Supervisor's report on the PhD. thesis by Mgr. Pavel Čížek

Mgr. Pavel Čížek has been working, during his PhD. study, on finding new solutions of the Einstein gravitational equations within the class of stationary and axially symmetric (and also orthogonally transitive) space-times. More specifically, his task has been to study how to describe a black hole encircled by a thin disc, while *not* restricting to a static case, i.e. allowing for *rotation* of the system. Such a problem is interesting theoretically as well as relevant astrophysically. Actually, general relativity is non-linear, so a simple superposition does not work and unexpected features may occur in space-times generated by multiple sources. However, mainly interesting would be to have, at last, a practical method how to study, analytically, a non-linear interaction between *dragging effects* from two or more rotating sources. On the astrophysical side, the black-hole–disc systems are very probably important in such high-energy (and strong-field) sources like active galactic nuclei, X-ray binaries or gamma-ray bursts.

Pavel has tackled the problem following two approaches: i) a suitable perturbation of Schwarzschild space-time, ii) application of the Belinski-Zakharov inverse-scattering technique. In the first case, he was able to obtain closed forms for the Green functions of the two crucial metric functions, namely a gravitational potential and a dragging angular velocity for an infinitesimally thin massive rotating ring encircling, symmetrically, a Schwarzschild black hole. This result, inspired by a "classical" paper by Will (1974) (where the Green functions were given in a series-expansion form), allowed him to proceed to extended "additional" sources (in case of which the Green functions have to be integrated over the source). As an illustration of the procedure, Pavel presents an explicit result for the first-order (linear) perturbation due to a thin finite annular disc with constant Newtonian surface density, including basic check of its physical properties.

The second part of the thesis is devoted, with the same aim, to the usage of the Belinski-Zakharov inverse-scattering technique. It is a common experience that the generating techniques which "introduce" a black hole into a given "seed" space-time yield unphysical outcomes – loosely speaking, the black hole is introduced without proper adjustment of the seed source (a thin disc in our case), which results in the occurrence of supporting lines or surfaces ("struts") along which matter is distributed with unphysical properties, ensuring that a given system of sources can remain stationary. Pavel tried to check whether there really is not any room for a regular result and confirmed that it is only the case within *static* solutions. This effort has also brought one particular new result, namely he was able to derive, in a closed form, "the second" metric function for a static system of a Schwarzschild black hole surrounded by a Bach-Weyl ring (up to now, this function has only been computed numerically).

The results presented in the first part of the thesis we submitted as a paper for *The Astrophysical Journal* and they has been accepted for publication (in the journal's *Supplement Series*) just a few days ago (on 17th August). Preliminary versions of these results (do not employing the closed form of the Green functions) were also published in several conference proceedings. Let me stress that the contribution of Pavel to the ApJ paper has been substantial, in particular, the closed-form Green functions for the perturbation of Schwarzschild are entirely his merit. (The Green function for potential is "obvious", because it just corresponds to a static, Bach-Weyl–ring external source, but the Green function for dragging stands for a new result.) Presently another student of mine continues working on the same topic and we are preparing another paper where the properties of the linear disc perturbation will be analyzed more

thoroughly. The study of the Belinski-Zakharov generating technique has not brought any publication yet.

My assessment of Pavel Čížek's work is a bit "torn", similarly as his PhD. study itself. Shortly after beginning of his doctorate, Pavel got employed and the work engagement has been preventing him from a long-term concentration. In recent years, Pavel has also been naturally devoting time to his family. Also, Pavel is a smart person, which is of course very positive, but sometimes it just makes a discussion with him slightly difficult, namely for someone who is not so deeply immersed in his momentary mathematical problem. I admit that sometimes the latter even applied to his supervisor, so some part of Pavel's PhD-study delay may have occurred because I did not always give him sufficient hints for further progress. On the other hand, he often did not follow my "practical" advices (and went on his own, more difficult way).

Be that as it may, I want to stress that Pavel finally surprised me positively by deciding to finish the thesis and by being really able to make it. Actually, it is a common experience that who works hard in a business sphere, and even has a family at the same time, is "lost for science". On the basis of our recent "sprint" with the thesis, I can say that Pavel is still intellectually tightly connected with science and keeps interested in mathematical and physical problems. I especially appreciate Pavel's knowledge and carefulness in differential equations and functional analysis, and his comprehension of the "structure" of a mathematical-physical problem (e.g., I have rarely met someone with so deep understanding of the – rather unintuitive – algorithms of "generating techniques"). Somewhat less perfect is his care for literature, so occasionally he re-derives results that had already been known, and is not able to properly discuss his results in the context of what have been obtained by others.

In conclusion, I think Pavel Čížek has shown a capability of independent scientific work and has also obtained interesting new results which can serve as a basis for further studies. I thus suggest to accept his thesis as fulfilling the requirements for the PhD. thesis, and to award him the PhD. title accordingly.

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