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The cost of carry model in stock index futures: theory and reality

Bachelor thesis

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Abstract

The thesis investigates the pricing efficiency of the commonly used cost of carry model in pricing stock index futures and its applicability on the German blue-chip index DAX and related futures contracts in recent years. The work considers the deviations of the observed futures prices from their theoretical counterparts as well as the fitness of the model through regression analysis. The results show that while there are many deviations from the fair values suggested by the model these are small in magnitude when compared with the potential transaction costs implying the contracts are efficiently priced. It is confirmed that there is a cointegrating relationship between futures and spot index values, however, given the regression analysis results the prices do not entirely follow the model design. The other part of the analysis focuses on the behaviour of the basis throughout the life of the relevant futures contracts. The results suggest that there is indeed a decreasing tendency towards the expiration of a contract, nevertheless, it is subject to considerable fluctuations. The paper also documents other factors that might impact stock index futures prices yet not included in the standard pricing formula.

JEL Classification	C12, C14, C22, G13
Keywords	stock index futures, futures pricing , cost of carry, basis, basis convergence, DAX index
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Abstrakt

Práce se zabývá efektivitou běžně užívaného modelu cost of carry na oceňování termínových kontraktů a možností jej aplikovat v rámci německého blue-chip indexu DAX a souvisejících termínových kontraktů během posledních let. Práce zvažuje jak odchylky pozorovaných cen termínových kontraktů od jejich teoretických hodnot, tak vhodnost modelu pomocí regresní analýzy. Výsledky ukazují, že i přes mnoho odchylek od správných hodnot navržených modelem jsou tyto deviace malé ve srovnání s potenciálními transakčními náklady, což naznačuje, že kontrakty jsou efektivně oceněné. Kointegrační vztah mezi termínovými a spotovými cenami indexu je potvrzen, nicméně podle výsledků regresní analýzy ceny zcela neodpovídají konceptu modelu cost of carry. Další část analýzy se soustřeďuje na chování báze během období příslušných termínových kontraktů. Výsledky naznačují, že báze opravdu vykazuje klesající tendenci směrem k expiraci kontraktu, nicméně podléhá značným výkyvům. Práce také představuje další faktory, které mohou ovlivnit ceny indexových termínových kontraktů a které však nejsou zohledněny ve standardním oceňovacím vzorci.

JEL klasifikace	C12, C14, C22, G13
Klíčová slova	indexové termínové kontrakty, oceňování termínových kontraktů, cost of carry, báze, princip konvergence, DAX index
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Declaration of Authorship

I hereby proclaim that I wrote my bachelor thesis on my own under the leadership of my supervisor and that the references include all resources and literature I have used.

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Prague, 3 July 2017

Signature

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Bachelor Thesis Proposal

Author	Marika Němcová
Supervisor	prof. Ing. Oldřich Dědek, CSc.
Proposed topic	The cost of carry model in stock index futures: theory and reality

Topic characteristics

The cost of carry model is the standard model for pricing futures contracts. It defines the relationship between futures and spot prices implying the price changes are perfectly correlated and the difference between these two values is assumed to be decreasing as the contract approaches its expiration. This phenomenon is known as the basis convergence. The classic cost of carry model is, however, limited by considerably strict assumptions of perfect markets, which actually confronts the reality.

In my thesis, I would like to focus on the field of stock index futures and compare the model suggestions with the actuality. An area of particular interest is the futures contracts based on the German stock index DAX. Firstly, based on daily closing prices, I will examine the deviation of the actual futures contract prices from the fair values suggested by the model. Based on the results I would like to provide discussion on other models that loosen the restrictions of the cost of carry and describe other factors that could have significant influence on the futures price. These could provide better theoretical fair values leading to a more efficient reflection of the underlying market. Secondly, I will look into how well the basis convergence at maturity is achieved.

There are many studies detailing a comparison between the cost of carry model and various other pricing models, including Hsu and Wang model. Furthermore there is literature on DAX index futures pricing, mostly regarding taxes, such as Fink and Theissen (2014). The purpose of the thesis is to compare the theoretical model with the real behaviour of the futures prices and to summarize some other factors that might be relevant in stock index futures pricing.

Hypotheses

1. The standard cost of carry model assuming perfect markets is no longer applicable.
2. There are other significant factors contributing to stock index futures pricing.
3. Futures prices converge with underlying index values when approaching the contract maturity and the exchange is involved in completing this target.

Methodology

Firstly, the magnitude of the relative mispricing in percentage of current spot index price is measured. In order to see how well the cost of carry model explains the price behaviour the fitness of the model is tested by regressions analysis using the actual futures prices. For this analysis it is needed to employ unit root and stationarity tests to confirm the cointegration relationship between spot and futures prices. Regarding basis convergence a trend analysis is run to obtain the effect of time to maturity of a futures contract.

Outline

1. Introduction
2. Literature review
3. Theoretical background
4. Methodology
5. Empirical results
6. Other important factors involved in stock index futures pricing
7. Conclusion

Core bibliography

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Acronyms

AC	autocorrelation
ADF	Augmented Dickey-Fuller
AIC	Akaike information criterion
AR	autoregressive
ATHEX	Athens Stock Exchange
BIC	Bayesian information criterion
CET	Central European Time
CNX	Credit Rating Information Services of India Limited and the National Stock Exchange of India
DAX	Deutscher Aktienindex (German stock index)
DF	Dickey-Fuller test
EMMI	European Money Markets Institute
EONIA	Euro OverNight Index Average
EURIBOR	Euro Interbank Offered Rate
FTSE	Financial Times and Stock Exchange
FWB	Frankfurter Wertpapierbörse (Frankfurt Stock Exchange)
HAC	heteroscedasticity and autocorrelation consistent
HKEx	Hong Kong Exchanges and Clearing Limited
IBEX	Spanish Exchange Index
IDE	integrated development environment
KOSPI	Korea Composite Stock Price Index
KPSS	Kwiatkowski-Phillips-Schmidt-Shin
LM	Lagrange Multiplier
MMI	Major Market Index
Nifty	National Stock Exchange Fifty (India)
Nikkei	Nihon Keizai Shimbun
NW	Newey-West
NYSE	New York Stock Exchange
OIS	overnight index swap

OLS	ordinary least squares
S&P	Standard and Poor's
SD	standard deviation
SET	Stock Exchange of Thailand
SGX	Singapore Exchange
UK	United Kingdom
US	United States
WIG	Warsaw Stock Exchange Index

1 Introduction

The rapid expansion of financial futures trading resulted in the introduction of stock index futures. In 1982 the Chicago Mercantile Exchange was the first to release trading in index futures and these types of futures contracts soon expanded outside the USA to the European area. Since then stock index futures have been an important and growing part of the financial markets and they gained popularity for several reasons. One of the primary factors was risk management as market participants obviously wished to reduce their risk at low cost. By providing liquidity and lower transaction costs potential risks in the cash market could be offset by index futures positions.

The effectiveness of a futures contract depends on its ability to reflect the underlying index and therefore the spot-futures relationship and the pricing performance of respective markets have been of interest to many financial analysts and researchers. Futures contracts can be priced on the basis of arbitrage meaning we can derive the price or the range of prices at which investors will not be able to establish positions with both the futures contract and the underlying concurrently in order to make riskless profit with no initial investment. The cost of carry model, first introduced by Cornell & French (1983), is considered as the standard pricing model of stock index futures and will be the centre of this study. The model is, however, based on rather unrealistic assumptions demanding perfect capital markets with no transaction costs, taxes and constraints on short sales, equal lending and borrowing rates or a certain risk-free rate. Over the years many academics have attempted to model the futures price under various assumptions and economic conditions trying to incorporate other factors that might significantly influence the index futures price and explain the mispricing reported in some index futures contracts.

This study will focus on the German blue-chip index DAX and its futures contracts covering the period 2012-2016. DAX is composed of the 30 largest German companies and belongs to the most popular worldwide underlyings for index derivatives as indicated by their high trading volumes. The core area of the thesis is the cost of carry model. To investigate its appropriateness in terms of pricing efficiency, first, the magnitude of the mispricing is measured. In order to see how well the cost of carry model explains the price behavior the fitness of the model is tested by regressions analysis using the actual futures prices. In anticipation of the poor model performance the thesis provides a summary of other important factors and models which might provide further explanation with regards to futures pricing.

Further part of the investigation is dedicated to the relationship between the spot and futures prices and the basis convergence suggested by the cost of carry model. The study looks at the suggested pattern of the model and the actual behavior of the prices over the life of a contract.

The rest of the thesis is organised as follows. The second chapter summarizes the literature dealing with the cost of carry model in pricing stock index futures and related testing of its suitability. It also presents some authors that proposed modifications of the standard model that have shown to be relevant. The next chapter introduces the theoretical background of stock index futures pricing. The cost of carry model, the related pricing formula and the basis convergence as well as the concept of normal backwardation and contango are explained. The chapter four provides some technical information regarding the DAX index and its futures contracts. The fifth chapter is dedicated to the methodology used in evaluating the relative mispricing of futures contracts and the fitness of the standard model. After the description of the dataset in section six, the following chapter discusses the empirical results. Afterwards chapter eight focuses on other factors that proved to be relevant in index futures pricing but that are not included in the standard pricing formula. The last chapter concludes the thesis and the appendix covers the tables and figures that are not included in the text.

2 Literature review

The pricing of stock index futures has been a point of interest for many academics and practitioners. There are numerous studies evaluating the pricing efficiency of the cost of carry model and comparing its performance with other modified models incorporating various factors that might have an influence on stock index futures price.

Cornell & French (1983) developed the cost of carry formula for pricing stock index futures under the assumptions of perfect markets and no-arbitrage argument. Later, Bühler & Kempf (1995) tailored the formula, which was primarily aimed at price indices, for performance or total return indices.

The cost of carry model has been percieved as the standard model for pricing stock index futures, however, plenty of studies report the mispricing of contracts when observed prices are compared to the theoretical fair values suggested by this model. Bühler & Kempf (1995) studied the price relation between the DAX index and its futures and they detected undervaluated contracts. Moreover, they found that the absolute value of the undervaluation increases with time to maturity for all contracts under the study. Investigating arbitrage opportunities in German markets they indicated a large number of arbitrage signals which disappear quickly for contracts with shorter time to maturity suggesting arbitrageurs exploit these signals rapidly but only in contracts with nearest delivery.

Similarly to German markets, Fassas (2010) reports deviations from the fair price of FTSE/ATHEX-20 index futures contracts suggesting profitable arbitrage opportunities exist in Greek markets. His findings also show that dividends, volatility, liquidity and short-selling restrictions influence the extent of mispricing. Many other studies found an undervaluation of the stock index futures contracts. MacKinlay & Ramaswamy (1988), Bhatt & Cakici (1990) investigated the US markets, Yadav & Pope (1990) examined the pricing efficiency of the UK markets, Lim (1992) confirmed the undervaluation of the contracts for Japanese markets.

Contrarily some research supports the standard pricing model.

Butterworth & Holmes (2000) examined the pricing of the FTSE 100 and FTSE mid 250 index futures contracts traded in the UK and found that the deviations from the fair prices are rather small in magnitude. With the estimation of the round-trip transaction costs based on Yadav & Pope (1990) the mispricing range was within arbitrage limits and contracts were efficiently priced. Furthermore, it was found that the futures trading in the FTSE mid 250 contract, which was designed to support the FTSE 100 with tracking of medium and smaller capitalization stocks in the UK, is associated with an improvement in the pricing performance of the FTSE 100 contract as indicated by a reduction in the occurence of arbitrage opportunities in the respective market.

Tharavanij (2012) tested the cost of carry model in relation to the SET50 futures traded on Thailand Futures Exchange and found that the model explains the SET50 futures very well. The paper also investigated the Granger causality between futures and spot prices and discovered that only spot prices lead futures values. Among other papers that report no significant deviations of futures prices from their theoretical fair values belong Cornell (1985) with the US market investigation and Bailey (1989) that examined Japanese markets.

Given the mixed results many researchers tried to explain the occurence of mispricing and suggest other models that might prove to be more efficient than the standard pricing model. Furthermore, there is copious literature stating that real capital markets are imperfect and the arbitrage mechanism does not complete which contradicts the cost of carry model design. Brenner et al. (1990), Gay & Jung (1999), Twite (1998) and Andreou & Pierides (2008) support the argument that real capital markets are imperfect.

Several authors focused on the taxation of dividends and tried to incorporate the tax effect in the valuation formula. Kempf & Spengel (1993) argued that investors' marginal tax rates have an influence on the fair stock index futures price and therefore add the correct dividend payment to the pricing equation. Considering additional costs to arbitrage, Janssen & Rudolph

(1995) modelled, apart from the tax treatment of dividends, transaction costs and interest rate taxation. Other refinements of the theoretical valuation formula in different tax systems can be found in Bamberg & Dorfleitner (2002) and Weber (2005).

The most recent study of the tax effect on stock index futures prices was conducted by Fink & Theissen (2014). They argue that the different taxation of dividends in the spot and futures market in Germany causes the violation of the cost of carry model. After analysing three significantly different historical tax regimes in Germany during the period 1990-2011 they validated that the mispricing stems from the dividend taxation and derived a DAX future valuation formula with the relevant tax effect. The severest mispricing was found in the *Vollanrechnungsverfahren* period, an imputation tax system that was in force until the end of 2000, where the marginal investor encounters the biggest gap between the index-assumed dividend payments and the dividend payments after tax. Fink & Theissen (2014) state that over the last 20 years the daily mispricing and arbitrage opportunities in the DAX futures contract were reduced as a result of the systematic change in the taxation of dividends. They concluded that more generic taxation rules lead to the easier and more accurate pricing of the DAX futures contracts.

Investigating the NYSE stock index futures, Hemler & Longstaff (1991) found that market volatility has an impact on stock index futures price and developed a pricing model with stochastic interest rate and market volatility. The influence of stock market volatility was confirmed in other papers too. Fung & Draper (1999) examined the Hong Kong Hang Seng Index futures and found the relation between the size of mispricing and market volatility, the results of Andreou & Pierides (2008) suggest the mispricing is caused by transaction costs, volatility and time to maturity. Among the most recent studies Wang (2011) tested the pricing of the SGX FTSE Xinhua China A50 and HKEx H-share Index futures and reported that the high price volatility of the two underlying Chinese stock markets undermines the efficiency of the standard cost of carry model in favor of the Hemler & Longstaff (1991) model. Manu & Narayana (2015) confirms that the Hemler & Longstaff (1991) model evinces the better pricing performance when compared with the cost of carry model for CNX Nifty futures and Bank Nifty futures of National Stock Exchange, India. On the other hand, CNX IT futures were found to be better priced by the standard pricing model.

Short selling constraints represent another variable that was tested for having explanatory power in the context of stock index futures pricing. The study of Fung & Draper (1999) on the Hong Kong Hang Seng Index futures contracts was conducted over the three distinct short selling regulatory regimes in Hong Kong. They concluded that the relaxing of the short sale constaints reduced the mispricing of stock index futures contracts and accelerated market adjustment. Similar findings are reported in Gay & Jung (1999) who found that the underpricing of Korean stock index futures was affected by the transaction costs and restrictions on short sales. Fassas (2010) confirms the effect of short sellling regulation as mentioned previously.

Transaction costs are another factor that is not present in the simple cost of carry formula and that leads to the formation of a no-arbitrage band rather than a single equation as expanded upon later. Aragó et al. (2003) examined daily closing prices of the IBEX 35 futures contracts during the period 1996-1997 covering a dramatic reduction in the transaction costs of trading IBEX 35 stock index futures in January 1997. They found that the correlation between the spot and futures prices increased.

Many researchers measure the market imperfections individually and when deriving a modified pricing model they seldom incorporate multiple market imperfections together. Nontheless, Hsu & Wang (2004) developed an imperfect market pricing model which is based on arguments of an incomplete arbitrage mechanism and real capital markets not being perfect. Instead of separating the individual effects, the model incorporates the factor of price expectation (expected growth rate) which reflects the effects of all market imperfections between the stock index futures market and its underlying spot market when implementing arbitrage activities. Empirically, the fact that the degree of market flaws influence the futures price and may impede the implementation of arbitrage mechamism in immature markets with higher degree of market defects is confirmed in several papers. While Brooks et al. (1999) support the cost of carry model pricing applicability in the well-established FTSE 100 and S&P 500 index futures markets with low extent of market imperfections, Gay & Jung (1999) and their study on Korean stock index futures market show the effect of market defects in immature markets with a relatively higher degree of imperfections. Wang (2007) compares the pricing performance of the cost of carry model and the imperfect market model for four Asian stock index futures markets covering the period 1997-2005. The paper examines both mature and imature markets and captures the Asian financial crisis in 1997-1998. The results confirm that market imperfections play an important role in determining the stock index futures prices for immature markets and turbulent periods. The cost of carry model proved to provide a more accurate pricing for a mature market of Nikkei 225, immature markets such as the SGX and KOSPI 200 futures markets were better priced by the imperfect market model. Moreover, the futures contracts during the Asian crisis period were better described by, again, the imperfect market model.

The relatively recent study Manu (2015) examines the pricing efficiency of the Hemler and Longstaff model and the Hsu and Wang model when applied on three futures indices of National Stock Exchange, India. The results show that the Hsu and Wang model outperforms the other model for all three futures markets and additionally suggest that average daily trading volume might influence the pricing error in Indian markets. Based on the empirical results and the fact that the standard cost of carry model does not include the price expectation parameter, practitioners should identify the degree of market imperfections for the markets in which they participate first before selecting the correct pricing model.

With regard to basis convergence, the literature focuses primarily on commodity markets that assume delivery of a commodity rather than on cash-settled futures contracts since the convergence of futures and spot prices of an index is acomplished automatimally on derivatives exchanges. Nontheless, there are some papers investigating stock index futures markets. Beaulieu (1998) investigated the S&P 500 and the MMI. Using daily data of the period 1985-1991 it was found that the variance of the basis decreased as maturity approached. Low et al. (2002) found similar results for the variance of the log-basis when examining the Nikkei 225 data of the period 1986-1996. Another studies are focusing on the factors that influence the basis, since the predictibility of the basis is of special importance for hedgers. While investigating the S&P 500 index futures, Chen et al. (1995) suggested that the basis decreases as the volatility of the stock market rises. Roll et al. (2007) found the mutual relationship between the basis and the liquidity of the stock market. Other factors influencing the basis include stochastic risk-free rates or turnover of the underlying asset on the market as suggested by Wu et al. (2011). Marcinkiewicz (2013) examined the Warsaw Stock Exchange and its WIG20 index futures. The study did not detect any impact of these factors excepting the positive relationship between the basis and the volatility which contradicts the findigns of previous studies. Surprisingly it was found that the the time to maturity had very little effect on the size of the basis with both positive and negative signs. The values of the basis strongly depended on the past observations which the author explained by the predominance of speculators in the Polish stock index futures market.

3 Theoretical background

3.1 The cost of carry model

The cost of carry model determines the fair, arbitrage-free price of a futures contract and defines the relationship between the spot and futures market price as per Hull (2006). It states that the futures price is determined by the relative costs of buying an asset with deferred delivery in the futures market versus buying the asset in the spot market with immediate delivery and carrying the asset in the inventory. When the stock index is considered we can either buy the stocks involved in the index immediately or enter a stock index futures contract with deferred settlement. The former would provide us with dividend payments, while with the latter we would save the proceeds needed for immediate purchase of stocks that could otherwise be invested at a money market interest rate. Therefore we can define the net carrying cost advantage of deferring the delivery of stocks as $r_f - d$, i.e. the difference between the risk-free rate and the dividend yield per period. This advantage must be offset by a differential between the futures price and the spot price. This is achieved when the following holds:

$$F_0 = S_0(1 + r_f - d), \tag{3.1}$$

where F_0 is the futures price, S_0 is the spot price, r_f is the risk-free rate and d represents the dividend yield from the stock. This is the so-called spot-futures parity theorem or the cost of carry relationship which states that the theoretical fair price of an index futures contract should be equal to the spot index price adjusted for the cost of carrying the index over the remaining life of the contract. The price difference between the two markets should then be equal to the cost of carry of the underlying.

Any deviation from this parity would give rise to a risk-free arbitrage opportunity. If the futures price were higher than the implied fair price, traders could short a futures contract and buy the stocks underlying the index at the spot price in an attempt to capitalize on that mispricing. This way traders would bid up prices in the spot market and bid down the futures price until the parity is satisfied. If the futures price were to be lower than the theoretical price the reverse could be done, i.e. shorting the stocks underlying the index and taking the long position in the futures contract. Therefore according to theory arbitrageurs should ensure that any deviation of the price of futures transaction from their fair values is soon eliminated.

3.2 Stock index futures pricing formula

One of the first models for valuation of stock index futures was derived by Cornell & French (1983). According to their model the fair, arbitrage-free price for a price index is found as follows:

$$F(t,T) = S_t e^{r_t(T-t)} - \sum_{i=1}^{N} D_i e^{r_t(T-t_i)}, \tag{3.2}$$

where $F(t,T)$ is the price of the index future at time t with maturity at time T, S_t is the value of the stock index at time t, r_t is the interest rate per year for lending or borrowing money for period $[t,T]$, $(T-t)$ is time to maturity of the futures contract and the last component presents the dividend payment at time t_i compounded to the maturity date that are deducted from the index.

As mentioned before, however, this cost of carry model relies on many assumptions. First, the model requires perfect capital markets meaning there are no transaction costs, no short sale restrictions, assets can be perfectly divided, interest rates for lending and borrowing are equal and are non-stochastic and orders are executed instantaneously. Furthermore, margin requirements are ignored and arbitrageurs are not restricted in terms of the size of their arbitrage positions. Also all dividends payments from the underlying stock portfolio that took place until the expiration of the contract are known. Lastly, the model does not take tax effects into consideration.

DAX is, however, a performance index. Measuring the total performance of the index stock portfolio it is adjusted for stock price changes that are results of subscription rights, stock splits and mainly dividend payments. Dividend payments are reinvested into dividend paying stock and once a year the index is adjusted and re-invested dividends are distributed to the involved companies proportionally to their market capitalization. This suggests that arbitrageur has to follow the reinvestment strategy to avoid unbalanced arbitrage positions i.e. reinvesting dividend payment into the dividend paying stocks and rebalancing the portfolio once a year is needed so that their stock portfolio increases in the same way as the index does. An important assumption here is that there are no tax effects that would result in different dividend payments.

As the increasing number of total shares of stocks in the index portfolio is offset exactly by the decrease in stock price (the book value per common share is diluted) the total value of the index portfolio remains unchanged by dividends. These are the arguments of Bühler & Kempf (1995) who moderately adjusted the cost of carry formula for futures contracts based on performance indices. The no-arbitrage relationship is as follows:

$$F(t,T) = S_t e^{r_t(T-t)}. \tag{3.3}$$

3.3 Basis convergence

Another area the cost of carry model covers is the notion of the basis convergence. As stated in Sutcliffe (2006) the basis refers to the difference between the spot price of the underlying index and its futures price. The sign of the basis depends on the cost of carry discussed above. Negative cost of carry represents the situation where funding costs needed to acquire the basket of index stocks are higher than the dividends resulting from the cash position. The futures contracts are simply more attractive and therefore trade at higher prices than the underlying index. When calculated as spot less futures the basis is negative and this phenomenon is referred to as *contango*. On the other hand, positive cost of carry implies that the dividends exceed the financing costs of the underlying index. When entering a futures position investors forego the income on the cash market instrument therefore futures prices are below the spot index level. When, again, quoted as spot less futures the basis is positive and the situation is called *backwardation*.

Regardless of whether the basis is negative or positive the cost of carry model suggests that when a contract approaches maturity the futures price converges to the spot price of the underlying index, in other words, the basis converges to zero. At expiration date the prices should be equal. If this is not the case the convergence should be completed with the help of arbitrage. If the futures price is higher than the index value during the delivery period traders might exploit the arbitrage opportunity by shorting a futures contract, buying the asset in the spot market and making the delivery. Clearly, profit would be made as price of the index is higher in the futures market than in the cash market. The simple law of supply and demand implies that this process would push the futures prices down. If the futures price is lower than the index value at expiration, the reverse holds.

3.4 Normal backwardation and contango

The term *backwardation* should not be interchanged with *normal backwardation*, which relates to the relationship between the current futures price and expected future spot price. Normal backwardation refers to a situation where the futures price is below the expected spot price at maturity and it is argued by Keynes (1930) to be usual for futures markets. When hedgers hold, on average, short futures positions and want to transfer risk efficiently to speculators, the speculators have to hold long futures positions. To induce speculators to take on price risk that commercial traders do not wish to take they have to be compensated in the form of risk premium and this in turn requires the futures price to be below the market's average opinion about the future spot price. And as the basis converges to zero when nearing maturity the futures price has to increase over time making gains for the longs and losses for the shorts. Yet the traders in short positions are willing to bear these losses to ensure against uncertain prices and therefore to reduce risk. The modern approach to explaining this relationship is based on the relationship between risk and expected return. Obviously, the higher the risk of an investment, the higher the expected return demanded by an investor.

When the return from an investment is positively correlated with the stock market, i.e. the one that comprises the systematic risk, the investor requires a higher expected return than the risk free rate of interest for bearing this risk. The stock index is indeed correlated with the market and as such needs to be traded for lower price on the futures market when compared to expectations about the future spot price. Speculators are compensated in the form of positive expected profits and short side investors are willing to suffer expected losses in order to lower the portfolio risk.

The opposite situation, where the current futures price is above the expected future spot price, is known as *contango*. On average, hedgers hold long futures positions and risk-averse speculators are motivated to enter short positions in the futures market by being offered expected return higher than riskless rate. This is achieved when the futures price now is higher than the expected spot price at maturity. As the basis narrows with maturity the futures price is expected to decline over time which favours speculators in short positions and disadvantages the long position traders.

3.5 Arbitrage and basis convergence in reality

As previously mentioned, the cost of carry model suggests that any deviation of futures price from its theoretical fair value is soon eliminated because of arbitrage activities. In reality it is not so straightforward and entering arbitrage positions might not always be feasible. One of the main reasons behind this are transaction costs that accompany arbitrage implementation. These can be slippage, commissions, fees or bid-offer spreads. Therefore, after allowing for transaction costs, relationship between the futures price and the underlying index will not be expressed by the single fair price but rather a fair range of prices that represent a band within which a profitable arbitrage is not possible as per Sutcliffe (2006). Arbitrage strategy will be then feasible only when the absolute size of any mispricing sufficiently exceeds the transaction costs that are associated with the trade.

The heterogeneity of investors complicates the situation even more as the same rules do not necessarily need to apply to everyone. Obviously, transaction costs might differ among various investors. Another example could be taxation of dividends in Germany which differs between domestic and foreign investors towards whom the procedure tends to be more discriminative. As a result foreign investors are not exposed to favourable conditions as German banks are and the arbitrage might prove to be unfeasible[1]. Moreover, regulations imposed on capital markets play the role. All these factors that exist in real markets and which the cost of carry model is unable to account for change the stock index futures price and widen the arbitrage window within which futures prices might fluctuate without triggering arbitrage activity. Furthermore, the no-arbitrage band is not the same for all investors.

Regarding the price convergence at maturity of the futures contract, arbitrage is, again, assumed to be efficient and eliminate any discrepancies between the futures and cash market. But in reality some short-term mispricing may persist. According to Kazmi (2011) excessive speculation and price manipulation might lead to non-convergence and illiquid markets. Exchanges can help to ensure convergence by assisting in arbitrage, keeping price manipulation at minimum or supressing excessive manipulation. In contracts they can implement some structural mechanisms and design criteria which may be various margins and position limits. One of the examples is a spot margin which constitutes rather sharp increases in margin requirements just before the contract maturity. This instrument forces a trader to forecast his expectations of the future and take action, this means they will either stay in the position or close it out. Following Sutcliffe (2006) position limits belong to other instruments used for achieving price convergence. By limiting the number of contracts, which a trader can enter into, potential price manipulation and extensive manipulation are prevented therefore unexpected and large price fluctuations can be potentially avoided.

The mechanism of price convergence is crucial especially for contracts that assume delivery of a commodity. The lack of convergence between the futures

[1]The information was provided by one of the employees of Czech National Bank.

and spot prices might lead to the failure of the contract itself as there could be e.g. insufficient amounts of commodity that could be delivered under the terms of the contract. Stock index futures, however, are settled in cash. A buyer of an index future is entitled to any appreciation in the index over the index futures price, a seller is entitled to any depreciation in the index under the specified future value. With respect to cash-settled contracts the convergence is automatic and the final price is determined by an exchange settlement procedure that might vary from exchange to exchange.

4 Institutional settings

4.1 DAX index and related futures contracts

DAX (*Deutscher Aktienindex*) is the equity index tracking the performance of the 30 largest and most important German companies. These companies are selected on the basis of exchange turnover and market capitalization traded on the Frankfurt Stock Exchange (*FWB*) as per the exchange documentation (Deutsche Böerse AG (2017)). As the included stocks represent around 80% of market capitalization listed in Germany the index is often perceived as the benchmark for German stock market and the indicator for the performance of the German economy as a whole. The index introduced in 1987 started with a base of 1000 index points and was moving close to 12,000 in early 2017. This is covering various sectors among which automotive, banking, chemicals and industrials can be found. As well as being representative and well-diversified the DAX serves as the underlying for index derivatives and is highly liquid as indicated by the high trading volumes of index futures and option contracts.

The most important characteristic of the DAX used in this study is that it belongs to total return (performance) indices. As stated in Fink & Theissen (2014) a total return index assumes that dividends are reinvested and therefore it shows the actual performance of an investment in the index portfolio. With regards to DAX, dividends paid by German companies in the index are reinvested into a dividend-paying stock. Once a year the index is adjusted and re-invested dividends are distributed to the involved companies proportionally to their market capitalization. This property can be visible when interpreting the cost-of-carry pricing formula.

The DAX futures contracts were traded for the first time in 1996. They soon became popular, highly liquid products. There are four maturity dates in a year - March, June, September and December whereas contracts with the three nearest maturity dates are traded. The main contract specifications are provided in Table 1.

Table 1: Contract specifications

Contract name	DAX Futures
Product ID	FDAX
Underlying	DAX
Index type	Total return in EUR
Contract value (per index point)	EUR 25
Minimum price change (index points)	0.5
Tick value	EUR 12.5
Contract terms	up to 9 months
Delivery months	March, June, September, December
Settlement	Cash settlement
Final settlement price	Based on Xetra intraday auction (13:00 CET)
Last trading day	Third Friday of the maturity month
Final settlement day	Third Friday of the maturity month

Source: Eurex (2017)

4.2 Arbitrage implementation

Regarding DAX futures trading, direct transaction costs are €0.50 per contract comprising a trading and clearing fee while a volume rebate makes transaction costs even cheaper when large volumes are traded. Nevertheless, transaction costs which are variable only account for a small proportion of real costs. The main costs are in establishing the network to either execute the arbitrage, or, in case of market makers, to prevent the arbitrage. This fixed costs that can't be neglected in overall marginal costs of trading represent large amounts of money paid in either latency or better models. In case a trader decides to trade without exchange connectivity then they have to pay a broker a commission per trade or contract.

Another factor which the arbitrage opportunity and risk depend on is the tick size, i.e. the minimum price increment of a trading instrument. With smaller tick sizes there are more increments in which an instrument's price can move hence more trading opportunities, i.e. with higher price granularity the arbitrage is easier. In this context FDAX can be considered granular enough[2].

4.3 Final settlement procedure

The final settlement of DAX futures takes place on the 3rd Friday of each maturity month. Eurex establishes the final settlement price according to the value of the underlying index based on Xetra auction prices of the component shares constituting the index. This is done by the Xetra intraday auction which together with opening and closing auctions serves as means of price discovery and strengthened liquidity. The auction price is determined according to the principle of highest volume transacted meaning it is the price at which the highest executable order volume is apparent. The prices determined in auctions are a result of strong demand and supply conditions and therefore they are perceived as very reliable. Eurex, where the DAX futures are traded, is not involved in enforcing the basis convergence[3].

[2]The information was provided by one of the employees of Eurex Exchange.

[3]The information was provided by one of the employees of Eurex Exchange.

5 Methodology

This thesis investigates the applicability of the cost of carry model in two ways. First, the magnitude of mispricing is calculated to see the degree of deviation of the actual futures prices from their theoretical counterparts and the direction of the misalignment. The second part investigates the fitness of the cost of carry model by regression analysis using the actual futures prices. Data processing is implemented in R programming language using RStudio IDE.

5.1 Mispricing

The mispricing series is calculated according to the formula suggested by Butterworth & Holmes (2000) as the difference between the actual futures price F_t and its theoretical fair price at time t divided by the value of the spot index underlying the futures contract S_t:

$$M_t = \frac{F_t - (S_t e^{r_t(T-t)})}{S_t}. \tag{5.1}$$

As the price difference is normalized with respect to the index value we talk about relative mispricing. If the cost of carry model describes the realized prices well, the average mispricing should not be significantly different from zero. Positive mispricing implies the contract is overvalued and in a world with no transaction costs this would automatically lead to the arbitrage activity of buying the stocks and entering short futures positions. On the other hand, negative mispricing signifies the undervaluation of futures contract that in turn would induce arbitrageurs to purchase index futures and short the stocks when no transaction costs are assumed. However, in the real world transaction costs need to be considered to decide whether an arbitrage move would yield any profits.

To confirm the findings of mispricing the statistical significance needs to be tested. The particular methods used have to be chosen on the basis of the properties of the data and therefore the data characteristics need to be examined first. In particular, the normality and autocorrelation are considered. To test the normality assumption the Shapiro-Wilk test and the Anderson-Darling test are employed. Introduced by Shapiro & Wilk (1965) who recommended the use of their test statistic with rather smaller samples, i.e. up to samples of size 20, Royston (1982) extended the test to the version that is applicable to larger samples. The Shapiro-Wilk procedure tests the null hypothesis that the sample of data comes from a normally distributed population against the alternative claiming otherwise. This test is accompanied by another commonly used normality test - the Anderson-Darling - that tests the same null hypothesis.

To test the hypothesis of the mean of a mispricing series being zero, i.e. the null $H_0 : \mu = 0$ against the two-sided alternative $H_A : \mu \neq 0$, the standard one-sample t-test can be employed given normality and the independence of observations are satisfied. If the contract data is normally distributed yet autocorrelated the Newey-West standard errors that are robust to heteroscedasticity and autocorrelation are used.

Nonetheless, the normality does not necessarily have to always be met. In the event of the normality assumption being violated a non-parametric test that does not require normally distributed underlying population needs to be exercised. The non-parametric test applied in this thesis is a one-sample rank test Wilcoxon signed-rank test that tests a similar hypothesis about the median of a mispricing series with symmetric distribution, i.e. $H_0 : \theta = 0$ against the two-sided alternative $H_A : \theta \neq 0$. To calculate the Wilcoxon signed rank statistic suggested in Bartoszynski & Niewiadomska-Bugaj (1996) the absolute differences $V_i = |X_i - 0|$ for $i = 1, ..., n$, where X_i are particular observations of a mispricing series and 0 is a hypothesized value of the median, need to be calculated. Considering the underlying distribution is continuous it is assumed that all V_i's are distinct and none equals 0. After arranging V_i's in increasing order the ranks $R_1, R_2, ..., R_n$ are assigned to them, with rank 1 assigned to the smallest V_i. Additionaly we define $\eta = +1$ if $X_i > 0$ and $\eta = -1$ otherwise. Then the test statistic is specified as

$$S_n = \sum_{i=1}^{n} \eta_i R_i \tag{5.2}$$

and its asymptotic variance is defined as

$$Var(S_n) = \frac{n(n+1)(2n+1)}{6}. \tag{5.3}$$

For large n $\frac{S_n}{\sqrt{Var(S_n)}}$ has approximately a standard normal distribution.

The Wilcoxon signed-rank test requires the independence of observations as the T-test does. If this assumption is violated along with the normality another approach needs to be employed. We could test the hypothesis that there is no mispricing by setting the equation (5.1) to zero:

$$\frac{F_t - (S_t e^{r_t(T-t)})}{S_t} = 0. \tag{5.4}$$

After rearranging we get the following equation which can be used in regression analysis:

$$\frac{F_t}{S_t} = e^{r_t(T-t)}. \tag{5.5}$$

More specifically, a regression with a constant can be run where $\frac{F_t}{S_t}$ is a dependent variable and $e^{r_t(T-t)}$ is an independent variable:

$$\frac{F_t}{S_t} = \beta_0 + \beta_1 e^{r_t(T-t)} + \varepsilon_t. \tag{5.6}$$

In the final step a joint hypotheses of $H_0 : \beta_0 = 0, \ \ \beta_1 = 1$ is tested. As each contract sample only contains approximately 60 observations and therefore is relatively small, a Lagrange Multiplier test needs to be employed instead of the classical F-test as it does not require the normality of the error terms. To correct for heteroscedasticity and autocorrelation the heteroscedasticity and autocorrelation consistent (HAC) standard errors are used.

5.2 Regression analysis

As described in the theoretical part of the thesis the long-run relationship between stock index futures price and the underlying index value is expressed as

$$F(t,T) = S_t e^{r_t(T-t)}, \tag{5.7}$$

where $F(t,T)$ is the price of the index future at time t with maturity at time T, S_t is the value of the stock index at time t, r_t is the risk-free rate for period $[t,T]$ and $(T-t)$ is time to maturity of the futures contract. This exponencial formula can be transformed to a linear model. After taking a natural logarithm on both sides of the equation (5.7) and rearranging the terms the following equation could be obtained:

$$\ln F_t = \ln S_t + r_t(T-t). \tag{5.8}$$

The equation (5.8) can be written for the regression purposes as

$$\ln F_t = \beta_0 + \beta_1 \ln S_t + \beta_2 r_t(T-t) + \varepsilon_t, \tag{5.9}$$

where $\ln F_t$ is the logarithm of the actual futures price at time t, $\ln S_t$ is the logarithm of the index price at time t and $r_t(T-t)$ is the product of the risk-free rate at time t and the time to maturity. Finally ε_t is an error term. If the model fits the data well, then $\beta_0 = 0$, $\beta_1 = 1$ and $\beta_2 = 1$ in the regression equation (5.9).

Order of integration

Before the estimation procedure time series properties of the variables should be determined. By performing unit root and stationarity tests the degree of integration of the individual series can be assessed. A series is said to be integrated of order 0, denoted as $I(0)$, when the series is "a stationary, weakly dependent time series process that, when used in regression analysis satisfies the law of large numbers and the central limit theorem" (Jeffrey et al. (2009), p. 850). A time series that is integrated of order 1, $I(1)$, needs to be first-differenced to result in a stationary process. In general, the $I(d)$ series have to be differenced d times in order to produce an $I(0)$ process.

The notion of stationarity is important since classical regression properties hold only for weakly dependent time series (Jeffrey et al. (2009)). If all series involved in the regression analysis were $I(0)$ the ordinary least squares procedure (OLS) could be used. However, the estimation with highly persistent time series characterized by a unit root can lead to misleading results if the classical linear model (CLM) assumptions are violated. In many cases, the usual large sample normal approximations in time series regression analysis are no longer valid and if we regress the $I(1)$ variables on each other we might encounter spurious regression problem when using OLS. A possible solution could be the transformation into a weakly dependent process by taking the first differences of the $I(1)$ variables and subsequently using these differences in the modelling process instead of the levels. However, as stated in Brooks (2008), this approach is not appropriate if there is some long-term relationship between the variables since pure first difference models have no long-run solution and therefore limits the number of questions that can be asked and satisfactorily answered . Fortunately, in one case the spurious regression problem can be overcome and $I(1)$ variables can be used in levels. A regression involving $I(1)$ variables is not spurious if the series are cointegrated. The notion of cointegration is introduced in the next subsection. For the purpose of determining the order of integration of the involved time series augmented Dickey-Fuller (ADF) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests are applied.

As the name suggests the ADF test is an augmented version of the Dickey-Fuller test (Dickey & Fuller (1979)) that is based on the null hypothesis that there is a unit root in the autoregressive representation of the time series. More precisely, the test examines the null hypothesis $H_0 : \theta = 1$ against the one-sided alternative $H_A : \theta < 1$ in the following AR(1) model:

$$y_t = \theta y_{t-1} + u_t, \;\; t = 1, 2, ... \tag{5.10}$$

In practice, for the sake of easier computation and interpretation the equation (5.11) is used. After subtracting y_{t-1} from each side of the equation (5.10) and defining $\psi = \theta - 1$ we could obtain

$$\Delta y_t = \psi y_{t-1} + u_t, \;\; t = 1, 2, ... \tag{5.11}$$

and test the null hypothesis of $H_0 : \psi = 0$. The DF test regression can also take the forms with an intercept or with an intercept and a deterministic trend. The model of the interest then would be

$$\Delta y_t = \psi y_{t-1} + \mu + \lambda t + u_t, \;\; t = 1, 2, ..., \tag{5.12}$$

where μ is a constant and λt is a deterministic trend. Since the null hypothesis is one of non-stationarity the test statistic does not follow the usual t-distribution under the null and the central limit theorem that is behind the asymptotic standard normal distribution for the t-statistic does not apply even in large samples. Therefore a new set of critical values was derived for the usual t-statistic of the coefficient of interest. The critical values for unit root t test have been tabulated by several authors, the ones suggested by Banerjee et al. (1993) are given in Table 19 in Appendix. It is evident that the DF critical values are bigger in absolute terms implying more evidence against the null hypothesis is required when testing for unit root than under standard t-tests.

The classical Dickey-Fuller test, however, is valid only if u_t is white noise, i.e. the process with zero mean, constant variance and zero autocovariances, except at leag zero. In other words it is assumed that u_t is not autocorrelated, but if there is autocorrelation in the dependent variable of the test regression this woud not be the case. To absorb any dynamic structure contained in the dependent variable Δy_t and hence to ensure that u_t is not serially autocorrelated the test regression can be augmented with the lagged changes Δy_{t-h} as follows:

$$\Delta y_t = \mu + \lambda t + \psi y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-i} + u_t, \;\; t = 1, 2, ..., \tag{5.13}$$

where p is the number of the lagged changes of the dependent variable. This is the augmented Dickey-Fuller test. The null hypothesis of a unit root stays the same, i.e. $H_0 : \psi = 0$ versus the alternative $H_A : \psi < 0$, and the same critical values as in the case of the DF test are used. The optimal number of lags of the dependent variable can be determined either by the frequency of the data or according to information criteria such as Akaike information criterion (AIC) or Bayesian information criterion (BIC) as stated in Brooks (2008). Since the daily financial data are used in this thesis the lag selection based on the frequency of the data does not supply an obvious answer and therefore an information criterion approach is more reliable.

As a supplement to the ADF unit root test the KPSS test developed by Kwiatkowski et al. (1992) is performed. The KPSS test, named after its authors Kwiatovski, Phillips, Schmidt and Shin, tests a null hypothesis that a time series is stationary against the alternative of the presence of a unit root. Kočenda & Černý (2015) recommend to use the KPSS along the ADF as the latter detects only the unit root and the absence of the unit root does not necessarily implies the stationarity of a series. The KPSS test assumes that a time series tested for trend stationarity, y_t, can be decomposed in the sum of a deterministic trend β_t, a random walk r_t and a stationary error ε_t as follows

$$y_t = \beta t + r_t + \varepsilon_t, \;\; r_t = r_{t-1} + u_t, \tag{5.14}$$

where u_t are normal $i.i.d.$ with a zero mean and variance σ_u^2. When testing for the level stationarity the deterministic trend is left out from the equation. The initial value r_0 is assumed to be fixed and serves as an intercept. The null of the stationarity of a series is equivalent to the hypothesis that $\sigma_u^2 = 0$ that ensures that $r_t = r_0$ for all t, i.e. the random walk has zero variance. To test this hypothesis the LM test is used. The KPSS test involves estimating the regression $y_t = \alpha_0 + e_t$ or $y_t = \alpha_0 + \beta_t + e_t$, depending on whether the trend stationatiry or level stationarity is examined, by OLS. The residuals $\hat{e}_t$ from the estimated regression are then used to compute the LM statistics, either for level stationarity η_μ or for trend stationarity η_τ as

$$\eta_{\mu/\tau} = T^{-2}\frac{1}{s^2(l)}\sum_{t=1}^{T} S_t^2, \;\; \text{where} \;\; S_t = \sum_{i=1}^{t} e_i \;\; \text{and}$$

$$s^2(l) = T^{-1}\sum_{t=1}^{T} e_t^2 + 2T^{-1}\sum_{s=1}^{l} w(s,l)\sum_{t=s+1}^{T} e_t e_{t-s}, \text{ where}$$
$$w(s,l) = 1 - s/(l+1). \tag{5.15}$$

S_t above is the partial sum process of the residuals e_t from the estimated equation, $s^2(l)$ is the estimator of the long-run variance of the residuals e_t, and $w(s,l)$ is the so-called Bartlett spectral window[4]. The variance estimator $s^2(l)$ is a function of the parameter l and as l increases the estimator begins to control for possible autocorrelation in the residuals e_t. Commonly the test is performed for l in the range of 0 to 8. Lastly, the LM statistics η_μ or η_τ is compared with the asymptotic critical values for the KPSS test. The table with the respective critical values is given in Table 20 in Appendix.

Cointegration

As mentioned before a regression with variables that are $I(1)$ could be meaningful when the time series involved are cointegrated. In most cases, when two $I(1)$ series are linearly combined the result is also an $I(1)$ process. However, it could be the case that some linear combination of non-stationary variables forms a stationary $I(0)$ process. This situation suggests that there is a long-run equilibrium relationship between the variables, in other words they are cointegrated. The formal definition given in Engle & Granger (1987) is that "the components of the vector x_t are said to be cointegrated of order d, b, denoted $x_t \sim CI(d,b)$, if all components of x_t are $I(d)$ and if there exists a nonzero vector α so that the linear combination of the components of x_t, i.e. $z_t = \alpha' x_t$, is $I(d-b),\ \ b > 0$", where the vector α is the so-called cointegrating vector. In practice, mostly the case of $C(1,1)$ variables forming the $I(0)$ stationary process is encountered.

One of the most common ways the cointegration can be examined is with the residual-based approach suggested by Engle & Granger (1987). After ensuring that the variables involved in the model are $I(1)$ by the application of a unit root test the regression is estimated using the standard

[4]For more detail, see Kwiatkowski et al. (1992).

OLS method. Subsequently the residuals are saved and tested for stationarity. More specifically, given the model with two variables $y_t = \hat{\alpha} + \hat{\beta} x_t$ and its residuals $\hat{u}_t = y_t - \hat{\alpha} - \hat{\beta} x_t$, the DF or ADF test is applied on the residuals using the regression of the form $\Delta \hat{u}_t = \psi \hat{u}_{t-1} + \nu_t$, where ν_t is an *i.i.d.* error term. When deciding about stationarity of the residuals another table of critical values needs to be used as the test is now being conducted on the residuals of an estimated model rather than on a series of raw data which results in the change of the distibution of the test statistic. Their critical values are larger in magnitude than the classical DF critical values. The ones given by Davidson, MacKinnon et al. (1993) are presented in Table 21 in Appendix. In addition to the ADF test the KPSS test is performed to provide more evidence about stationarity of the residuals. If non-stationarity of the residuals is rejected the variables are said to be cointegrated. The estimated model $y_t = \alpha + \beta x_t + \varepsilon_t$, where α and β are model parameters and ε_t is the zero mean stationary process, is then referred to as the cointegrating regression and the usual OLS procedure consistently estimates the regression paratemers. The cointegrating regression represents the long-run relationship between the variables implying they might deviate from their equilibrium in the short run but in the long run they return back together. If no cointegrating relationship was found the regression is levels is not appropriate and rather the first-differenced model should be employed as per Jeffrey et al. (2009).

Frequently, the cointegrated variables are further processed in an error correction model that describes the short-run dynamics between the corresponding variables. These models may incorporate first-differenced and lagged levels of cointegrated variables as well as lagged values of the respective differences. As another variable the lagged residuals from the cointegrating regression are included and are referred to as an error correction term. However, this procedure is beyond the scope of this thesis since the centre of interest is just the potential long-run equilibrium between the variables included in the model.

5.3 Basis convergence

The last part of the model analysis focuses on the phenomenon of basis convergence. As mentioned previously the basis is the difference between the spot and the futures price of an asset at time t, i.e. $S_t - F_t$. As well as considering the basis magnitude and direction the mutual progression of the prices over time is considered. Since it is assumed that the prices converge on the way to contract maturity the basis is expected to decline over time. To inspect this event and identify a trend the absolute value of basis is regressed on time to maturity in days:

$$|Ba_t| = \alpha + \beta Mat\_day_t + \varepsilon_t, \tag{5.16}$$

where the estimated coefficient $\hat{\beta}$ is of interest. Finally, the KPSS test for trend stationarity is applied to investigate the potential stationarity of the basis around a deterministic trend and therefore the smoothness of the realization of basis convergence.

6 Data

The data on futures prices and index spot prices were acquired from Thomson Reuters. Interest rates were supplied by EMMI (*European Money Market Institute*).

The DAX futures contracts expire on the third Friday in March, June, September and December respectively. The thesis embraces the index futures contracts that matured in years 2013-2016 covering the period from the first trading day of the Mar 2013 contract (Jun 18, 2012) to the expiration day of the Dec 2016 contract (Dec 16, 2016). As there are four contracts per year, this study includes 16 contracts with DAX and DAX futures closing prices collected on a daily basis.

As a proxy for risk-free rate when calculating the fair futures price the Euribor (*Euro Interbank Offered Rate*) rates are employed. The Euribor rates are deemed the most important reference rates in the European money market. They are based on the average rates of interest at which the so-called panel banks borrow funds from one another. The panel banks are the banks with a first class credit standing that transact the largest volumes in the Eurozone money markets. The dataset consists of 1 week, 2 week, 1 month, 2 month, 3 month, 6 month, 9 month and 12 month Euribor daily rates and a 1-day European interbank interest rate Eonia (*Euro Overnight index Average*). When evaluating a future contract two of the aforementioned rates which match the maturity of the futures contract best are interpolated linearly to obtain the corresponding risk-free rate.

As the trading volume is significantly higher for contracts that are nearest to maturity the dataset used for the regression analysis and calculating mispricing series employs only contracts which are nearest to expiration and are assumed to be rolled over at maturity. This comprises in total 1006 trading days. Obviously, when investigating the basis convergence all trading days within the life of a contract are included. On average one contract trades for 189 days.

7 Empirical results

7.1 Mispricing

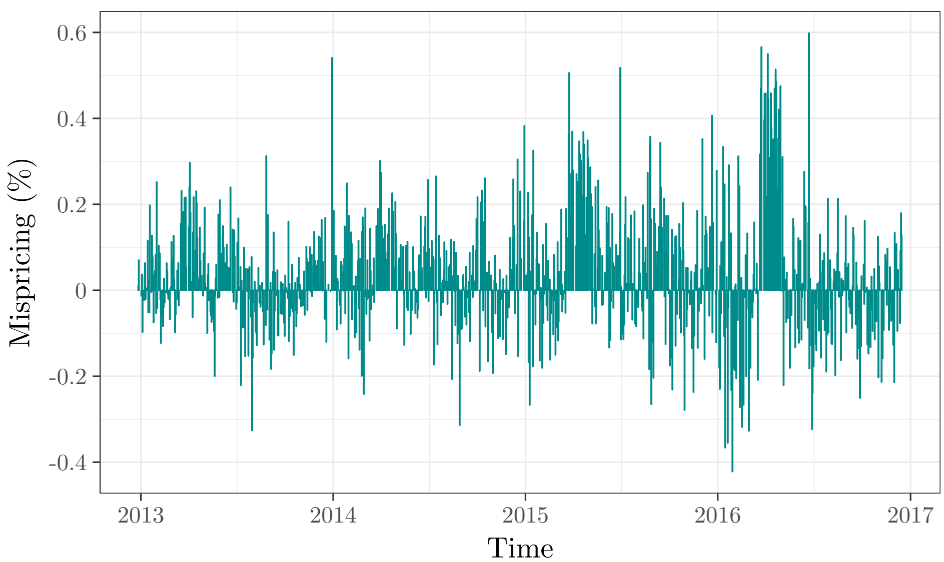
To detect any deviations from fair prices, the relative mispricing of the futures contracts is calculated according to the equation (5.1). The theoretical prices were derived using the standard cost of carry formula as given in the equation (3.3). Tables 2, 3, 4 and 5 provide summary statistics of the contracts that matured between years 2013 and 2016 and Figure 1 shows the daily mispricing series over the studied period. As mentioned earlier the results are shown for the contracts that are nearest to maturity.

If the model describes the data well the average mispricing should not be significantly different from zero. The average size of misprising ranges in absolute value between 0.01% and 0.17%. As visible in Figure 1 the daily mispricing tends to remain within the -0.4-0.4% range. Table 6 provides a closer look at the actual magnitude of mispricing by comparing the size of deviations with various benchmark levels of total round-trip transaction costs. The mean of absolute mispricing values tend to remain below 0.22%. The mispricing indeed remains within the -0.4-0.4% band with less than 2% of the observations surpassing these limits whereas the contracts maturing between March 2013 and March 2015 and in the second half of 2016 mostly exhibit deviations below 2% of the spot index price.

However, June contracts are the exception since apart from the year 2014 they show larger and more frequent deviations from fair values when compared with other maturities which confirms the similar findings of Fink & Theissen (2014). This might be connected to the dividend payments as German companies pay dividends usually once a year during summer months as opposed to e.g. S&P 500 and its quarterly payments. The accumulation of dividend payments between April and June apprears to lead to more deviations. This might be explained by the nature of total return indices. As noted previously, in the DAX calculation the dividends are assumed to be reinvested fully in the same basket of shares which leads to the fact that, other things being equal, the DAX futures price will rise a little each year. If investors do not decide to fully invest their dividends into the same basket the short-term discrepancies may arise. Nevertheless, even these do not surpass 0.6% of the index value which implies that the DAX futures contracts are efficiently priced and the index arbitrage is rather rare in German markets.

Even though the markets and the model appear to be pretty efficient it should be pointed out that the deviation of e.g. 0.2% represents approximately €375-620 per contract due to the relatively large size of FDAX.

Regarding the direction of mispricing the futures contracts are mostly overpriced as premiums occur on 596 (59.24%) occasions and discounts on 394 (39.17%) occasions leaving the rest being zero, specifically at contracts' expiration dates. This contradicts the findings of Bühler & Kempf (1995) who detected mostly undervalued contracts.



Source: Own construction based on Thomson Reuters data.

Figure 1: Mispricing December 2012 - December 2016

Table 2: Summary statistics of contracts maturing in 2013

		N	Mean	SD	Max	Min
Mar 2013	Overpricing	30	0.07	0.06	0.25	0.00
	Underpricing	24	-0.05	0.03	-0.00	-0.12
	Total	55	0.02	0.07	0.25	-0.12
Jun 2013	Overpricing	52	0.11	0.06	0.30	0.01
	Underpricing	14	-0.06	0.03	-0.01	-0.20
	Total	67	0.07	0.07	0.30	-0.20
Sep 2013	Overpricing	26	0.08	0.06	0.31	0.01
	Underpricing	38	-0.08	0.03	-0.00	-0.33
	Total	65	-0.02	0.07	0.31	-0.33
Dec 2013	Overpricing	35	0.06	0.05	0.17	0.00
	Underpricing	29	-0.05	0.04	-0.00	-0.15
	Total	65	0.01	0.07	0.17	-0.15

Source: Own calculations based on Thomson Reuters data.

Table 3: Summary statistics of contracts maturing in 2014

		N	Mean	SD	Max	Min
Mar 2014	Overpricing	35	0.10	0.10	0.54	0.01
	Underpricing	24	-0.07	0.06	-0.00	-0.24
	Total	60	0.03	0.12	0.54	-0.24
Jun 2014	Overpricing	49	0.10	0.07	0.30	0.00
	Underpricing	12	-0.05	0.04	-0.00	-0.13
	Total	62	0.07	0.09	0.30	-0.13
Sep 2014	Overpricing	40	0.08	0.06	0.27	0.00
	Underpricing	24	-0.08	0.07	-0.00	-0.31
	Total	65	0.02	0.10	0.27	-0.31
Dec 2014	Overpricing	38	0.09	0.08	0.30	0.00
	Underpricing	25	-0.08	0.06	-0.01	-0.19
	Total	64	0.02	0.11	0.30	-0.19

Source: Own calculations based on Thomson Reuters data.

Table 4: Summary statistics of contracts maturing in 2015

		N	Mean	SD	Max	Min
Mar 2015	Overpricing	31	0.11	0.09	0.38	0.01
	Underpricing	28	-0.08	0.07	-0.01	-0.27
	Total	60	0.02	0.12	0.38	-0.27
Jun 2015	Overpricing	50	0.20	0.11	0.51	0.00
	Underpricing	10	-0.06	0.04	-0.00	-0.13
	Total	61	0.15	0.14	0.51	-0.13
Sep 2015	Overpricing	45	0.12	0.11	0.52	0.01
	Underpricing	19	-0.09	0.07	-0.02	-0.27
	Total	65	0.06	0.14	0.52	-0.27
Dec 2015	Overpricing	37	0.10	0.08	0.35	0.00
	Underpricing	27	-0.10	0.07	-0.00	-0.28
	Total	65	0.01	0.12	0.35	-0.28

Source: Own calculations based on Thomson Reuters data.

Table 5: Summary statistics of contracts maturing in 2016

		N	Mean	SD	Max	Min
Mar 2016	Overpricing	25	0.13	0.12	0.41	0.00
	Underpricing	35	-0.15	0.12	-0.00	-0.42
	Total	61	-0.03	0.18	0.41	-0.42
Jun 2016	Overpricing	47	0.25	0.18	0.57	0.00
	Underpricing	14	-0.08	0.07	-0.00	-0.22
	Total	62	0.17	0.21	0.57	-0.22
Sep 2016	Overpricing	31	0.09	0.11	0.60	0.00
	Underpricing	33	-0.10	0.07	-0.00	-0.32
	Total	65	-0.01	0.13	0.60	-0.32
Dec 2016	Overpricing	25	0.07	0.05	0.18	0.00
	Underpricing	38	-0.09	0.06	-0.00	-0.25
	Total	64	-0.03	0.10	0.18	-0.25

Source: Own calculations based on Thomson Reuters data.

Table 6: Mispricing versus transaction costs levels

	Ave Abs.	$\lvert M_t \rvert > C$					
		0.1%	0.2%	0.3%	0.4%	0.5%	0.6%
Mar 2013	0.0560	8	1	0	0	0	0
Jun 2013	0.0971	25	10	0	0	0	0
Sep 2013	0.0786	20	3	2	0	0	0
Dec 2013	0.0555	12	0	0	0	0	0
Mar 2014	0.0876	20	3	1	1	1	0
Jun 2014	0.0899	26	4	1	0	0	0
Sep 2014	0.0748	17	4	1	0	0	0
Dec 2014	0.0857	23	6	1	0	0	0
Mar 2015	0.0920	22	5	2	0	0	0
Jun 2015	0.1723	40	24	10	1	1	0
Sep 2015	0.1118	30	9	4	1	1	0
Dec 2015	0.0952	27	6	1	0	0	0
Mar 2016	0.1396	31	19	8	2	0	0
Jun 2016	0.2112	39	28	21	12	3	0
Sep 2016	0.0941	23	5	2	1	1	0
Dec 2016	0.0820	20	4	0	0	0	0

Source: Own calculations based on Thomson Reuters data.

7.2 Statistical significance

To confirm the findings of the mispricing investigation the usual one-sample t-test, its modification with Newey-West standard errors and the Wilcoxon signed-rank test are performed. First, the individual contracts data are tested for normality using the Shapiro-Wilk and the Anderson-Darling tests. The results of which along with the size of autocorrelation at lags 1 and 10 are presented in Table 7. The null hypothesis of normality is not rejected at 5% level of significance in 13 out of 16 contracts as indicated by high p-values given by both normality tests. For the contract maturing in March 2014 the null hypothesis is not rejected at 5% level of significance by the Anderson-Darling, however, there is evidence against the normality in the Shapiro-Wilk test results. Given these contradictory results, the normality is not assumed for this dataset and therefore the Wilcoxon signed-rank test needs to be applied as it was for the contracts where the normality was rejected by both tests.

Regarding dependence of observations in the samples, the data mostly do not exhibit strong autocorrelation excepting the June contracts in all periods and the September 2013 contract with slightly higher autocorrelation at lag 10. For all these contracts the t-test with Newey-West standard errors is employed apart from the more complicated June 2016 contract for which neither the t-test nor the Wilcoxon signed-rank test can be run since there is not enough evidence that the sample comes from the normal distribution and it also lacks independence of observations. Therefore in this case auxiliary regression and joint hypothesis testing are performed as described in the methodology overview.

Table 7: Shapiro-Wilk and Anderson-Darling normality tests and autocorrelation

	Shapiro-Wilk		Anderson-Darling		AC[1]	AC[10]
	W	*p-val*	A	*p-val*		
Mar 2013	0.9664	0.1267	0.3996	0.3522	-0.0632	-0.0988
Jun 2013	0.9798	0.3469	0.5584	0.1437	0.4850	0.2778
Sep 2013	0.9826	0.4914	0.3361	0.4977	-0.0245	0.2259
Dec 2013	0.9880	0.7838	0.2298	0.7994	0.0707	0.1772
Mar 2014	0.9312	0.0022	0.6084	0.1088	-0.0656	-0.0397
Jun 2014	0.9933	0.9826	0.1229	0.9866	0.2925	0.2872
Sep 2014	0.9666	0.0765	0.6333	0.0946	0.0983	0.2011
Dec 2014	0.9786	0.3303	0.4731	0.2350	0.1244	0.0274
Mar 2015	0.9776	0.3356	0.4645	0.2463	-0.1382	-0.1346
Jun 2015	0.9846	0.6396	0.2793	0.6350	0.5253	0.3535
Sep 2015	0.9738	0.1818	0.4794	0.2269	0.1054	-0.1528
Dec 2015	0.9911	0.9248	0.1888	0.8977	-0.0064	0.0089
Mar 2016	0.9841	0.6146	0.3815	0.3899	-0.0481	0.0877
Jun 2016	0.9496	0.0128	1.1442	0.0050	0.5807	0.3037
Sep 2016	0.9105	0.0002	0.8640	0.0251	0.1137	-0.0721
Dec 2016	0.9912	0.9323	0.1473	0.9642	0.1279	-0.0075

Source: Own calculations based on Thomson Reuters data, using R software.

The results of the t-tests and the Wilcoxon signed-rank tests are provided in Table 8 and 9, respectively. The results of the LM test applied in the case of the June 2016 contract are presented in Table 10. There is no indication of statistically significant evidence at 0.05 alpha level against the null hypothesis of the mean, or the median, being zero on 10 out of 16 (62.5%) occasions hence the null cannot be rejected. In the other 6 (37.5%) cases, including the June 2016 contract, the data do provide evidence against the null which can therefore be rejected at 5% level of significance. The situation of no mispricing predominates in this analysis, however, it was not possible to statistically confirm that the average size of mispricing is not significantly different from zero in 37.5%. This corresponds to the later contracts under the study and the contracts maturing in June that exhibit larger and more frequent deviations.

Table 8: Statistical significance by application of T-test

	N	Mean	SD/SD_{NW}	T-stat	df	$p-val$
Mar 13	55	0.0159	0.0730	1.6164	54	0.1118
Jun 13	67	0.0722	0.1727_{NW}	3.4192	66	0.0011
Sep 13	65	-0.0168	0.1005_{NW}	-1.3446	64	0.1835
Dec 13	65	0.0119	0.0700	1.3724	64	0.1747
Jun 14	62	0.0689	0.1320_{NW}	4.1072	61	0.0001
Sep 14	65	0.0177	0.0980	1.4565	64	0.1501
Dec 14	64	0.0248	0.1116	1.7754	63	0.0807
Mar 15	60	0.0180	0.1232	1.1332	59	0.2617
Jun 15	61	0.1533	0.2481_{NW}	4.8275	60	0.0000
Sep 15	65	0.0574	0.1392	3.3226	64	0.0015
Dec 15	65	0.0147	0.1207	0.9797	64	0.3309
Mar 16	61	-0.0306	0.1818	-1.3132	60	0.1941
Dec 16	64	-0.0289	0.0974	-2.3739	63	0.0207

Source: Own calculations based on Thomson Reuters data, using R software.
Note: In case of high autocorrelation, Newey-West standard errors were applied.

Table 9: Statistical significance by application of Wilcoxon signed-rank test

	N	Median	W-stat	Var	z-score	$p - val$
Mar 14	60	0.0186	482	73810	1.7741	0.0760
Sep 16	65	-0.0040	-342	93665	-1.1175	0.2638

Source: Own calculations based on Thomson Reuters data, using R software.

Table 10: LM test for the June 2016 contract

	N	df_{ur}	q	Chisq	$p - val$
LM test	62	60	2	69.91	0.0000

Source: Own calculations based on Thomson Reuters data, using R software.
Note: Testing $H_0 : \beta_0 = 0, \beta_1 = 1$ in the equation (5.6).

7.3 Cointegration

It is sensible to believe that there is a long-run relationship between futures prices and the underlying index values. These are obviously prices for the same asset yet at different points in time, i.e. for future and immediate delivery, and therefore they might be cointegrated. This long-run equilibrium relationship is given by the cost of carry model.

The notion of cointegration could be used to estimate the equation (5.9) as the regression involves natural logarithms of both futures and spot prices of the index. The additional variable included is the product of interest rate and the corresponding time to maturity. First, the non-stationarity of variables and so their order of integration is examined by the ADF and the KPSS tests that check for a unit root and level stationarity, respectively. The ADF tests applied involve a regression with a drift and a trend as in the equation (5.13). After the estimation of the equation (5.9) by OLS the residuals are tested as well using the same tests, however, this time the ADF applies a different regression as described in the methodology part. The results are given in Table 11. It is evident that both natural logarithm of futures prices and natural logarithm of index values are $I(1)$ as the null hypothesis of a unit root is not rejected when the ADF test is executed on levels but is rejected when the test is run at first-differenced variables. Moreover, the KPSS test confirms the results by rejecting the null of level stationarity in case of the level variables and not rejecting the null when first differences are tested. However, the tests of the variable $r_t(T-t)$ produce conflicting results. The null of a unit root is rejected in case of both levels and first differences. Regarding the KPSS the null of stationarity is rejected when testing the levels, but is not rejected when first-differenced variable is examined. Hence these results imply that the variable indeed becomes stationary when first-differenced, however, does not contain a unit root and therefore is somewhere between an $I(1)$ and $I(0)$ process. This potentially constitutes a problem for cointegration testing since it is required that all the variables involved in the regression are integrated of the same order. Nonetheless, this condition is not that strict if there are at least two $I(1)$ variables in the regression[5].

The last step in investigating a potential cointegrating relationship comprises estimating the model of interest and testing its residuals for unit root and stationarity. As shown in Table 11 the null of a unit root was rejected at 1% significance level and the KPSS results indicate the stationarity as there was not enough evidence to reject the null hypothesis. This outcome suggests the residuals are stationary and the variables are cointegrated. Therefore the equation (5.9) can be consistently estimated by OLS. The estimated coefficients of the equation (5.9) are summarized in Table 12. To correct for heteroscedasticity and autocorrelation the HAC standard errors are used. The coefficient on the natural logarithm of the spot price is strongly significant and very close to unity which follows the model design. Nonetheless, the coefficient on the second independent variable, $r_t(T-t)$, is significant only on 10% level of significance and does not correspond to what the model suggests. The constant is close to zero and statistically significant at 5% level of significance. The F-test is applied to test the joint hypothesis $H_0 : \beta_0 = 0,\ \ \beta_1 = 1$ and $\beta_2 = 1$. The results in Table 13 show

[5] Based on consultations with the Finance and Capital Markets Department of IES.

that this hypothesis is strongly rejected which undermines the model design. Therefore it can be concluded that only the size and the direction of the effect of the spot price is as suggested by the model.

Table 11: ADF unit root and KPSS stationarity tests

		ADF	KPSS
$\ln F_t$	level	-2.1855	7.7871***
	Δ	-10.1999***	0.0570
$\ln S_t$	level	-2.1950	7.8044***
	Δ	-10.1280***	0.0568
$r_t(T-t)$	level	-5.8308***	8.6966***
	Δ	-10.3854***	0.0511
$\hat{\varepsilon}_t$	level	-22.9340***	0.1046

Source: Own calculations based on Thomson Reuters data, using R software.
Note: *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

Table 12: The cost of carry model regression estimation

	N	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	R^2
Model	1006	-0.0145**	1.0016***	2.9545*	0.99
		(0.0068)	(0.0007)	(1.5959)	

Source: Own calculations based on Thomson Reuters data, using R software.
Note: Figures in parentheses are HAC standard errors. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

Table 13: Joint hypothesis testing by application of F-test

	N	$\mathrm{df_{ur}}$	q	F	$p-val$
F test	1006	1003	3	14.13	0.0000

Source: Own calculations based on Thomson Reuters data, using R software.
Note: Testing $H_0 : \beta_0 = 0, \beta_1 = 1, \beta_2 = 1$ in the equation (5.9).

7.4 Basis convergence

The last part of the analysis was dedicated to the basis convergence. Figures 2-9 show the development of the daily closing spot and futures prices[6] over the life of a contract for index futures maturing in years 2015 and 2016. Obviously, it is not that straightforward to identify a decreasing manner of the basis in these graphs. In most cases the futures price follows the spot index value throughout the whole period and there is a little sign of the basis narrowing when the contract approaches its maturity. The narrowing tendency is best visible in contracts maturing in December 2015, March 2016 or September 2016. Figures 10-17 show the observed basis over the life of a contract and the difference between the spot and the futures price suggested by the model. Even though the basis appears to be decreasing over time in many cases it is subject to considerable fluctuations. Large fluctuations are noticeable for example in the June 2015, the September 2015 and the June 2016 contracts.

Table 14 provides summary statistics of the basis along with Table 22 in Apendix that shows the same for the basis in absolute value expressed as the percentage of the current spot index price. As already noted the basis of some contracts appears to be unstable with large span of values. Generally though the size of the fluctuations remain below 1% of the spot index price. In majority of cases the basis is negative meaning the futures price is above the spot price except the contract that matured in December 2016 where positive basis prevails.

The results of the estimation of the regression equation (5.16) are presented in Table 15. The beta coefficients representing the effect of the time trend, or more precisely the time to maturity of the contract in days, are all positive and significant indicating the basis increases with time to maturity as suggested by the model. The size of the effect generally ranges between 0.02 and 0.15 implying the basis increases by 0.02-0.15 index points with additional day to

[6]The last closing spot index price does not correspond to the intraday spot price, i.e. the final settlement futures price, in these graphs.

maturity.

Finally, the KPSS test results are provided in Table 16. In majority of the contracts the basis was not found to be trend stationary as indicated by low p-values. At 0.05 alpha level only the basis of the September 2014 and March 2016 are trend stationary followed by the March 2013 and March 2015 contracts where the basis appears to be fairly close to a trend stationary process. These results confirm the basis convergence evinces substantial fluctuations and the convergence process is not that smooth as one would expect.

As mentioned previously in the literature review, movements in the basis might be subject to other market factors as many researchers have suggested. The unstable and unpredictable basis represents the higher risk of losses and might undermine the efficiency of hedging strategies as explained in Sutcliffe (2006). It is always better for investors to know the regularities of the basis. Based on these facts the basis might be examined further to estimate the effects of spot market volatility, liquidity, interest rate and others, nevertheless, this is beyond the scope of this text.

Source: Own construction based on Thomson Reuters data.

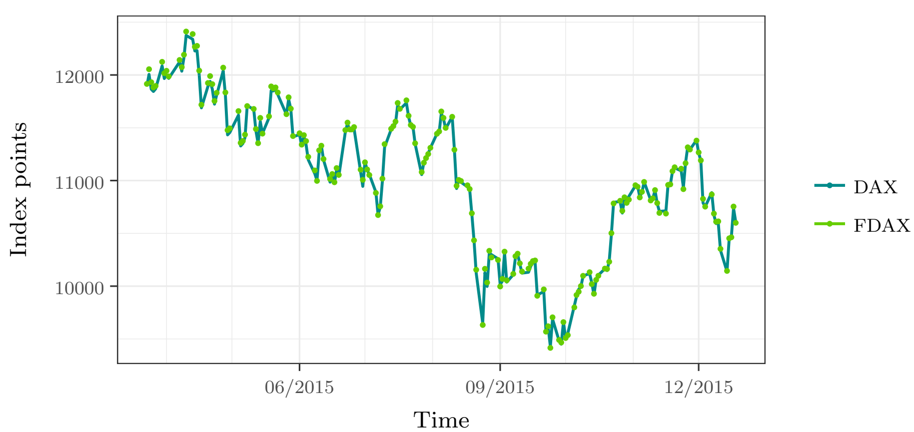
Figure 2: Basis convergence - March 2015 contract

Source: Own construction based on Thomson Reuters data.

Figure 3: Basis convergence - June 2015 contract

Source: Own construction based on Thomson Reuters data.

Figure 4: Basis convergence - September 2015 contract



Source: Own construction based on Thomson Reuters data.

Figure 5: Basis convergence - December 2015 contract

Source: Own construction based on Thomson Reuters data.

Figure 6: Basis convergence - March 2016 contract

Source: Own construction based on Thomson Reuters data.

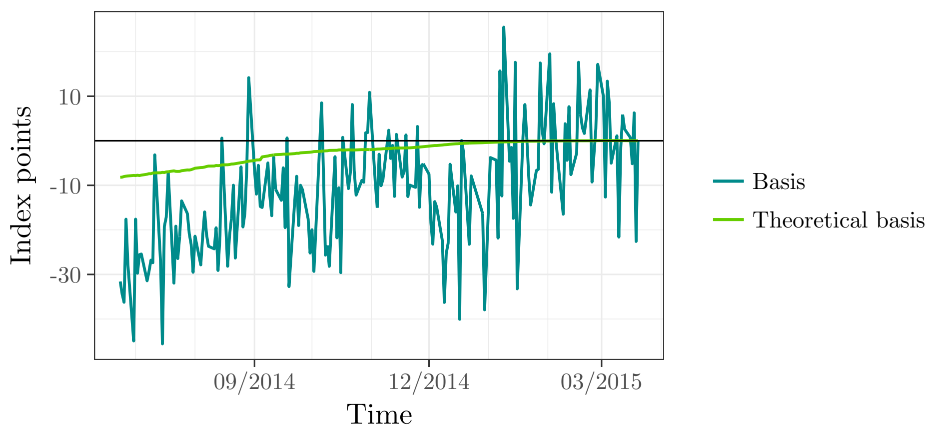
Figure 7: Basis convergence - June 2016 contract

Source: Own construction based on Thomson Reuters data.

Figure 8: Basis convergence - September 2016 contract

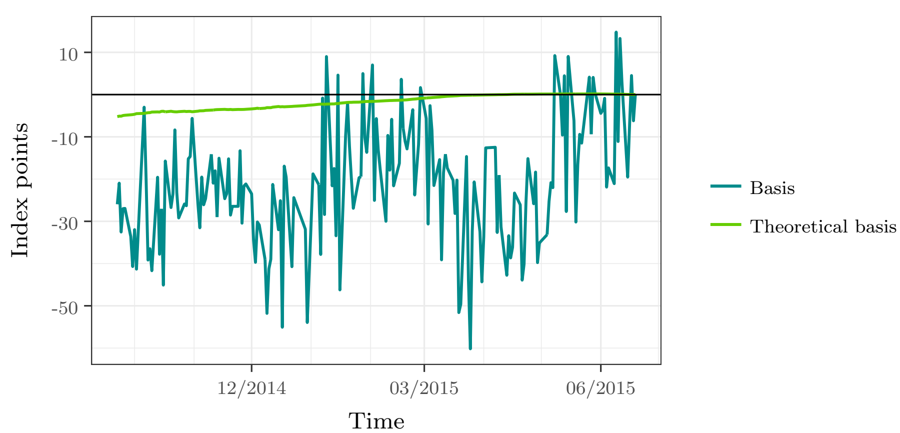
Source: Own construction based on Thomson Reuters data.

Figure 9: Basis convergence - December 2016 contract



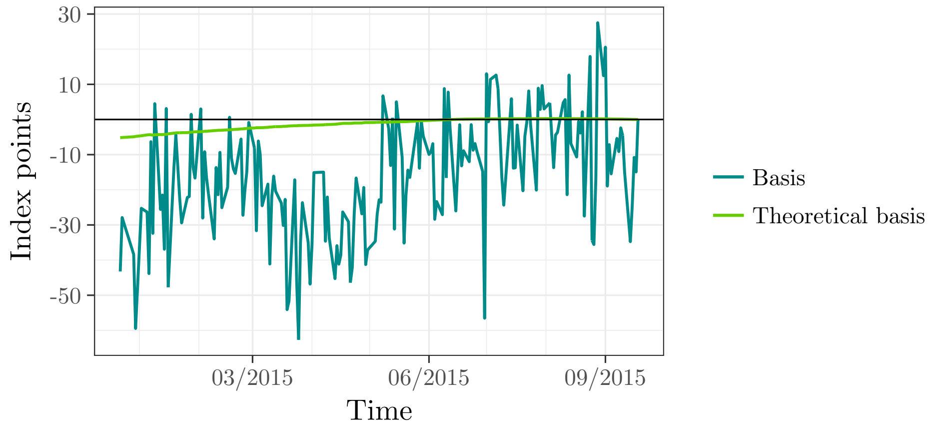
Source: Own construction based on Thomson Reuters data.

Figure 10: Basis - March 2015 contract



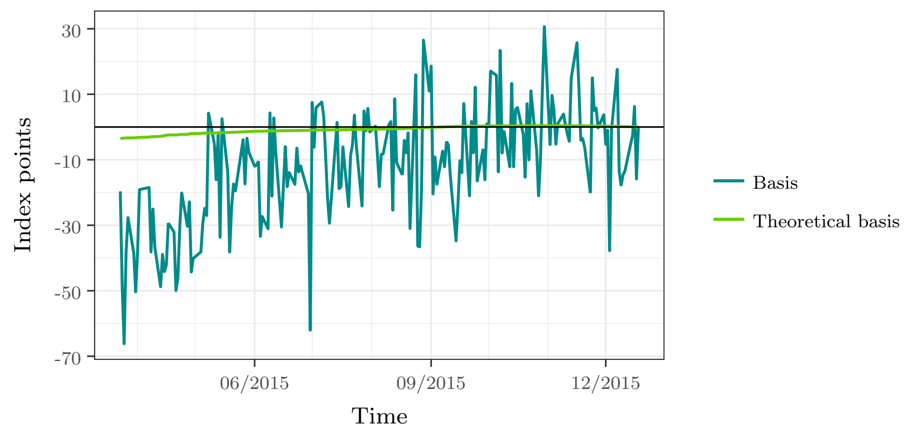
Source: Own construction based on Thomson Reuters data.

Figure 11: Basis - June 2015 contract



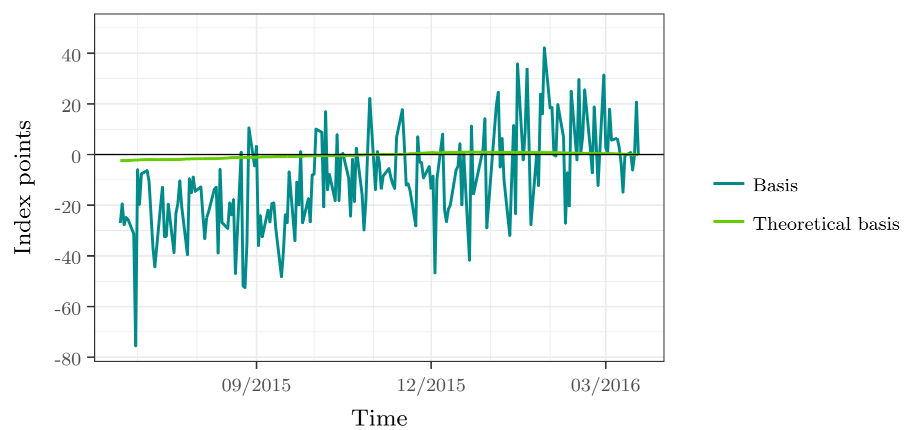
Source: Own construction based on Thomson Reuters data.

Figure 12: Basis - September 2015 contract



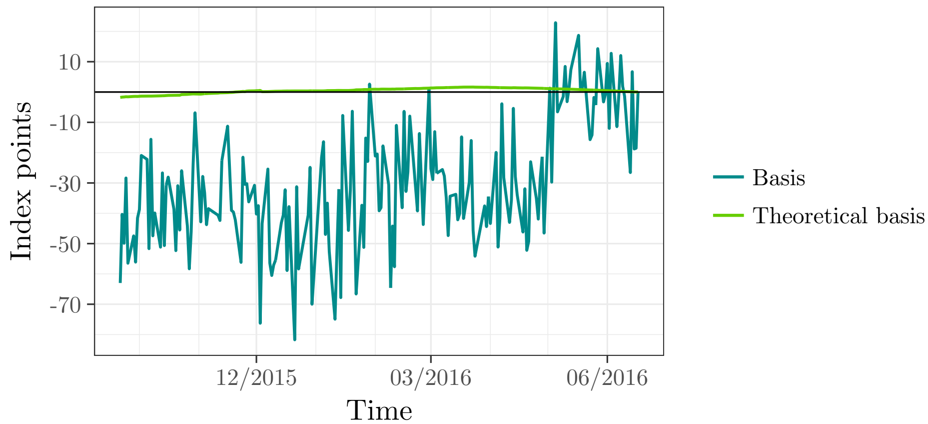
Source: Own construction based on Thomson Reuters data.

Figure 13: Basis - December 2015 contract



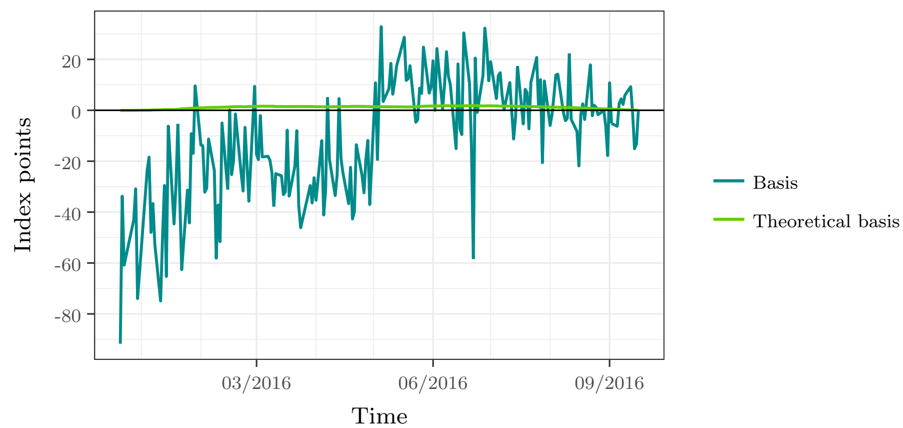
Source: Own construction based on Thomson Reuters data.

Figure 14: Basis - March 2016 contract



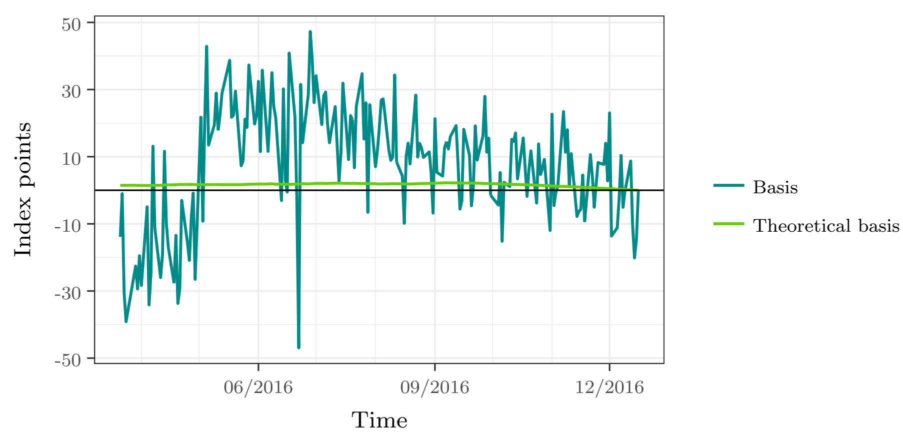
Source: Own construction based on Thomson Reuters data.

Figure 15: Basis - June 2016 contract



Source: Own construction based on Thomson Reuters data.

Figure 16: Basis - September 2016 contract



Source: Own construction based on Thomson Reuters data.

Figure 17: Basis - December 2016 contract

Table 14: Summary statistics of the basis

	N	Mean	SD	Max	Min	$Ba > 0$	$Ba < 0$
Mar 13	190	-5.02	7.09	10.14	-27.81	49	140
Jun 13	187	-10.97	7.83	16.70	-32.35	15	171
Sep 13	187	-10.00	11.25	26.47	-38.95	36	150
Dec 13	197	-9.16	11.17	19.47	-44.76	44	151
Mar 14	190	-11.66	10.59	23.08	-53.34	24	165
Jun 14	187	-20.92	13.44	11.60	-70.84	12	174
Sep 14	187	-16.83	16.37	29.67	-79.34	25	161
Dec 14	191	-11.41	13.71	25.17	-49.09	38	152
Mar 15	189	-11.05	13.47	25.50	-45.59	42	146
Jun 15	185	-21.07	14.56	14.79	-60.18	16	168
Sep 15	186	-16.03	16.63	27.53	-62.68	29	156
Dec 15	191	-11.61	17.37	30.64	-66.18	49	141
Mar 16	191	-10.03	18.70	42.11	-75.53	56	134
Jun 16	186	-30.05	20.41	22.86	-81.73	18	167
Sep 16	188	-10.80	23.07	32.86	-91.73	67	119
Dec 16	191	7.70	17.42	47.28	-46.94	132	58

Source: Own calculations based on Thomson Reuters data.
Note: Ba denotes the basis of a contract.

Table 15: Trend analysis of the basis

	$\hat{\alpha}$	$\hat{\beta}$	R^2
Mar 13	4.1605*** (0.7446)	0.0195*** (0.0047)	0.08
Jun 13	6.8573*** (0.8760)	0.0358*** (0.0056)	0.18
Sep 13	4.5519*** (0.9676_{HAC})	0.0620*** (0.0065_{HAC})	0.35
Dec 13	1.0122 (1.2642_{HAC})	0.0745*** (0.0093_{HAC})	0.43
Mar 14	7.8449*** (1.1156)	0.0403*** (0.0071)	0.14
Jun 14	7.4460*** (1.5137_{HAC})	0.1045*** (0.0127_{HAC})	0.45
Sep 14	-0.0609 (1.1919_{HAC})	0.1429*** (0.0093_{HAC})	0.61
Dec 14	1.9830 (1.6608_{HAC})	0.0918*** (0.0121_{HAC})	0.45
Mar 15	5.7571*** (1.1560_{HAC})	0.0616*** (0.0077_{HAC})	0.23
Jun 15	15.8372*** (2.5439_{HAC})	0.0465** (0.0136_{HAC})	0.08
Sep 15	10.4019*** (2.1866_{HAC})	0.0620*** (0.0158_{HAC})	0.12
Dec 15	4.3028** (2.0124_{HAC})	0.0887*** (0.0179_{HAC})	0.27
Mar 16	9.1878*** (1.7205)	0.0569*** (0.0109)	0.12
Jun 16	16.0539*** (2.5135_{HAC})	0.1145*** (0.0154_{HAC})	0.25
Sep 16	2.7660* (1.5191_{HAC})	0.1262*** (0.0145_{HAC})	0.36
Dec 16	7.1003*** (1.0579_{HAC})	0.0645*** (0.0087_{HAC})	0.22

Source: Own calculations based on Thomson Reuters data, using R software.

Note: Figures in parentheses are standard errors, where needed HAC standard errors are applied. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

Table 16: KPSS trend stationarity test for the basis

	KPSS	$p - val$
Mar 13	0.1462**	0.0499
Jun 13	0.2878***	<0.0100
Sep 13	0.1966**	0.0173
Dec 13	0.3679***	<0.0100
Mar 14	0.2737***	<0.0100
Jun 14	0.6268***	<0.0100
Sep 14	0.0982	>0.1000
Dec 14	0.3937***	<0.0100
Mar 15	0.1476**	0.0486
Jun 15	0.2024**	0.0151
Sep 15	0.2640***	<0.0100
Dec 15	0.3273***	<0.0100
Mar 16	0.0558	>0.1000
Jun 16	0.3564***	<0.0100
Sep 16	0.3966***	<0.0100
Dec 16	0.6322***	<0.0100

Source: Own calculations based on Thomson Reuters data, using R software.

Note: *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

8 Other relevant factors in stock index futures pricing

As already indicated in the theoretical and literature review sections of this thesis the cost of carry is based on rather strict assumptions and the pricing formula might lack some factors that may have an impact on the index futures price and explain further the mispricings of contracts. This fact raises questions about the size of the effect of violation of particular assumptions on the validity of the cost of carry and whether some modified, more appropriate relationship between the spot and futures index prices can be derived.

8.1 Different risk-free interest rates and transaction costs

It might be the case that an arbitrageur faces different riskless borrowing and lending rates of interest which contradicts the assumption of single interest rate. Moreover, arbitrageurs incur transaction costs and these might be different for going short and going long. Table 17 provides some examples of estimated round trip transaction costs that were acquired from Sutcliffe (2006). After allowing for different rates of interest and transaction costs the futures price will not be described by an equality but rather it will be allowed to fluctuate within a no-arbitrage band without creating index arbitrage opportunities. It can be assumed that the investor can borrow at r_b and lend money at r_a, where $r_a < r_b$. Furthermore, he or she incurs transaction costs $c_{S,l}$ when buying shares in the index, $c_{S,s}$ when shorting these shares, $c_{F,l}$ when going long in the futures contracts and $c_{F,s}$ when selling the futures. Transaction costs might differ between traders. The costs used for the no-arbitrage band are the ones of the marginal trader, usually the market makers in the underlying shares. Then the corresponding no-arbitrage band is as follows:

$$(S - c_{S,s} - c_{F,l})(1 + r_a - d)^t \;\leq\; F \;\leq\; (S + c_{S,l} + c_{F,s})(1 + r_b - d)^t, \quad (8.1)$$

where S is the spot price of the index, F is the futures price, d is the dividend yield over lifetime of futures contract as % of current index value and t is the time to maturity of the contract. The relevant arbitrage activities associated with the situation where the futures price is above the upper limit F_h, i.e. the right side of the equation (8.1) and when the futures price is below the lower limit F_l, i.e. the left side of the equation (8.1), after allowing for transaction costs are presented in Table 18. When being inside this band the futures price does not induce any arbitrage activities. Outside the no-arbitrage band the riskless arbitrage opportunities emerge.

As noted previously, in reality the derivation of the no-arbitrage window and implementation of the arbitrage itself is much more complicated.

Table 17: Round trip transaction costs for index arbitrage: some examples

Author	Index future	Round trip trans. costs
Billingsley & Chance (1988)	US futures	1%
Robertson (1990)	FTSE 100	1.85% (institutions) 0.90% (market makers)
Yau, Schneeweis & Yung (1990)	Hang Seng	1.96%
Liffe & LTOM (1991), Liffe (1991)	FTSE Eurotrack 100	1.67% (long in stocks, short in futures)
		2.42% (short in stocks, long in futures)
Chung, Kang & Rhee (1994)	Nikkei St. Av. (Osaka)	2.7% (institutions) 0.8% brokers

Source: Sutcliffe (2006).
Note: Costs expressed as percentage of the index spot price.

Table 18: Arbitrage with different risk-free rates of interest and transaction costs

	$F < F_l$		$F > F_h$	
	Position	Cashflow	Position	Cashflow
1	Buying futures	$-c_{F,l}$	Selling futures	$-c_{F,s}$
2	Shorting stocks	$S - c_{S,s}$	Borrowing spot price at r_b	$S + c_{F,s} + c_{S,l}$
3	Lending money at r_a	$-(S - c_{S,s} - c_{F,l})$	Buying stocks	$-(S + c_{F,s} + c_{S,l})$
4	Taking delivery of futures	$-F$	Delivering futures	F
5	Returning borrowed stocks + dividens	$-S((1+d)^t - 1)$	Paying back loan	$-(S + c_{F,s} + c_{S,l})(1 + r_b)^t$
6	Collecting loan	$(S - c_{S,s} - c_{F,l})(1 + r_a)^t$	Receiving dividends	$S((1+d)^t - 1)$
Sum		$(S - c_{S,s} - c_{F,l})(1 + r_a - d)^t - F > 0$		$F - (S + c_{F,s} + c_{S,l})(1 + r_b - d)^t > 0$

Source: Own construction based on Sutcliffe (2006)

8.2 Taxes

Another parameter that the standard cost of carry formula is short of appears to be the taxation of dividends. While investigating German futures market, in particular the DAX futures prices, Fink & Theissen (2014) derived a modified cost of carry pricing formula that corrects for the missing tax effect.

The model of Fink & Theissen (2014) defines r as the interest expenditure and D as the dividend payments in the period of the time to maturity, $(T-t)$, s_k denotes the tax on all capital gains and losses from stock and futures transactions, s_z is the tax levied on the interest income and expenses, and finally the dividends are taxed with the rate of s_d. For the sake of simplification, a symmetric taxation of gains and losses is assumed. The formula for the theoretical futures price is obtained based on the no-arbitrage cost of carry argument. Assuming an arbitrage that involves a long position in the stock index and a short position in the corresponding futures, first an investment into a portfolio of stocks, S_0, that replicates the index is required. This is financed with a credit at the beginning of the arbitrage activity. Simultaneously the equivalent amount of stock index futures contracts is shorted. According to Fink & Theissen (2014) the dividends that are paid by the index stocks until expiration cause tax payments to differ both at the index level and the personal portfolio level. As a result these differences have to be financed with supplementary credits. At maturity the following needs to be paid: capital gains tax on the gains and losses of the index stocks portfolio, $(1-s_k)$, taxes on the profit of the future contract, $(F-S_T)$, at the rate of s_k, taxes on the reinvested dividends at the rate of s_d, finally the repayment of the loan on the index stocks as well as the supplementary loans on dividend adjustment costs. From these payments the investor's interest tax credit at the rate of s_z is subtracted. The cash flows are exactly zero both after the transactions at the beginning and at the end of the arbitrage activity. After adding the cash flows at maturity and solving for the futures price, F, the following cost of carry formula that considers the effects of taxes is obtained[7]:

$$F = S_0\left(1 + r\frac{1 - s_z}{1 - s_k}\right) - [s_k - s_d - r(T - t)s_d(1 - s_z)]\frac{1}{1 - s_k}D. \quad (8.2)$$

Empirical results from Fink & Theissen (2014) confirmed that the changes in the dividend taxation had an effect on the stock index futures prices. Hence, taxes represent another challenge to the validity of the simple cost of carry model.

8.3 Hemler & Longstaff model with market volatility and stochastic interest rate

Hemler & Longstaff (1991) proposed other forces that may play a role in stock index futures pricing and therefore contradict the standard no-arbitrage formula. Based on empirical studies, their general equilibrium model, which incorporates market volatility and stochastic interest rate may account for mispricings indicated by the cost of carry model and is supposed to be more efficient especially in highly volatile periods.

The model involves fitting the following regression equation:

$$L_t = \alpha + \beta_1 r_t + \beta_2 V_t + \varepsilon_t, \quad (8.3)$$

where $L_t = \ln \frac{F_t e^{q(T-t)}}{S_t}$ is the natural logarithm of the dividend-adjusted futures/spot price ratio, S_t is the value of the underlying stock index, F_t denotes the theoretical futures price of interest, r_t is the risk-free rate of interest, V_t denotes the market volatility, q is the constant annual dividend yield, $T - t$ is the time to maturity, α, β_1 and β_2 are the coefficients of the regression equation (8.3), and ε_t is normally distributed error term with zero mean. The estimated coefficients are then substituted to the general equilibrium model to predict the dividend-adjusted futures/spot price ratio, L_t, from which the theoretical futures price, F_t, can be subsequently obtained. Hence the stock index futures price depends on the volatility of returns on the index, the spot index price, risk-free rate of interest, dividend yield, and time to maturity. Whilst remaining in this framework, the cost of carry

[7] The detailed arbitrage table and calculations can be found in Fink & Theissen (2014).

model predicts $\alpha = \beta_2 = 0$ and $\beta_1 = T - t$ which is the time to maturity of the contract.

The only parameter in the Hemler and Longstaff model that can't be directly observed is the volatility of the index returns, V_t. To estimate the variance of the index returns, commonly the equally weighted moving average method is applied to the past spot index returns[8].

8.4 Hsu & Wang model with the degree of market imperfections

As was already sketched in the literature review part, Hsu & Wang (2004) introduced the stock index futures pricing model with the concept of the degree of market imperfections.

The imperfect market model considers a hedged portfolio P that is made up of one unit of spot index and x units of futures contracts while assuming there is initially no requirement of cash outflow for futures contracts and that the underlying stock index price follows a geometric Wiener process. Then the rate of return of the hedged portfolio, P, is given as follows:

$$\frac{dP}{P} = (w_f u_f + u)dt + (w_f \sigma_f + \sigma)dZ, \tag{8.4}$$

where $w_f = \frac{xF}{S}$, F is the index futures price, S is the spot price of the index, u and σ are the constant expected growth rate and the constant volatility of the stock index, respectively, u_f and σ_f are the instantaneous expected return on futures and the instantaneous standard deviation of return on futures, respectively, and finally dZ denotes the geometric Wiener process. If $w_f = -\frac{\sigma}{\sigma_f}$ then $w_f \sigma_f + \sigma = 0$ in the equation (8.4) and this would mean that the return of portfolio P is certain and therefore the hedged portfolio is risk-free. However, to keep it riskless w_f needs to be continuously rebalanced until the expiration of the contract. Figlewski (1989) argues that building risk-free portfolio and continuously rebalancing the positions is possible only in perfect markets unlike in imperfect markets where portfolio that can't be riskless earns some expected rate of return that might be different from the risk-free rate.

[8] For more details, see Manu & Narayana (2015).

Then the authors obtain u_p, the instantaneous expected rate of return of the hedged portfolio, and σ_p, the coefficient of Wiener process dZ in the equation (8.4) as

$$w_f u_f + u = u_p, \tag{8.5}$$

$$w_f \sigma_f + \sigma = \sigma_p. \tag{8.6}$$

From the equations (8.5) and (8.6) a partial differential equation is obtained as follows:

$$\frac{1}{2}\sigma^2 S^2 F_{ss} + u_\alpha S F_s + F_t = 0, \tag{8.7}$$

where $u_\alpha = [(u_p - q) - (u - q)\frac{\sigma_p}{\sigma}]/(1 - \frac{\sigma_p}{\sigma})$ is the price expectation parameter with the component $\frac{\sigma_p}{\sigma}$ reflecting all the effects of market imperfections, in other words, the degree of market imperfections. Finally the solution of the partial differential equation (8.7) is given by[9]

$$F_t = S_t e^{u_\alpha (T-t)} \tag{8.8}$$

which is the Hsu and Wang imperfect market pricing model. This model assumes a continuous constant dividend yield q paid during the life of the contract. However, it can be modified in terms of the index that pays irregular lumpy dividends. If the capital market is perfect then $\sigma_p = 0$ and $u_p = r$ which corresponds to the cost of carry model.

The only parameter in the Hsu and Wang pricing model that needs to be estimated is the price expectation parameter, u_α. Hsu & Wang (2004) proposed the implied $u_{\alpha,t-1}$ as follows:

$$u_{\alpha,t-1} = \frac{1}{T - (t-1)} \ln \frac{F_{t-1}}{S_{t-1}}. \tag{8.9}$$

8.5 Some factors that proved to be unimportant

The cost of carry model assumes that the dividend payments from the underlying stocks that took place until the expiration of the contract are known. However, the dividends might not be certain. Yadav & Pope (1990) examined this assumption on the FTSE 100 index and found that a 50%

[9] For more detailed derivation of the model, see Hsu & Wang (2004).

variation in dividends led to a change of only 0.3% in the no-arbitrage futures price. Yadav & Pope (1994) examined the effects of dividend risk in terms of both the size of the dividend and the payment date. They concluded that the estimate of dividends used in the price calculation does not have any significant effect and that the dividend certainty assumption is not that important. For performance indices, such as the DAX index, Bühler & Kempf (1995) concluded that there is no dividend risk.

Another assumption of the standard pricing model is that assets can be perfectly divided. When constructing an arbitrage transaction with the number of futures exactly offsetting the basket of shares in the index, it might be needed to hold shares or futures in fractional quantities which is not possible. This problem tends to be smaller if each futures contract has a small nominal value and the total value of the arbitrage transaction is large. Nontheless, according to Sutcliffe (2006) the effect of the violation of the perfect asset divisibility assumption in real capital markets does not have any major impact and can be omitted.

The cost of carry pricing formula applies rather to forward than futures contracts since it does not allow for marking to the market. Futures positions are settled on a daily basis and traders' accounts are adjusted accordingly which is not the case of forwards. Cox et al. (1981) suggested that if riskless interest rates are certain futures and forwards prices are identical. Therefore the prices will not be exactly equal if interest rates are stochastic. Nonetheless, many economic studies concluded that the difference between the prices of forwards and futures is economically insignificant as per Sutcliffe (2006).

There are other factors that might influence the stock index futures price and therefore disrupt the no-arbitrage argument. More delicate ones concerning initial margin, index weights, proceeds on short sales or risk of default can be found in Sutcliffe (2006). Empirically it was found that deviations from the perfect markets assumptions are rather of little consequence or when needed the new relationship between futures and spot index prices can be derived.

9 Conclusion

The aim of the thesis was to examine the pricing efficiency of the cost of carry model and its applicability on the German performance index DAX and related stock index futures contracts covering the period 2012-2016. The work investigated the suitability of the theoretical model confronted by the real behaviour of futures prices while complementing the literature with more updated dataset.

The analysis showed that the DAX futures contracts exhibit minor mispricing. While there are many deviations from the theoretical fair values they tend to remain in the range of -0.4-0.4%. The standard t-test and the non-parametric Wilcoxon signed-rank test confirmed that the mean mispricing is not significantly different from zero in most cases of examined contracts. Given the small magnitude of the deviations it can therefore be concluded that the futures contracts are efficiently priced and any index arbitrage activity is rather rare. Further to this, the June contracts appear to have a special position since the size and the occurrence of the June contracts' mispricing was found to be larger when compared with other maturities. This phenomenon can be related to the dividend payments that accumulate between April and June.

The ADF and the KPSS tests confirmed that futures prices and spot index values follow the $I(0)$ process and that they are cointegrated. Regression analysis showed that except the effect of spot index the variables do not entirely follow the model suggestions implying the cost of carry can only provide an approximation of the futures price.

Regarding the basis convergence, it was identified that there is a decreasing tendency in the basis towards a contract's expiration and that the futures and spot prices merge automatically at maturity. Nevertheless, even though the size of the basis remains low relative to the spot index price, it is subject to many fluctuations. This was confirmed by the KPSS test that failed to identify a trend stationarity in the basis.

The thesis also introduced some pricing models that do not demand some of the restrictive assumptions of the cost of carry and summarized some factors which empirically proved to be relevant with the mention of the assumptions of perfect markets which in practice turned out to be negligible. Minor discrepancies in German markets can be explained as most likely by transaction costs and taxation of dividends.

The work suffers from some limitations. First, the exact round-trip transaction costs of DAX index arbitrage are not provided hence, despite the low magnitude of mispricing, the results should be interpreted with caution. Secondly, as a proxy for risk-free rate the EURIBOR rates were used which might not apply to all investors. Another proxy which can be employed is an overnight index swap (OIS) rate for shorter maturities of futures contracts. Lastly, as the work uses closing prices some discrepancies may arise because of non-contemporaneous trading.

The research of stock index futures pricing can be extended further. First, the exact round-trip transaction costs for the DAX index arbitrage could be estimated. To infer if the mispricing can actually trigger some arbitrage activity it would be beneficial to determine the lower and upper bound of the no-arbitrage window. Furthermore, as previously suggested, the movements in the basis might be investigated with respect to market forces to support hedging strategies.

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Appendix

Table 19: Asymptotic critical values for unit root T test

Significance level	1%	5%	10%
Model with constant and no trend	-3.43	-2.86	-2.57
Model with constant and trend	-3.96	-3.41	-3.12

Source: Banerjee et al. (1993).

Table 20: KPSS test critical values for η_μ and η_τ

Significance level	1%	5%	10%
η_μ	0.739	0.463	0.347
η_τ	0.216	0.146	0.119

Source: Kwiatkowski et al. (1992).
Note: η_μ and η_τ denote statistics for level and trend stationarity, respectively.

Table 21: Asymptotic critical values for cointegration testing

Significance level	1%	5%	10%
Critical value	-3.90	-3.34	-3.04

Source: Davidson, MacKinnon et al. (1993).

Table 22: Summary statistics of the basis as percentage of current spot index value

	Mean	SD	Max	Min		Mean	SD	Max	Min
Mar 13	0.10	0.08	0.44	0.00	**Mar 15**	0.15	0.11	0.47	0.00
Jun 13	0.15	0.09	0.45	0.00	**Jun 15**	0.21	0.13	0.58	0.00
Sep 13	0.16	0.10	0.50	0.00	**Sep 15**	0.17	0.12	0.61	0.00
Dec 13	0.14	0.12	0.57	0.00	**Dec 15**	0.14	0.12	0.57	0.00
Mar 14	0.15	0.10	0.56	0.00	**Mar 16**	0.16	0.12	0.69	0.00
Jun 14	0.23	0.14	0.74	0.00	**Jun 16**	0.31	0.18	0.78	0.00
Sep 14	0.20	0.15	0.83	0.00	**Sep 16**	0.20	0.17	0.87	0.00
Dec 14	0.15	0.11	0.51	0.00	**Dec 16**	0.15	0.11	0.50	0.00

Source: Own calculations based on Thomson Reuters data.