Evaluation of RNDr. Martin Tancer's habilitation thesis

This habilitation thesis is based on nine papers written by Martin Tancer. They are all very recent ones, two of them written by him alone and seven of them with coauthors. Their topic is very broad including combinatorics, combinatorial geometry, topology and computational complexity. The motivation however is quite common: how topology interplays with combinatorics and combinatorial geometry, and also, the algorithmic aspects of algebraic topology. The results in these papers are new and represent significant development and add novel understanding to the problems in this field. I want to state here, in the beginning, that based on this thesis the applicant should be appointed as an associate professor.

Now I will evaluate some of the results in more detail. The paper [6] (on the list of nine papers in the thesis) is about the algorithmic decidability of the question $\texttt{Embed}_{k\to d}$, that is, whether a given k-dimensional simplicial complex can be (piecewise linearly) embedded into $\mathbb{R}^d$. Theorem 1.1 of [6] states that $\texttt{Embed}_{2\to 3}$ are $\texttt{Embed}_{3\to 3}$ algorithmically decidable. This is an important new result, and is part of the team work initiated by Jiří Matoušek on the complexity of topological problems.

Embeddability is a central question in topology and it appears again in [3] and [4]. For instance in [4] a far-reaching generalization of Helly's classical theorem is proved. It states that, for non-negative integers b, d there is an integer $h(b, d)$ such that the following holds. Suppose $\mathcal{F}$ is a finite collection of subsets of $\mathbb{R}^d$ with the property that for every (proper) subfamily $\mathcal{G}$ of $\mathcal{F}$, the ith Betti number of $\bigcap \mathcal{G}$ is bounded by b for every $i = 0, 1, \ldots, \lceil \frac{d}{2} \rceil - 1$. Then the Helly number of $\mathcal{F}$ is at most $h(b, d)$. Helly numbers are always interesting and the real surprise here is that the condition only goes up to half the dimension of the underlying space $\mathbb{R}^d$.

A central question about the intersection pattern of convex sets in $\mathbb{R}^d$ is the so-called collapsibility of its nerve. Collapsibility of simplicial complexes comes up in other areas, for instance in discrete Morse theory. Theorem 1 of [9] says that it is NP-complete to decide whether a 3 or more-dimensional simplicial complex is collapsible. This beautiful result is a vast generalization of the one of Magouyres and Francés treating the case when a 3-dimensional complex collapses to a 1-complex. The proof is a remarkable construction of Bing's house with 3 rooms (by Martin Tancer without coauthors), a modification of the famous Bing's house with two rooms.

The papers [2], [5], and [7] deal with curves and graphs on surfaces. For instance [5] is about extending Fáry's theorem on straight line segment representation of planar graphs on other surfaces. Theorem 1 in [5] says that the 2-sphere, 2-dimensional projective plane, the 2-dimensional torus, and the Klein bottle can be endowed with a *universal shortest path metric*, meaning that any graph embeddable there can be embedded in such a way that the edges are shortest paths in this metric. This is a remarkable achievement. In [7] there are two families of simple curves on a manifold M, with no crossings in either family. The question is, under what conditions can one entangle the two families (or at least reduce the number of crossings between them) by a boundary preserving homeomorphism $M \to M$. The paper gives interesting new bounds on the number of crossings in question.

Summarizing, this thesis contains several important new results on the use of algebraic topology in combinatorics, combinatorial geometry, and on the computational complexity of the related topological questions. It shows clearly that Martin Tancer is a talented and hard working young mathematician, who is able to conjecture and prove, and who has contributed significant results to these fields and who will continue to do so. I want repeat what I said in the beginning,that, based on this thesis and on his achievements in general, the applicant should be appointed as an associate professor at Charles University.

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