LINKÖPING UNIVERSITY

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Report on the Ph.D. thesis

Properties of Sobolev mappings by **Tomáš Roskovec**

This thesis consists of three papers, of which two have appeared (in good international mathematics journals) and the third recently has been submitted. Roskovec is the sole author of all three papers. In all the papers he studies properties of Sobolev mappings.

In the first paper he studies how the optimal Sobolev embedding exponent, as a function of p, can depend on the domain. Sobolev (in the 1930s) showed that the optimal embedding exponent is smooth for Lipschitz domains, while Gol'dshtein and Gurov (1994) constructed an example where it is not smooth. Roskovec takes this further and produces examples where it is discontinuous. It is quite surprising that such examples exist.

The second and third papers both deal with the Luzin (N) condition for Sobolev mappings from (an open subset of) $\mathbf{R}^n$ to $\mathbf{R}^m$. The Luzin (N) condition says that a set of measure zero is mapped to a set of measure zero. It prevents the mapping from creating "something" from "nothing", and is an important condition in many situations both within mathematics and in applications.

In the third paper two types of mappings are studied, general mappings in the Sobolev space $W^{k,p}$ and homeomorphisms in $W^{k,p}$. When $k = 1$ there are positive and negative results completely characterizing when such mappings always satisfy the Luzin (N) condition: For mappings this happens if and only if $p > n$, while for homeomorphisms if and only if $p \geq n$. Roskovec fully extends both of these characterizations to the higher-order cases $k > 1$. The positive results are relatively easy, but creating suitable counterexamples is far from trivial. The second paper deals with a sharpening of the counterexample for mappings belonging to $W^{1,n}$.

The most important things in mathematics are theorems, but examples are also important. In particular counterexamples are very important in order to know what the limits for the theorems really are, and especially when one looks for generalizations of older results. In all three papers Roskovec shows great ingenuity in creating complicated counterexamples.

Sobolev space theory is at the core of the modern treatment of partial differential equations, but is also important in other areas. Roskovec's results deepen our understanding both of the applicability as well as the limitations of Sobolev mappings, which is important both within in mathematics as well as for applications.

Taking all this into account, I think this thesis makes a significant contribution which deserves to be awarded a Ph.D. degree.

Yours sincerely,

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