramatic fall in asset prices after the housing bubble burst. Such matter was seriously treated by the Federal Reserve and soon inflation rate was characterized by an upward trend, thus normalizing and being almost exactly at the target for the remaining period.

In table 4.3 can be found a summary of the descriptive statistics for inflation and in figure 4.4 can be found the plot of data.
Table 4.3: US Inflation Rate Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Max (MM/YY)</th>
<th>Min (MM/YY)</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>1.9308%</td>
<td>2%</td>
<td>2.9% (09/06)</td>
<td>0.6% (10/10)</td>
<td>0.4378%</td>
</tr>
</tbody>
</table>

Source: Author’s computations, E-views.

Figure 4.4: US Inflation Rate

Source: Author’s computations, E-views.

4.1.2 European Union

Euronext\(^3\) is the largest stock market in European Union with a market capitalization of EUR 3.6 trillion. For this study has been considered to use the N100 index representing the 100 biggest and most valued companies in the Euronext.

The selected index has an average closing price of EUR 796.22 and a median of EUR 809.95, which is very close to the mean. In addition, the maximum value of EUR 1068.23 was observed very recently during September 2017, while the minimum value of EUR 469.65 was observed during the global financial crisis on January 2009. It is important to recall that such developments are also very similar to the US case, thus indicating the magnitude and severity of the global financial crisis.

From the available data, it is easy to agree on the fact that prices are almost at the same level as they used to be during the pre-crisis period. In addition, the data reveal the significant drop during the global financial crisis, but also the negative impact of the European sovereign debt crisis during late 2009 till 2013. But, regardless, of the crisis and their severity, the N100 index has been going through a very promising trend in the recent years.

Regarding its returns, they are averaged at 0.39%, with a median of 0.93% thus indicating a lot of dispersion during the sample period. Such statement is further

\(^3\) https://www.euronext.com/en/equities
supported by the maximum value of 12.31% encountered in March 2009 and the minimum value of -14.73% encountered during September 2008. The minimum value also coincides with the failure of “Lehman Brothers”, thus indicating the impact of such failure into Euronext returns. Moreover, even the standard deviation value serves as a further confirmation of the formerly explained volatility.

The descriptive statistics for N100 closing prices and returns are summarized in table 4.4. In addition, in figures 4.5 and 4.6 are plotted the data.

Table 4.4: Euronext 100 Historical Prices and Returns Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Max (MM/YY)</th>
<th>Min (MM/YY)</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>N100 Price</td>
<td>796.2248</td>
<td>809.95</td>
<td>1068.23 (09/17)</td>
<td>469.65 (01/09)</td>
<td>148.5957</td>
</tr>
<tr>
<td>N100 Returns</td>
<td>0.39179%</td>
<td>0.931777%</td>
<td>12.3187% (03/09)</td>
<td>-14.73338% (09/08)</td>
<td>3.972807%</td>
</tr>
</tbody>
</table>

Source: Author’s computations, E-views.

Figure 4.5: Euronext 100 Closing Prices 01/05 - 12/17 (EUR)

Source: Author’s computations, E-views.

Figure 4.6: Euronext 100 Returns

Source: Author’s computations, E-views.
Shadow Rate

Shadow rate measures for EU are almost walking side by side with little differences like in the case of US as well. The measure by Wu & Xia is again with a smaller range and less volatile than the measure developed by Krippner. The former one has a mean of 0.023% and a median of 0.056%, which are not very informative on the matter due to the fact that our sample includes periods where the European Central Bank refinancing rate used to be positive. As a consequence, the shadow rate under those conditions was positive as well, but our interest strives in the period when the refinancing rate hit the ZLB. Thus the maximum value of 4.27% observed in August 2008 – initiation of the global financial crisis – represents the pre-ZLB period, while the minimum value of -5.5% observed during May 2017 represents our period of interest.

The latter rate developed by Krippner represents similar developments, but with a higher variation as it was claimed earlier. Its mean and median with respective values of -0.41% and 0.27% are much further compared to the former rate. In addition, the maximum value of 4.35% is similar to the previous measure and also the period when it was observed differentiates by one month as this one was observed in July, while the other one in August 2008. Regarding the minimum value, it is found to be -7.6% in October 2016, indicating a more differentiated approximation of the end of the yield curve. Such dispersion and volatility are further confirmed by the high standard deviation of the series.

When the shadow rates are plotted against the refinancing rate, it is easily observable that they almost walk side by side in normal times when the latter one is positive. In periods when interest rates hit the ZLB shadow rates are displayed as a reflection of refinancing rate under normal times.

The summary of descriptive statistics for shadow rates is summarized in table 4.5. In addition, in figures, A.3 and A.4 in Appendix A can be found the plot for each shadow rate and in figure 4.7 can be found the plot of shadow rates against ECB refinancing rate.

Table 4.5: EU Shadow Rates by Wu & Xia and by Krippner Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Max (MM/YY)</th>
<th>Min (MM/YY)</th>
<th>Std. Dev.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Shadow Rate</strong></td>
<td><strong>Wu &amp; Xia</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.023341%</td>
<td>0.056389%</td>
<td>4.27851% (08/08)</td>
<td>-5.502042% (05/17)</td>
<td>2.721954%</td>
<td></td>
</tr>
<tr>
<td><strong>Shadow Rate</strong></td>
<td><strong>Krippner</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.415347%</td>
<td>0.275481%</td>
<td>4.356119% (07/08)</td>
<td>-7.636271% (10/16)</td>
<td>3.183327%</td>
<td></td>
</tr>
</tbody>
</table>

*Source: Author’s computations, E-views.*
Inflation developments in the European Union are more dispersed and volatile while comparing to the United States. Even though averaged at 1.57%, with a median of 1.75%, which are close to the ECB’s target of 2% with +/- 1% difference, inflation behavior has been concerning. Concerning for the fact that inflation reached values as high as 4.1% during July 2008 and values as low as -0.7% during July 2009. Such developments respectively coincide with the global financial crisis “eve” and sovereign debt crisis “eve”. With such a high volatility in just one year, Europe got hit by a fear of deflation and the economy seemed to be more fragile than ever. Moreover, the measure of standard deviation also serves as a confirmation of the volatility issue.

Furthermore, the trend remained unclear as there have been periods when inflation was headed upwards and then immediately changing direction downwards as it happened during the sovereign debt crisis. The recent situation seems to be more promising for a stable inflation rate at a target of 2%, but the insecurities are still present with inflation undershooting its target.

In table 4.6 are summarized the descriptive statistics for the variable of inflation, while in figure 4.8 are plotted the data in order to offer a visual inspect of the time series.

<table>
<thead>
<tr>
<th>Mean</th>
<th>Median</th>
<th>Max (MM/YY)</th>
<th>Min (MM/YY)</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>1.576282%</td>
<td>1.75%</td>
<td>4.1%</td>
<td>-0.7%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(07/08)</td>
<td>(07/09)</td>
</tr>
</tbody>
</table>

Source: Author’s computations, E-views.
4.1.3 United Kingdom

For the case of United Kingdom, this study employs the LSE index, as the best measure of asset prices in this country. LSE\(^4\) represents a market capitalization of USD 3.61 trillion, thus ranking it among the most highly capitalized indexes.

On average the closing price is found to be USD 1316.91, with a median value of USD 879.8, which gives an indication for non-normal distribution of the series. Interesting is the maximum value of USD 3935.683 observed during August 2008 and the minimum value of USD 309.4254 observed during March 2005. A similar situation of the latter one has been observed during March 2009, even though the entire period from late 2007 till late 2012 has been characterized by low closing prices, clearly due to consecutive crises. Surprising is the standard deviation of the time series, which is more than doubled while comparing with the former cases of S&P 500 or N100 indexes. This serves as a clear indication of asset prices volatility during the sample period.

Moreover, it is obvious from the data plot that the UK suffered the consequences of the global financial crisis, but even though it is part of EU, its development during the sovereign debt crisis have not been similar to the latter at all. Also while the picture is similar for US and EU regarding prices behavior during the financial crisis, UK prices exhibited a different trend. Closing prices have been increasing more and more after the crisis and have maintained their trend till nowadays. Recent prices are found to be up to 2.5 times higher compared to the pre-crisis period.

\(^4\) Data are available at: https://www.stockmarketclock.com/exchanges/lse
Even when calculating the returns it is easily observable that the returns averaged at 1.86%, with a mean of 2.17% is much higher compared to the former cases, thus confirming once again the previous statements. But, even though the trend and returns are much higher, it is important to stress out that such results have been associated with a very high level of volatility. The maximum value of 32.44% represents the upper extreme achieved during April 2009 when the UK was experiencing a recovery from the major hit of global financial crisis. The minimum value of -35.28% observed in October 2008, on the other hand, represents the lower extreme, thus picturing a very extreme range of values. Moreover, standard deviation serves as a confirming measure of such dispersion in the time series.

In table 4.7 are presented the summary of descriptive statistics for the closing prices and returns of LSE index. Moreover, figures 4.9 and 4.10 present the plot for each of the series, with the aim of visualizing the developments during the sample period.

**Table 4.7: London Stock Exchange Historical Prices and Returns Descriptive Statistics**

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Max (MM/YY)</th>
<th>Min (MM/YY)</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSE Price</td>
<td>1316.916</td>
<td>879.8088</td>
<td>3935.683 (08/17)</td>
<td>309.4254 (03/05)</td>
<td>966.5683</td>
</tr>
<tr>
<td>LSE Returns</td>
<td>1.868186%</td>
<td>2.172962%</td>
<td>32.44930% (04/09)</td>
<td>-35.28390% (10/08)</td>
<td>9.000678%</td>
</tr>
</tbody>
</table>

*Source: Author’s computations, E-views.*

**Figure 4.9: London Stock Exchange Closing Prices 01/05 - 12/17 (USD)**

*Source: Author’s computations, E-views.*
Shadow rate by Wu & Xia for the case of UK is averaged on -1.035%, with a median of -2.5%. Again following the same reasoning like in the former case, what is within our interest are the maximum and minimum values. The maximum value of 6.05% observed during July 2007, represents the shadow rate in normal times, which is close to UK interbank rate. The minimum value of -6.5% observed during March 2013, on the other hand, represents the approximation of interbank rate during the ZLB period.

Approximately the same developments are observed in the Krippner’s computation, regarding the maximum and minimum values. Interesting are the differences in mean, but the interpretation of such measure will not be able to reveal any relevant information except methodological differences. Again as expected the standard deviation measure reveals also a high volatility in the series.

While plotted against the interbank rate, it can be observed that during the pre-crisis period the latter one together with the shadow rates exhibits the same behavior. Of course, the post-crisis developments reveal differences in the measures where the reverse bell shape of Krippner’s rate is tighter than Wu & Xia’s rate.

In table 4.88 are summarized the main descriptive statistics for the shadow rates. Moreover, figures A.5 and A.6 in Appendix A display the plotted series of shadow rates for each of the computations. Furthermore, figure 4.11 below plots the shadow rates against the interbank rate in order to visualize the differences.
Table 4.8: UK Shadow Rates by Wu & Xia and by Krippner Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Max (MM/YY)</th>
<th>Min (MM/YY)</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shadow Rate</td>
<td>-1.035891%</td>
<td>-2.507890%</td>
<td>6.054334%</td>
<td>-6.509898%</td>
<td>4.187218%</td>
</tr>
<tr>
<td>Wu &amp; Xia</td>
<td></td>
<td></td>
<td>(07/07)</td>
<td>(03/13)</td>
<td></td>
</tr>
<tr>
<td>Shadow Rate</td>
<td>0.547318%</td>
<td>0.091541%</td>
<td>5.944855%</td>
<td>-6.755506%</td>
<td>3.281587%</td>
</tr>
<tr>
<td>Krippner</td>
<td></td>
<td></td>
<td>(07/07)</td>
<td>(04/13)</td>
<td></td>
</tr>
</tbody>
</table>

Source: Author’s computations, E-views.

Figure 4.11: UK Shadow rates Vs. Interbank Rate

Source: Author’s computations, E-views.

Inflation

The inflation rate in the UK is averaged on 2.22% during the sample period, with a median of 2.4%, which compared to the former cases is higher. Its range of values as well is more differentiated, with a maximum value of 4.8% observed during September 2008 and a minimum value of 0.2% observed during October 2015. Interesting is the fact that while US and EU passed their deflation concerns during the post-crisis period, UK experienced a different pattern. Once the global financial crisis hit the economy a significant drop in inflation was experienced, but such situation did not continue for too long. Immediately a significant increase was experienced from late 2009 till late 2011. But the situation was about to change as the late 2011 inflation peaked at 4.5% and through the policies, it exhibited a significant drop to close to 2% during late 2012 and maintained this level till late 2013. After these developments, inflation followed a downward trend and in 2015 was observed the minimum value mentioned earlier. From the dramatic fall of 2015 inflation has recovered and fluctuates around the 2% target with small deviations.
In table 4.9 are summarized the main descriptive statistics for inflation, while figure 4.12 visualizes the time series.

**Table 4.9: UK Inflation Rate Descriptive Statistics**

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Max (MM/YY)</th>
<th>Min (MM/YY)</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>2.229487%</td>
<td>2.4%</td>
<td>4.8%</td>
<td>0.2%</td>
<td>0.971146%</td>
</tr>
</tbody>
</table>

Source: Author’s computations, E-views.

**Figure 4.12: UK Inflation rate**

Source: Author’s computations, E-views.

### 4.2 Variable’s Testing

In this section, variables will be subject to stationarity and multicollinearity testing. Such procedure will reveal the final form of variables in order to perform the empirical assessment correctly.

#### 4.2.1 Unit root Test

In order to test for the presence of unit roots, two tests been employed in this study. The first one is “Augmented Dickey-Fuller Test” (ADF) and the second one is “Kwiatkowski–Phillips–Schmidt–Shin” (KPSS) test. In table 4.10 can be found the summary of unit root testing for each of the variables and respective testing techniques.

Based on ADF, the first three variables used to measure returns are stationary as their p-values are zero. This means that there is enough evidence to reject the null hypothesis of unit root presence. In addition, the former conclusion is supported only partially from the KPSS test as only the test values of Euronext 100 and London Stock Exchange are smaller than the critical value of 0.146. As a result, there is not
enough evidence to reject the null hypothesis of stationarity for these two variables. Unfortunately, KPSS suggests the presence of unit root for the variable of S&P 500 and finds it stationary only under the first difference.

Regarding the variables of shadow rate, it is easily understood from both tests that such series are non-stationary. As a result according to the ADF, they can all become stationary under first differencing, but according to the KPSS, the US shadow rates and UK's Krippner rate are not stationary under first differencing as well.

Lastly, the variables of inflation as well are found to be non-stationary in levels by both ADF and KPSS. But under first differencing such issue is corrected and unit roots disappear. Such conclusion for ADF can be easily drawn from the probability values, which are zero and for KPSS from the test values which are lower than the critical value.

Apart from the conclusions suggested from these tests we need to take into account an important detail before processing with the final form of this thesis's variables. For the non-stationary variables, the solution seems to be the first differencing, but such transformation would clearly remove any rich dynamics on the data. What this thesis is trying to achieve is an understanding of monetary policy impact on the stock returns and in order to satisfy such intention, it is necessary to carry this investigation by leaving the variables in their levels. Of course, this action has been interpreted differently in literature as there are many advocates and opponents as well.

One important insight extracted from the studies of Toda & Yamamoto (1995) and Ghassan (2011) is that a possible solution to the problem might be the appropriate number of lags. Such proposal might yield correct results, but this is also constrained by the complexity of the model. In our case, the model is very complex and the number of lags used in this situation is limited to the computation feasibility by making the choice somehow arbitrary.

It is important to emphasize at this stage that we are interested in the nature of the relationship and not in point estimates, thus the non-stationarity issue should not constitute a major concern as stated by Sims (1980) and Sims, Stock, & Watson (1990). In addition to that, as expressed earlier the intention is the observation of the nature of the relationship, which will mainly be measured through impulse response functions. Basically, this comes at a mutual point with Amisano & Giannini (2012), stating that dynamic responses of non-policy variable due to a shock from policy variables could be captured simply by Cholesky decomposition. Moreover, it is
argued that this is achievable as such method applies a recursive structure in the model and this is one of the scenarios employed in this thesis, thus making the estimation feasible. In order to conclude the variable’s final form, it is suggested to carry the empirical estimation by leaving the variables at their initial states. Such decision is motivated by the intention of capturing dynamics which cannot be exploited under first differencing of the variables of interest.

Table 4.10: Stationarity Tests (ADF&KPSS)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Augmented Dickey Fuller Test (ADF) (P-value)</th>
<th>Kwiatkowski–Phillips–Schmidt–Shin (KPSS) Test (Test Value)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Levels</td>
<td>1st Difference</td>
</tr>
<tr>
<td>S&amp;P 500 Returns</td>
<td>0.000</td>
<td>0.178034</td>
</tr>
<tr>
<td>N100 Returns</td>
<td>0.000</td>
<td>0.094483</td>
</tr>
<tr>
<td>LSE Returns</td>
<td>0.000</td>
<td>0.06887</td>
</tr>
<tr>
<td>U.S. Wu &amp; Xia Shadow Rate</td>
<td>0.7476</td>
<td>0.000</td>
</tr>
<tr>
<td>U.S. Krippner Shadow Rate</td>
<td>0.6942</td>
<td>0.000</td>
</tr>
<tr>
<td>E.U. Wu &amp; Xia Shadow Rate</td>
<td>0.9622</td>
<td>0.000</td>
</tr>
<tr>
<td>E.U. Krippner Shadow Rate</td>
<td>0.8059</td>
<td>0.000</td>
</tr>
<tr>
<td>U.K. Wu &amp; Xia Shadow Rate</td>
<td>0.583</td>
<td>0.000</td>
</tr>
<tr>
<td>U.K. Krippner Shadow Rate</td>
<td>0.4748</td>
<td>0.000</td>
</tr>
<tr>
<td>U.S. Inflation Rate</td>
<td>0.1524</td>
<td>0.000</td>
</tr>
<tr>
<td>E.U. Inflation Rate</td>
<td>0.1655</td>
<td>0.000</td>
</tr>
<tr>
<td>U.K. Inflation Rate</td>
<td>0.2584</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Source: Author’s computations, E-views.

4.2.2 Multicollinearity

In tables 4.11, 4.12 and 4.13 are summarized the correlations between the variables used in this study. Such investigation aims to identify and correct for any multicollinearity issue that may arise due to explanatory variables correlation.

According to the below correlation matrixes, there is no significant evidence suggesting for possible multicollinearity in the dataset. The variables are poorly correlated among others and the maximum correlation coefficient barely exceeds 0.5. The only strongly correlated coefficients are the two measures of shadow rate, which does not constitute a problem as they first are dependent variables and secondly will be used separately rather than together.
### Table 4.11: Correlation Matrix US

<table>
<thead>
<tr>
<th>Variables</th>
<th>Wu &amp; Xia Shadow Rate</th>
<th>Krippner Shadow Rate</th>
<th>Inflation</th>
<th>S&amp;P 500 Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wu &amp; Xia Shadow Rate</td>
<td>1</td>
<td>0.908</td>
<td>0.1015</td>
<td>0.1017</td>
</tr>
<tr>
<td>Krippner Shadow Rate</td>
<td>0.908</td>
<td>1</td>
<td>0.379</td>
<td>0.0134</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.1015</td>
<td>0.379</td>
<td>1</td>
<td>-0.177</td>
</tr>
<tr>
<td>S&amp;P 500 Returns</td>
<td>0.1017</td>
<td>0.0134</td>
<td>-0.177</td>
<td>1</td>
</tr>
</tbody>
</table>

*Source:* Author’s computations, E-views.

### Table 4.12: Correlation Matrix EU

<table>
<thead>
<tr>
<th>Variables</th>
<th>Wu &amp; Xia Shadow Rate</th>
<th>Krippner Shadow Rate</th>
<th>Inflation</th>
<th>S&amp;P 500 Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wu &amp; Xia Shadow Rate</td>
<td>1</td>
<td>0.968</td>
<td>-0.5747</td>
<td>-0.029</td>
</tr>
<tr>
<td>Krippner Shadow Rate</td>
<td>0.968</td>
<td>1</td>
<td>-0.572</td>
<td>0.000206</td>
</tr>
<tr>
<td>Inflation</td>
<td>-0.5747</td>
<td>-0.572</td>
<td>1</td>
<td>-0.2205</td>
</tr>
<tr>
<td>S&amp;P 500 Returns</td>
<td>-0.02907</td>
<td>0.000206</td>
<td>-0.2205</td>
<td>1</td>
</tr>
</tbody>
</table>

*Source:* Author’s computations, E-views.

### Table 4.13: Correlation Matrix UK

<table>
<thead>
<tr>
<th>Variables</th>
<th>Wu &amp; Xia Shadow Rate</th>
<th>Krippner Shadow Rate</th>
<th>Inflation</th>
<th>S&amp;P 500 Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wu &amp; Xia Shadow Rate</td>
<td>1</td>
<td>0.932</td>
<td>-0.5367</td>
<td>-0.0056</td>
</tr>
<tr>
<td>Krippner Shadow Rate</td>
<td>0.932</td>
<td>1</td>
<td>-0.474</td>
<td>-0.0277</td>
</tr>
<tr>
<td>Inflation</td>
<td>-0.5367</td>
<td>-0.474</td>
<td>1</td>
<td>-0.1021</td>
</tr>
<tr>
<td>S&amp;P 500 Returns</td>
<td>-0.0056</td>
<td>0.0277</td>
<td>-0.1021</td>
<td>1</td>
</tr>
</tbody>
</table>

*Source:* Author’s computations, E-views.
5 Empirical findings

The empirical chapter will be divided into three parts, which will summarize the main findings from the cases of United States, European Union, and the United Kingdom. In each of the three parts, the analyses will be focused on discussing the residual's volatility for each of the equations produced, the impulse response functions and the forecast distributions. All of the formerly mentioned analyses components will be evaluated against the simple VAR findings, thus aiming to reveal the added value and findings robustness of time-varying parameter (TVP) VAR.

5.1 United States Case

Initiating the analyses, a general focus is attributed to the simple VAR results whose output for both Wu & Xia and Krippner rate are displayed in table A.1 and figure A.7 in Appendix A. It is important to understand that analyzing such results is beneficial for three main reasons. Firstly, the simple VAR provides an overall picture of what can we expect as it is easily computable and understandable. Secondly, it would help determine the difference on the output with our main model as it does not allow time variation in parameters. Thirdly, it helps to compare the differences in residual's standard deviation, which under the basic model is constant while under the time-varying model it is expected to exhibit time variation.

Back to the main VAR output, it is easily understood that inflation keeps a negative approach towards S&P 500 returns. In addition, it is observable that its persistence does not differ much through the first and second lag, with only a slightly lower impact on the latter. Moreover, none of the inflation lags are found to be significant at 5% significance level. Such findings are in the same line with Gultekin (1983), Geske & Roll (1983), Boyd, Levine, & Smith (2001) and Bordo, Duecker, & Wheelock (2008). Furthermore, while observing the output from the next model with Krippner rate as the dependent variable, the conclusions look different. Different in the sense that in the first lag the impact of inflation remains negative, but strangely it becomes three times more negative with a value of -1.87 compared to -0.57 in the former case. In addition to that, the second lag transforms into a positive value of 0.6, which is very surprising compared to the former case.

Shifting the focus from inflation to shadow rate itself, it is found that this variable exhibits a positive impact with a value of 0.86 on its first lag towards S&P 500 returns, while in the second lag it becomes negative with a value of -0.89. Similar
developments are observed even when we use Krippner’s shadow rate instead of Wu & Xia’s, but what changes in this case is the size of coefficients, which respectively become 1.24 and -1.17 for the first and second lag. Such difference could be expected as when we analyzed the data in the former chapter it was clearly observed a higher volatility in Krippner’s shadow rate. Such outcome can be only understood if we take into account the studies by Black (1995), Wu & Xia (2015) and Krippner (2016). As among the firsts who discussed the idea of shadow rate, Black (1995) implied that the nominal rate could not be negative as people would still have the option to keep their money in cash. As a result of its definition of nominal rate as shadow real rate plus inflation, it was clearly expected that shadow rate would be positive and would walk side by side with the interest rate in normal times when the latter one would be positive. In times when nominal interest rate would hit the ZLB, the shadow rate would simply become negative by serving as a reflection of how nominal interest rate would look in normal times. Having this information in mind and recalling the findings from Rigobon & Sack (2003) or Gregoriou, Kontonikas, MacDonald, & Montagnoli (2009), it is understood that the relationship between nominal interest rate and stock market returns is negative. As a consequence, the impact of shadow rate in normal times would be negative as it is observed in the second lag. That is why during the crisis the monetary policy authorities would lower the interest rates so they could inject liquidity. While such instrument is no longer possible due to the ZLB, the shadow rate would be expected to follow the same path as normal rate, but as we can observe it has reversed the relation by becoming positive. This is normally something that can be observed only in the recent period as it is approaching zero with a positive trend.

While having discussed the initial output from VAR models, it is important to start analyzing the findings from the time-varying models as well. In figure 5.1 are plotted the standard deviation results of each of the three equations produced by our model for the Wu & Xia shadow rate measure, while the results from Krippner models are plotted in figure A.8 in appendix A. The figures themselves in both cases indicate a significant time variation in residuals. If we would compare it to a simple VAR, the graph for the latter one would be a simple straight constant line. Interesting from the output is that there are significant differences when comparing both shadow rate measures. Such differences are observed in terms of fluctuations band where the Krippner rate seems to fluctuate more, thus exhibiting larger variation compared to the alternative measure. Such difference can be clearly observed in the other graphs as well, but with a more constrained band. These graphs serve as a major support to the allowance for stochastic volatility in our model, thus reconfirming the implications of Primicieri (2005) and Nakajima, Kasuya & Watanabe (2011).
Empirical findings

Figure 5.1: Posterior Means: Standard Deviation for Wu & Xia Model's Residuals (US)

This figure represents the residual's standard deviation for TVP-VAR estimations with Wu & Xia shadow interest rate. The light green line in the middle represents the normal standard deviation, while the lower and upper orange lines represent the 0.16 and 0.84 percentiles. In addition, the x-axis stands for the time period and the numbers stand for the months. Note that the Wu & Xia shadow rate sample is restricted from 2005 to 2015 due to the fact that it ceases to exist when the interest rates are above 0.25%.

Source: Author’s computations, R-studio.

Another important result from these models are definitively the impulse response functions constituting the most significant output from which we can determine the response of certain variable towards shocks from variables of interest. In order to indicate the differences among the models, both simple VAR’s and TVP-VAR’s results are plotted against one another under three different scenarios. The first scenario stands for a one unit shock (one standard deviation) to only one of the residuals elements, aiming to see the developments in the dependent variable, while comparing to a case when \( u_t \) is a vector of zeros. In addition, the second scenario determines the impact of a one-unit shock to a part of the error term's elements. This scenario uses the so-called Cholesky decomposition of the variance-covariance matrix. The third scenario, on the other hand, developed by Del Negro & Primicieri (2015) is similar to the former scenario, but it sets the elements of \( \Sigma_t \) on their average values for the study period. The output from the formerly mentioned scenarios has been plotted in figure 5.2 below for the Wu & Xia and in figure A.9 in the Appendix A for the Krippner model.

For the first scenario, it can be easily observed that a one unit shock of Wu & Xia shadow rate exhibits a negative impact on S&P 500 returns, but the persistence of the shock does not seem to be quite strong. Krippner's shadow rate, on the other hand, represents a stronger impact initially, which apparently dies out very soon by becoming less and less significant in the coming periods. While comparing both of them with the basic VAR impulse responses it is obvious that there are substantial differences. Firstly, the simple VAR fails to indicate significant developments of the shock and this is basically due to the lack of time variations. And secondly, it fails to
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Indicate any relevant response as the line clearly stays near to zero. As discussed earlier when analyzing the coefficients, such developments are quite normal due to the negative relationship between interest rate and stock market returns implied by the literature.

Scenario two and three almost display similar dynamics like in the first case, when speaking for Wu & Xia. But in these scenarios, it is interesting to see that the effect seems to be more pronounced as the line is slightly more distanced from zero. Also, the picture helps to indicate that the impact of the shock almost remains constant for many periods and gets reduced very slowly. This result is in the same line with the findings by Gali & Gambetti (2015) and Jansen & Zervou (2017) and Pascal (2018). On the other hand, Krippner's measure presents more interesting findings as in both scenarios can be detected an initial positive impact of shadow rate on stock returns. Such impact lasts only for a very short period and turns out to become negative later on, but compared to the first scenario the magnitude of the shock seems to be slightly more moderate. The initial positive impact, in this case, is quite temporary and associated with a large amount of uncertainty as indicated by the wideband. What is important in this case is the rest of the development, which is in the same line to the former model and the literature as well.

While the impact of a shadow rate shock is clarified, it would be relevant to see the developments in terms of inflation shocks. As it can be observed in the IRFs output, for the Wu & Xia models the impact is generally negative. This finding serves as an additional confirmation of the former results on the simple VAR, but relevant in these results is the magnitude of reaction towards a shock of this kind. In each of the scenarios, the TVP and simple VAR lines meet on average after 15 periods, but their path is not similar at all. The former one reflects a significant response of stock returns towards an inflation shock, which apparently remains strong for a significant period of time. The latter one on the other hand basically acts on the contrary as it indicates a weak response initially, followed by a stronger response in the coming periods.

Regarding the Krippner model, we can observe a different pattern as the impact is more similar to simple VAR when speaking for the second and the third scenarios. Under the first scenario, the TVP-VAR indicates a positive response of stock market returns towards an inflation shock. Such development apparently lasts for less than 4 periods as the impact soon becomes negative. Still, it is important to highlight that the magnitude of its negativity is much smaller than in the Wu & Xia model.
One additional indication that this IRFs provide is the fact that while comparing shocks with one another there seems to be a higher uncertainty surrounding the inflation shock. This can be observed by seeing the wideness of the shaded area representing the 0.05 and 0.95 quantiles. The shadow rate shock, on the other hand, has a narrower band, thus indicating less uncertainty in the model. Such finding for inflation shocks is common in the literature, but what else they reveal is the confirmation of this variable as a potential factor in impacting the stock market returns. This is in the same line with the studies of Bean (2007), Roger (2009) and Williams (2009), which have acknowledged inflation as a potential variable in calibrating the asset prices.

**Figure 5.2: Wu & Xia TVP-VAR’s Vs. Simple VAR’s Core Impulse Response Functions (US)**

This figure summarizes the core impulse response functions (IRFs) that are of significant interest for this thesis. In addition, the TVP-VAR IRFs are plotted together with simple VAR IRFs, where the former one is represented by the solid black line while the latter one by the dashed black line. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

*Source:* Author’s computations, R-studio.

While for sure the former analyses provides interesting insights regarding the behavior of the variables under a one unit shock, it would be relevant to consider some interesting developments on certain dates. More concretely, it has been selected
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the 43rd month of our sample corresponding to July 2008, the 85th month corresponding to March 2012 and the 115th month corresponding to August 2014, the recovery period. The choice of these dates is somehow arbitrary, but the motivation could be clearly understood from their respective period's developments. So, the aim in here is to have a picture of how the experience of an inflation shock or shadow rate shock would impact this thesis samples in times of crisis and in times of prosperities. In addition, it would help to illustrate the relevance of monetary policy before and after the ZLB. All the plots for Wu & Xia are summarized in figure 5.3 and the impulse response functions are built based on the second scenario.

Starting with the shadow rate, unlike determined earlier when considered the general shock without focusing on specific dates, the findings are somehow different. In part a) of figure 21 can be observed the response difference during periods of financial crisis and periods of prosperity. More concretely, on July 2008 period of our sample, the response to one unit shock in stock returns is initially negative and remains so just in the first period, by following a strictly upward path later. In addition, after becoming positive it tends to maintain the sign as the effect dies slowly in the coming periods. Similar developments can be observed during the period of March 2012 and during the period of August 2014, but the magnitude differs substantially. This means that the effect is strongly pronounced during the crisis and becomes less and less relevant under growth periods.

Moreover, in parts b) and c) can be observed the significant differences between July 2008 – March 2012 and July 2008 – August 2014 impulse response functions. These graphs support the former claim on recession and growth developments of one unit shock. As it was expected the difference is more substantial among the latter pair as the August 2014 period is long after the global financial crisis, while the March 2012 period only indicates the initial stages of the recovery when the economy was still weak. In addition, the difference between March 2012 and August 2014 impulse response functions indicates the similarity of their respective period's results. Such indication can also be taken by viewing the distance between the 16th and 84th percentiles, which are very wide in part b) and c) while becoming very narrow in d).

Shifting from shadow rate to inflation, again the dynamics observed in here are not similar to the general shock observed earlier. Inflation follows basically the same logic as shadow rate in terms of transforming its impact from negative to positive in all the examined periods. In addition, its impact is also more pronounced during the crises and diminishes under economic prosperity, thus exhibiting similar behavior to the former variable. Moreover, it is interesting to see that even though the
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Dates represent different developments the difference among impulse response functions is not that relevant. Having said this, we can confirm such claim by looking at parts f), g) and h), which clearly support such argument as the distance among the 16th and 84th percentile is very small. The main contribution and inference received from this part is the fact that the pre and post-crisis developments are quite similar, thus confirming the continued relevance and effectiveness of monetary policy.

Having observed the Wu & Xia models, it would be beneficiary to shift our focus to the Krippner models as well so we can determine if there are different developments. Indeed when observing the impulse response functions in figure A.16 in Appendix A, part a) for the shadow rate, we notice that the impact is very different compared to the Wu & Xia models and also not similar to the original impulse response functions as well. The impact in here is substantial as well, but in the same time more coherent, meaning that the lines for March 2012 and August 2014 are very similar, while the July 2008 line is able to join the former ones just after 10 periods. In addition, while the two latter periods lines represent a more pronounced effect, the July 2008 period line of the global financial crisis is less negative. Moreover, all of them seem to follow an upward trend after the first 2 periods, indicating their path toward transforming into a positive impact. The small differences among the lines can be seen also in parts b), c) and d) which according to the y-axis range are very narrow.

Inflation, on the other hand, follows quite a similar impact like in the former case. Almost all of the lines are exhibiting same path and only the July 2008 period line stands slightly lower compared to the others. This difference actually constitutes the main mismatch compared to the former model's developments. According to such pattern, the impact of an inflation shock would be less pronounced during the crisis than during the prosperity period. Such outcome, of course, needs deeper assessment as there may be other significant factors on that date shaping the line. Moreover, like in the former case, the similarity of the lines can be observed in parts f), g) and h), which indicate low distance among percentiles and small range in the y-axis as well.

Figure 5.3: Wu & Xia Impulse Response Functions on 43rd (July 2008), 85th (March 2012) and 115th (August 2014) Periods (US)
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This figure summarizes the IRFs for the 43rd, 85th and 115th months of the dataset for Wu & Xia models. Figure 21 a) contains the plots of shadow rate IRFs for each of the periods. Part b) indicates the difference between 43rd and 85th shadow rate IRFs, associated with 16th and 84th percentiles. Part c) indicates the difference between 43rd and 115th shadow rate IRFs, associated with 16th and 84th percentiles. Part d) indicates the difference between 85th and 115th shadow rate IRFs, associated with 16th and 84th percentiles. Part e) contains the IRFs of inflation for each period of interest. Part f) indicates the difference between 43rd and 85th inflation IRFs, associated with 16th and 84th percentiles. Part g) indicates the difference between 43rd and 115th inflation IRFs, associated with 16th and 84th percentiles. Part h) indicates the difference between 85th and 115th inflation IRFs, associated with 16th and 84th percentiles.

Source: Author’s computations, R-studio.

The last part of the U.S. analysis is focused on building an out of sample forecast distribution. For this case we have chosen to build such distribution for S&P 500 returns and such forecast will be constructed for two out of sample periods. The results are displayed in figure 5.4 below where the light green line indicates the TVP-VAR, while the black line indicates the simple VAR.

When considering the figures for Wu & Xia, it is obvious that not much difference can be observed as both lines are almost overdrawn on one another. This means that the variance, in this case, is very similar to the wideness of the distribution is almost the same. Such outcome comes as unexpected in the sense that based on the methodology simple VAR's variance is averaged over the sample period. Time-varying model, on the other hand, focuses on the fact that the system variables have developed lower dispersion over time.

While the outcome was unexpected in the former case, Krippner model provides the expected figure. Such figure clearly reflects a more concentrated TVP-VAR and a more dispersed VAR. this means that the variance is smaller in the advanced model, which is something desirable in our case. In addition, the logarithmic values at the bottom of each graph serving as a performance measure speak in favor of the TVP-VAR in the first period’s forecast distribution and in the next period speak in favor of simple VAR, but with a very slight difference.
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Figure 5.4: Forecast Distribution for 2 Periods (US)

This figure summarizes the out-of-sample forecast distributions for both Wu & Xia and Krippner models. The green lines indicate the TVP-VAR models while the black line indicates the VAR models. 

Source: Author’s computations, R-studio.

5.2 European Union Case

Starting the analysis from the simple Wu & Xia VAR results we are able to identify an initial negative impact of inflation on N100 returns. Such impact, which seems to be strong with a value of -1.43, could not be found to be significant at any significance level. In addition, in the second lag, the coefficient changes its sign and becomes positive with a value of 0.63, but is not found to be significant at any level as well. While observing the Krippner model results and contrasting them with the former ones in terms of inflation, it is easily observable that there is a significant change in terms of values and significance. Inflation remains negative in the first lag, but increases to -2.76 and becomes significant at 5% significance level. Moreover, the second lag like in the previous case turns positive with a value of 1.5 and loses its significance. As the sample contains periods under which the developments can be considered to some extent abnormal, we could still conclude on a generally negative impact of inflation in stock market returns. In addition to that, the studies by Borio & Lowe (2002) or Stock & Watson (2003) indicate that the reverse impact could be possible as inflation may be triggered by certain developments in credit levels and asset market.

The shadow rate, on the other hand, is found to exert a negative impact on stock returns with a value of -0.107 in the first lag and a positive impact of 0.037 in the second lag. Such impacts could not satisfy the significance rules at any level. Moreover, on the contrary to the former model, Krippner model results present different dynamics. In this case, the shadow rate exerts a positive impact in the first lag, followed by a negative impact on the second lag. But even though the dynamic
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seems to be reversed in any of the cases we could not prove the significance. As explained in the initial parts of the chapter, the dynamics of the shadow rate should be understood based on the dynamics of the normal interest rates. A negative relationship was expectable among interest rates and stock market returns and this has been early assessed in the study by Mads (1989). (See table A.1 and Figure A.7 in Appendix A)

Shifting from simple VAR to the analysis of TVP-VAR’s posterior means, the results for Wu & Xia are displayed in figure 5.5 below, while for Krippner in the figure A.17 in Appendix A. Starting with Wu & Xia model, we can easily detect a significant presence of time variation in the residuals. Such variation seems to be more pronounced in the initial periods of the sample, which clearly coincide with the initiation of the global financial crisis. During the rest of the periods, the variation is still maintained, but as the lines indicate the band narrows. This explanation suits the three equation's posterior means. While comparing these developments with the ones in the Krippner model, we can see that the patterns are quite different. Even though the presence of time variation is evidential, it displays different behavior of residuals. Such indication can be extracted from each of the graphs and again it serves as a reconfirmation of the presence of stochastic volatility implied by Primicieri (2005) and Nakajima, Kasuya, & Watanabe (2011).

Figure 5.5: Posterior Means: Standard Deviation for Wu & Xia Model's Residuals (EU)

<table>
<thead>
<tr>
<th>S.D Shadow Rate</th>
<th>S.D Inflation Rate</th>
<th>S.D S&amp;P 500 Returns</th>
</tr>
</thead>
</table>

This figure represents the residual’s standard deviation for TVP-VAR estimations with Wu & Xia shadow interest rate. The light green line in the middle represents the normal standard deviation, while the lower and upper orange lines represent the 0.16 and 0.84 percentiles. In addition, the x-axis stands for the time period and the numbers stand for the months.

Source: Author’s computations, R-studio.

Having gone through the standard deviations of the posterior means, it would be relevant to get to the core analyses of the impulse response functions. Like in the US case even here the impulse response functions for Wu & Xia presented in table 5.6 indicate the responses of stock market returns towards one unit shock originating from shadow rate and inflation in each of the three scenarios explained earlier in the
chapter. In addition, together with the TVP-VAR results can be observed the simple VAR's impulse response functions represented by the dashed black line.

Analyzing initially the shock of shadow rate in Wu & Xia model, we get an indication that the result is in the same line with the literature when speaking of its negativity. In addition, the persistence of its impact seems to be long lasting as even after 20 periods the line keeps a decreasing trend by becoming more distanced from zero. Similar patterns can be observed in scenarios 2 and 3. The Cholesky decomposition in 2 indicates a more moderated response, suggesting that the impact will remain negative for a long period of time. In the same time, its results suggest a lower uncertainty as the band indicated in light black is much narrower compared to the former case. In the Krippner model, different developments can be observed in terms of magnitude and in terms of impact sign as well. Initially, the impact seems to be positive, even though it starts decreasing rapidly and just after 4 periods transforms into negative by maintaining a decreasing trend for the rest of the periods. In scenarios 2 and 3 as well can be observed the same behavior. (See figure A.18 in Appendix A) In comparison to the former case, what drags attention in here is the fact that the light black band is narrower, thus indicating less uncertainty. Moreover, interesting as well are the simple VARs, which basically behave similarly to the former case by being positive and in contradiction to the literature. Such findings indicate the effectiveness of monetary policy even under the ZLB, thus confirming the unconventional monetary policy relevance claimed by Baumeister & Benati (2012) and Lima, Vasconcelos, Simão, & de Mendonça (2016).

While considering the inflation shock, it is easy to identify significant differences among the basic VAR and TVP-VAR in Wu & Xia model. These differences have materialized significantly in terms of impact’s magnitude and impact’s sign. Across all of the scenarios can be indicated that the impact of inflation on stock returns is in the same line with the literature like TVP-VAR suggests. In addition, the simple VAR exhibits a very short negative impact on the first period, which is followed later on by a positive transformation and tends to remain so for the rest of the periods. Moreover, the influence seems to be built under large levels of uncertainty as suggested by the band surrounding the TVP-VAR's lines. When shifting to Krippner model other indications can be extracted as the behavior seems to be more determinable. In both VARs the line remains below zero, meaning that this time they are consistent with the literature. The TVP-VAR represents large uncertainty initially as the impact even though negative, is likely to vary, but the situation changes more significantly over the course. Basic VARs, on the other hand, maintain a more constant behavior by staying close to the zero line and meeting with the other model just after 15 periods on average. The large amount of uncertainty
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explains the reasons why Roger (2009), Williams (2009), Beningo & Paciello (2010), Coibion, Gorodnichenko, & Wieland (2012) strongly argue on different inflation strategies under the ZLB.

**Figure 5.6: Wu & Xia TVP-VAR’s Vs. Simple VAR’s Core Impulse Response Functions (EU)**

![Figure 5.6: Wu & Xia TVP-VAR’s Vs. Simple VAR’s Core Impulse Response Functions (EU)](image)

This figure summarizes the core impulse response functions (IRFs) that are of significant interest for this thesis. In addition, the TVP-VAR IRFs are plotted together with simple VAR IRFs, where the former one is represented by the solid black line while the latter one by the dashed black line. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

*Source*: Author’s computations, R-studio.

Following the impulse response functions analysis like in the US case, we consider the response of stock market returns towards one unit shock on July 2008, March 2012 and August 2014 periods of the sample for the Wu & Xia model. While the motivation for such choice has been elaborated in the former parts of this chapter, we shift directly to the results plotted in figure 5.7 below. In part a) are displayed the three impulse response functions for each of the respective periods under the second scenario. As it can be observed, a one unit shock from shadow rate is initially going upwards very sharply, but this lasts for less than 2 periods as after that the trend decreases its slope even though it keeps going upwards. Apparently, in here the July 2008 period exhibits a greater shock compared to the other periods and such difference can even be observed in parts b), c) and d), which represent the respective
differences for the three possible impulse response functions pairs. As it could be expected the biggest difference observed in part c) belongs to the July 2008-August 2014 pair, which respectively indicates the global financial crisis and post-crisis periods. In addition, the similarity among the July 2008-March 2012 pair indicates the European sovereign debt crisis, which followed right after the global financial crisis. This is the main reason why the difference among their impulse response functions presented in c) is so small with 16th and 84th percentiles forming a very narrow band. Interesting in these cases is the positive impact of the shock, which was found to be negative in the former situation. Under such conditions, it is hard to determine the reasons behind such impact. Of course, there are claims in the literature like in the study by Gregoriou, Kontonikas, MacDonald, & Montagnoli (2009), which state that even though interest rate has a negative impact on stock market returns in normal periods, it ceases to exhibit the same influence during the crisis by transforming into positive. While such study might explain the development for the July 2008 and March 2012 periods corresponding respectively to the global financial crisis and to the European sovereign debt crisis, it fails to explain the post-crisis period.

Shifting from shadow rate to inflation rate the outcome seems to be of a very moderated magnitude. The impulse response functions plotted in part e) exhibit similar trend with July 2008 period leading the severity of the shock's response, followed by the March 2012 and August 2014 period shocks. Due to the low magnitude of reaction the difference among impulse response functions is very small, as it can be observed in parts f), g) and h). Unexpected in this case is definitively the sign of the impact, which is positive and contradicts the previous findings with regard to the inflation shock. Regardless of the sign issue, the biggest benefit, in this case, is for sure the confirmation of monetary policy consistency across periods, thus disregarding the presence of ZLB and its associative conditions.

Continuing the former analyses with the Krippner model's results in figure A.25 in Appendix A, it is obvious that the magnitude of the shadow rate shock has changed substantially for each of the periods compared to the former case. Unfortunately, the direction of the impact stays the same, even though its increasing trend tends to last longer and starts to decrease only after 7 periods. In this case, the July 2008 period impulse response function remains below others by being significantly distanced from the August 2014 period shock, thus indicating lower severity and persistence as well. The August 2014 period shock unlike in the former case seems to be above all others by tripling its magnitude. On the other hand, the March 2012 period shock corresponding to the European sovereign debt crisis remains distant from the former shock's impulse response function and is almost
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identical to the July 2008 period's shock. The differences and similarities among them can be easily observed in parts b), c) and d), which indicate the differences for each of the pairs. Such outcome comes in disagreement with the former explanation in the Wu & Xia model related to the crisis developments and their association with the respective shocks. If we would see figure A.4 in Appendix A, Krippner shadow rate had exhibited a significant upturn in the 115th period, corresponding to mid-2014, while maintaining a decreasing trend in the former periods. As a result of that, the shock at this period may present these unexpected dynamics, which contradict with the former shadow rate case due to different patterns.

Regarding inflation, the impulse response functions summarized in part e), indicate similar developments among the July 2008 and March 2012 periods, while the August 2014 period line stands slightly above the former one's lines. In addition, the initial impact is strongly positive and last just for 3 periods as after the line changes direction by following a downward trend. In comparison to the former shock of shadow rate, this one dies faster and has a lower persistence. Parts f), g) and h) of figure A.25 in Appendix A indicate the similarity among impulse response functions as their respective lines are almost zero, with 16th and 84th percentiles forming a very narrow band in each case. In a sense, such developments and similarity represent a kind of coherence when studying inflation originating shocks.

Figure 5.7: Wu & Xia Impulse Response Functions on 43th (July 2008), 85th (March 2012) and 115th (August 2014) Periods (EU)

This figure summarizes the IRFs for the 43rd, 85th and 115th months of the dataset for Wu & Xia models. Figure 21 a) contains the plots of shadow rate IRFs for each of the periods. Part b) indicates the difference between 43rd and 85th shadow rate IRFs, associated with 16th and 84th percentiles. Part c) indicates the difference between 43rd and 115th shadow rate IRFs, associated with 16th and 84th percentiles. Part d) indicates the difference between 85th and 115th shadow rate IRFs, associated with
16th and 84th percentiles. Part e) contains the IRFs of inflation for each period of interest. Part f) indicates the difference between 43rd and 85th inflation IRFs, associated with 16th and 84th percentiles. Part g) indicates the difference between 43rd and 115th inflation IRFs, associated with 16th and 84th percentiles. Part h) indicates the difference between 43rd and 85th inflation IRFs, associated with 16th and 84th percentiles.

Source: Author’s computations, R-studio.

Having discussed the basic VAR and TVP-VAR outputs in terms of expectations, impacts, and theoretical analyses, we are left with the part of building a forecast distribution. Such distribution is plotted in figure 5.8 for both Wu & Xia, and Krippner models. Starting with Wu & Xia model, we observe a very similar distribution among the TVP-VAR represented in light green line and simple VAR represented in black line. In addition, the TVP-VAR line seems to be narrower compared to the simple VAR line, thus representing a smaller variance for the former one, even though the difference is very small. Such conclusion can be also supported by the logarithmic scores below each of the graphs, which speak in favor of TVP-VAR.

While considering the Krippner model, the picture provides clearer results as TVP-VAR, in this case, is more concentrated. The bell shape of VAR is wider and as such represents a higher variation compared to the other model. Such claim once again is supported by the logarithmic scores at the bottom of each graph speaking in favor of TVP-VARs.

**Figure 5.8: Forecast Distribution for 2 Periods (EU)**

This figure summarizes the out of sample forecast distributions for both Wu & Xia and Krippner models. The green lines indicate the TVP-VAR models while the black line indicates the VAR models.

Source: Author’s computations, R-studio.
5.3 United Kingdom Case

Starting with the VAR analysis, the initial impact of shadow rate on stock market returns for the case of UK is found to be positive according to the output from Wu & Xia model. In addition, such positivity constrained only within the first lag as the sign soon becomes negative when we get to the second lag. The behavior of this kind is observed earlier as well in the other cases described in this chapter. When shifting to the output of Krippner model, the results seem to be reversed with the first lag being negative and the second one being positive. Moreover, the latter model's coefficients are much smaller compared to the former one, thus indicating a more moderated impact. (See table A.1 and figure A.7 in appendix A)

While having an indication for the reflection of policy variables impact on non-policy ones, we shift to the output of TVP-VARs by initially considering the posterior means for Wu & Xia in figure 5.9 below. Like expected and observed in the former cases as well even in here there are significant time variations in residuals. In addition, the standard deviation plots indicate similarity among shadow rate and inflation rate plots, even though the latter one represents a larger band among its percentiles. The LSE returns standard deviation, on the other hand, represents a large amount of volatility as it can be indicated by the y-axis. Moreover, the Krippner model, plotted in figure A.26 in appendix A, indicates a richer pattern with significant time variation in residuals. A similar pattern is displayed in the all plots, but what drags attention is the smaller band of fluctuations in the LSE standard deviation plot. While recalling from the former cases the UK's developments based on these graphs are quite similar to the US.

Figure 5.9: Posterior Means: Standard Deviation for Wu & Xia Model's Residuals (UK)

This figure represents the residual's standard deviation for TVP-VAR estimations with Wu & Xia shadow interest rate. The light green line in the middle represents the normal standard deviation, while the lower and upper orange lines represent the 0.16 and 0.84 percentiles. In addition, the x-axis stands for the time period and the numbers stand for the months.

Source: Author’s computations, R-studio.
Having gone quickly through the simple VAR’s output and TVP-VARs posterior means, we shift to the core results from Wu & Xia impulse response functions plotted in figure 5.10 below. Like in the former cases the dashed line represents the simple VAR's impulse response functions, while the solid line represents the TVP-VAR's impulse response functions. Starting with the Wu & Xia model in scenario 1, we can detect from the graph that a one unit shock from shadow rate causes a negative impact on stock market returns as indicated by TVP-VAR line. Such impact seems to be very pronounced in the initial periods but dies fast as only after 5 periods you could see it almost attached to the zero line. Unfortunately, the simple VAR's line is less informative as it seems to be developed entirely close to the zero line by being positive. While considering the second scenario of Cholesky decomposition we can identify similar patterns with TVP-VAR indicating a moderated impact and VAR indicating a weak response which remains constant after the first 5 periods. Similar developments are observed in the third scenario as well.

Moreover, while considering the Krippner model in figure A.27 in Appendix A, we are able to identify quite homogenous developments with respect to the former case. Such developments are alike in terms of patterns displayed in the graphs but remain more moderated in terms of magnitude as suggested by the y-axis values in the first and third scenarios. Interesting in this situation is the behavior of VAR impulse response functions, which even though maintain the same trend like in the former model in the short-run, their behavior changes significantly in long-run by transforming their impact's sign to negative. Again such transformation does not make them alike to the TVP-VAR's output but gives an indication for the long-run developments of the stock market returns. Such findings confirm the relevance of monetary policy in impacting stock market returns as claimed in the study of Lima, Vasconcelos, Simão, & de Mendonça (2016) and find partial support in the study of Borio & Hofmann (2017) which claim for a loss in efficacy.

Shifting from shadow rate to inflation rate shocks, it is important to highlight the similarity across models in the Wu & Xia case. Such development deserves to be highlighted as it is rarely seen over the number of cases discussed by this thesis. In addition, the impact remains significantly negative during the entire periods and across the three scenarios. Moreover, it is worth stating that the severity and persistence of the shock are weak as the effect is moderated in the beginning and seems to die out very fast. In the Krippner model the behavior of simple VAR is consistent with the former case and does not change too much, but the TVP-VAR represents a very different pattern. It initiates as a positive impact by lasting for less than two periods as it follows a very steep downward trend. Such downward trend continues for several periods by preserving a significant magnitude and by
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maintaining a relevant persistence. Moreover, it seems to die out just after 15 periods on average, thus appearing to be long-lasting. Another implication from these graphs is the amount of uncertainty at the beginning of the impact as indicated by the light black band.

**Figure 5.10: TVP-VAR’s Vs. Simple VAR’s Core Impulse Response Functions (UK)**

This figure summarizes the core impulse response functions (IRFs) that are of significant interest for this thesis. In addition, the TVP-VAR IRFs are plotted together with simple VAR IRFs, where the former one is represented by the solid black line while the latter one by the dashed black line. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

*Source*: Author’s computations, R-studio.

Having discussed and compared the general IRFs across models and scenarios, it is time to shift to the specific analyses of shocks from the July 2008, March 2012 and August 2014 periods of the sample. Figure 5.11 summarizes the results of Wu & Xia model impulse response functions on these dates and their respective differences as well under the scenario of Cholesky decomposition. Part a) indicates a positive impact of shadow rate for each of the respective period's shocks. Such finding is not surprising as we have faced and discussed it in the former parts of this chapter. As expected the magnitude of the July 2008 period shock, corresponding to the global financial crisis, is of larger scale compared to the other cases. In addition, the shock seems to have long-lasting effects as its persistence is obvious.
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from the increasing trend of the line during the entire 20 periods. Below the July 2008 line is drawn the March 2012 one, which to some extent was expected to be more distanced from the August 2014 period shock as it corresponds to the European sovereign debt crisis. Apparently, such development does not have any relevant impact on UK stock returns as suggested by the impulse response function line.

When considering the differences among the lines, which are displayed in parts b), c) and d), we are able to indicate that the most significant dissimilarity can be observed in the former two parts belonging respectively to July 2008-March 2012 and July 2008-August 2014 pairs.

While considering the development of inflation rate shocks in the respective periods of investigation, the impact seems to be quite meaningful as it is negative in every case. In addition, the magnitude is dispersed similarly to the former shock in terms of periods with the July 2008 period representing the biggest severity of the shock, followed by the March 2012 and lastly by the August 2014 periods. Moreover, the severity of the shock seems to have a short relevance of only 5 periods on average, as later on the effect seems to die out rapidly. Again alike to the shadow rate shock, the inflation rate developments for March 2012 and August 2014 periods are quite homogenous, thus indicating once again the similarity among periods and lack of influence from the European sovereign debt crisis.

Again when it comes to the differences, we can observe them in parts f), g) and h), which picture it for each of the possible pairs. As expected the main differences are among the July 2008-March 2012 and July 2008-August 2014 pairs, reflecting the behavior heterogeneity during the crisis and non-crisis periods.

Extending the former analyses to the Krippner model we focus once again on the behavior of stock market returns in response to a one unit shock by shadow rate. With the impulse response functions summarized in part a) of figure A.34 in Appendix A, we can see that the pattern looks similar in terms of impact's sign, but different in terms of period's shocks magnitudes. While the August 2014 period remains at approximately similar levels like in the former case, the March 2012 period shock has caused the impulse response function line to slightly increase. On the other hand, the July 2008 period line has experienced a significant drop. The drop is so significant that the two latter mentioned periods seem to be overdrawn above one another. As such they indicate similar behavior during the global financial crisis and European sovereign debt crisis. Apart from these differences, the pattern is similar in all other aspects with an initial positive impact continuing for several periods and dying out after 10 periods on average. Moreover, the differences among
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the pairs, plotted in parts b), c) and d), indicate the similar developments and serve as a supportive argument to the former description.

When it comes to the inflation shock we see a totally different pattern compared to the former case in terms of magnitude and sign as well. The former model indicated a negative impact of the inflation shock, which is a typical finding in literature, but the current model has turned such observation upside down by turning positive. Interesting is also the fact that the positive impact is scaled across periods with the August 2014 period on top, followed by March 2012 and July 2008 periods. Such ranking is also reflected in the differences plots in parts f), g) and h), which seems to have developed quite similarly.

Figure 5.11: Wu & Xia IRFs on 43th (July 2008), 85th (March (2012) and 115th (August 2014) Periods (UK)

This figure summarizes the IRFs for the 43rd, 85th and 115th months of the dataset for Wu & Xia models. Figure 21 a) contains the plots of shadow rate IRFs for each of the periods. Part b) indicates the difference between 43rd and 85th shadow rate IRFs, associated with 16th and 84th percentiles. Part c) indicates the difference between 43rd and 115th shadow rate IRFs, associated with 16th and 84th percentiles. Part d) indicates the difference between 85th and 115th shadow rate IRFs, associated with 16th and 84th percentiles. Part e) contains the IRFs of inflation for each period of interest. Part f) indicates the difference between 43rd and 85th inflation IRFs, associated with 16th and 84th percentiles. Part g) indicates the difference between 43rd and 115th inflation IRFs, associated with 16th and 84th percentiles. Part h) indicates the difference between 43rd and 85th inflation IRFs, associated with 16th and 84th percentiles.

Source: Author’s computations, R-studio.

The forecast distributions plotted in figure 5.12 serve as an indication of accuracy and variance of TVP-VAR in comparison to the simple VAR. In addition,
such evaluation is available for two periods and its evaluation is supported by the logarithmic scores as well at the bottom of each graph.

For the Wu & Xia model, it is easily observable that the TVP-VAR's distribution is much more concentrated. In addition, the VAR line indicated in black seems to be very dispersed. Such observation provides clear insights on a larger variety on VAR's distribution. In support of such claim is the logarithmic score as well, which speaks in favor of TVP-VAR as well by reflecting a substantial difference among the models. Moreover, the Krippner model, on the other hand, provides more relaxed dynamics as the distribution lines of the models seem to be overdrawn above one another. Such claim clearly finds support on the values of logarithmic scores, which suggest a significant similarity. As a result, this case leads to the conclusion that both models represent similar variation.

**Figure 5.12: Forecast Distribution for 2 Periods (UK)**

This figure summarizes the out of sample forecast distributions for both Wu & Xia and Krippner models. The green lines indicate the TVP-VAR models while the black line indicates the VAR models. 

*Source: Author’s computations, R-studio.*
6 Conclusion

This thesis main aim is to confirm the relevance of monetary policy in explaining stock market returns even after the interest rate hit the so-called ZLB. In addition, it investigates the claim that the ZLB does not make a difference by making use of the concept of shadow interest rate. Moreover, this thesis tries to identify if the measure of shadow rate is relatively significant in restoring the breach of consistency caused when the interest rate hit the ZLB.

In order to test for the formerly stated claims, we have made use of vector autoregressive methodology with time varying-parameters and stochastic volatility. This methodology constitutes one of the main contributions of this thesis as its joint application with the variable of shadow interest rate has not been assessed earlier in the literature. In addition, this thesis has considered both measures of shadow interest rate by Wu & Xia and Krippner with the aim of providing robust and reliable results. Analyzes has been applied to three main samples which respectively are United States, European Union, and the United Kingdom. The inclusion of multiple samples is motivated by the aim of providing a comparative analysis and revealing the characterizing dynamics for each case, thus vouchsafing an additional contribution to the literature.

The results provide a very relevant panorama of how the impact of monetary policy is reflected in the stock market returns. Initially, the United States case represents a negative relation between shadow rate and stock market returns in both Wu & Xia and Krippner models. Such indication, which is clearly in the same line with the literature, also implies that the shadow rate represents a continuity of interest rates in normal times. In addition, it can be claimed that this variable serves as a connecting bridge in explaining prior and post-crisis monetary policy impact on stock market returns. While providing continuity, the shadow rate has been interpreted and explained based on its relation and connection to the normal rates. Moreover, when considering its impact during and post-crisis, the indication that we get is unclear. Firstly the Wu & Xia model produces a positive impact from 1 unit shock of shadow rate at each of the chosen periods, thus going against the results obtained when considering 1 unit shock in general. Krippner model, on the other hand, remains quite consistent by representing a negative impact of 1 unit shock from shadow rate in each of the respective periods.
Regarding the European Union developments, it can be easily stated that they are quite similar to the case of United States as well. Such conclusion is strongly supported by the output of impulse response functions, indicating a negative impact from 1 unit shock in the shadow rate for each of the respective models. Interesting can be considered the fact that to some extent the magnitude of the effect is smaller, but at the same time uncertainty is higher as well. While the general developments are similar, the shadow rate shocks on specific dates partly serve as symmetrical reflections of the former case. The impact of one unit shock from shadow rate turns out to have a positive impact as suggested by the Wu & Xia and Krippner models. In addition, the response magnitudes are quite similar to Wu & Xia model’s output in the former case.

Moreover, the United Kingdom case as well represents quite similar dynamics. Again like in the former cases the impact is generally negative, but the magnitude is smaller and is associated with a narrower band of fluctuations as well, thus suggesting less uncertainty. In addition, the dynamics of 1 unit shock in different periods of the sample follow similar behavior like in the former cases by remaining consistent in this aspect.

Lastly, these results provide consistency in 3 main aspects. Firstly, the literature claims for a negative relationship between interest rate and stock market returns and the shadow rate represents a bridge on maintaining that relationship. Secondly, the post-transition developments from conventional tools to unconventional tools can be explained by making use of this variable, thus preserving the continuity of monetary policy communication. Thirdly, such findings support the policymakers by providing an important incentive on measuring the impact of monetary policy for stock market returns through the shadow rate.


Appendix A: Tables and Figures not presented in the main text

Figure A.1: Wu & Xia US Shadow Rate

Source: Author’s computations, E-views.

Figure A.2: Krippner US Shadow Rate

Source: Author’s computations, E-views.
Appendix A: Tables and Figures not presented in the main text

Figure A.3: Wu & Xia EU Shadow Rate

![Chart showing the Wu & Xia EU Shadow Rate from 2006 to 2016.](chart.png)

Source: Author’s computations, E-views.

Figure A.4: Krippner EU Shadow Rate

![Chart showing the Krippner EU Shadow Rate from 2006 to 2016.](chart.png)

Source: Author’s computations, E-views.
Figure A.5: UK Wu & Xia Shadow Rate

Source: Author’s computations, E-views.

Figure A.6: UK Kripner Shadow Rate

Source: Author’s computations, E-views.
This figure summarizes all the orthogonal impulse response functions of the simple VAR for the case of US, EU and UK. The solid black line stands for the VAR output, while the dashed red line indicates the 16th and 84th percentiles. Regarding the notation, “ssr” stands for the shadow interest rate, “inf” stands for the inflation rate, “spret” stands for S&P 500 returns, “nret” stands for N 100 returns and “lseret” stands for London Stock Exchange returns.

Source: Author’s computations, R-studio.
This table summarizes the VAR results for the case of US, EU and UK. In the table “ssr” stands for shadow rate, “ret” stands for the respective stock markets returns and “inf” stands for the inflation rate.

*Source:* Author’s computations, R-studio.
Figure A.8: Posterior Means: Standard Deviation for Krippner Model's Residuals (US)

<table>
<thead>
<tr>
<th>S.D Shadow Rate</th>
<th>S.D Inflaton Rate</th>
<th>S.D S&amp;P 500 Returns</th>
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This figure represents the residual's standard deviation for TVP-VAR estimations with Krippner shadow interest rate. The light green line in the middle represents the normal standard deviation, while the lower and upper orange lines represent the 0.16 and 0.84 percentiles. In addition the x-axis stands for time period and the numbers stand for the months.

Source: Author’s computations, R-studio.

Figure A.9: Krippner TVP-VAR’s Vs. Simple VAR’s Core Impulse Response Functions (US)

This figure summarizes the core impulse response functions (IRFs) that are of significant interest for this thesis. In addition, the TVP-VAR IRFs are plotted together with simple VAR IRFs, where the former one is represented by the solid black line while the latter one by the dashed black line. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

Source: Author’s computations, R-studio.
Appendix A: Tables and Figures not presented in the main text

Figure A.10: Wu & Xia Impulse Response Functions under Scenario 1 (US)

Under this scenario the impulse response functions represent the response of the variables under 1 unit shock to the vector \( u(t) \) for the Wu & Xia model of US. The x-axis indicates the number of periods ahead, while the y-axis indicates the magnitude of the impact. In addition, the solid black line indicates the TVP-VAR response to the shock and its persistence change across periods. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

Source: Author’s computations, R-studio.

Figure A.11: Wu & Xia Impulse Response Functions under Scenario 2 (US)
Under this scenario the impulse response functions represent the response of the variables under Cholesky Decomposition 1 unit shock to some of the elements of e(t) for the Wu & Xia model of US. The x-axis indicates the number of periods ahead, while the y-axis indicates the magnitude of the impact. In addition, the solid black line indicates the TVP-VAR response to the shock and its persistence change across periods. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

Source: Author’s computations, R-studio.

**Figure A.12: Wu & Xia Impulse Response Functions under Scenario 3 (US)**

Under this scenario the impulse response functions represent the response of the variables under Cholesky Decomposition 1 unit shock to the average values of the elements of e(t) for the Wu & Xia model of US. The x-axis indicates the number of periods ahead, while the y-axis indicates the magnitude of the impact. In addition, the solid black line indicates the TVP-VAR response to the shock and its persistence change across periods. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

Source: Author’s computations, R-studio.

**Figure A.13: Krippner Impulse Response Functions under Scenario 1 (US)**
Under this scenario the impulse response functions represent the response of the variables under a 1 unit shock to the vector $u(t)$ for the Krippner model of the US. The x-axis indicates the number of periods ahead, while the y-axis indicates the magnitude of the impact. In addition, the solid black line indicates the TVP-VAR response to the shock and its persistence change across periods. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

*Source:* Author’s computations, R-studio.

**Figure A.14: Krippner Impulse Response Functions under Scenario 2 (US)**

Under this scenario the impulse response functions represent the response of the variables under Cholesky Decomposition 1 unit shock to some of the elements of $e(t)$ for the Krippner model of the US. The x-axis indicates the number of periods ahead, while the y-axis indicates the magnitude of the impact. In addition, the solid black line indicates the TVP-VAR response to the shock and its persistence change across periods. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

*Source:* Author’s computations, R-studio.
Appendix A: Tables and Figures not presented in the main text

Figure A.15: Krippner Impulse Response Functions under Scenario 3 (US)

Under this scenario the impulse response functions represent the response of the variables under Cholesky Decomposition 1 unit shock to the average values of the elements of e(t) for the Krippner model of US. The x-axis indicates the number of periods ahead, while the y-axis indicates the magnitude of the impact. In addition, the solid black line indicates the TVP-VAR response to the shock and its persistence change across periods. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

Source: Author’s computations, R-studio.

Figure A.16: Krippner Impulse Response Functions on 43th (July 2008), 85th (March 2012) and 115th (August 2014) Periods (US)
This figure summarizes the IRFs for the 43th, 85th and 115th months of the dataset for Krippner models. Figure 21 a) contains the plots of shadow rate IRFs for each of the periods. Part b) indicates the difference between 43th and 85th shadow rate IRFs, associated with 16th and 84th percentiles. Part c) indicates the difference between 43th and 115th shadow rate IRFs, associated with 16th and 84th percentiles. Part d) indicates the difference between 85th and 115th shadow rate IRFs, associated with 16th and 84th percentiles. Part e) contains the IRFs of inflation for each period of interest. Part f) indicates the difference between 43th and 85th inflation IRFs, associated with 16th and 84th percentiles. Part g) indicates the difference between 43th and 115th inflation IRFs, associated with 16th and 84th percentiles. Part h) indicates the difference between 43th and 85th inflation IRFs, associated with 16th and 84th percentiles.

Source: Author’s computations, R-studio.

**Figure A.17: Posterior Means: Standard Deviation for Krippner Model's Residuals (EU)**

![Figure A.17: Posterior Means: Standard Deviation for Krippner Model's Residuals (EU)](image)

This figure represents the residual’s standard deviation for TVP-VAR estimations with Krippner shadow interest rate. The light green line in the middle represents the normal standard deviation, while the lower and upper orange lines represent the 0.16 and 0.84 percentiles. In addition the x-axis stands for time period and the numbers stand for the months.

Source: Author’s computations, R-studio.

**Figure A.18: Krippner TVP-VAR’s Vs. Simple VAR’s Core Impulse Response Functions (EU)**

![Figure A.18: Krippner TVP-VAR’s Vs. Simple VAR’s Core Impulse Response Functions (EU)](image)
This figure summarizes the core impulse response functions (IRFs) that are of significant interest for this thesis. In addition, the TVP-VAR IRFs are plotted together with simple VAR IRFs, where the former one is represented by the solid black line while the latter one by the dashed black line. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

Source: Author’s computations, R-studio.

Figure A.19: Wu & Xia Impulse Response Functions under Scenario 1 (EU)

Under this scenario the impulse response functions represent the response of the variables under 1 unit shock to the vector u(t) for the Wu & Xia model of EU. The x-axis indicates the number of periods ahead, while the y-axis indicates the magnitude of the impact. In addition, the solid black line indicates the TVP-VAR response to the shock and its persistence change across periods. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

Source: Author’s computations, R-studio.
Appendix A: Tables and Figures not presented in the main text

Figure A.20: Wu & Xia Impulse Response Functions under Scenario 2 (EU)

Under this scenario the impulse response functions represent the response of the variables under Cholesky Decomposition 1 unit shock to some of the elements of e(t) for the Wu & Xia model of EU. The x-axis indicates the number of periods ahead, while the y-axis indicates the magnitude of the impact. In addition, the solid black line indicates the TVP-VAR response to the shock and its persistence change across periods. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

Source: Author’s computations, R-studio.

Figure A.21: Wu & Xia Impulse Response Functions under Scenario 3 (EU)
Under this scenario the impulse response functions represent the response of the variables under Cholesky Decomposition 1 unit shock to the average values of the elements of $e(t)$ for the Wu & Xia model of EU. The x-axis indicates the number of periods ahead, while the y-axis indicates the magnitude of the impact. In addition, the solid black line indicates the TVP-VAR response to the shock and its persistence change across periods. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

*Source:* Author’s computations, R-studio.

**Figure A.22: Krippner Impulse Response Functions under Scenario 1 (EU)**

Under this scenario the impulse response functions represent the response of the variables under 1 unit shock to the vector $u(t)$ for the Krippner model of EU. The x-axis indicates the number of periods ahead, while the y-axis indicates the magnitude of the impact. In addition, the solid black line indicates the TVP-VAR response to the shock and its persistence change across periods. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

*Source:* Author’s computations, R-studio.

**Figure A.23: Krippner Impulse Response Functions under Scenario 2 (EU)**
Appendix A: Tables and Figures not presented in the main text

Under this scenario the impulse response functions represent the response of the variables under Cholesky Decomposition 1 unit shock to some of the elements of e(t) for the Krippner model of EU. The x-axis indicates the number of periods ahead, while the y-axis indicates the magnitude of the impact. In addition, the solid black line indicates the TVP-VAR response to the shock and its persistence change across periods. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

Source: Author’s computations, R-studio.

Figure A.24: Krippner Impulse Response Functions under Scenario 3 (EU)

Under this scenario the impulse response functions represent the response of the variables under Cholesky Decomposition 1 unit shock to the average values of the elements of e(t) for the Krippner model of EU. The x-axis indicates the number of periods ahead, while the y-axis indicates the magnitude of the impact. In addition, the solid black line indicates the TVP-VAR response to the shock and its persistence change across periods. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

Source: Author’s computations, R-studio.
Figure A.25: Krippner Impulse Response Functions on 43\(^{\text{th}}\) (July 2008), 85\(^{\text{th}}\) (March 2012) and 115\(^{\text{th}}\) (August 2014) Periods (EU)

This figure summarizes the IRFs for the 43th, 85th and 115th months of the dataset for Krippner models. Figure 21 a) contains the plots of shadow rate IRFs for each of the periods. Part b) indicates the difference between 43th and 85th shadow rate IRFs, associated with 16th and 84th percentiles. Part c) indicates the difference between 43th and 115th shadow rate IRFs, associated with 16th and 84th percentiles. Part d) indicates the difference between 85th and 115th shadow rate IRFs, associated with 16th and 84th percentiles. Part e) contains the IRFs of inflation for each period of interest. Part f) indicates the difference between 43th and 85th inflation IRFs, associated with 16th and 84th percentiles. Part g) indicates the difference between 43th and 115th inflation IRFs, associated with 16th and 84th percentiles. Part h) indicates the difference between 43th and 85th inflation IRFs, associated with 16th and 84th percentiles.

Source: Author’s computations, R-studio.

Figure A.26: Posterior Means: Standard Deviation for Krippner Model's Residuals (UK)

This figure represents the residual’s standard deviation for TVP-VAR estimations with Krippner shadow interest rate. The light green line in the middle represents the normal standard deviation, while the lower and upper orange lines represent the 0.16 and 0.84 percentiles. In addition the x-axis stands for time period and the numbers stand for the months.
Appendix A: Tables and Figures not presented in the main text

Source: Author’s computations, R-studio.

Figure A.27: Krippner TVP-VAR’s Vs. Simple VAR’s Core Impulse Response Functions (UK)

This figure summarizes the core impulse response functions (IRFs) that are of significant interest for this thesis. In addition, the TVP-VAR IRFs are plotted together with simple VAR IRFs, where the former one is represented by the solid black line while the latter one by the dashed black line. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

Source: Author’s computations, R-studio.

Figure A.28: Wu & Xia Impulse Response Functions under Scenario 1 (UK)
Appendix A: Tables and Figures not presented in the main text

Under this scenario the impulse response functions represent the response of the variables under 1 unit shock to the vector \( u(t) \) for the Wu & Xia model of UK. The x-axis indicates the number of periods ahead, while the y-axis indicates the magnitude of the impact. In addition, the solid black line indicates the TVP-VAR response to the shock and its persistence change across periods. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

*Source:* Author’s computations, R-studio.

**Figure A.29: Wu & Xia Impulse Response Functions under Scenario 2 (UK)**

Under this scenario the impulse response functions represent the response of the variables under Cholesky Decomposition 1 unit shock to some of the elements of \( e(t) \) for the Wu & Xia model of UK. The x-axis indicates the number of periods ahead, while the y-axis indicates the magnitude of the impact. In addition, the solid black line indicates the TVP-VAR response to the shock and its persistence change across periods. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

*Source:* Author’s computations, R-studio.
Figure A.30: Wu & Xia Impulse Response Functions under Scenario 3 (UK)

Under this scenario the impulse response functions represent the response of the variables under Cholesky Decomposition 1 unit shock to the average values of the elements of e(t) for the Wu & Xia model of UK. The x-axis indicates the number of periods ahead, while the y-axis indicates the magnitude of the impact. In addition, the solid black line indicates the TVP-VAR response to the shock and its persistence change across periods. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

Source: Author’s computations, R-studio.

Figure A.31: Krippner Impulse Response Functions under Scenario 1 (UK)
Under this scenario the impulse response functions represent the response of the variables under 1 unit shock to the vector $u(t)$ for the Krippner model of UK. The x-axis indicates the number of periods ahead, while the y-axis indicates the magnitude of the impact. In addition, the solid black line indicates the TVP-VAR response to the shock and its persistence change across periods. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

Source: Author’s computations, R-studio.

**Figure A.32: Krippner Impulse Response Functions under Scenario 2 (UK)**

Under this scenario the impulse response functions represent the response of the variables under Cholesky Decomposition 1 unit shock to some of the elements of $e(t)$ for the Krippner model of UK. The x-axis indicates the number of periods ahead, while the y-axis indicates the magnitude of the impact. In addition, the solid black line indicates the TVP-VAR response to the shock and its persistence change across periods. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

Source: Author’s computations, R-studio.

**Figure A.33: Krippner Impulse Response Functions under Scenario 3 (UK)**
Appendix A: Tables and Figures not presented in the main text

Under this scenario the impulse response functions represent the response of the variables under Cholesky Decomposition 1 unit shock to the average values of the elements of e(t) for the Krippner model of UK. The x-axis indicates the number of periods ahead, while the y-axis indicates the magnitude of the impact. In addition, the solid black line indicates the TVP-VAR response to the shock and its persistence change across periods. The figures include among others the 5, 25, 50, 75 and 95 percent quantiles of the impulse response functions.

Source: Author’s computations, R-studio.

Figure A.34: Krippner Impulse Response Functions on 43th (July 2008), 85th (March 2012) and 115th (August 2014) Periods (UK)

This figure summarizes the IRFs for the 43th, 85th and 115th months of the dataset for Krippner models. Figure 21 a) contains the plots of shadow rate IRFs for each of the periods. Part b) indicates the difference between 43th and 85th shadow rate IRFs, associated with 16th and 84th percentiles. Part c) indicates the difference between 43th and 115th shadow rate IRFs, associated with 16th and 84th percentiles. Part d) indicates the difference between 85th and 115th shadow rate IRFs, associated with 16th and 84th percentiles. Part e) contains the IRFs of inflation for each period of interest. Part f) indicates the difference between 43th and 85th inflation IRFs, associated with 16th and 84th percentiles. Part g) indicates the difference between 43th and 115th inflation IRFs, associated with 16th and 84th percentiles. Part h) indicates the difference between 50th and 85th inflation IRFs, associated with 16th and 84th percentiles.

Source: Author’s computations, R-studio.
Appendix B: Content of Enclosed DVD

There is a DVD enclosed to this thesis which contains empirical data and R-studio source codes.

- Folder 1: Source codes
- Folder 2: Empirical data