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Report on the doctoral thesis of M. Jakub Slavik

The doctoral thesis of M. Jakub Slavik, entitled "Evolutionary differential equations in unbounded domains", consists of four parts (papers).

The general theme of the thesis is the study of the asymptotic behavior, in terms of attractors, of PDE's in unbounded domains. Compared with bounded domains, the situation is much more involved when dealing with unbounded domains, due, essentially, to the lack of compactness in Sobolev injections. This yields that the proper functional setting is already an issue and one has to be very careful when introducing the functional spaces. One approach consists in working with locally uniform spaces. A second issue is the fact that the attractors may naturally have infinite dimension. However, one has to find a way to "measure" this infinite dimensionality, as an attractor has to be much thinner than the original phase space (otherwise, any bounded absorbing set would be an attractor, which is not at all reasonable). A proper tool in this direction is the Kolmogorov ϵ-entropy.

The first part of the thesis deals with a nonlocal equation with delay. Such a nonlocality brings additional difficulties. M. Slavik proves the well-posedness and the existence of the locally compact global attractor. He also gives estimates on the entropy of this attractor. An essential tool here (and in other parts of the thesis) is the method of short trajectories.

The second part is concerned with a general nonlinear reaction-diffusion equation and extends results due to S. Zelik and his co-authors. Here, M. Slavik proves the existence of an infinite-dimensional exponential attractor (again, one has to be careful in the definition of such a set as it has to be "thin" in some proper sense, i.e., in the sense of the Kolmogorov ϵ-entropy). Furthermore, a necessary and sufficient condition for the existence of such an attractor is given.

The third part of the thesis deals with a semilinear wave equation with a weak nonlinear damping. Note that such a situation is delicate to handle. M. Slavik proves the well-posedness and then studies the existence of a proper attractor. To do so, he needs to define proper functional spaces and, more precisely, he introduces a cone-version of locally uniform spaces. Again, the method of short trajectories is essential here.

The final part of the thesis is concerned with a strongly damped wave equation. Compared to the previous part, one no longer has a finite speed of propagation and one cannot

use this to construct the global attractor; one thus needs to adapt the proof of the so-called squeezing property.

In conclusion, M. Slavik has addressed an impressive number of very difficult problems. To solve them, he had to master a large number of tools, techniques and theories in PDE's and functional analysis which are often very delicate and difficult. All the results obtained in the thesis are original and should be of interest to the large community working on (infinite-dimensional) dynamical systems, but also more generally on PDE's. It is clear that M. Slavik has the ability for creative scientific work; I also believe that he has the ability for a very successful academic career should he decide to pursue in this direction.

This work constitutes in my opinion an excellent doctoral thesis. I thus strongly recommend that it is accepted and defended.

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