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Game Theory Approach
to Hostile Takeovers

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Abstract

This bachelor thesis investigates the field of hostile takeovers predominantly from the perspective of the management of the target company. Particular emphasis is placed on review of the defense strategies against hostile takeovers and ways in which they might be abused by the management. This thesis attempts to formulate a game-theoretic model describing the process of a hostile takeover as an extensive-form game with perfect information. Payoff functions for the game further in the thesis computed as Nash equilibria of bargaining problem with the current state of the game as a base for the utility gains. We then briefly discuss the implied relationship among the raider, management and the shareholders.

JEL Classification

C7; C72; G34

Keywords

hostile takeover; game in extensive form; defense strategy


**Abstrakt**

Tato bakalářská práce se zabývá oblastí nepřátelského převzetí společnosti převážně z pohledu managementu společnosti, která vystupuje jako kořist. Zvláštní důraz je kladen na sumarizaci obranných strategií proti nepřátelskému převzetí společnosti a na způsoby, kterými může management tyto strategie zneužívat. Tato práce se pokouší o formulaci teorie-herního modelu popisujícího proces nepřátelského převzetí společnosti v rozvinuté podobě a s úplnými informacemi. Výplatné funkce pro tuto hru jsou v této práci počítány jako Nashova ekvilibria vyjednávacího problému s aktuálním stavem hry jako výchozí pozicí pro nárůst užitků. Poté tato práce stručně disku- tuje implikované vztahy mezi útočníkem, managementem a akcionáři.

**JEL Klasifikace**

C7; C72; G34

**Klíčová slova**

hry v rozvinuté podobě; nepřátelské převzetí společnosti; obranná strategie
Declaration of Authorship

I hereby proclaim that I wrote my bachelor thesis on my own under the leadership of my supervisor and that the references include all resources and literature I have used.

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Preliminary scope of work

Hostile takeovers play an important role in the stock market as every publicly traded company is potentially vulnerable to them. The vast literature spans numerous attempts to create (hopefully successful) defensive strategies to prevent hostile takeover. We shall make an effort to utilize game theory to provide answers to the following research question: "What is the most efficient defensive strategy when facing a tender offer and how does this strategy affect the shareholders?" Our model shall be based on the recent results of Loyola and Portilla (2016). In the process of answering this question we shall focus on subsequent goals. In particular, we shall investigate an effective algorithm to defend against a hostile takeover in an optimal way. Further, we shall inspect whether the optimal defense strategy leads to a loss for the shareholders, whether there exists a defensive strategy which will not leave the company much weaker than it was before the raid, and last but not least, whether the equilibrium in a Hostile Takeover game is in favor of the raider. In this thesis we shall also include some case studies to illuminate the effects of the described theory.

In the light of the research hypotheses specified above, the results of this thesis might be useful in practical situations for a decision making process of putting hostile takeover defenses in place.
Outline

Introduction
Literature review and basic strategies on hostile takeovers
Games in extensive form
Game-theoretic model of hostile takeover
Case studies, results and discussion
Conclusion

Core bibliography

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1 Introduction

Hostile takeovers are a crucial composite of the corporate world as every publicly traded company is potentially vulnerable to them. McKinsey & Company (2007) outlined that only in the year 2006 over 100 hostile transactions took place, being cumulatively valued at over 520 billion USD. Even though the motivation of the target in such a hostile takeover to attempt to defend itself might not be obvious, as the shareholders of the target company tend to experience high positive abnormal returns during the takeover (Jensen and Ruback, 1983), the management of the target company seldom embraces the takeover which would likely lead to inevitable redundancy of their positions (Harris, 1990; Jarrell 1985).

Throughout the history numerous defense strategies have been developed to assist the board of the target company to either improve their negotiation position, or directly neutralize the raider’s ability to continue with the takeover attempt. A broad span of academic literature examined and evaluated these strategies separately, however a paucity of practical implications has been expressed. In addition, the available research has shown significant controversy is present in utilization of the strategies.

Consequently, the lack of conclusive designation of effectivity and suitability of defense strategies to specific scenarios led to scarcity of literature illuminating the grand scheme of managerial options of defense, thus virtual absence of a recommendation which could be made on behalf of defending the company against hostile takeovers is present. This thesis shall attempt to utilize game theory to provide further insight on optimal behavior of the management regarding their utility maximization, thus treating the impact on shareholder wealth as means and not objective.
The topic of hostile takeovers and defensive measures against them calls for various approaches. This thesis shall not argue with distinguished authors who undertook this topic from different angles (e.g., regulatory, shareholder wealth maximization) as it shall delimit itself to addressing hostile takeovers from the perspective of the management and subsequently evaluate the impact on shareholders.

Examples of previous research regarding systematic evaluation of defense strategies against hostile takeovers shall be found in section 2 of this thesis as well as brief introduction to hostile takeovers. Review of relevant literature regarding game theory along with a general introduction into games in extensive form shall be found in section 3 and section 4 shall make an effort to formulate a game theoretic model describing the process of a hostile takeover.
2 Hostile takeovers

This section of this thesis shall provide a brief summary on hostile takeovers, address the reasoning of the parties playing a role in this process, and finally review some of the most important defense strategies we are going to utilize in the model.

Hostile takeovers are generally considered to be any attempts to usurp corporate control on the takeover market (“a market in which alternative managerial teams compete for the rights to manage corporate resources”, Jensen and Ruback, 1983) which is performed against the will of the management of the target company. On the contrary, Schwert (2000) inclines to not interpreting the hostility in hostile takeovers as entrenchment, but rather as bargaining strategy. They might take the form of mergers, tender offers or a proxy fight (Jensen and Ruback, 1983).

Proxy fight (more precisely “proxy contest”) is described as a scenario in which an insurgent group attempts to occupy controlling portion of seats on the board of directors (Jensen and Ruback, 1983). Tender offer is a broadly used term describing an offer for purchase of shares which is made directly to the shareholders who decide on whether to accept it or not under their own discretion. Also referred to as a “hostile bid”.

Rationale behind the raider’s attack

First and foremost, we shall discuss the reasoning behind the raider initiating a takeover attempt. Even though the only reasonable explanation of a hostile bid is the perceived financial gain by the attacker, the means may vary. Schleifer and Vishny (2003), while not denying other findings, identify a connection between mispricing of a company by the market and
the probability of it being taken over.

Abreast with this phenomenon, other variables may be of key importance regarding the incentive for the takeover attempt. Holmstrom and Kaplan (2001) argue that the takeover activity also brings potential improvement of the effectivity of certain companies if operated as a single entity. By extension we can conclude that other than mispricing-related factors affecting the rationale behind a hostile takeover might be at play as for instance ensuring access to the target’s distribution channels, customer base or goodwill of the brand.

Harford (2005) provides evidence on economic, technological and regulatory shocks, in case of sufficient available outstanding liquidity, being responsible for merger waves. In accordance with the preceding, Martynova and Renneboog (2008) also conclude that specific shocks have causal relationship with merger waves. In addition, they even suggest that after the optimal stopping point of a merger wave is passed, there is still suboptimal merger activity which might be caused by “limited information processing, hubris or managerial self-interest”. This supports the theory that factors different from mispricing of companies also account for takeover activity.

It is also notable, that rationality of these attacks has been examined from multiple perspectives. According to Schwert (2000) in case of the bidder the assumption of rationality correlates with reality, although it has been shown that this might not be the case in every scenario (e.g. Millstone & Subramanian, 2007).

**Rationale behind the management’s defense**

Jensen and Ruback (1983) define corporate control as “the rights to determine the management of corporate resources”. As hostile takeovers are means through which corporate control is established, it seems evident that in the aftermath of a successful hostile takeover the raider is likely to either replace the current management as he would consider it to control the company in a sub-optimal manner or let the management of the acquiring company to take
control, thus rendering the current managerial positions redundant. This by itself provides a strong incentive for the management to attempt to prevent the takeover from taking place.

The board of directors of a company is typically composed of either managers who are under strong influence of major shareholders, or in some cases shareholders themselves. Thus, from this point onward this thesis shall refer to the board, controlling shareholder or a group of controlling shareholders and the management interchangeably. As a result, numerous subsequent factors of the management’s decision to defend the company from a hostile bid arise. The board of directors is likely to manage the target company in a manner consonantly with their stances, values and presumably social responsibility, which may all be violated in case a corporate raider would acquire the company and retain only a fraction of his interest, reselling the remainder for parts.

Complementary to this, Ruback (1987) argues that the solemn fact of a hostile bid might be recognized as a sign of incompetence of the management to operate the company effectively, as it indicates unfulfilled potential. Thus, the board of the target company may perceive such offer to be a sign of weakness and be inclined to defend its position.

**Position of the shareholder**

There is a broad agreement among academic literature on the positive effect of hostile takeovers on target shareholder welfare which corresponds with the intuition of higher premia being offered during hostile takeovers, thus the target shareholders receiving abnormally high payments in comparison with the market value.

Many argue that the natural unwillingness of the management of the target to accept the takeover and thus grant their shareholders the benefit of thriving from the premia is undesirable. Jarrell (1985) deliberates the possibility of takeover defenses such as golden parachutes to be the solution to this problem as they provide an incentive for the management to negotiate
on behalf of the shareholders with their best interests in mind. In addition, Berkovitch and Naveen (1990) provide examples on how can “value reducing defense strategies” increase the benefit for shareholders of the target company.

2.1 Defense strategies

The following part of this thesis shall attempt to summarize the most important strategies used as defense against hostile takeovers. Furthermore, this part shall discuss the effects and consequences of the respective strategies in terms of market value of the company, effectivity and alternation of the negotiation power of the management. These defenses may be partially divided into preemptive and reactive takeover defenses. Preemptive defenses, such as poison pills or staggered boards, are those which may be already in place before the takeover attempt has started, while reactive defenses, such as targeted repurchase or litigation, refer to those which are executed succeeding the time of the initial bid.

Examining the defense strategies essential to formulation of a descriptive game theoretic model further in the thesis, as there is no standardized way to thoroughly describe the effects of the defense strategies on the takeover process and bargaining. The list of the strategies is not exhaustive, as some of the strategies were left out due to either limitations of the model, or insufficient available rigorous information on them (e.g. white knight defense strategy is not included, although this thesis was inspired by the work of Loyola and Portilla, 2016, which resembled mostly around this defense strategy).

2.1.1 Preemptive

Fair price

It is self-explanatory that many shareholders prefer not to sell their shares at the first instance a tender offer is made and utilize the time pressure forced on the raider to leverage a more favorable price negotiation posi-
tion (Comment and Jarrell, 1987). As a reaction to this phenomenon, the acquirer often creates an incentivized environment for the shareholders to forego their shares by utilizing so called *front end loaded* takeover. In such a case, the tender offer in the first stage includes provisions guaranteeing a second stage price for the final merger. The first-tier price is substantially higher than the second one, thus it increases the pressure on the shareholders to accept the original terms (Ruback, 1987) and refrain from free-riding into the second stage, as the expected value of it is below the original one (Jarrell and Poulsen, 1987).

Martin Lipton, a corporate lawyer, comments on the issue of tender offers followingly: “First, the special dynamics of a tender offer are such that the decision of shareholders is almost always a foregone conclusion - they will tender; therefore, it is misleading to speak of a free shareholder choice at all. The existence of an offer to acquire a controlling interest in a company makes it almost impossible for a shareholder in the target to prudently retain his shares ...”. This is often the case if the company is not protected by fair price amendments, as such approach is favorable for the raider.

Fair price amendments prevent such behaviour, unless it is negotiated with the management or the offer gets accepted by the supermajority of the shareholders and increasingly more companies tend to include some provisions in the spirit of fair price amendments into their charters. (Jarrell and Poulsen, 1987), and force the bidder to offer a blended price to all the shareholders (Ruback, 1987). This strategy does not impose any extension of costs on the raider, it does however force a different form of the offer, which removes the increased incentive for the shareholders to accept the offer as soon as possible. Although Comment and Jarrel (1987) provide empirical evidence about the fair price amendments having no measurable impact on the final premium at which the merger is performed, Ruback (1987) classifies it as mild, but at least somewhat effective strategy. The main point of implementation is not complete prevention of the takeover, as it rather aims at improving the negotiation baseline for the management of
the target company in case of a tender offer. The estimated average effect on stock prices caused by this preventive measure is -0.65 percent (Jarrell and Poulsen, 1987).

In the model, we are going to work with the premise, that such an anti-takeover measure is in some form indeed implemented, as this strategy is to some extent often also represented in the legal code and is heavily used in the cases of a purchase of the whole company.

**Golden parachutes**

Golden parachutes are a very common measure not only protecting managers from hostile takeovers, but also stabilizing their position inside of the company. These provisions usually grant the management certain benefits in case of termination of their employment, thus making it very costly to remove them from the position.

It is quite intuitive that the general public views these provisions as negative, and Bebchuk, Cohen and Wang (2014) report that the implementation of these contributes a decrease in stock value at the level of about 5 percent. What is more, they report the shareholder wealth to be also negatively affected by these measures, suggesting that “could be due to golden parachutes increasing managerial slack and/or to golden parachutes providing executives with incentives to go along with some acquisitions that do not serve shareholder interest”.

**Staggered Board**

The definition of the term *Staggered board* slightly varies among available literature (c.f. Ruback (1987), Bebchuk and Cohen (2005)), however the core remains unchanged. This strategy prevents complete instant replacement of the management by shifting the election periods for parts of the board of directors in such a way, that a majority can be obtained after a minimum of two non-simultaneous elections. The typical setting of such a provision is a division of the directorial positions into three classes and holding an annual
election, at which only one of the classes of directors is voted into position (Koppes et al, 1999).

Implementation of such election mechanism results in a long transitional interim during which the original management would still have control over the company, even after the takeover. The defenders of this provision argue, that it not only to some extent protects the company from potential hostile takeovers, but also grants substantial stability to the company (Koppes et al, 1999), as in case of turbulent environment among the shareholders the impact on the management of the company is reduced and the company itself is thus protected from the threat of fundamentally changing its long-term plans and strategies frequently. In the period between the years 1995 and 2002 around 60 percent of the companies in IRRC database did use some variation of staggered board. We might thus conclude this strategy to be a common phenomenon.

There are two ways in which this rule can be implemented into the company: As an amendment to the charter, or as an amendment to the bylaws. This distinction is very important mainly for the purpose of observation of the effects a staggered board has on value of a company. Even though Jarrell and Poulsen (1987) conclude that the effect is, although negative, statistically insignificant, in a more recent study, Bebchuck and Cohen (2005) show statistically significant lower market value of companies which did implement staggered board mechanism through the company’s charter. On the other hand, they also provided empirical evidence that bylaw-based staggered boards exhibit only very mild and marginally statistically significant negative effects on company value.

As for the effectiveness of this strategy, due to the generally lower level of rigidity among bylaws, bylaws-based staggered boards do not show vast potential to prevent a hostile bid. On the other hand, Ruback (1987) characterizes this strategy (when addressing the charter-based variation) as moderately effective.
Poison pill

Poison pill is one of the strategies which aim to deflect the potential raider even before the takeover attempt has begun. Their implementation is public and they are not primarily meant to be executed, rather to deter the potential acquirer by their implied activation after such an offer would be made. According to Bruner (1991) who has thoroughly described different variations of the poison pill in effect at the time, the core principle of this defense strategy is to provide an option to dramatically increase the present value of the current shares exercisable after a trigger event (which is usually either a new shareholder, or one of the current ones, obtaining more than a predetermined portion of the shares).

The two main variants of the poison pill are so called flip over and flip out poison pills (Ruback, 1987). In the flip in form, the shareholders may, after the trigger event, purchase common stock worth a multiple of the current price for a discount price, which is considered to be an exercise price, rather than the price of the stock. The raider is explicitly excluded from this right, thus as a natural consequence, the portion of shares the raider possesses dramatically decreases, the price of the remainder of the shares raider would want purchase dramatically increases and lastly there is created a large economic dilution of the raider (Bruner, 1991). The flip out form, on the other hand, endangers the raider directly: “This provision would have the effect of exhausting the target’s assets and making the takeover less attractive. If the shareholder declined to convert into target shares or to put the preferred stock back to the target, this preferred stock automatically carried the same conversion rights into the stock of the bidder ...” (Bruner, 1991).

Poison pill became popular after a confirmation of the legality of such amendments by Delaware Chancery Court ruling in 1985 (Ruback, 1987). Although Comment and Schwert (1995) state the following: “Our new evidence on how stock prices change with poison pill adoptions does not suggest an economically meaningful degree of deterrence.”, they still provide
implicative proof of either presence of deterrence, or negative information about ongoing negotiations. Moreover, they provide that the net effect of these amendments in combination with business laws. Their conclusion, “From this perspective, the evidence we provide about the actual takeover rate and premiums paid as a function of antitakeover devices outweighs the event study evidence in judging deterrence.” does suggest effectivity of this strategy. In addition, they state that development and implementation of these provisions made “complete deterrence feasible”. Bruner (1991) goes as far as concluding, that “From the standpoint of deterrence, the poison pill is virtually a sure thing.”

There is also no conclusive evidence pointing at a negative impact of such defensive method on stock prices, as the studies are usually inconclusive in the scale and significance of the effect (c.f. Comment and Schwert (1995), Ryngaert (1988)), however they agree on a decline in stock prices in case these provisions are activated.

2.1.2 Reactive

Crown jewel defense

Crown jewel defense is a popular designation of a specific variation of asset restructuring. This strategy is based on selling the most valuable assets of the company and thus appearing less attractive to the raider. As such, this strategy is very controversial and many consider it to be automatically harmful for the shareholders, although there seems to be no empirical evidence which would prove this theory conclusively. Due to being a special case of asset restructuring, the effect on the company value of this strategy is also very complicated to observe and separate from other noise, however has been shown, that utilization of such strategy often results in either disciplinary action, or general loss of confidence on the part of shareholders (Franks and Mayer, 1996).

Controversy of this strategy may be well represented by the approach
of the courts, as before the Revlon rule triggers\textsuperscript{1}, the managers may use this strategy to some extent, however after the trigger, their options are drastically restricted (Kurp, 1994).

**Dual class recapitalization**

Dual class recapitalization is a general corporate governance strategy which is based on issuance of stock with superior voting rights. Such a stock is then distributed among the shareholders, which are given the option to exchange this stock for the ordinary, common stock. Shareholders are incentivized to utilize this option, as the stock with superior voting rights does usually suffer from lower dividend returns and/or lower marketability (Ruback, 1987).

Another variation of this approach is issuance of preferred shares, which provide higher dividend returns in exchange for limited to no voting rights, which are issued as new stock and thus the capital investment can be increased without endangerment of the voting position of the current dominant shareholders.

Naturally, such a plan provides the managers with a disproportionate dominant voting rights and the general belief is that it creates environment for the managers to commit entrenchment (Dmitrov and Jain, 2006). As our model shows later in this thesis, without commenting on potential of the direct entrenchment during the time the management retains their managerial positions, it is a very dangerous strategy for the shareholders mainly in the case the management achieved deflection of one hostile takeover through this strategy and then faces another one, as it provides the management with very powerful tool to skew a substantial proportion of the premium by utilizing the bargaining power gained from utilization of this strategy.

Dmitrov and Jain (2006) conclude, that the dual class recapitalization strategy does not pose as an unreasonable risk of entrenchment of the managers and that it does not have any negative impact on the stock prices in in the following years. It is, however, important to note that their research is based on U.S. companies’ data, thus the rest of the world is not accounted

\textsuperscript{1}See section Litigation of this thesis
for in this conclusion. In addition, they specifically state that the returns diminish aggregately with non-native U.S. companies entering the market.

This strategy might be a very effective defense against hostile takeovers while it also might have little to none negative impact on the stock prices, as even in case of a successful bid for the shares with lower voting rights, the raider would not have the possibility to take control over the company (Ruback, 1987). While the potential of this strategy is large, it is very dependent on the case-by-case usage and can result in the opposite - being a great burden for the company, while providing very little security.

**Litigation**

This strategy is crucial for creating a proper model, as it limits the possible steps the managers may undertake to ensure keeping their positions and forces them to refrain from denying the shareholders an offered premium in case the offer has already been made.

As litigation is common phenomenon in the corporate world, it is also a common defense against hostile bids (Ruback, 1987). This becomes self-explanatory when the strict regulatory environment and rigid form of the guidelines for mergers and acquisitions are accounted for (Kurp, 1994). Winning the case, even though it is an obvious success in terms of deflection of a hostile takeover, may not be among the top relevant goals of litigation, as it might be used merely as a pure defense strategy and not as a winnable case.

The main objective of such scenario is typically to delay the negotiations, as the initial offer, or even rumors about the offer, attract other potential bidders to raise the offer, resulting in a bidding war which provides superior standpoint for the shareholders in the negotiations (Gilson and Kraakman, 1990). It is important to note that stalling is very effective after the initial offer has been made, for as aforementioned, it drains the raiders resources heavily. Another important potential positive outcome of litigation for the target company is the possibility of the raider himself raising the offer to
avoid this prolonged process and legal fees (Gilson and Kraakman, 1990). As for a third effect litigation may have on an ongoing hostile bid, it typically forces the raider to provide a broader disclosure accessible to the shareholders, thus often discredit the bidder in the eyes of the shareholders (Kurp, 1994).

Obviously, fully justified suits also play an important role in the world of M&A, however we can hardly speak of them as of defense strategies, as they constitute rather legal mechanisms separate from the takeover power play. It is notable that litigation in the other direction is also very common, as the raider is usually accusing the target company’s management of breaching their fiduciary duties. This resulted in the conception of so called Revlon rule, originating in the U.S. case law, which became a generally accepted legal guideline. Gilson and Kraakman (1990) characterize the rule as follows: “The short answer can only be: As long as shareholders remain free to choose between alternatives, the board actually facilitates an auction - and hence discharges its duty under Revlon - by offering shareholders an attractive alternative to an existing bid. The critical distinction is between the board’s freedom to offer shareholders an alternative and the board’s freedom to impose that alternative on shareholders. Any recapitalization transaction can be cast either in a form that may be implemented on management’s authority alone, as in Black & Decker and Bass Group, or in a form that requires shareholder approval, whether by tender or vote.” It is of high importance to note that the Revlon rule only applies to post-offer defenses and thus does not limit the preemptive measures management might be implementing. Kurp (1994) states that upon the trigger of the Revlon rule, target boards are no longer allowed to employ takeover defenses, unless they are made with the intent to improve the target shareholders value.

**Targeted repurchase and standstill agreement**

Targeted repurchase represents a post-offer anti-takeover measure consisting of buying the shares the bidder currently possesses back and not extending
this offer to the remaining shareholders (Ruback, 1987). It might be very tempting for the raider to exploit this option, as shall he accept, the profit would be completely risk free as opposed to continuation with the hostile takeover, which always involves certain degree of uncertainty.

Chang and Hertzel (2004) argue that a targeted repurchase from entities in their sample, which either are “known takeover specialists” or purchased the stock within a year before the repurchase, is significantly higher than otherwise. This confirms the intuition that substantially higher premium is required by the raider, when the targeted repurchase is used as a takeover defense, specifically 9.1 percent and 1.3 percent respectively. Ruback (1987) argues that the negative impact on company value is at the level of 3 percent, however is more than covered by the initial increase of the stock value which occurs after the initial takeover offer is made. In accordance with this Klein and Rosenfeld (1988) provide evidence on the overall effect being a 6.45 percent increase (excluding the interim period cumulative abnormal return), while the two-day repurchase abnormal return being a 3.27 percent decrease.

As for the effectivity of this defensive measure, it is one with vast potential, however no guarantee (Ruback, 1987). The success is fully dependent on the willingness of the raider to accept the offer, which by extension means it is dependent on the observable expected value it provides him.

Standstill agreements are more of a negotiation of an armistice than an actual defense. This strategy relies on coming to an agreement with the aggressor and convincing him to enter a contract which ensures preservation of status quo on his side in exchange for either several seats on the board of directors or votes of the management on a certain topic (Ruback, 1987).

The shareholder who attempted to initiate the takeover is then potentially less incentivized to continue with the hostile bid, as he might reach his goals through this agreement. As aforementioned, ownership of the company is often not the primary goal of a raider, as these conflicts are led over power within the company. Thus, this agreement may be perceived as a treaty rather than a defense. The effects on the stock prices seem to follow the
same logic as in the case of targeted repurchase (Ruback, 1987), as these two defenses are trivially very similar - both require cooperation on the side of the hostile bidder and could be characterized as quid pro quo.

**Final note on defense strategies**

A variety of unique defense strategies may be utilized by the target company to avoid the hostile takeover. While some are perceived to be very effective, such as poison pills, most of these defenses also bring up extremely sensitive matter of the position of the shareholders and their utilization may, even though neutralizing the attacker, lead to unavoidable exchange of the management inside the target company and thus resulting into an overall loss for the board. It is essential to keep the fact the current shareholders may also decide on exchanging the management themselves, thus not only regulatory restrictions apply while utilizing defense strategies against hostile takeovers.
3 Games in extensive form

3.1 Game theory

Game theory is an aggregate of analytical instruments which help us to understand the phenomena in the decision-making process during interaction of separate entities (Osborne and Rubinstein, 1994). We can find the origins of game theory as far as in the ancient times when soldiers on battlefields were facing a decision between fighting and fleeing to maximize their chance to survive. This situation, while being curious in itself - overlapping philosophy with analytical reasoning, also presents the soldier with the dilemma that his fellow comrades-in-arms are facing precisely the same decision, thus forcing the soldier to take their decisions into account alongside with his own preferences (Ross, 2014).

While following the footsteps of the fathers of game theory, we must not miss Antoine Augustin Cournot, who in the year 1838 formulated a theory which allowed for investigation and modelling of oligopolistic competition among entities. Today, we would call the methodology he utilized to analyze models rising from the aforementioned theory “Nash Equilibrium” which, to say the least, is quite surprising as Nash was publishing more than a century after Cournot. The reason is even though Cournot did use the approach before Nash, he did not generalize it to the broad solution methodology for non-cooperative game theory and rather only used a single application of it (Myerson, 1999).

The next milestone in development of game theory, or rather its predecessors, came with the mathematician Emile Borel who in the year 1921 published a paper in which he for the first time presented the term “method of game”, which should be viewed as a ruleset that would determine the next
step of a person given every possible circumstance. He did so by forming a matrix of every possible expected values for the players in the given game (Myerson, 1999).

Although Borel did already provide foundations for theory of games, it was not until John von Neumann who in 1927, for the first time in history, formulated a general model of extensive games - such that the players move in sequence after each other and are provided with imperfect information about the current state of the game (they do not have comprehensive information about the other players’ previous moves). The game itself is a mathematical term describing the totality of the rules which describe it, while the players (also agents) are the decision makers playing the game by choosing their actions according to the information they have (von Neumann and Morgenstern, 1953). Von Neumann followed Borel and after the formulation of the model defined a strategy for each player to represent a set of rules, which at every possible stage in the game when it is the players turn determines the respective players next move, as a function of his information at the given stage. An important distinction of his approach in this paper from later works lies in his restriction towards the players’ ability to choose a strategy exclusively before the start of the game. Nowadays, we would denote such a structure as the “normal form”, as von Neumann and Morgenstern did so for the first time in their work in 1944 (Myerson, 1999).

Von Neumann continued to investigate games, namely zero-sum games (which are such games that a gain in utility for one player automatically results in an equivalent loss in utility for another one), and in the year 1928 published the proof for the minmax theorem. As minmax solution to zero-sum games relies on the principle of one agent maximizing the minimum at which the game may conclude and the other agent minimizing the maximum at which the game may conclude, which has proven to be a key principle in zero-sum games, the proof of this theorem, stating that the minmax solution in the case of finite two-person zero-sum games exists and is a Nash equilibrium, showed to be invaluable to the development of game theory.
The very existence of game theory as a separate mathematical field is considered to begin with the revolutionary book “Theory of Games and Economic Behavior” by von Neumann and Morgenstern first published in 1944. It was the first work to summarize the knowledge and research made on game theory at the time and provided an axiomatic definition of an \(n\)-person game consisting of 21 axioms. Another giant contribution this work has done is fundamentally new approach to measurement of utility and its maximization problem, lying mainly in comparability solved by new axiomatic definition. Von Neumann and Morgenstern also proposed utilization of backward induction as a solution method, which has later proven to be invaluable. Last but not least, they have shown strategy to be a linking tool between games in normal and extensive form (which we shall discuss further in this thesis). Although this book was indeed a huge step forward, there was still a crucial shortcoming, as they fully focused on zero-sum games with transferable utility (Meyerson, 1999).

From this point onward, development of game theory moved at a fast pace, as John Forbes Nash Jr. published his work on a general solution to a two-agents bargaining model in 1950 in which he did not assume the property of transferability of utility. He elegantly utilized the individual’s utility theory presented by Neumann and Morgenstern and concisely described it on Mr. Smith’s utility from obtaining a new car. In combination with Nash (1953) he has proven the equilibrium in two-agent bargaining model under complete information is such that it maximizes the product gain of the utilities\(^2\) (Harris, 1990). He then continued in his next work to establish a general definition of equilibria in normal form games, while also proving the existence of such equilibria for randomized strategies in all finite normal form games (Meyerson, 1999). Nash (1951) also showed, that cooperative games may be simply reduced to non-cooperative ones via transferring the game into the phase of negotiation of the terms of cooperation. This way

\(^2\)This statement holds if the outcome to the bargain is a division in which both of the agents benefit from coming to an agreement and does not hold if at least one of the agents would be better off without the bargain coming to an agreement at all.
Nash managed to create a general solution concept of games.

Many different types of games were investigated in the field of game theory, as for instance aforementioned cooperative and non-cooperative games, where in the case of cooperative games the distinction from non-cooperative ones lies in the ability of agents to form coalitions and play the game “together”, which is especially useful in political economy, while non-cooperative games strictly forbid such behavior (used mainly in competitive environment when cooperation is either impossible, or not allowed, for example in poker). Another crucial distinction must be made according to the information which the agents possess. We know multiple ways of describing information (complete vs incomplete, symmetric vs asymmetric, etc.), however the most important one for this thesis is perfect and imperfect information. While perfect information assumption requires every agent at any point of the game to have full information about the current status of the game and payoff functions of his opponents, games with imperfect information do not impose this restriction (Rasmusen, 2006).

3.2 Formal definition of games in extensive form

For the purpose of this thesis we shall focus only at a narrow part of game theory, namely finite non-cooperative perfect information two agent dynamic games in extensive form with classical rationality assumption and simple bargaining theory. While there are different streams in game theory regarding approaches to rationality (Binmore, 2015), as the game we shall attempt to model consists of utility addressable by monetary indices, none of the issues arising in other cases do not need to concern us.

To continue with this thesis, we shall first properly define a game in extensive form. We must start by firstly defining a set $N = 1, 2, ..., n$ of agents. We then define a rooted tree, $T$, called the game tree. We now must assign every non-terminal node (also called a leaf) of the rooted tree $T$ to an agent $i \in N \cup \{0\}$, where the agent 0 represents nature (or the game itself) and is used to probabilistically determine the moves which the other
agents cannot affect. This way, we distribute the non-terminal nodes of the
rooted tree $T$ into $n + 1$ subsets $P^0$ through $P^n$, where the members of $P^i$
are denoted as the nodes of agent $i$. For each node in $P^0$ there must be
a probability distribution over its direct successors (which are the closest
nodes in the direction of the game). For the game to fulfill the attribute of
extensive form, it shall also hold that for each $i \in N$ a partition of $P^n$ into
$k^i$ information sets $U^i_1$ through $U^i_{k(i)}$ is such, that for each $j = 1, 2, ..., k(i)$:

1. every node in $U^i_j$ has the same number of direct successors and the
   sets of outgoing branches\(^3\) of different nodes in $U^i_j$ are corresponding
   one-to-one;

2. every directed path (meaning a sequence of nodes) in $T$ from the root
to a terminal node may go through $U^i_j$ at most once.

Every terminal node of the rooted tree is assigned an $n$-dimensional vector
$g(t)$ consisting of members $g^1(t)$ through $g^n(t)$, each of which represents a
payoff of the respective agent. The last necessary part of the formal definition
of a game in extensive form is the common knowledge among the players
about this system of the game (thus of the whole definition), which is often
referred to as complete information (Hart, 1992).\(^4\)

3.3 Examples

In this section of the thesis we shall present few examples of games to illu-
minate the meaning of the aforementioned. First, we shall briefly look at the
notoriously known example of Prisoners dilemma, which is a great example
of a normal form game. Two prisoners are facing a decision whether to con-
fess, or not. If none of the prisoners confesses, they will both be sentenced

\(^3\)A branch of node $x$ is a set of all successors to a direct successor of node $x$, including the direct
successor.

\(^4\)It is necessary to mention, that not all of the notation used in Hart, 1992 was preserved. For instance,
Hart distinguishes a player from an agent, as agents are in his definition making decisions on behalf of
players, however as in this thesis we are going to operate with “perfect recall” which refers to the player
remembering all of his previous choices made throughout the game, we may treat them as a common
entity, as the nature of the problem to be examined in this thesis does not allow for any other case.
to one year in prison. On the other hand, if both of them confess, they will be both sentenced to three years in prison and lastly if only one of them does confess and the other does not, the one who confessed will be freed and the other one will receive a sentence in the duration of four years. By examining this problem in a vacuum and assuming that the utility of the prisoners originates solemnly in the duration of the sentence, we can easily argue that each prisoner is incentivized to select the strategy “confess” over the strategy “not confess”, as it is superior - the prisoner will be better off regardless of which strategy does the other prisoner choose. Such a phenomenon is in game theory called “strongly dominated strategy”. From this simple thought, we can easily derive the conclusion, that the Nash equilibrium of this game will lie in the point with payoffs $(3; 3)$ and both prisoners will confess.

To illuminate simple bargaining problem, we may look at the pie division. Two players are bargaining on how will they split time spent in the bathroom, while the utility function of one player is $u_1 = t + \frac{1}{4}$, where $t$ is the proportion of the available he is going to receive and second player’s utility function is $u_2 = t - \frac{1}{8}$. The game is played under complete information, thus both players are aware of the structure of the game and the payoff. If we let $\varepsilon$ be the proportion of the pie which the first player receives, we can calculate the Nash equilibrium by maximizing the function

$$f(\varepsilon) = (\varepsilon \cdot t + \frac{1}{4}) \left( (1 - \varepsilon)t - \frac{1}{8} \right).$$

We may now find $\varepsilon^*$ by taking the first and second derivative and we simply obtain

$$\varepsilon^* = \frac{1}{2} - \frac{3}{16t},$$

which is the proportion of the time spent in bathroom attributed to the first player and $\frac{1}{2} + \frac{3}{16t}$ is the time attributed to the second player.

To show utilization of the backward induction, we may consider a simple “game” in the civil meaning in which two players are playing against each other under the traditional assumption of rationality on a playing field depicted in Figure 1. Player 1 begins at the root and chooses whether the game...
will continue to the right, or to the left. Then, at the next node, player 2 chooses right or left again and the final number is the outcome of the game. Now the goal of player 1 is to maximize the outcome number and the goal of player 2 is to minimize it. This “game” does translate well into the game in terms of game theory, as it is already represented in almost the same way as a zero-sum game in extensive form (Figure 1).

Figure 1: “Game”

We can consider the payoff function of the first player $g^1(t)$ equal to the outcome of the game $t$ and the payoff function of the second player $g^2(t) = -t$. Now starting at the bottom of the tree we can compare the outcomes with respect to the payoff of the second player and assign the value of the outcome he would choose to the node at which he is choosing - thus node $b$ becomes in the perspective of player 1 equivalent to a terminal node with outcome 1 and node $c$ equivalent to a terminal node with outcome 5. This way we have reduced the problem for the first player to only choose between those two values and as $g^1(t) = t$, we can easily conclude that the
game is going to be played as player 1 choosing right, player 2 choosing left and the game concluding at the terminal node  \( f \) providing payoffs \((5; -5)\).

Figure 2: Game in the extensive form

![Game Diagram]

This simple approach to solving particular games proposed already by von Neumann and Morgenstern at the very beginnings of game theory. What is more, Kuhn (1953) has proven utilizing backward induction that every finite game in extensive form does have a subgame perfect equilibrium. For this reason, our capability of solving such games is no longer reliant on us knowing how to solve such games, but rather on the computing power availability. To demonstrate this, we shall consider the Chinese game Go, which is a board game in which two players set stones on the playing field and attempt to capture the other player’s stones by completely surrounding them. While there are limited options for each player at any given stage of the game and the game itself is indeed finite in length, as the average branching factor (i.e. the average number of outgoing branches from certain node) in Go is 250 with average length of a game of 150 plies, the total size of the tree is approximately \(10^{360}\) (Allis, 1994). Thus, although this game is solvable, due to computing power restriction we are with current technological limit-
ations incapable of obtaining the solution (although of course game theory is still utilized in artificial intelligence algorithms combined with machine learning).

Finally, there are vast applications of different types of game theory across various fields including biology, economy (Rasmusen, 2006) or computer science (Roy, 2010).
4 Game-theoretic model of hostile takeover

Problem Setup

In this part of the thesis we shall address properties of the problem we are going to analyze, so that we may attempt to formulate a model based on game theory further in this thesis.

The problem may be perceived as utility maximization problem for the management at given parameters of the state of the world in the event of potential hostile attack on the target company under weak corporate governance. In the first stage of the problem, management has to decide on which preemptive defense strategies it wishes to implement and weight their potential profit out of every possibility available. Afterwards, the raider, who sees a potential added value from some synergies which would arise in case the target was overtaken by the raider, engages into evaluating whether he wishes to attempt a hostile takeover of the target company or to forfeit the opportunity and refrain from interfering with the target, in which case the game ends\(^5\). In the case he decides to proceed, the raider presents the shareholders with an tender offer in the amount of \(O\), which leads to triggering the Revlon rule, thus every offer which the management bargains at the end of the game must provide at least the same benefit to the shareholders\(^6\). Once the raider has decided on his further endeavors, the management

\(^5\)We shall assume that the synergies are only available in the case of the raider taking over the target company as a whole and would be completely lost in any other case, including the option of the target company taking over the company represented by the raider and utilization of the white knight defense strategy. This assumption may be justified either by inferior sources of financing available to the target, or by insufficient expertise of the management.

\(^6\)Although the Revlon rule is related to the monetary amount offered for the shares and not utility, as we are going to consider the utility of the shareholders to only consist of financial factors, we may treat them as if they were one.
has the choice on which reactive strategies are to be put in place and thus influence the expected future market value of the company, the position of the management in the target company (in terms of proportional decline of the added private benefit for the management) and potentially influence the price and terms at which the sale would take place, if at all. The raider then faces an option to abandon the takeover or to continue with the attempt and initiate bargaining with the management of target company. In case of sale of the company, the management is going to be let go. It is reasonable to assume, that such structure would be common knowledge among the entities taking part in this problem, as such setup does not alter the real scenario anticipated by the author of this thesis, apart from assigning the events into a firm timeline framework.\(^7\)

Although in the section regarding defense strategies in this thesis we have discussed several empirical observations of market value declines as effect of utilized strategies as well as several other parameters of defense strategies, we are going to apply them as variables, due to the results provided by the research may not be suitable for determining the values in specific cases.\(^8\)

In addition to assuming the agents are aware of the structure of the problem, we are also going to assume they will play the game specified further under perfect information. Despite the potential inapplicability of results from academic research as values of the variables, it is feasible to expect the companies either initiating or being targeted by a hostile takeover to have the resources to perform a thorough analysis unveiling these values preceding committing to formation of their respective strategy of choice. We

\(^7\)The assignment does not conflict with the real scenario in which the events may have various durations, as the order of them shall remain unchanged. The negotiation between the raider and a shareholder representative may be unrealistic, however was included to represent the Revlon rule, as in case the shareholders of the target company would get a better offer without any further interference of the management, they are entitled to it and any interference of the management would be considered breaking their fiduciary duties.

\(^8\)While not arguing with the aggregate results provided by the research reviewed in the section regarding defense strategies, we cannot be sure the values are generally representative, as the boards of the companies which were examined may have (and it is likely that they did) have analyzed the expected effects of utilization of a strategy and only implementing it in case the expectations were feasible, thus possibly creating a selection bias.
will also assume the raider has sufficient available financing at least at the value of his perceived premium.

The management of the target company is going to derive their utility from some premium of private benefit $B$ which they consume as a result of controlling the target company\(^9\) and value of the shares of the proportion of the target company they own, or in case of successful takeover from the proportion of the added value which the raider sees in the target company appertaining them combined with other monetary transfers resulting from utilization of some of the defense strategies. The raider derives his utility from the proportion of amount of the added value arising from potential synergies which is going to associate to him and potential transfers from the company to the raider in case of an unsuccessful takeover. Lastly, the shareholders (who are not part of the game, however we must take them into account during the calculations) of the target company derive their utility from the premium at which the shares are going to be purchased in the case of sale of the company and from the market value of the shares in the case of preservation of status quo.

**Defense strategies**

Utilization of a staggered board may decrease the market value of the target company by $\alpha_1$ percent and will also impose extra costs on the raider (e.g. through unrealized changes in the target company throughout the waiting period) in the amount of $a$. Poison pill is a very specific strategy which may completely prevent any attempt of the hostile takeover and for feasibility of the model we are going to treat it as a strategy which may only be implemented by itself and it is going to increase the amount of shares raider needs to purchase $\lambda$-times. It will however decrease the market value of the target company by $\alpha_2$ percent. Implementing golden parachutes will cause decline of the market value of target company by $\alpha_3$ and a monetary transfer

\(^9\)B might be understood as the difference between the private benefit the management receives at the target company and the highest private benefit it would receive in another company in which could employ it.
from the raider (although the amount would be paid by the target company as an entity, we can account the transfer to the raider as he is going to be the owner of the company) $g$ representing the amount at which are golden parachutes set. This amount would also be transferred from the shareholders of the target company to the managers in case the game would not result into a takeover, however the managers would be let go.

Dual class recapitalization provides increased bargaining power of the management, thus increasing the price at which the management would be willing to sell their shares to the raider by $\gamma^{10}$, however also decreases the market value of shares owned by the managers by $\theta$ in case the takeover would not take place. In case of implementation of targeted repurchase or a standstill agreement, the effect will only take place in case of the takeover resulting in no sale and will be represented as a transfer $t$ from the private benefit consumed by the management to the raider. Lastly, utilization crown jewel defense is reflected by $c$, representing the decrease in the premium at which the raider is willing to buy the company and importantly $\alpha_6$, representing the percentage decline in the market value of the company.

In addition, all the defense strategies increase the probability $\delta$ of the management being let go in case the game will resolve into not selling the target company. To simplify the notation let $I$ be a set of all defense strategies utilized in the respective directed path from the root to the terminal node of the extensive game model. We are going to treat the variables

$$\forall i \in I : \alpha_i > 0, a \geq 0, \delta_i \geq 0, \lambda \geq 0, g \geq 0, \gamma \geq 0, \Theta \geq 0, O \geq 0, t \geq 0$$

describing the impact of the defense strategies as exogenous and with the properties $X - a - g - c - \gamma \geq \tau$, where $\tau > 0$ and $O \leq \frac{X}{2}$. As aforementioned, these variables are known to the agents. The defense strategies and their impact are summarized in Table 1.

$^{10}\gamma$ expresses the lump sum of an extra premium available only to the management.
<table>
<thead>
<tr>
<th>Name</th>
<th>i</th>
<th>Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Staggered board</td>
<td>1</td>
<td>Decrease in the market value by $\alpha_2$ percent</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Extra costs for raider $a$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Increase in the probability of the management</td>
</tr>
<tr>
<td></td>
<td></td>
<td>being let go in case of an unsuccessful takeover by $\delta_1$</td>
</tr>
<tr>
<td>Poison pill</td>
<td>2</td>
<td>Decrease in the market value by $\alpha_2$ percent</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Increasing the total number of shares by $\lambda$-multiple</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Increase in the probability of the management</td>
</tr>
<tr>
<td></td>
<td></td>
<td>being let go in case of an unsuccessful takeover by $\delta_2$</td>
</tr>
<tr>
<td>Golden parachutes</td>
<td>3</td>
<td>Decrease in the market value by $\alpha_3$ percent</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Transfer from the raider or the shareholders of the target to the</td>
</tr>
<tr>
<td></td>
<td></td>
<td>management $g$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Increase in the probability of the management</td>
</tr>
<tr>
<td></td>
<td></td>
<td>being let go in case of an unsuccessful takeover by $\delta_3$</td>
</tr>
<tr>
<td>Dual class recapitalization</td>
<td>4</td>
<td>Increase in the price for raider for the portion of the target company</td>
</tr>
<tr>
<td></td>
<td></td>
<td>owned by management $\gamma$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Decrease in the market value of the proportion of the shares of the</td>
</tr>
<tr>
<td></td>
<td></td>
<td>target company owned by the management $\theta$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Increase in the probability of the management</td>
</tr>
<tr>
<td></td>
<td></td>
<td>being let go in case of an unsuccessful takeover by $\delta_4$</td>
</tr>
<tr>
<td>Targeted repurchase &amp;</td>
<td>5</td>
<td>Transfer from the private benefit of the management to the raider $t$</td>
</tr>
<tr>
<td>standstill agreement</td>
<td></td>
<td>Increase in the probability of the management</td>
</tr>
<tr>
<td></td>
<td></td>
<td>being let go in case of an unsuccessful takeover by $\delta_5$</td>
</tr>
<tr>
<td>Crown jewel</td>
<td>6</td>
<td>Decrease in the market value by $\alpha_3$ percent</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Decrease in the premium perceived by the raider by $c$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Increase in the probability of the management</td>
</tr>
<tr>
<td></td>
<td></td>
<td>being let go in case of an unsuccessful takeover by $\delta_6$</td>
</tr>
</tbody>
</table>

**Utility functions**

With the information about the structure about the problem we shall now continue by assigning the entities in this problem utility functions. Firstly,
the raider whose utility beyond the scope of the takeover we may normalize to 0, as it will not have any effect on the problem, is interested in maximizing the remainder of the premium created by synergies. Let $\varepsilon$ be the proportion of the remainder of the premium assigned to the shareholders of the target company and $\beta_M > 0$ be the proportion of the target company owned by the management. Utility function of the raider thus may be written as follows:

$$u_R = \begin{cases} (1 - \varepsilon)(X - a - c - g - \gamma), & \text{takeover} \\ t, & \text{else} \end{cases}$$

The target shareholders’ utility is affected by the proportion of the remainder of the premium they receive in the case of a successful takeover taking place and in the case of the sale not happening, they will be affected by the drop in the market value of the target company and the potential of letting the management go in case the company has golden parachutes provision included. Let $M$ be the market value of the target company before the game. Then utility function of the shareholders may be expressed as

$$u_S = \begin{cases} \varepsilon(1 - \beta_M)(X - a - c - g - \gamma), & \text{takeover} \\ -\left(\sum_{i \in I} \delta_i\right)g + (1 - \beta_M)\left(\prod_{i \in I} 1 - \alpha_i\right)M, & \text{else} \end{cases}$$

Finally, the utility of the management is in the case of the takeover determined by the amount of the premium which is going to be assigned to them, any potential further increase in price for shares held by the management achieved by dual class recapitalization and consumption of the potential golden parachutes. In the case of sale of the target company on the other hand, the management gains utility from the market value of the portion of the shares it owns, the private benefit weighted by the probability of being let go because of the utilized defense strategies and decreased by the transfers made due to utilization of targeted repurchase & standstill agreement. Thus, the utility function of the management holds as follows:

$$u_M =$$
\[
\begin{cases}
\varepsilon \beta_M (X - a - c - g - \gamma) + g + \gamma, & \text{takeover} \\
\left(1 - \sum_{i \in I} \delta_i\right) (B - t) + \left(\sum_{i \in I} \delta_i\right) g + \theta \beta_M \left(\prod_{i \in I} (1 - \alpha_i)\right) M, & \text{else}
\end{cases}
\]

4.1 Formulation of the model

In this section of the thesis we are going to attempt to formulate a representative model for the problem setup in previous section. We already have a set of two agents, the management $M$ and the raider $R$. Our stage-by-stage setup from the previous section does form a rooted tree, there are no cycles present in the tree and all the non-terminal nodes (as we are going to evaluate the bargaining problems first and then treat their solutions as payoffs to the model of the game in extensive form) are divided among the agents. Due to the assumption of perfect information the complete description of the game is indeed common knowledge, however for the game in extensive form (Figure 3) we are about to analyze we need to compute the payoffs.

Figure 3: Extensive form representation of the game

Node stage 1-1 is the root of the tree. Please note that Figure 3 is not a complete graphical representation of the game in extensive form, but merely
a part of it. All the branches which are not displayed in Figure 3 (indicated by the dashed line) are however in the same form as the outgoing branch of node stage 3-3 going through node stage 4-6. All nodes across the stages marked with an odd number are nodes which represent the outcome in which the target company is not sold and all nodes across the stages marked with an even number represent the outcome in which the takeover is successful. It is necessary to mention that for some amounts of the tender offer the game tree is going to be reduced (e.g. if no preemptive strategies were used and the tender offer is $O = \frac{X}{2}$, utilization of any defense strategy would be a breach of fiduciary duties of the management, as it would necessarily result into reduction of the price the shareholders are eligible to according to the Revlon rule).

**Bargaining**

We are going to find a general solution of the bargaining problem by including all the possible variables which may occur with respect to the defense strategies chosen and while describing the game in extensive form treat all the variables which are attributed to the defense strategies which were not used as 0.

To identify the equilibrium of the bargain, we are first going to find $\epsilon^*$ representing the proportion of $X - a - c - g - \gamma$ attributed to the shareholders of the target company including the management in case the takeover takes place and then compare the utility such a sale provides to the agents with their alternatives in case no agreement was reached. We are going to search for such $\epsilon$, which maximizes the product of gains in utility of the agents at the stage of the game at which the raider is making the decision whether to bargain or not.

\[
(1-\epsilon)(X-a-c-g-\gamma) - t \left[ \epsilon \beta_M (X-a-c-g-\gamma) + g \left( 1 - \sum_{i \in I} \delta_i \right) + \gamma + t \right]
- \left(1 - \sum_{i \in I} \delta_i \right) B + (1+\theta) \beta_M \left(1 - \prod_{i \in I} (1 - \alpha_i) \right) M
\]
By taking the derivative with respect to $\varepsilon$ we obtain the expression

$$
\beta_M(X - a - c - g - \gamma)^2 - 2\varepsilon\beta_M(X - a - c - g - \gamma)^2 - \\
-\beta_M(X - a - c - g - \gamma)t - (X - a - c - g - \gamma)\ast

\left[ g\left(1 - \sum_{i \in I} \delta_i\right) + \gamma + t - \left(1 - \sum_{i \in I} \delta_i\right)B + (1 - \theta)\beta_M\left(1 - \prod_{i \in I} 1 - \alpha_i\right)M \right]
$$

and as $(X - a - c - g - \gamma) \geq \tau > 0$ and $\beta_M > 0$ we can simply observe that by setting the expression equal to zero, we obtain

$$
\varepsilon^* = \frac{1}{2} - \frac{t}{2(X - a - c - g - \gamma)} - \\
g(1 - \sum_{i \in I} \delta_i) + \gamma + t - (1 - \sum_{i \in I} \delta_i)B + (1 - \theta)\beta_M(1 - \prod_{i \in I} 1 - \alpha_i)M \\
2\beta_M(X - a - c - g - \gamma)
$$

At this point we must find the payoffs arising from such $\varepsilon^*$. We can simply substitute $\varepsilon$ in the gains utility for $\varepsilon^*$.

$$
\pi_R = \frac{1}{2} \left[ X - a - c + g\left(\frac{1 - \sum_{i \in I} \delta_i - \beta_M}{\beta_M}\right) + \gamma\left(\frac{1 - \beta_M}{\beta_M}\right) + t\left(\frac{1 - \beta_M}{\beta_M}\right) \right]

+ \frac{1 - \theta)\beta_M(1 - \prod_{i \in I} 1 - \alpha_i)M - (1 - \sum_{i \in I} \delta_i)B}{\beta_M}
$$

$$
\pi_R = \frac{\beta_M}{2} \left[ X - a - c + g\left(\frac{1 - \sum_{i \in I} \delta_i - \beta_M}{\beta_M}\right) + \gamma\left(\frac{1 - \beta_M}{\beta_M}\right) + t\left(\frac{1 - \beta_M}{\beta_M}\right) \right]

+ \frac{(1 - \theta)\beta_M(1 - \prod_{i \in I} 1 - \alpha_i)M - (1 - \sum_{i \in I} \delta_i)B}{\beta_M}
$$

However, the payoffs will take this form only in the case both parties are better off than if no agreement was reached, thus we must now decide whether both of the potential payoffs are positive, as in every other case, no agreement is going to be reached. We may describe the equilibria of this bargaining game as
and thus we may continue with the formulation of the model of the game in extensive form.

Adjusting the model

Before assigning the payoffs to the terminal nodes, we shall first reduce the options of the management to only available defense strategies. To evaluate whether the option to choose a given defense strategy is going to be available to the management, we must compute the utility of the shareholders and compare it with the original tender offer.\[11\] We may do this simply by substituting \( \varepsilon^* \) for \( \varepsilon \) in the utility function.

\[
\hat{a}_z = \begin{cases} 
\frac{1-\beta_M}{2} \left( X - a - c - g - \gamma - t \cdot \frac{(1-\sum_{i \in I} \delta_i)\gamma + e - (1-\sum_{i \in I} \delta_i)\gamma + g(1-\prod_{i \in I} 1-\alpha_i)^M}{\beta_M} \right), & \pi_R > 0 \land \pi_M > 0 \\
(1-\beta_M)\prod_{i \in I} 1-\alpha_i)M - (\sum_{i \in I} \delta_i)g, & \text{else}
\end{cases}
\]

Now it is obvious, that the management is not allowed to choose any strategy which would conclude into preventing the takeover after the raider has proposed his initial tender offer if \( \forall i \in I : \alpha_i > 0 \), the expression

\[
(1-\beta_M)\left( \prod_{i \in I} 1-\alpha_i \right)M - (\sum_{i \in I} \delta_i)g < (1-\beta_M)M
\]

\[11\]We are again performing this step to ensure the legal aspect does not breach the logical connection of the model to a real-world situation. We are essentially making a tradeoff, as on one hand we are improving accuracy of the model in the sense of the management not possessing godlike powers, on the other hand however, we are also, in extreme cases, reducing the ability of the management to influence the outcome enough.
certainly holds. Thus firstly, to decide which outgoing branches may follow from each of the nodes in Stage 3 (representing the choices on sets of defensive strategies the management can utilize), we must find only such branches, for which the subgame perfect equilibrium payoffs fulfill the condition $u^*_s \geq 0$. In the special case of no strategies being implemented, instead of removing the branch which lead to the bargaining, we will swap the payoff for one representing the original tender offer. In the next step, we must eliminate all branches originating in the root of the tree and potentially leading to a node in Stage 3 which has no outgoing branches. To clarify, we are not solving the game, as we are rather adjusting the model to reflect the possibilities and available choices determined the values of the exogenous variables.

Payoffs

If we now keep the naming of the nodes depicted in Figure 3, resetting to 1 at the beginning at each stage, we may attribute payoffs to all the terminal nodes which we did not eliminate while adjusting the model. We may obtain the payoffs in terms of utility for even terminal nodes branching from nodes in Stage 4 by substituting the $\varepsilon^*$ we found while solving the bargaining problem for $\varepsilon$ in the utility functions. As these nodes represent a successful takeover, we might calculate the utility not as the whole utility function, but rather only as the part regarding a successful takeover. If we let $j \in N$ be the number of the terminal node in stage 4, we obtain

$$\bar{r}_{2j}^R = \frac{1}{2} \left[ \frac{X - a - c - g - \gamma + t \left( \frac{1 + \beta_M}{\beta_M} \right) + g(1 - \sum_{i \in I} \delta_i) + \gamma + t - (1 - \sum_{i \in I} \delta_i)B + (1 + \theta)\beta_M(1 - \Pi_{i \in I} 1 - \alpha_i)M}{\beta_M} \right]$$

and
\[
\hat{\Pi}_M^{2j} = \frac{1}{2} \left[ \beta_M (X - a - c - g - \gamma) - t(1 + \beta_M) + g \left( 1 + \sum_{i \in I} \delta_i \right) + \gamma + \left( 1 - \sum_{i \in I} \delta_i \right) B \right. \\
+ \left. (1 - \theta) \beta_M \left( 1 - \prod_{i \in I} 1 - \alpha_i \right) M \right]
\]

As for the odd terminal nodes branching from nodes in stage 4, we may simply use the part of the respective utility regarding the situation in which hostile takeover does not happen.

\[
\hat{\Pi}_M^{2j+1} = \left( 1 - \sum_{i \in I} \delta_i \right) (B - t) + \left( \sum_{i \in I} \delta_i \right) g + \theta \beta_M \left( \prod_{i \in I} 1 - \alpha_i \right) M
\]

and \(\hat{\Pi}_R^{2j+1} = t\). Also we add for the very first terminal node in stage 4 the payoffs \(\hat{\Pi}_M^1 = \frac{\beta_M X}{2}\) and \(\hat{\Pi}_R^1 = \frac{X}{2}\).

Next of the model remaining undescribed are nodes stage 3-9; 10. These nodes are the result of utilization of the poison pill defense strategy. We are going to use the insight provided by the research on this topic and utilize \(\lambda\) as a multiplier of the number of outstanding shares. As this strategy in general serves as an ultimate protection against hostile takeovers, in our model we have set it up in such a way, it may be only used as a singleton. We can therefore simply derive the utilities arising from the use of this strategy by taking \(\lambda\) times \(\alpha_2 M\) as the price for which the shares are going to be purchased. Thus, by taking \(s\) as a sale and \(n\) as no-sale for the indices we obtain

\[
\hat{\Pi}_M^S = \beta_M \alpha_2 M \lambda \\
\hat{\Pi}_R^S = X - \alpha_2 M \lambda \\
\hat{\Pi}_M^N = (1 - \delta_2) B - \beta_M \alpha_2 M \\
\hat{\Pi}_R^N = 0
\]

### 4.2 Results

We do not attempt to provide a fully general solution to the model, as it has already been proven that such solution exists and can be found utilizing the
backward induction method. Identification of the equilibria in the general form is overly complex in terms of “for which interval of which parameter are we going to get which result”. Unfortunately, either is the premise we are working with, that the companies do analyze potential effects of application of hostile takeovers wrong, or the data is not publicly available. It would be rather curious, if no such analysis was taking place, thus it is more likely that in the interest of protection of company secrets such documents are kept private.

This model uncovers the relationship between defense strategies against hostile takeovers and the effects it has on the shareholders’ wealth, as well as illuminates the different interests of the shareholders and the management. If the model resembles the reality at least to some extent, dual class recapitalization seems to be a very dangerous tool in the hands of a management in a company with weak corporate governance. Although it has not proven to be malicious to its full potential in this model, if we categorized this strategy differently - as a preemptive strategy - the ability of the management to divert the premium at which the company could be purchased is vast. If used strategically, it could even be acceptable in the legal merits, as tender offer would not yet be presented.

The structure of the payoffs also confirms a very interesting phenomenon, as the shareholders do need the management’s representation in negotiations to increase the proportion of the premium over the market value they are going to receive, however simultaneously it is in their best interest to limit the powers of the management to the least extent possible, as only strong corporate governance may stop the management from deciding on the distribution of the premium under their own discretion, provided the information gap between the management and the shareholders.
5 Conclusion

In this thesis we have discussed hostile takeovers and mainly defense strategies against them. We have also covered games in extensive form and bargaining problem. We attempted to formulate a game-theoretic model which describes the process of a hostile takeover. We did succeed in utilizing bargaining equilibria as payoffs in the model of a hostile takeover and thus uncovered some of the impacts defense strategies have on the utility of the shareholders of the target company, the raider and the management. Although this thesis failed to provide clear answers to the questions posed, it did provide some insight on the difference of interests between the shareholders and their managers along the way. The introduction of the Revlon rule to the model also brought some insight on the way hostile takeovers are being resolved, as legal boundaries do almost force the parties to engage into negotiations and thus break the nature of true hostility in the takeover. This thesis, however, did by no means fully uncover the process of hostile takeover from game theoretic perspective, as many of the flaws which presented themselves during the formulation of the model could be fixed by implementing elements from behavioral game theory and access to more relevant data. Finally, this thesis made an effort to demonstrate the interaction among the raider’s, shareholders’ and management’s wealth through the payoff functions of the extensive form model of a hostile takeover.
Bibliography


