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August 16, 2017

Dear Colleagues,

This is my evaluation report of the doctoral dissertation entitled EXTENSION PROPERTIES OF GRAPHS AND STRUCTURES, by RNDr Pavel Klavik.

The dissertation represents an impressive package of very strong results which are already having significant impact on the general area of well-structured graphs, in particular geometrically representable graphs. Here his innovative approach and elegant results pioneered a new direction of research that has blossomed into a popular research topic.

The dissertation is written in a very pleasing manner, with emphasis on intuition and motivation, with well chosen examples and illustrations. It also emphasizes connections to other parts of mathematics and presents the results in the framework of a larger picture; he will typically survey an entire area to place the results in a proper context. Additionally, there are a number of excellent open problems that will make the dissertation a valuable resource.

There are many strong and deep results included in the dissertation, with intricate proofs that delve into the fine structure of the graphs in question. In each case, Klavik explores many natural variants and extensions, covering the area with a full range of related results.

Here are some of the best results (and their variants):

1. A Fulkerson-Gross-like characterization of when a partial representation of an interval graph can be extended to a full representation. This means the characterization is in terms of an ordering of maxcliques, and hence can be adapted, via a fine analysis of re-ordering PQ-trees, to a linear-time algorithm to test the extendability. (Theorem 3.22 and Proposition 3.4.3)

These fundamental results are complemented by studies of extension problems for related geometrically defined graphs, such as proper and unit interval graphs, chordal graphs, circle graphs, permutation graphs, and so on. This study has been embraced enthusiastically by a number of other researchers who continue the work, e.g. [20,243].

2. A Lekkerkerker-Boland-like characterization of the same class of extendable partial representations of interval graphs. This means the characterization is in terms of forbidden induced subgraphs, of course with possible partial representation. A particularly attractive feature of these forbidden subgraphs is their compact description using induced paths, which

results in a finite list of obstructions. (This is useful even for the original Lekkerkerker-Boland characterization.) As a byproduct of this beatiful result, the author gives a linear-time certifying algorithm to test extendability of partially represented interval graphs, using modified PQ-trees (MPQ-trees). The intricate analysis inherent in this chapter is particularly impressive. (Theorem 2.2.1 and Theorem 2.2.3)

3. A linear-time algorithm to compute the minimum nesting of an interval graph, i.e., the minimum k such that there exist an interval representation with no tower of $k+1$ intervals $I_0 \subset I_1 \ldots \subset I_k$ with proper inclusions. This also uses MPQ-trees and an intricate dynamic programming algorithm. (Section 5.3)

This popular concept is one natural generalization of proper interval graphs. There is another natural generalization that stems from the fact that proper interval graphs coincide with unit interval graphs. It would have been nice to have similar algorithm (or NP-completeness result) for interval graphs representable with k interval lengths. Unfortunately this problem remains open. However, it is shown that the extension problems for such interval graphs are NP-complete even for $k = 2$.

4. A Jordan-like recursive characterization of automorphism groups of planar graphs. (Theorem 8.2.10) This is an enhancement of Babai's celebrated result on this topic, with a more concrete description of the groups. As an application, the author derives Negami's theorem that regular quotients of planar graphs are projective planar. (Theorem 10.4.4)

Further enhancements give Jordan-like characterizations of automorphism groups of outerplanar graphs, interval graphs, proper interval graphs, circle graphs, permutation graphs, and so on. (As a nice byproduct, it turns out that, for instance, the automorphism groups of interval graphs coincide with those of trees.) All these results are based on a study of decompositions of graphs into 3-connected pieces, given in Chapter 7.

5. NP-completeness results for list isomorphism problems restricted to bipartite graphs, split graphs, strongly chordal graphs, self-complementary graphs, and many other classes. The author argues that when a graph class is known to be complete for the graph isomorpism problem, the proofs can usually be converted into a proof of NP-completeness of the list isomorphism problem (as in the above examples).

Similarly, the nicer combinatorial isomorphism algorithms, such as the algorithm for isomorphism of trees, say, can be enhanced to solve the corresponding list isomorphism problem. This is done for planar graphs, interval graphs, circle graphs, and permutation graphs, to name a few graph classes. However, the more group-theoretic isomorphism algorithms don't extend, and for instance while the isomorphism problem for cubic graphs is polynomial, the list isomorphism problem is shown to be NP-complete. The list isomorphism problem was pioneered by Lubiw, but Klavik's results have re-focused attention on this interesting problem.

6. An FPT algorithm to decide whether for given graphs G, H one regularly covers the

other. (Chapter 11) A detailed discussion of regular graph covers in Chapter 10 enables this interesting algorithm. Unfortunately, this is not sufficient to find a polynomial time algorithm (except if a proposed map is given), and in fact it is shown that the problem is at least as hard as the graph isomorphism problem.

I recommend acceptance of the dissertation. Dr Klavik has demostrated he is very capable of doing independent research. He is already accepted as one of the leading researchers in the area of geometrically representatable graphs, and one of the pioneers of research on representation extension problems.

Sincerely,

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