

Extension Properties of Graphs and Structures

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The main motivation for graph drawing and geometric representations is finding ways to visualize graphs efficiently to make their structure as understandable as possible. In this thesis, we are concerned with structural properties which are implied for graphs having certain geometric representations. We study two types of geometric representations: intersection representations in which the vertices are represented by geometric sets while the edges are encoded by their intersections, and planar embeddings of planar graphs which are drawing of graphs into the plane without crossing edges. The existence of geometric representations can be used to deduce additional information about graphs. The main idea of this thesis is to ask what extra information can be deduced from the structure of all possible geometric representations.

In Part I, we study the partial representation extension problems for intersection representations. Aside from the graph, the input also gives a partial representation, which prescribes a representation of an induced subgraph. We ask whether this partial representation can be extended to a full representation of the input graph without altering the predrawn sets. I introduced this problem in 2010 in my Bachelor's thesis. We survey the state-of-the-art results for many graph classes. We concentrate on interval graphs and prove both structural and algorithmic results for the partial representation extension problem.

In Part II, we study algebraic properties of graphs, namely their automorphism groups, the graph isomorphism problem and regular graph covering. The main structural tool is the 3-connected reduction which decomposes each graph G into 3-connected components. We are mostly concerned with planar graphs, but some of our results apply to general graphs. In 1867, Jordan described an inductive characterization of the automorphism groups of trees. We describe the first Jordan-like inductive characterization of the automorphism groups of planar graphs. Also, we study the list restricted graph isomorphism problem for variety of graph classes and parameters. For regular graph covering, we describe all regular quotients of planar graphs and construct an FPT algorithm for testing regular graph covering for planar inputs.