

Report of the supervisor for the dissertation

V. Tuček

### Invariant differential operators for 1-graded geometries

The basic question behind the dissertation by V. Tuček is a generalization of a geometric construction of the so called BGG sequences, which was described in a full generality for simple Lie groups and their finite dimensional representations, to the case of unitary highest weight modules. There are two substantially different cases to consider depending on infinitesimal character of Verma modules involved in a particular BGG sequence, which can be either regular or singular (a special case being the odd dimensional conformal case, where the character is regular but not integral). The cases of regular infinitesimal character are closely connected to a truncation of the standard BGG sequence for a finite dimensional representation and to the notion of Kostant modules. The case of singular infinitesimal character is predicted by the Enright-Shelton theory describing an equivalence of categories between singular and regular cases for different flag manifolds. Both these different cases are illustrated nicely in the Chapt. 3 in explicit discussion of nilpotent cohomology of unitarizable highest weight modules.

The dissertation does not bring the complete answer to the problem but it contains a treatment of several interesting and important topics related to the basic question described above. The first chapter of the dissertation contains a very short and dense summary of some notions needed later. Topics treated here are the translation principle, the Shapovalov form, the Enright-Shelton theory, Kostant modules and globalization of Harish-Chandra modules.

In Chapt. 2, there are two different topics containing new results. The first one is an elementary treatment of one of the two exceptional cases of Hermitian symmetric spaces based on the description of the real octonionic projective plane. The second topic is an explicit calculation of singular vectors (determining the corresponding homomorphism of generalized Verma modules) using the so called F-method in the case of Grassmannians.

Chapt 3. deals with unitary highest weight modules and their nilpotent cohomology. In Sect. 3.1, there is a review of their classification and Sect. 3.2 contains a description of the corresponding nilpotent cohomology case by case. It is quite complicated and nontrivial combinatorial question. To handle it, there is a computer program developed (and listed in App. B)

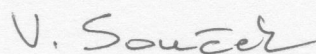
in the dissertation, which is quite useful. App. A contains tables of results for low ranks. This is an important part of the thesis. The description is complete in the conformal cases, where majority of cases belong to regular infinitesimal character. A lot of interesting cases belonging to the singular infinitesimal character are present in the case of Grassmannians and in the symplectic case.

The dissertation contains a substantial amount of interesting material from different fields of mathematics. It is clearly visible from it that V. Tuček is a broadly educated mathematician who is able to treat the considered problem from different angles. A style of writing of the thesis is very condensed but the description is formulated in a quite precise and consistent way, which is nice to read. It would be helpful, however, for the reader to have more comments and explanations included to the text. The dissertation also contains misprints making the reading of the text more difficult.

It took a long time to write the dissertation. It is due to the complexity of the problem as well as to the fact that it was partly written during the internal study and finished during the external period of the study when the author was already working in a full time job. An important point to say is that V. Tuček was able to work on the topic which I gave him quite independently and my influence on the work was very limited. It shows that he is developed during his PhD study into a strong and independent mathematician who is able to work in many parts of mathematics.

To summarize, the dissertation contains interesting new results in an important direction of the study of invariant differential operators and I recommend it for the defense.

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Vladimír Souček