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Paris, le 24 mars 2007

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Dear Colleagues,

It is a pleasure to me to present my report on the doctoral thesis of Mr. Zdeněk DVOŘÁK, entitled:

"ASYMPTOTICAL STRUCTURE OF COMBINATORIAL OBJECTS".

The subject of this thesis is an emerging concept in the theory of graphs. Specifically, Mr. Dvořák studies how the property of a class of graphs to have depth r minors' average degree bounded by a function of r (that is: the *bounded expansion* property) is related to other graph parameters. Such a property allows particular graph decompositions, which are convenient to study structural properties of graphs, and to obtain efficient algorithms.

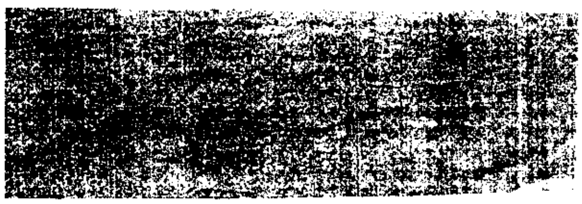
Chapter 3 abounds in new theoretical results relating graphs parameters (acyclic chromatic number, arrangeability, greatest average density, size of vertex separators) to the exclusion of small subdivisions of specific types of graphs (graphs with high chromatic number, graphs with high minimum degree, expanders). For instance, Mr. Dvořák proves that a d-degenerate graph without 1-subdivisions of a graph with minimum degree d has its arrangeability at most equal to $4d^2(4d+5)$. It follows that graphs without subdivisions of the complete graph K_p have arrangeability $O(p^6)$, what improves the best known bound of p^8 obtained by Rödl and Thomas. In the same spirit, Mr. Dvořák show how bounding the game chromatic number by the maximum chromatic number of allowed 1-subdivisions would solve the conjecture of Zhu and Dinsky, which asserts that the hereditary game chromatic number and the acyclic chromatic number are bounded by a function of each other.

Chapter 4 concerns complexity issues. Mr. Dvořák first extends the NP-completeness result obtained by Boedlander et al. for the problem of determining the maximum degeneracy over the minors of a graph to the case where considered minors have depth at most r, and proves that even the "simple" problem of deciding $\nabla_1(G) \geq 2$ is NP-complete. Mr. Dvořák then gives a polynomial algorithm to compute $\nabla_r(G)$ for graphs with bounded tree-width, which he describes in the particular where $r = 1$. The remaining of the chapter is devoted to a polynomial time approximation algorithm, which allows Mr. Dvořák to improve the best known bounds for low tree depth coloring numbers (in terms of the ∇_r's) in Chapter 5.

Chapter 5 is devoted to tree-depth and subgraph colorings. Mr. Dvořák first improves the bound on the order of forbidden subgraphs of the class of the graphs with tree-depth at most k from a tower function to a double exponential. Then, Mr. Dvořák applies the approximation algorithm given in Chapter 4 to bound $\chi_p(G)$ by some polynomial of degree $O(8^p)$ on $\nabla_{2^{p-1}-1}(G)$, to be compared with the original dependency of the form $f(\nabla_{p^p}(G))$. Although a better dependency has been obtained very recently by X. Zhu as a bound of the form $f(\nabla_{2^p-3}(G))$ (for $f(x) \approx (2x)^{[(2^{p-2}-1)!]^2 \, 2^{2^{p-2}+p-3}}$) by a probabilistic argument, Mr. Dvořák's arguments might likely be a way to improve this later bound. Last, Mr. Dvořák studies the problem of induced subgraph coloring, introduces the induced upper chromatic number and proves for this invariant properties similar to the ones satisfied by tree-depth.

Let me conclude with saying that I am very much impressed by the precision and the thoughtfulness of the theoretical development for this thesis, as well as the (maybe insufficiently emphasized) power of the results established in its course. To me, there is no doubt that according to international academic standards this thesis satisfies all requirements for a doctoral thesis in the field of Mathematics, and, in fact, an **excellent** one.

Best Regards,



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