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Review of the Habilitation Thesis
Linear and Exact Extended Formulations
of Hans Raj Tiwary, M.Sc., Ph.D.

Professor Jiří Sgall, the Chair of the Habilitation Board, has asked me to act as an opponent of the habilitation thesis of Hans Raj Tiwary, M.Sc, Ph.D. I have gladly accepted this invitation. In order to provide my review in a timely manner, in the following I will try to give a concise assessment of Hans Raj Tiwary's scientific merits in his habilitation thesis.

I have known Hans Raj Tiwary personally since his time as a postdoc in Berlin between October 2008 and May 2010. During this time he was employed as a postdoc within the DFG Research Training Group "Methods for Discrete Structures" whose faculty I belonged to. As he was mainly working with the groups of Günter Ziegler and Günter Rote at FU Berlin, however, I did not directly interact with him. Within the past 5 years I have followed the remarkable work of Hans Raj Tiwary and his collaborators in the area of extended formulations with great interest.

The habilitation thesis of Hans Raj Tiwary builds upon ten publications that he coauthored between 2012 and 2016. Six of these ten papers have been published in international scientific journals in the areas of applied mathematics and theoretical computer science. These journals are unanimously very good and the list contains absolute top journals such as the *Journal of the Association for Computing Machinery* (JACM), *Mathematical Programming* (MathProg), and *Discrete & Computational Geometry* (DCG). One paper has been published in the proceedings of a very good international workshop and the remaining three are available on the *ArXiv* and have been submitted for publication.

In the following I give a short overview of the research area and some of the main contributions that can be found in the habilitation thesis.

The study of extended formulations has been initiated in the beginning of the 1990s with a paper by Yannakakis who – annoyed by other researcher's repeated attempts to publish papers 'proving' P=NP via intricate polynomial-size linear programming formulations of NP-hard problems such as the traveling salesman problem (TSP) – proved that there is no such LP formulation satisfying a certain symmetry condition. The problem to get rid of the latter symmetry restriction remained open until the 2015 landmark JACM paper "Exponential Lower Bounds for Polytopes in Combinatorial Optimization" by Fiorini, Massar, Pokutta, Tiwary, and de Wolf. In this paper it is proven that the TSP does not allow for a polynomial-size LP formulation, putting an end to any attempt to prove P=NP this way. In my opinion, this is an absolutely outstanding result, one of the most significant contributions in the area of mathematical optimization and complexity theory within the past years.

In terms of the geometry of polyhedra the result states that there is no family of polytopes of polynomially bounded dimension and number of facets whose projection to an appropriate subspace is the TSP polytope, i.e., the convex hull of the incidence vectors of all TSP tours. The authors show that the same result holds for the cut polytope and the stable set polytope. The proofs of these results rely on a novel connection between semidefinite programming reformulations of LPs and one-way quantum communication protocols.

Subsequently, in the 2015 MathProg paper "On the Extension Complexity of Combinatorial Polytopes", Avis and Tiwary extend these results via a lifting argument to three dimensional matching, subset sum, and further NP-complete problems. Moreover, they succeed in extending the exponential extension complexity of the cut polytope of general graph to special graph classes by developing a respective relationship between graphs and their minors.

Another outstanding contribution is the 2015 MathProg paper "Extended formulations, nonnegative factorizations, and randomized communication protocols" by Faenza, Fiorini, Grappe, and Tiwary. They extend Yannakakis' observation on the connection between extended formulations and communication complexity from deterministic protocols to randomized protocols. This extension can be crucial for obtaining small extended formulations which is exemplarily demonstrated for the perfect matching and spanning tree polytopes.

Last but not least I would like to mention Tiwary's very recent, unpublished paper "Extension Complexity of Formal Languages" where he extends the concept of extension complexity from polytopes to formal languages and proves interesting closure properties of languages admitting compact extended formulations. Moreover, he develops a sufficient machine characterization of compact languages leading not only to lower bounds in streaming models but also to upper bounds on the extension complexity of polytopes.

Overall, these and the other results described in the habilitation thesis demonstrate Tiwary's outstanding expertise and scientific merits in this recent and very active field of research. He is internationally recognized as one of the few main players in this field and has thus gained considerable international visibility. He maintains connections to all important researchers in his areas of interest and collaborates very successfully with several of them. Moreover, his publications and, in particular, his habilitation thesis are very well written and presented and he also gives very inspiring talks.

I am therefore glad to strongly recommend to appoint Hans Raj Tiwary as an associate professor at the Charles University.

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