The thesis compiles my contributions to the tilting theory, mainly in the setting of a module category over a commutative ring. We give a classification of tilting classes over an arbitrary commutative ring in terms of data of geometrical flavor - certain filtrations of the Zariski spectrum. This extends and connects the results known previously for the noetherian case, and for Prüfer domains. Also, we show how the classes can be expressed using the local and Čech homology theory. For 1-tilting classes, we explicitly construct the associated tilting modules, generalizing constructions of Fuchs and Salce. Furthermore, over any commutative ring we classify the silting classes and modules. Amongst other results, we exhibit new examples of cotilting classes, which are not dual to any tilting classes - a phenomenon specific to non-noetherian rings.