

Examiner’s Report
on the PhD Dissertation of Pavel Klavík:
“Extension Properties of Graphs and Structures”

This unusually long but also extraordinarily rich dissertation is based on (at least) 8 journal versions of publications co-authored by Pavel, with varying cast of co-authors. It is centered around graph representations that can be called geometric and around their properties related to symmetries and coverings, in both cases with emphasis on algorithmic aspects.

In Chapter 1 the author gives a very broad introduction to the topics. The exposition includes excursions into selected streams of research within and also outside mathematics where geometric representations of graphs arise naturally. Particular representations are then surveyed in great detail in subsequent subsections, followed by a narrative summary of author’s results on partial representation extensions and on symmetries and coverings.

The bulk of the dissertation is then split into two parts accordingly, focusing on a detailed presentation of selected results from full-length articles and conference papers co-authored by Pavel. In part I, dealing with extensions of partial representations of graphs, an exhaustive state-of-the-art description is followed by a wealth of results on finding extensions of partial interval representations fast, minimal obstructions for such extensions, and interval graphs of limited nesting. Part II, devoted mainly to symmetries and coverings, begins with an overview of relevant knowledge and continues with results on reductions to 3-connected graphs, automorphism groups of planar graphs, list-restricted graph isomorphisms, 3-connected reduction for regular graph covers, and finally on algorithmic aspects of regular graph covers.

I have no doubts about quality of the results presented. I would like to highlight the very detailed characterization of automorphism groups of planar graphs stemming from the material contained in chapters 7 and 8, as well as the results on the complexity of problems related to graph coverings in chapters 10 and 11. In both cases one can without exaggeration speak about substantial advance of the current state of knowledge in these areas, the first case being a great improvement over a classical result of Babai and in the second case opening up completely new directions of research as well.

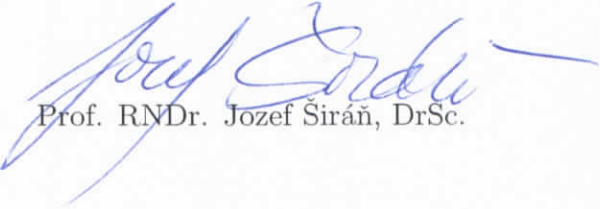
The dissertation is written in what I would describe as a ‘user-friendly manner’, combining narrative style with rigour. The typesetting, a thoughtful layout and elaborate illustrations add to an overall favourable impression. In passing the author often drifts into quite detailed discussions on related topics and consequences, demonstrating deep understanding of the material. Chapters and sections in which new results are presented contain also a number of open problems. Versatility of topics and related open problem are the likely reasons why Pavel did not choose to conclude his dissertation with a separate concluding chapter.

My only reservation concerns referencing. While at the beginning of every chapter that presents new stuff the author states which of his papers the results have been extracted from, individual results in sections and subsections are no longer referenced. I give, say, Theorem 2.2.1 on page 61, as one example for all. Although on page 59 in the line before the heading of 2.2.1 there is a quotation of 4 papers, it would be good to know which result (and in which paper) Theorem 2.2.1 actually is; this information is also absent on page 150 containing a proof of Theorem 2.2.1 (based on a lot of preceding auxiliary results presented in chapter 4). In my opinion it would have been better if the author included more detailed references to the paternity of the presented results, say, in the usual form ‘Theorem 2.2.1 [s, Theorem t]’.

I should also mention that the dissertation is written in very good English in general. Although one can find places in need of correction here and there (such as on page 6 [lines 4,5] – thesis’, and the end of the sentence; on page 197 [line -15] – *to* recognize, or for *recognizing*; on page 292 [line 5] – *locally* bijective; page 310 [line 3] – Membership in NP instead of Belonging to NP; page 337 [lines 1,2] – word order, to name just a few) but these are more than tolerable, given their proportion of appearance to the length of the entire text. The same comment applies to typos; there some (such as on page 128 [1st line of 4.2.2] the reference should have been to section 2.2, pages 187 and 288 with the name Weisfeiler misspelled, page 333 [line -7] – arising, page 334 [page -4] led or leads, again to bring up just a few) but I reiterate that such instances are rare.

On the whole I am impressed by Pavel’s dissertation and the results presented therein. He has certainly demonstrated ability to conduct independent research of very good quality. I am more than happy to **recommend** that Pavel be conferred the degree of PhD in ‘Discrete Models and Algorithms’ (research area 141 in the Nomenclature of the Czech Republic) on the basis of this dissertation.

Bratislava, 9. 8. 2017.



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