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Opponent review of the habilitation thesis of RNDR. Martin Tancer, Ph.D

Martin Tancer has handed in a habilitation thesis comprising a total of 320 pages. Most of this impressive work consists of reprints of 9 articles authored respectively coauthored by the candidate. These papers have been submitted to ArXiv between November 2013 and October 2016. Six of the papers have corresponding conference versions that appeared at the Symposium on Computational Geometry (SoCG) and the Symposium on Graph Drawing (GD) respectively. Three of the full versions have already appeared, and three others are accepted for publication. All the journals chosen for publication have a very good reputation. According to Google-Scholar the 9 papers have been cited more than 50 times.

The topics of the 9 papers mostly fall in the realm of Geometry and Topology mostly with a connection to Computability and Combinatorics. In the introductory first chapter of the thesis Martin Tancer offers two different taxonomies to group the papers according to their content, however, the order of reprints in the thesis remains misterious.

For my personal prospective the three papers circling around drawings of graphs on surfaces are the most acessible and interesting contributions of the thesis. The oldest of these papers is entitled *Untangling two systems of noncrossing curves*, it deals with two systems A and B of curves which live on a surface with boundary such that all endpoints of curves are on the boundary and each of the systems is free of crossings. The aim is to find a homeomorhism ϕ such that A and the transformed system $\phi(B)$ have few crossings. It is shown that $O((|A| + |B|)^4)$ crossings suffice. The proof combines some surgery type arguments to deal with surfaces of higher genus with a simple result about simultaneous embeddings of planar graphs. In *shortest path embeddings of graphs on surfaces* the authors deal with embeddings of graphs on surfaces such that all edges are represented by shortest paths. Straight line drawings of planar graphs are an instance of such drawings. It is shown that the same surface S may have two metrics such that all S-planar graphs are shortest paths embeddable with respect to metric 1 but not with respect to metric 2. In fact the simplest surface for which this is shown is the Klein bottle. For higher genus g it remains open whether a surface of genus g admits a universal metric, i.e., a metric where all embeddable graphs are shortest path embeddable. It is shown, however, that every orientable surgace of genus S admits a metric where all embeddable graphs can be embedded such that all edges consist of $O(g)$ pieces which are shortest paths. The third paper

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in this group is *a direct proof of the strong Hanani-Tutte theorem on the projective plane*. The strong Hanani-Tutte theorem says that a graph can be embedded in the projective plane if and only if it has a drawing in the projective plane such that all independent pairs of edges have an even number of crossings. The corresponding statement for the plane goes back to Hanani (1934) and Tutte (1970), their proofs rely on the forbidden subgraph characterization of planar graphs. A major advance was obtained by Pelsmajer et al. (2005) who reproved the theorem using a redrawing scheme that results in a crossing free drawing. The strong Hanani-Tutte theorem on the projective plane was obtained by Pelsmajer et al. (2009) by analysing drawings of the forbidden minors for the projective plane. This line of proof is prohibited for other surfaces because the list of forbidden minors is not known. The paper under consideration may open the door towards further Hanani-Tutte type theorems because it develops redrawing tools to prove the result.

The next three papers

- *Embeddability in the 3-sphere is decidable*
- *On generalized Heawood inequalities for manifolds: a van Kampen-Flores-type nonembeddability result*
- *Bounding Helly numbers via Betti numbers*

deal with embeddability between topological spaces with a computational respectively combinatorial twist. Measured by the impact on the community the first of these papers may be the most important of the thesis. It fills one of the enthralling gaps in the understanding of the complexity landscape for embeddings.

The two papers

- *Shellability of the higher pinched Veronese posets*
- *Recognition of collapsible complexes is NP-complete*

are concerned with proof techniques for certificating that a simplicial complex is 'easy'. In the first of the papers a new variant of shellability is introduced and applied to so called pinched Veronese posets. The result has applications in Algebra. The second paper indicates that certificating that a simplicial complex is 'easy' will remain as task that requires creative combinatorial work.

Last but not least there is the paper *On Betti numbers of flag complexes with forbidden induced subgraphs* it expands on previous work and gives for all graphs H optimal or near optimal bounds for the total Betti number of clique complexes of graphs with a forbidden H-subgraph.

With this thesis Martin Tancer has clearly proved his mathematical talent and maturity. I recommend without hesitation to accept the thesis and promote Martin Tancer to associate professor.

(Stefan Felsner)