The purpose of this thesis is to construct a numerically solvable quantum mechanical model describing the dynamics of the indirect mechanism of the dissociative recombination process of a molecular cation by electron impact. The model also describes vibrational excitation of a molecular cation by electron impact. The solution of this model is carried out by implementing a combination of finite elements, discrete variable representation and exterior complex scaling methods. This is then specifically applied to the dissociative recombination and vibrational excitation of  $H_2^+$  by an incoming electron. The results can be used to test the accuracy of approximative methods and the programs expanded to cover the cases of other diatomics.