

Opponent's review on Master thesis "Tomaszewski's conjecture" by Tomas Toufar

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This is an opponent's review on the thesis "Tomaszewski's conjecture" by Tomas Toufar submitted as a Master thesis at Charles University in May 2018. In fact, this is a revised version of my previous review of an earlier version of the same thesis which was submitted in January 2018 (but no defense took place at that time, as I was told). Toufar corrected most of the comments I had on the previous version.

Contents

Tomaszewski's conjecture from 1986 is a problem in combinatorics/discrete probability. It asserts that for any unit vector $a \in \mathbb{R}^n$ for at least 2^{n-1} of the 2^n vectors $\epsilon \in \{-1, +1\}^n$ we have $|\sum_i a_i \epsilon_i| \leq 1$. This is an intricate problem. One source of difficulty is that if the conjecture holds true then the set of extremal vectors a is likely to be quite complicated. In the thesis, Toufar summaries the state of the art and answers in positive one particular case of the conjecture. Namely, he proves that the conjecture holds for vectors of the form $a = (x, y, y, y, \dots, y)$, where x is fixed but arbitrary and $y = \sqrt{\frac{1-x^2}{n-1}}$, for n sufficiently large. The proof of this statement proceeds by a careful estimation of binomial coefficients coming directly from a reformulation of the conjecture. It needs to be said that while the proof is fairly technical (ca 7 pages) it is completely pedestrian. That is, certain bounds on the binomial coefficients are taken from literature (Section 3) and careful but rather unsophisticated analytic tools (e.g., comparing derivatives of one-variable functions) are then used.

I believe that the form and the contents fulfills requirements for a Master thesis at Charles University. Let me point out the main points to support this.

- The thesis deals with a problem in combinatorics. This field is well within Toufar's study branch *Discrete Models and Algorithms*.
- It contains a new result.
- Relevant literature is listed in accordance with standards in the field.
- The thesis is well structured.

That said, I do not think this is a particularly strong thesis. The new result is not strong, is obtained using fairly standard techniques. In such a case I would expect a somewhat broader scope.

On a scale *excellent* / *good* / *satisfactory* / *unsatisfactory* I consider the submitted thesis **good**.

Comments

1. Inequalities in text should be written in words. For example on page 2, line 5, “value >1 ” should be “value more than 1”.
2. Already in the introduction it should be said why is the constant $1/2$ best possible (if true), ideally with several examples.
3. section “Chebyshev type inequality” on page 2: “For a general random variable with variance 1 this is not true; consider the random variable attaining values $-1, 0, 1$, each with probability $1/3$.”-I do not understand this. The random variable given does not have variance 1, so it cannot be a counterexample. And then: “However, the statement is true if we consider a linear combination of mutually independent random variables assuming values -1 and 1 with equal probabilities.” — I would appreciate a comment why this is so.
4. The sentence “Von Heymann [2010] also mentions percolation in connection with this formulation, though he does not provide any further details.” does not carry any information (except, that the author has not found these supposed connections himself).
5. first paragraph of section 1.2.2: The symbol Φ for the cumulative density function of the standard normal distribution is used here but introduced only on page 4.
6. page 20, line 1 of Section 6.4: This sentence does not make any sense to me. What are “the worst case vectors” and how does it relate to any attempt for “uniformization”?

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