Title: Baire and Harmonic Functions

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Abstract: The present thesis consists of six research papers. The first four articles deal with topics related to potential theory, Baire–one functions and its important subclasses, in particular differences of semicontinuous functions. The first paper is devoted to the stability of the Dirichlet problem for which a new criterion in terms of Poisson equation is provided. The second paper improves the recent result obtained by Lukeš et al. It shows that the classical Dirichlet solution belongs to the $B_{1/2}$ subclass of Baire–one functions. A generalization of this result to the abstract context of the Choquet theory on functions spaces is provided. Finally, an abstract Dirichlet problem for the boundary condition belonging to the class of differences of semincontinuous functions is discussed. The third paper concentrates on the Lusin–Menshov property and the approximation of Baire–one and finely continuous functions by differences of semicontinuous and finely continuous functions. It provides an exposition of topologies (various density topologies as well as the fine topologies in both linear and non–linear potential theory) where this technique can be used together with several new proofs. In the fourth article, we provide a generalization of the characterization of differences of bounded semicontinuous functions by H. Rosenthal from metric to a general Hausdorff topological space. As an application, we provide a new proof that the classes of differences of both bounded and unbounded semicontinuous functions are perfect. The last two papers deal with two open problems from the general topology, continuum theory. First, we provide an affirmative answer to a problem of J. J. Charatonik, W. J. Charatonik and J. R. Prajs whether there is a chain of dendrites without monotone supremum. Second, we show a negative answer to a question of Y. Ohsuda whether if X is an arcwise connected continuum with a free arc and with the fixed set property for monotone onto maps then X is a simple closed curve.

Keywords: Baire–one functions, Harmonic functions, Dirichlet problem, Dirichlet solution, Lusin–Menshov property, Differences of semicontinuous functions, Continuum theory