

Referee's report on the PhD Thesis of Tomáš Jaki

“d-Frames as algebraic duals of bitopological spaces”

d-frames are a nice byproduct of the work of Achim Jung and Drew Moshier, motivated by theoretical computer science, on Stone-type and Priestley-type duality theories for bitopological spaces. They are the algebraic duals of classical bitopological spaces under a duality that encompasses several famous dualities such as Stone duality and Priestley duality (as well as the dual adjunction between topological spaces and frames and the corresponding duality between sober spaces and spatial frames). Apart from that, d-frames look also interesting because, in a way similar to the case for the (dual) adjunction between frames and spaces that motivates point-free topology, d-frames can be thought of as point-free generalised bispaces in which many bitopological properties can still be treated (like e.g. normality, regularity, zero-dimensionality and compactness).

The thesis under assessment is an outstanding contribution to the theory of d-frames. In fact, until now, little was known about the fundamental properties of the category of d-frames. This thesis fills in those missing categorical aspects of the theory. Namely, it is shown (in Chapter 3) that the category of d-frames is complete and cocomplete (and explicit constructions are presented for quotients and coproducts, the hard cases) and that it has the extremal epi-mono factorization system. Further, it is shown that free d-frames do exist. This together with quotients yields presentations of d-frames by generators and relations, a wonderful tool provided by the switch from topology to algebra. This is discussed in detail in Chapter 3.

Another relevant new contribution to the theory is the Vietoris construction for d-frames (Chapter 4), which generalises the corresponding Vietoris constructions that are known in some subcategories of the category of d-frames.

Then, Chapter 5 presents some applications of the theory of d-frames to the theory of stably compact spaces. Besides providing new bitopological proofs of some old results in the theory of stably compact spaces, the d-frame Vietoris construction yields here new results concerning powerspace constructions for stably compact spaces.



Finally, in Chapter 6, the relation between d-frames and logic is further investigated. Here a connection between bispaces and a paraconsistent logic allows the author to develop a suitable geometric logic for d-frames.

I am impressed by the mathematical maturity and knowledge the defendant reveals with this carefully written dissertation. Misprints are scarce, which makes the reading very pleasant. The inclusion of an appendix with the background material on set theory, order theory, category theory and point-free topology looks very helpful and shows the care the author puts in the communication of his ideas and results to a broader audience. I have only a (very) small criticism regarding the presentation of the material in Chapter 2, “Bispaces and d-frames”: for someone like me with background on biframes rather than d-frames, several questions came to my mind after reading the basics on d-frames that made me spend some time thinking on the relationship between the two categories (e.g. on how to construct the patch frame that produces the natural functor from d-frames to biframes); only later on I found that all those questions were carefully treated in the thesis by O. Klinke [Kli12]. So I think that a brief note mentioning this would be very helpful for some readers.

To sum up, as the author clearly mentions, the material presented in Chapters 3-6 is original, as well as Theorem 2.6.11 in Chapter 2. Part of these results has been already published and I am aware that the other part is being prepared for publication. I am sure they will have impact in the future development of the theory of d-frames.

In conclusion, this is a very good thesis, and its author most certainly deserves the degree. My recommendation is that the thesis be accepted unamended.

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