CHARLES UNIVERSITY

FACULTY OF SOCIAL SCIENCES

Institute of Economic Studies

Bachelor thesis

2017 **Jakub Šedivý**

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Pairs trading at CEE markets

Bachelor thesis

Prague 2017

Author: Jakub Šedivý
Supervisor: Ing. Aleš Maršál

Academic Year: 2016/2017

Bibliographic note

ŠEDIVÝ, Jakub. *Pairs trading at CEE markets.* Prague 2017. 39 pp. Bachelor thesis (Bc.) Charles University, Faculty of Social Sciences, Institute of Economic Studies. Thesis supervisor Ing. Aleš Maršál

Abstract

We investigate the use of investment strategy called pairs trading on small-sized equity markets located in Central Eastern Europe. Pairs trading is self-financing trading strategy that identifies two stocks based on their historical relationship, and makes profit on their short-term relative mispricing, since the strategy relies on their convergence into the long-term equilibrium. The objective of this thesis is to compare two different methods of pairs trading, distance method based on minimizing the sum of squared deviations between normalized historical prices and cointegration method using daily data from June 2008 to March 2017. We examine whether any of those method is profitable on Prague Stock Exchange, Bucharest Stock Exchange and Budapest Stock Exchange and can be used on such markets with high industry diversity. Our findings were not stastically different from zero in all but one case and majority of average returns was negative. In comparison to US and Finnish equity markets the strategy falls behind. Even though we identified some cointegrated pairs, their profitability was more than questionable and further investigation showed that small equity markets such as the ones we have studied are not a good fit for pairs trading strategy.

Abstrakt

Zabýváme se využitím investiční strategie nazývané párové obchodování na malých kapitálových trzích ve střední a východní Evropě. Párové obchodování je samostatně financovaná obchodní strategie, která identifikuje dvě akcie na základě jejich dlouhodobého vztahu a vydělává na jejich krátkodobém vychýlení z relativního ocenění, jelikož spoléhá na jejich zpětnou konvergenci do dlouhodobého ekvilibria. Cílem této práce je porovnat dvě různé metody párového obchodování. Jde o metodu vzdáleností založenou na minimalizování součtu čtvercových odchylek mezi normalizovanými historickými cenami a kointegrační metodu pomocí denních dat od června 2008 do března 2017. Zkoumáme, zdali je některá z těchto metod výnosná na Pražské, Bukurešťské nebo Budapešťské burze cenných papírů a může se použít na trzích s tak vysokou různorodostí průmyslových odvětví. Naše výsledky nebyly statisticky rozdílné od nuly v žádném, až na jeden případ, a většina průměrných výnosů byla negativní. I přesto, že jsme identifikovali některé kointegrované páry, jejich výnosnost byla více než pochybná a další zkoumání ukázala, že malé kapitálové trhy jako ty, které jsme analyzoval,i nejsou vhodné pro párové obchodování.

Keywords

Pairs trading, cointegration, mean-reversion, statistical arbitrage, spread, Prague Stock Exchange, Budapest Stock Exchange, Bucharest Stock Exchange, algorithmic trading.

Klíčová slova

Párové obchodování, kointegrace, návrat ke střední hodnotě, statistická arbitráž, rozpětí, Burza cenných papírů Praha, Burza cenných papírů Budapešť, Burza cenných papírů Bukurešť, algoritmické obchodování.

Declaration of Authorship

I hereby proclaim that I wrote my bachelor thesis on my own under the leadership of my supervisor and that the references include all resources and literature I have used.

I grant a permission to reproduce and to distribute copies of this thesis document in whole or in part.

Prague, 30 July 2017

Jakub Šedivý

Acknowledgment

I would like to express my gratitude to my supervisor Ing. Aleš Maršál for his helpful advice and comments. His guidance helped me in all the time of research and writing of this thesis.

I also thank my fellow classmates for their help and cooperation during all the exhaustive home assignments and school projects and mental breakdowns we have experienced together.

Last but not least I would like to thank my parents and friends for the unceasin encouragement, support and attention they gave me throughout my studies.

Bachelor's Thesis Proposal

Institute of Economic Studies
Faculty of Social Sciences
Charles University in Prague

Author's name and surname: Jakub Šedivý

E-mail: 54066942@fsv.cuni.cz

Phone: +420 605 450 230

Supervisor's name: Aleš Maršál

Supervisor's email: ales.marsal@seznam.cz

Proposed Topic:

Pairs trading in CEE

Preliminary scope of work:

Research question and motivation

This thesis should familiarize reader with pairs trading and its methodologies and different approaches to pairs trading. I will try to identify which pairs of stocks are cointegrated in CEE region and develop adequate strategy to enter and leave the position. The computations will be based on the recent data from Bloomberg. Thesis should serve as useful tool for reader to show him possible ways of pairs trading step by step, because nowadays it is much more common to see algorithmic trading on stock market and therefore it should be important to increase awareness of this topic.

Contribution

Pairs trading at the Prague Stock Exchange has been already examined (11. in bibliography) with findings, that profits from this type of trading are not statistically significant from zero. The results were compared to US equity market. However this work will try to extend those findings with hopefully different pairs of stocks and another point of view across whole CEE, not only on PSE. The output of this thesis could be used to gain basic knowledge of trading on the stock and it should serve as an example of possible methods using pairs trading.

Methodology

Research will be based on the data downloaded from Bloomberg, which will be analysed later by using MATLAB software and creating my own algorithmic trading strategy.

Outline

1. Introduction
2. Pairs trading and its methodologies
3. Data analysis (interpretation of the data and introducing the developed model)
4. Discussion of results
5. Conclusion

List of academic literature:

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Author **Supervisor**

List of Acronyms

ADF Augmented Dickey-Fuller

AIC Akaike Information Criterion

AR(1) Autoregressive Process of Order 1

BIC Bayesian Information Criterion

BSE Budapest Stock Exchange

BVB Bucharest Stock Exchange

ECM ECM Real Estate Investments Ag SA

KB Komerční Banka as

NYSE New York Stock Exchange

OLS Ordinary Least Squares

PSE Prague Stock Exchange

REIT Real Estate Investment Trust

SSD Sum of Squared Deviations

US United States

VIG Vienna Insurance Group AG Wiener Versicherung Gruppe

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1 Introduction

Pairs trading, one of the popular trading strategies, has been an important driver behind trading since 1980s, when it was introduced by Nunzio Tartaglia and his quant group in Morgan Stanley (Vidyamurthy, 2004). The fundamental idea of pairs trading is very simple, we need to find two stocks whose prices have some historic relationship, and when the spread between them widens by larger price than usual, we sell (short) the winner and buy (long) the loser (Mori and Ziobrowski, 2011). The strategy relies on reverting behavior of the stocks into the original equilibrium, since they have mentioned relationship, and the investor will profit from returning into the original position. The profits from such trading depend on potential mispricing and the time it takes to revert into the equilibrium. The larger the mispricing the greater the potential return, and when this event occurs multiple times, the more opportunities for the investor.

Since the birth of pairs trading, various quantitative methods have been developed and used to pairs trading in the existing literature. Three commonly used methods are: distance method, cointegration and stochastic spread method. Regardless of their wide use among investors and hedge funds on Wall Street, the literature on this topic has attracted much less attention. Nonetheless, there are few empirical studies which examine the pairs trading and its effectiveness on stock exhanges. Till today, the distance-approach study written by Gatev, Goetzmann and Rouwenhorst (2006) has been broadly used as a sample for number of empirical studies. There exist couple of more studies, however most of them follow Gatev, Goetzmann and Rouwenhorst (2006).

Contribution of this thesis is to analyse whether any of the methods of distance and cointegration is profitable or even usable at Prague Stock Exchange, Budapest Stock Exchange or Bucharest Stock Exchange. The distance method mainly follows Gatev, Goetzmann and Rouwenhorst (2006) which chooses a pair by finding the securitites that minimizes the sum of squared deviations between the two normalized price series. Cointegration method is based on the paper of Engle and Granger (1987). We identify which method earns better results and if the trading could be used in those markets from standpoint of reward and risk. We compare our results with Gatev, Goetzmann and Rouwenhorst (2006) in order to assess the importance of the number of shares listed, investor's attention which is linked to the size of the equity market and other factors which might affect earning distribution.

This bachelor thesis is organized as follows. Section 2 provides literature review. Section 3 describes the data and construction of periods which we use in the empirical part. Following section summarizes the theory that is necessary for cointegration approach. Next section focuses on methodology of constructing pairs, calculating returns and constructing the information ratio. Section 6 states the possible outcomes of empirical analysis which we could obtain. Following section provides summary of empirical results and their possible explanation, and comparison among single markets together with differences with US presented by Gatev, Goetzmann and Rouwenhorst (2006). The last section provides the conclusion of the thesis.

2 Literature Review

First work that empirically tested the pairs trading strategy is the one written by Gatev, Goetzmann and Rouwenhorst (1999). They used the sum of squared deviations (SSD) strategy between normalized historical prices to form pairs. They also bootstrapped random pairs in order to distinguish pairs trading from pure mean-revesion strategies. They wanted to show, that simple mean-reversion is not the only driving force of the profits. If it was, it would suffice to match pairs randomly and it would still generate positive returns. This was not the case and consequently, study found an alternative way to explain profits of the strategy. Excess returns indicate that pairs trading profits from temporary mispricing of close substitutes. The authors wrote the extension of the paper (Gatev, Goetzmann and Rouwenhorst, 2006) where they extended the testing period by five years. Paper rules out several explanations for the pairs trading profits, including mean reversion, unrealized bankruptcy risk, and the inability of arbitrageurs to take advantage of the profits because of short-sale constraints. It also discovered decreasing returns in pairs trading in the latest years of the study which are explained by increased hedge fund activity, or by the alternative view, that the abnormal returns are a compensation to arbitrageurs for enforcing the "Law of One Price".

During those couple of years another authors published their theoretical works with different approaches to pairs trading. Vidyamurthy (2004) developed a framework for forecasting using the cointegration method and analysed the mean reversion of the residuals. Elliott, Hoek and Malcolm (2005) provided an analytical framework for stochastic spread method by proposing a mean-reverting Gaussian Markov chain model for the spread which is observed in Gaussian noise. These two approaches, together with SSD approach presented by Gatev, Goetzmann and Rouwenhorst (2006) are now main methods used by practitioners. A few years later, Zeng and Lee (2014) compared cointegration method and SSD method from the perspective of profitability, and found out that cointegration method is slightly more profitable than SSD method even with increasing transaction costs. They also invented their own optimal rule which outperforms those two methods. However, their work is limited, because they assumed model parameters to be constant, which is unrealistic in the long-run. The same result has been obtained by Huck and Afawubo (2015), they also compared different triggers, like two and three standard deviations to initiate the trade.

Nevertheless, these studies (with exception of Zeng and Lee (2014)) do not include empirical evidence, and even Zeng's and Lee's (2014) empirical part is limited. Since Gatev, Goetzmann and Rouwenhorst (2006) several works studied pairs trading on US markets, such as paper written by Mori and Ziobrowski (2011) which compares pairs trading on New York Stock Exchange (NYSE) and U.S. Real Estate Investment Trust (REIT). They discovered that trading with REIT results in larger profits accompanied with smaller risk, which is caused by presence of more good candidates for pairs trading in the REIT market with more close substitutes due to relatively high homogeneity among REIT stocks.

Another research on pairs trading profitability was written by Jacobs and Weber (2015) where they investigate what drives pairs trading profitability. They performed an empirical analysis of several world markets to identify that abnormal returns are larger in countries with higher average idiosyncratic volatility, as well as in countries with large stock markets relative to their economic size. Their findings also indicate that the type of news leading to pair divergence, the dynamics of investor attention as well as the dynamics of limits to arbitrage are important drivers of the stategy's time-varying performance. Next paper commenting profitability of pairs trading is from Do and Faff (2010), in which they provide evidence of declining profits just as Gatev, Goetzmann and Rouwenhorst (2006). They also proposed additional metrics to form pairs to achieve higher profitability - incorporating industry homogeneity and frequency of historical reversal in the price spread. Papadakis and Wysocki (2007) discovered that pairs trades are frequently triggered around financial announcements and these announcements negatively affect the profitability of pairs trading in comparison to trades that were triggered other time. There needs to be mention that all of these works used one year formation period, which might affect profitability too (Huck, 2013).

Unfortunately, so far only few published papers replicated and tested SSD method outside the US market. Broussard and Vaihekoski (2012) empirically tested pairs trading on the Finnish stock market and found out that pairs trading is there even more profitable than in the US. However, they argue that majority of their pairs portfolio is represented by multiple share classes of the same stock, while Gatev, Goetzmann and Rouwenhorst (2006) used pairs formed by stocks of different companies. Lei and Xu (2015) tested the cointegration method with dual-listed Chinese shares.[1]They found that cointegration approach generally outperforms the benchmark standard deviation approach, just as Zeng and Lee (2014) did. Another works considering different markets than US are limited just to thesis and non-published works.

[1]Shares were listed on both share markets in China, Shanghai and Hong-Kong.

3 Data

We use a daily data from Thomson Reuters Wealth Manager for the Prague Stock Exchange (PSE), Budapest Stock Exchange (BSE) and Bucharest Stock Exchange (BVB) from the beginning of June 2008 till the end of March 2017. The number of stocks listed on the PSE varied between 10 and 13, on the BSE between 12 and 17 and on the BVB between 12 and 17, respectively, during the sample period. The number of pairs that could be potentially created rises quadratically and is computed by following formula

$$P_N = \binom{N}{2} = \frac{N!}{2!(N-2)!}$$

where N is the total number of stocks listed and suitable for trading in certain period. The range of number of pairs formed is between 45 and 136, which compared to Gatev, Goetzmann and Rouwenhorst (2006) or Broussard and Vaihekoski (2012) are really low numbers and might limit our computations.

Similar to Gatev, Goetzmann and Rouwenhorst (2006), our implemetation of the pairs trading strategy proceeds in two stages. First, pairs of the stocks are chosen for trading using the formation period. Second, trades are made on the pairs during the trading period which follows the formation period. While Gatev, Goetzmann and Rouwenhorst (2006) use twelve month period of data for formation and six month period for trading, we will use fifteen month period for formation and six month period for trading. We use fifteen months instead of twelve, because together with SSD method we use cointegration method, which requires longer period for more accurate identification of cointegrated pairs (Hakkio and Rush, 1991). Also, Gatev, Goetzmann and Rouwenhorst (2006) used time periods which were rolled forward by one month, we roll periods forward by six months. The trading period of six months is chosen so that the selection process is recent and round trips have time to occur using a reasonable opening trigger.

Structures of our Stock Exchanges are extremely diversified with following industry groups:[2] Banks, Capital Goods, Collective Investments, Com-

[2]Industry groups are taken from Thomson Reuters Wealth Manager.

mercial & Professional Services, Consumer Durables & Apparel, Consumer Services, Diversified Financials, Energy, Food, Beverage & Tobacco, Insurance, Materials, Media, Pharmaceuticals, Biotechnology & Life Sciences, Real Estate, Telecommunication Services and Utilities. As Stock Exchanges we study rank among quite small equity markets, we cannot compose pairs mainly from the same industry group, as is common and proven practice. Do and Faff (2010) show that industry homogeneity brings higher profits, thus our findings might be affected by this reality.

Further, we define two restrictions on stocks to incorporate them in the empirical analysis:

1. Each stock must be listed at least during the whole formation period and all over the following trading period, i.e. a whole 21-month period.
2. Each stock is traded on every business day in order to avoid illiquidity and therefore biased results.

Imposing of these criteria results in not incorporating some of our data. For example, criterion 1 is not met by Kofola CeskoSlovensko as, Moneta Money Bank as on PSE or Fondul Proprietatea on BVB.[3] The first two mentioned do not fulfill criterion 1 nor for single period, therefore we cannot include them in our study, while the Fondul Proprietatea does not fulfill criterion 1 for the first five periods, for the rest of our analysis we can include this stock without problems. Criterion 2 is not met by the only stock, the ECM, that was not traded on 15 June 2011 and since 21 June 2011 while it was yet listed on the PSE. Some stocks were traded in single units during certain dates, which might signal illiquidity as well, however we stated criterion 2 for zero trades. The exact dates we used for the formation period and for the trading period together with the number of stocks for each equity market and several other informations are in Table 10. The names of the companies listed on PSE, BSE and BVB together with the sample periods in which they were used can be found in Tables 7, 8 and 9.

[3] These are not the only stocks that do not fulfill criterion 1.

4 Theory

For us to be fully able to understand cointegration method, we need to clarify some key properties of time series data. We need to define some basic features such as covariance stationarity, integration and cointegration. Also, we present test for stationarity and show what problems we can obtain if the test is positive. Finally, we describe the approach of Engle and Granger (1987).

4.1 Covariance Stationarity

A stationary time series process is one whose probability distributions are stable over time in the following sense: If we take any collection of random variables in the sequence and then shift that sequence ahead h time periods, the joint probability distribution must remain unchanged (Wooldridge, 2015). Covariance stationarity is weaker form of this process, however, it is fully adequate for our intentions. A stochastic process $\{x_t := 1, 2, \dots\}$ with finite second moment $[E(x_t^2) < \infty]$ is covariance stationary if its mean is constant across time (1), its variance is constant across time (2), and the covariance between x_t and x_{t+h} depends only on the distance between the two terms, h, and not on the location of the initial time period, t (3):

$$E(x_t) = \mu \tag{1}$$

$$Var(x_t) = \sigma^2 \tag{2}$$

$$Cov(x_t, x_{t+h}) = \gamma_i \quad \forall i \geq 1 \tag{3}$$

To show an example of stationary and nonstationary process we picked a stock of AAA Auto Group from PSE from period dating from June 6, 2008 until July 4, 2013. We compare plot of its stock prices with plot of the first differences of its stock prices in Figure 1.

Graph in the left top corner of Figure 1 clearly shows a trending behavior and for that reason we can say that it is nonstationary process. This claim is supported by the histogram below, which shows that different sequences of data show different means, which violates condition 1. On the contrary,

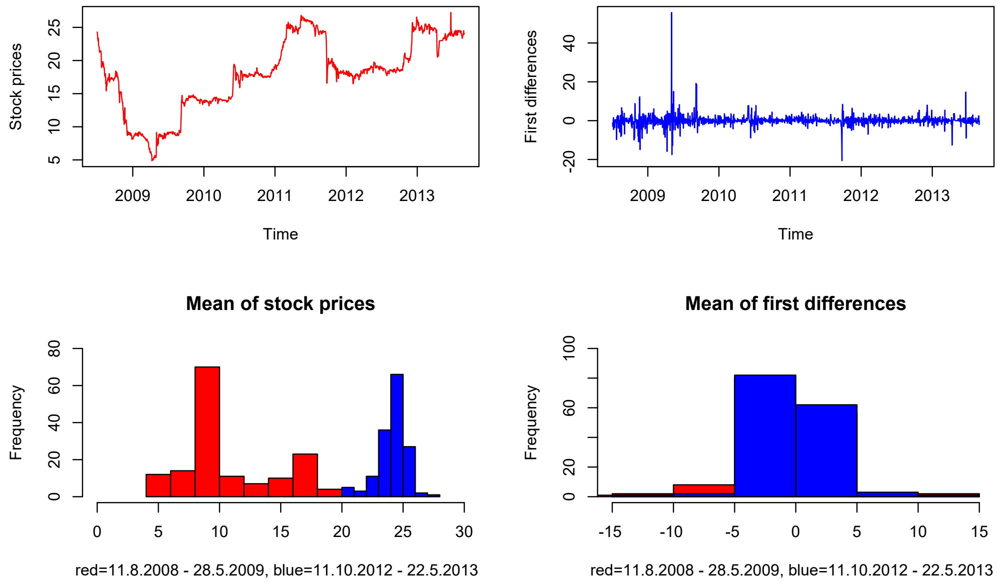

Figure 1: Nonstationary and Stationary Process with its Histograms

the time series with first differences shown in the top right corner of Figure 1 displays completely different behavior, where it fluctuates around constant value, even zero, and does not show trending behavior. From histogram in the bottom right corner of Figure 1 we can see that mean in this case stays more or less the same. From this we can say that first differences are covariance stationary process.

Of course there are plenty of other and more reliable ways to determine whether a time series is stationary or nonstationary. The most favorite test is the one developed by Dickey and Fuller (1979), where they test for a unit root in time series. This test is also used in the paper of Engle and Granger (1987). There are three cases of this test and each depends on our alternative hypothesis and presence of the drift or a time trend.

Testing for the unit root starts with a simple AR(1) model:

$$y_t = \alpha + \rho y_{t-1} + e_t, \quad t = 1, 2, \ldots, \tag{4}$$

where e_t is assumed to be i.i.d. with zero mean, constant variance and is independet of y_0. We are interested in the value of ρ, and time series has the unit root if, and only if, $\rho = 1$.[4] Therefore, our null hypothesis is that time series has the unit root, which means that $H_0 : \rho = 1$ and we are interested in the one-sided alternative $H_1 : \rho < 1$. We use one-sided alternative, because the alternative $H_1 : \rho > 1$ would imply that y_t is explosive.

A more common and convenient equation for carrying out the unit root test is to substract y_{t-1} from both sides of (4) and to define $\theta = \rho - 1$:

$$\Delta y_t = \alpha + \theta y_{t-1} + e_t, \tag{5}$$

and estimate (5) by OLS. Now our null hypothesis is $H_0 : \theta = 0$ against $H_1 : \theta < 0$. If we reject H_0, we can conclude that time series does not have the unit root. In the other case, time series has the unit root and is non-stationary. This method has become known as the Dickey-Fuller test. The problem with this test is that under the null hypothesis, y_t is nonstationary, which means that the t statistic does not have an approximate standard normal distribution even in large sample sizes. Therefore we have to use different t statistic than we are used to. We use the usual t statistic for $\hat{\theta}$, which we obtained from Banerjee, Dolado, Galbraith and Hendry (1993):

		Significance level		
Variant		1%	5%	10%
$\Delta y_t = \theta y_{t-1} + e_t$	no constant	-2.5658	-1.9393	-1.6156
$\Delta y_t = \alpha + \theta y_{t-1} + e_t$	no trend	-3.4336	-2.8621	-2.5671
$\Delta y_t = \alpha + \lambda t + \theta y_{t-1} + e_t$	with trend	-3.9638	-3.4126	-3.1279

Table 1: Asymptotic Critical Values for Unit Root t Test

The tests above are only valid if e_t is white noise. Especially, e_t will be autocorrelated if there was autocorrelation in the dependent variable of the regression (Δy_t) which we have not modeled. The solution is to "augment" the test using j lags of the dependent variable. The alternative model has

[4] If $\alpha \neq 0$ and $\rho = 1$, time series is a random walk with a drift. The unit root with the drift behaves quite differently than the one without the drift. Nevertheless, it is normal to leave α unspecified under the null hypothesis.

now the following form:

$$\Delta y_t = \alpha + \lambda t + \theta y_{t-1} + \sum_{j=1}^{m} \gamma_j \Delta y_{t-j} + e_t. \qquad (6)$$

The same critical values from the Table 1 are used as before. A problem now arises in determining an optimal number of lags of the dependent variable Δy_t. There are two ways possible. First, we can decide on the number of lags based on the frequency of the data, or secondly, we can use information criteria. Information criteria are measures of the goodness fit of an estimated statistical model and the most usual ones are Akaike information criterion (AIC), Bayesian information criterion (BIC) or Hannan-Quinn information criterion. Limitation of this test is in the number of lags we include, because the more lags we have, the more observation we lose and hence this results in the lower power of the test. The method based on (6) is known as the augmented Dickey-Fuller (ADF) test.

Now we have developed a formal way to prove that our graphs in Figure 1 are stationary and nonstationary, respectively. When we run the augmented Dickey-Fuller test on both series, we get quite straightforward results implicating that we were absolutely right. The statistics[5] for our ADF tests are following:

Process	Lag order	Dickey-Fuller statistic	p-value
Stock prices	10	-2.9868	0.1606
First differences	10	-9.4262	0.01

Table 2: Augmented Dickey-Fuller test for Stock Prices and its First Differences

From Table 2 we can easily see that Dickey-Fuller statistic is in case of first differences much lower. From presented p-values we can surely say that stock prices are indeed nonstationary and first differences are not and we can reject the null hypothesis of the unit root. In our case we had to difference stock prices only once, however, that is not always the case. There is an order of integration, denoted $I(d)$, which is the minimum number of times a

[5] R software printed value 0.01 in case of the first differences. However, it was warning us that the value is smaller than that.

series must be differenced. In our example the stock prices are integrated of order one, or $I(1)$, and first differences are integrated of order zero, or $I(0)$.

The information about stationarity or nonstationarity is quite signignificant in regression analysis, because highly persistent time series can lead to very misleading results if classical linear model assumptions are violated. For example, regression of nonstationary series can show us significant relationship, when in reality, the series are not related. This phenomenon is called spurious correlation and it is important to account for time trend when we encounter such problem.

Apparently, variables with one order of integration should be used in regression analysis with considerable awareness. However, the problem of nonstationarity is inevitable in the field of economics, moreover in financial data such as stock prices we use throughout this theses. For this reason we need to introduce an approach which will ensure that we have qualitative results when we regress $I(1)$ variables one on another. This approach was presented by Engle and Granger (1987) in their paper and is called cointegration.

4.2 Cointegration

Cointegration and the ideas behind it can be formalized in the following definition formulated by Engle and Granger (1987):

Definition 1. The components of the vector x_t are said to be *cointegrated of order d, b,* denoted $x_t \sim CI(d, b)$, if (i) all components of x_t are $I(d)$; (ii) there exists a vector $\alpha(\neq 0)$ so that $z_t = \alpha' x_t \sim I(d-b), b > 0$. The vector α is called cointegrating vector.

When we concentrate on the case where $d = 1, b = 1$ and two-dimensional vector x_t, it gives us linear combination $z_t = \alpha_1 x_{1t} - \alpha_2 x_{2t}$, where both x_{1t} and x_{2t} are $I(1)$ processes, which will generally result in derived series $z_t \sim I(1)$. Theoretically speaking, this means that these series may drift apart from each other and zero-crossings would be very rare. However, there may occur such α, that linear combination z_t will result in $I(0)$ process, and

z_t will rarely drift far from zero if it has zero mean and z_t will often cross the zero line, such event is called mean reversion. If this α exists, then we can say that x_{1t} and x_{2t} are cointegrated. Throughout this theses we will focus on case when $x_t \sim CI(1,1)$, because any other situation is irrelevant and also beyond the scope of the text.

Focusing on vector α, in pairs trading this parameter represents cointegration between individual stocks. In practice it means that when we short one dollar of x_{1t}, then we should long α_2 dollars of x_{2t} and vice versa. The α is therefore ratio in which we should hold our position on the market.

Similarly as for the unit root we can test for cointegration. Engle and Granger (1987) suggest several tests like cointegrating regression Durbin-Watson test, Dickey-Fuller test, augmented Dickey-Fuller test, restricted vector autoregression test or others. However there is one test that stands above all those and it is based on a two-step procedure of Engle and Granger named after them: Engle-Granger test for cointegration.

When we suspect two variables to be cointegrated, we firstly test if they have the unit root. Assuming both series have unit roots (both are non-stationary) we find approximate relationship by an OLS regression (7) and saving the residuals $\hat{\epsilon_t}$.

$$x_{1t} = \alpha_0 + \alpha_2 x_{2t} + \epsilon_t \tag{7}$$

Then we test residuals $\hat{\epsilon_t}$ for stationarity via ADF test (6), which in our two variables case looks like this:

$$\Delta\hat{\epsilon}_t = \gamma\hat{\epsilon}_{t-1} + \upsilon_t. \tag{8}$$

There also exists augmented version of Engle-Granger test which includes lags of residuals. This test is basically test for the unit root of the least squares residuals. Null hypothesis for this states that our variables are not cointegrated, then if we reject the null of the unit root then we cannot reject that the two variables cointegrate. The problem with this test is that the coefficient α_2 needs to be estimated and the potential bias is transfered into the second step (8). Because of this we obtain non-standard distributions of test-statistics, which are different from those used in ordinary unit root tests like ADF, and so we need to use different critical values. For this reason we display asymptotic critical values from Hamilton (1994):

Variant		Significance level 1%	5%	10%
$x_{1t} = \alpha_2 x_{2t} + \epsilon_t$	no constant	-3.39	-2.76	-2.45
$x_{1t} = \alpha_0 + \alpha_2 x_{2t} + \epsilon_t$	no trend	-3.96	-3.37	-3.07
$x_{1t} = \alpha_0 + \lambda t + \alpha_2 x_{2t} + \epsilon_t$	with trend	-3.98	-3.42	-3.13

Table 3: Asymptotic Critical Values for the Cointegration Test

5 Methodology

Our implementation of pairs trading strategy consists of two stages as stated. We form pairs over fifteen month period (the formation period) and trade them in the following six month period (the trading period). Fifteen month period is chosen dependently on our cointegration approach and six months period is chosen in accordance with Gatev, Goetzmann and Rouwenhorst (2006), but in general it is chosen that the selection process is recent and round-trips have time to occur using a reasonable opening trigger. Both periods have remained our horizons throughout the whole study.

5.1 Distance Method

Similar to Gatev, Goetzmann and Rouwenhorst (2006), we have selected pairs on the basis of minimized sum of squared deviations (SSD) between the two normalized price series. This approach is used because it best approximates the description of how traders themselves choose pairs, because they try to find two stocks whose prices move together, according to Gatev, Goetzmann and Rouwenhorst (2006). The methodology follows the subsequent steps:

1. Over the formation period, we construct a daily returns index for each stock as a ratio of stock price to its value on the previous day:

$$R_{it} = \frac{P_{it}}{P_{it-1}} - 1 \tag{9}$$

   This formula's interpretation is approximate percentage return per day (regularly used in economic literature).

2. Normalized returns are computed by substracting sample mean and then divided by their sample standard deviation:

$$\widehat{R_{it}} = \frac{R_{it} - \mu_i}{\sigma_i} \tag{10}$$

3. Cumulative total returns are computed by adding the normalized re-

turns $\widehat{R_{it}}$ and by their cumulative summation:

$$R_i^c = \sum_{t=1}^{T} \widehat{R_{it}} \tag{11}$$

4. Such series are then used in the equation of the SSD:

$$SSD_{i,j} = \sum_{t=1}^{T} (R_{it}^c - R_{jt}^c)^2 \qquad \forall i \neq j \tag{12}$$

5. Values obtained from equation (12) are then ordered and five pairs with the lowest SSDs are chosen for trading. The rest of the pairs is not considered throughout the rest of the analysis.

Once we have got pairs through distance method, we need to introduce the spread as the extent to which the stocks are relatively mispriced between themselves. The spread is the distinction between cumulatively summed normalized returns of stocks that form each pair. Normalization is performed slightly differently than in equation (10), because we compute it in the trading period. Therefore we carry the mean and the standard deviation cumputed in the formation period over to the trading period and we normalize the returns in the trading period, using the equation:

$$\widehat{R_{it}} = \frac{R_{it}^T - \mu_i^F}{\sigma_i^F}, \tag{13}$$

where F and T indicate in what period the values were acquired. These returns are summed by cumulative summation, and the spread is described by the difference between the series of individual stocks. The possibility of look-ahead bias rules out using mean and standard deviation obtained right in the trading periods.

5.2 Cointegration Method

From Definition 1 defined by Engle and Granger (1987) and application to the case $CI(1,1)$, it is visible that stock prices need to be $I(1)$ (nonstationary) processes to have some potential cointegration. Based on the time periods we test, where we have both stock prices decline (usually in periods dating to the half of 2009) and also their growth, we choose equation that implements a constant as well as a trend:

$$y_t = \alpha + \lambda t + \rho y_{t-1} + \sum_{j=1}^{5} \gamma_j \Delta y_{t-j} + e_t, \quad t = 1, 2, \ldots \tag{14}$$

$$\Delta y_t = \alpha + \lambda t + \theta y_{t-1} + \sum_{j=1}^{5} \gamma_j \Delta y_{t-j} + e_t, \quad t = 1, 2, \ldots \tag{15}$$

In the equations (14) and (15) y_t stands for time series of the stock prices and t is the trend. Also, we set the maximum number of lags to five (length of the usual trading week), allowing the error terms to be autocorrelated. The actual number of lags is determined via AIC, which selects the optimal number of lags according to the minimum AIC value (16). We use AIC instead of BIC simply because AIC penalizes the number of parameters less strongly than BIC. For the choice of lag terms AIC is therefore more appropriate since we face the compromise between the smaller dataset and danger of autocorrelation, where the second is more serious. Also, our formation periods usually consist of about 313 data points, hence losing of maximally 5 observations is quite insignificant in the resulting power of the test.

$$AIC = 2k - 2\log(\hat{L}) \tag{16}$$

The null hypothesis for the ADF test for unit root based on (15) is $H_0 : \theta = 0$ and the alternative hypothesis is $H_1 : \theta < 0$. Under the null hypothesis, stock prices y_t have the unit root and under the alternative they are trend-stationary. We use one-tailed version of the test and we use significance level of 5%, which means we have to use asymptotic critical value $c = -3.4126$ (see Table 1). Our null hypothesis is rejected, if t statistic on parameter $\hat{\theta}, t_{\hat{\theta}}$, is lower than our critical value c. We need to be thorough to correctly identify trend-stationary time series, because $I(1)$ is not the same property as trend-stationarity, thus each stock that is recognized as such process is excluded from further analysis. If we did not include time trend in equation (15), we would have incorrectly recognized the trend-stationarity as nonstationarity in some series.

Next step in identifying of cointegrated pairs according to augmented Engle and Granger test is performing an OLS regression (17). Our time series have naturally nonzero mean, because they are stock prices, so we need to include constant term in our regression. Because of the previous step we use only nonstationary stocks, so we do not include time trend in equation (17). y_t and x_t are time series of our stock prices.

$$y_t = \mu + \beta x_t + e_t \tag{17}$$

After estimating (17) we obtain residuals in the form $\hat{\mu} + \hat{e}_t = y_t - \hat{\beta} x_t$, which we save for further testing. However, residuals are not the only interesting thing about this regression, we also save $\hat{\beta}$, because it is the potential cointegration parameter that we will use in trading periods as a ratio in which we should hold our position in the market in case the pair is detected with cointegration. The $\hat{\mu}$ is the equilibrium value and $\hat{e}_t$ is time series fluctuating around zero mean. Just as in the case of ADF test (15) we include five lags of residuals in order to avoid autocorrelation in cointegration test:

$$\Delta e_t = \theta e_{t-1} + \sum_{j=1}^{5} \gamma_j \Delta e_{t-j} + \epsilon_t \tag{18}$$

Similarly to ADF test (15), we use AIC to set the optimal number of lags included and also, the same null and alternative hypotheses apply (that is $H_0 : \theta = 0$ and $H_1 : \theta < 0$) for the augmented Engle and Granger test. As we include constant, but not a time trend in our model, the critical value for testing our hypotheses is $c = -3.37$, which can be found in Table 3. Pairs, which are detected with cointegration, are the ones with t statistic lower than our critical value, that is $t_{\hat{\theta}} < c$.

Since the markets we test are quite small, we wish for not rejecting H_0 while testing for the unit root, so we are left with as many stocks as possible, and in testing for cointegration we are looking for rejecting the null. That is because then we are left with more cointegrated pairs, which might possibly result in the increase of the number of trades, which consequently positively influences the power of results. However, if we detect cointegration, it does not mean we will use a pair in our analysis, because we also require $\hat{\beta}$ to be positive. Estimator $\hat{\beta} < 0$ tells us, that when we should execute the trade, we should long both stocks or short both stocks, respectively, which is not a principle of contrarian investment strategy. Therefore all pairs that does not satisfy the condition of $\hat{\beta} > 0$ are not considered for the rest of the analysis.

After obtaining the cointegrated pairs, we have to introduce the spread as a fundamental metric for trading. The estimators $\hat{\beta}$ and $\hat{\mu}$ need to be estimated in formation period and used in trading period in order to prevent us from falling in a trap called look-ahead bias. If we would use estimators obtained in trading period instead of formation period, we would not simulate real conditions on the market, because we would use information that are not known at the moment during the trading period. We form trading spread between cointegrated stocks by the following equation:

$$\hat{e}_t^T + \hat{\mu}^F = y_t^T - \hat{\beta}^F x_t^T, \tag{19}$$

where F and T stand for either trading period or formation period and x_t and y_t stand for time series of stock prices just as in equation (17).

5.3 Trading Rules

In either cases, distance method and cointegration method, we use oscillations around equilibrium value of the spread as a criterion when to open or close position. When, and in some cases if, the spread surpassed preestablished value, the mispricing between two stocks in a pair drifted far away from its historical mean, which is an indication for us to enter the trade. The process of obtaining the equilibrium value of the spread and value of thresholds that are an indication to start a trade follow the similar principle in both methods: they are based on standard deviations computed from values of the spread in the formation periods.

The equilibrium value of the spread in case of the distance method is estimated as a mean of the spread in the formation period. Standard deviation is acquired from the same spread. We start the trade when the distance become equal or larger than two standard deviations in both directions from the mean, and clear the position when the distance returns to it, i.e. when the spread of the pair intersect the value again. To be thorough, we sell the spread[6] if, at time t, the relative mispricing crosses the value of two standard deviations above the mean (Equation 20), and buy the spread at time $t+r$ (Equation 21):

$$R_{it}^T - R_{jt}^T = E(R_{it}^F - R_{jt}^F) + 2\sqrt{Var(R_{it}^F - R_{jt}^F)}, \quad i \neq j \tag{20}$$

$$R_{it+r}^T - R_{jt+r}^T = E(R_{it+r}^F - R_{jt+r}^F), \quad i \neq j, \tag{21}$$

where T and F express whether the values were obtained from trading or formation period.

Similarly, we buy the spread when, at time t, the mispricing reaches the value of two standard deviations below the mean (Equation 22), and buy the spread at time $t+r$ (Equation 23):

$$R_{it}^T - R_{jt}^T = E(R_{it}^F - R_{jt}^F) - 2\sqrt{Var(R_{it}^F - R_{jt}^F)}, \quad i \neq j \tag{22}$$

$$R_{it+r}^T - R_{jt+r}^T = E(R_{it+r}^F - R_{jt+r}^F), \quad i \neq j \tag{23}$$

We refer to $E(R_{it+r}^F - R_{jt+r}^F)$ as the closing value. The gain on the trade is the accumulative change in the spread, $2\sqrt{Var(R_{it}^F - R_{jt}^F)}$.

In case of cointegration method we consider the long-run equilibrium value of the linear combination $y_t - \hat{\beta}x_t$ is $\hat{\mu}$. From the residual series we obtain the standard deviation of this linear combination: $\sigma = \sqrt{Var(\hat{\mu} + \hat{e}_t)}$. Likewise in the distance method, trading signals are predetermined on the two standard deviations (2σ) from the long-run equilibrium in both directions (Equation (24)), and position is cleared when the spread of the pair intersects the value $\hat{\mu}$ (Equation 25)):

$$y_t - \hat{\beta}x_t = \hat{\mu} \pm 2\sigma \tag{24}$$

$$y_{t+r} - \hat{\beta}x_{t+r} = \hat{\mu} \tag{25}$$

Pairs can undergo more round-trips[7] than just one, or they may undergo none in case the prices do not diverge by more than two standard deviations

[6] To sell the spread says that we sell the overpriced stock and buy the underpriced one.

[7] Round-trip is a situation, when the pair diverges and converges again, pairs can undergo this event more than once in the trading period.

in the trading period, no matter which method is used. When a pair is open on the last day of trading period, we clear the position no matter the convergence, which may result in the loss instead of a profit.

Pairs trading, such as many other investment strategies on equity markets, is subject matter to a bid-ask bounce (Jegadeesh and Titman, 1995), which sways computed profits and therefore we need to approach this reality to obtain more realistic results. We often see two different stock prices on equity markets such as stock exchanges. The lower price, the bid price, is the price a buyer is willing to pay for a security in the exact moment. The higher price, the ask price, is the amount a seller is willing to receive for his security at the exact moment. Both prices are always quoted together and the difference between them is referred as the bid-ask bounce. The smaller the difference the more liquid and efficient the market is. Since in pairs trading we always have a winner and loser in each trade, it is probable, that winners' price is traded for an ask amount and losers' price for a bid amount, which creates higher divergence in prices than it truly is. To reduce the impact of the bias, we start the trade of a pair one day after obtaining the trading signal and close the position one day after the mean-reversion of the spread. This approach should treat pairs trading in more realistic way from perspective of transaction costs and potential difficulties in executing the trade (we are not always able to sell or buy all stocks immediately). Gatev, Goetzmann and Rouwenhorst (2006) and Broussard and Vaihekoski (2012) use the same approach as well and they use this decrease in profits compared to no-delay strategy as an approximate calculation of transaction costs.

5.4 Computation of Returns

For a matter of simplicity and easier interpretation of results we assume, that in periods where no position was opened we earn zero return on capital.

To compute the returns of a pair of stocks over the entire trading period using distance method, we will follow Gatev, Goetzmann and Rouwenhorst (2006) and Broussard and Vaihekoski (2012). We need to accumulate weighted daily returns from long and short positions using following equation:

$$r_{pt} = w_{1t} r_t^L - w_{2t} r_t^S, \tag{26}$$

where w_{1t} and w_{2t} are daily weights of returns, and r_t^L and r_t^S are daily returns for the positions where L and S represent the long and short position, respectively. The weights are initially assumed to be one after which they change accordingly to the changes in the value of the stocks: $w_{it} = w_{it-1}(1+r_{it-1})$. This computation gives the same result as the one done in Gatev, Goetzmann and Rouwenhorst (2006)[8]:

$$r_{pt} = \frac{\sum_{i \in P} w_{it} r_{it}}{\sum_{i \in P} w_{it}} \tag{27}$$

if the weights are adjusted accordingly.

Because we expect our initial investment to be zero, r_{pt} from equation (26) can be defined as daily excess returns. We compute possible profits of individual pairs throughout the whole holding period, we add equation (26) over all days we hold the position, which results in cumulative total excess returns.

To compute the returns of a pair of stocks using the cointegration method we will follow Vidyamurthy (2004):

$$r_{pt} = [\log(P_{t+i}^L) - \log(P_t^L)] - \beta[\log(P_{t+i}^S) - \log(P_t^S)], \tag{28}$$

which we can simply modify into the following form:

$$r_{pt} = [\log(P_{t+i}^L) - \beta \log(P_{t+i}^S)] - [\log(P_t^L) - \beta \log(P_t^S)], \tag{29}$$

where β is the cointegration coefficient. We need to mention that equations (28) and (29) refer to the case when we long the spread. We may experience during some trading periods that a stock in pair will be once undervalued and in the same trading period during another round-trip it will be overvalued, or vice versa. This reality results in the fact that the spread will be once

[8]Note that there was a small typographical error in equation (2) in Gatev, Goetzmann and Rouwenhorst (2006).

bought and the next time sold or the other way around. When we short the spread, the equation will have the following form:

$$r_{pt} = -[\log(P^S_{t+i}) - \log(P^S_t)] + \beta[\log(P^L_{t+i}) - \log(P^L_t)], \tag{30}$$

where the ratio in which we trade the stocks is the same. Because of the trade signals we predetermined, we cannot hold both long and short position in the spread of one pair in the same day.

As a risk-adjusted performance measurement we can choose from two mainly used ratios, the Sharpe ratio and the information ratio, we chose the latter for several reasons. The Sharpe ratio measures what reward an investor could expect for investing in a risky asset instead of a risk-free asset and it is scaled by a risk of that asset, while the information ratio tells an investor how much excess return is generated from the amount of excess risk taken relative to the benchmark (Kidd, 2011). In other words, the Sharpe ratio tells us what portion of a portfolio's performance is associated with risk taking and the information ratio can tell whether a manager outperformed his or her benchmark on a risk-adjusted basis. Also, the information ratio works with equity indices, whereas the Sharpe ratio with risk-free rate, which works for the information ratio in our case. The information ratio has the following form:

$$IR_p = \frac{\bar{R}_p - \bar{R}_B}{\hat{\sigma}_{p-B}}, \tag{31}$$

where $\bar{R}_p$ represents the portfolio return, $\bar{R}_B$ is the return of the benchmark, which in our case are equity indices, and $\hat{\sigma}_{p-B}$ is the standard deviation of the difference between the portfolio and its benchmark, it can also be written as $\sqrt{Var(\bar{R}_p - \bar{R}_B)}$.

We need to know what values the information ratio shows and what interpretation to assess to them. Grinold and Kahn (2000) contended that top-quartile active equity managers generally have information ratios of 0.50 or higher. However, commonly used practise is that the information ratio of 1.0 is rated as exceptional, 0.75 as very good, and 0.50 as good.

6 Potential Scenarios

The following section describes potential scenarios which may occur in our results throughout the analysis of our three stock markets. The scenarios take into account specifications of those stock markets such as their size, wide diversification among stocks etc. Due to these specifics our results are hardly comparable to the results from empirical studies done on the US and Finnish equity markets, because they differ in many other aspects like volumes traded, efficiency or industry diversity. As we did not take these features into account in our methodology, it is quite probable that they will affect pairs formation and consequently returns obtained from such trading. We anticipate three different scenarios for distance method as well as for cointegration method. These scenarios are then displayed in Table 4.

6.1 Scenario 1

In the Scenario 1 we expect returns to be positive for both methods, distance and cointegration. We are looking for the similar results presented by Gatev, Goetzmann and Rouwenhorst (2006) and Broussard and Vaihekoski (2012), which are significantly positive. Therefore the majority of returns will be positive and significantly different from zero, even if the trade needs to be closed on the last day of the trading period. We will also implement metric of average number of days the positions were opened in order to complete the trade. Broussard and Vaihekoski (2012) reported average number of days the pairs were traded to be approximately 23 days, opposed to Gatev, Goetzmann and Rouwenhorst (2006) who reported round-trips longer than 55 days. Because of this discrepancy we decided to state the average number of days the pair is traded to be lower than 55 days as was the case of Gatev, Goetzmann and Rouwenhorst (2006), which represents approximately half of our trading period.

In cointegration method we expect returns to be positive. We base our hypothesis on the small size of equity markets, where stocks are more likely to be interconnected and are prone to similar events irregardless industry diversity. Therefore we expect strong links between individual stocks which consequently will form cointegrated pairs, which we anticipate to be at least 10% of all possible pairs each period. The average number of days the trade requires will be 55 days at maximum.

6.2 Scenario 2

In the Scenario 2 for the distance method we expect the number of days pairs need to completion of round-trip to be above our limit from Scenario 1. As a result a lot of our pairs will be closed on the last day of the trading period which will sway the average returns downward. However, we still anticipate returns to be positive, but not as significantly as in the Scenario 1, or they could be zero.

The cointegration method in the Scenario 2 will identificate some cointegrated pairs, but their count will be quite slim. Because of the diversification of the equity markets we will not expect returns to be positive due to the industry shocks and non-convergence of the stocks.

6.3 Scenario 3

Methodology chosen for our pairs trading using distance method is constructed in a way that no matter the stocks and their differences, we will always choose five pairs which we will trade, by the lowest SSD. Hence it is possible that this SSD is the lowest, but still pretty high and it results in situations where our chosen pairs are not correlated or similar at all. This reality causes situations that when the trade is triggered by the divergence of the stocks, they will not converge again but instead they will continue to wander-off even more or remain open till the end of the trading period where they need to be liquidated no matter the convergence. Such events will naturally lead to the average negative returns as well as to the average number of trading days above our stated threshold of 55 days.

Because of the small size of equity markets we study and their industry diversity, which will cause industry shocks, we will not be able to find any cointegrated pairs which will lead to inapplicability of cointegration method on such markets. If we find any cointegrated pairs, they will not be profitable and their average trading days will be well above our 55 days threshold.

	Distance method		Cointegration method		
	Returns	Average days	Returns	Average days	Cointegrated pairs
Scenario 1	$\overline{r_{it}} > 0$	$\overline{days} < 55$	$\overline{r_{it}} > 0$	$\overline{days} < 55$	Yes, 10%
Scenario 2	$\overline{r_{it}} \geq 0$	$\overline{days} > 55$	$\overline{r_{it}} < 0$	$\overline{days} > 55$	Yes, max 5%
Scenario 3	$\overline{r_{it}} < 0$	$\overline{days} > 55$	$\overline{r_{it}} < 0$	$\overline{days} > 55$	No

Table 4: Potential Scenarios

7 Empirical Results

7.1 Distance Method Trading

Table 5 shows descriptive statistics of the percentage returns per trade for the top five pairs traded using the distance method for three different equity markets, Prague Stock Exchange (PSE), Budapest Stock Exchange (BSE) and Bucharest Stock Exchange (BVB). Returns are calculated using daily cumulative return indices. The five pairs were chosen using the SSD method in all equity markets.

	PSE	BSE	BVB
Total number of trades	89	93	90
Average excess return	-0.0049	-0.0116	0.0006
Standard error	0.0204	0.0145	0.0163
t-statistic	-0.24	-0.80	0.04
Average information ratio	0.1252	-0.0657	0.0283
Average number of days the pair is open	60.29	53.06	51.2
Share of round-trips[9]	29.21%	36.56%	42.22%
Excess return distribution			
Median	-0.0101	-0.0193	0.0023
Standard deviation	0.1927	0.1398	0.1543
Skewness	0	0.56	0.04
Excess kurtosis	1.44	0.6	0.39
Minimum	-0.6451	-0.3059	-0.3740
Maximum	0.5926	0.4499	0.4467
Share of negative observations	51.69%	54.84%	50%

Table 5: Trade Returns and Their Distribution for Distance Method Trading

As we can see in Table 5, the descriptive statistics are really similar for all three equity markets. The average trading returns are negative for PSE and BSE and almost zero for BVB. However, they are not statistically different from zero for any market, which we can see from all three low t-statistics.

[9] The percentage of trades that were finished during the period, the rest must have been liquidated on the last day of the period.

The 10% critical value for 90 degrees of freedom is 1.662. From looking in Table 10 we would probably expect the lowest average return to be in the market with the lowest average number of stocks traded, which would be PSE in our case and the highest average return to be in the market with the highest average number of stocks traded, BSE. This would be because we would have more pairs to choose from and it would be possible to find ones that would have real relationship. Unfortunately, it is not the case and from looking on the average number of days the pair is open, which is in the case of PSE higher than our preestablished threshold of 55 days, we are forced to reject Scenario 1 for PSE. Considering the high share of negative observations in all markets and really low information ratios, which signals no active returns on the investments. We will reject Scenario 1 for BSE as well, because the scenario states positive outcomes. For BVB we will not reject the Scenario 1, because we have lower average number of days the pair is open than our preestablished threshold and the lowest of all three markets and also the highest average excess return. Nevertheless, we have to take this with caution since the t-stastic is only 0.04. Our conclusions are also supported by the share of round-trips, which signals that the average number of days would be longer than what we have obtained, because the positions were forced to close and therefore have lower number of days they were open than they would actually have had. It also signals, that the more round-trips we have completed, the higher the return might be, when we compare PSE and BVB.

Scenario 2 is the case for all three equity markets, since all of the excess returns are not stastically significant and the average number of days the pair is open fluctuates around the threshold of 55 days. We also have share of negative observations around 50%, which helps us in not rejecting Scenario 2. BSE is on the line between Scenario 1 and Scenario 2, but when we take into account the lowest average excess returns with the highest t-stastic, we clinge more to Scenario 2 than Scenario 1, although Scenario 2 states higher number of days than we obtained from our analysis. Finally, Scenario 3 cannot be reject for PSE for sure, since we have negative excess returns with more than half of negative observations and higher average number of days the pairs are traded than 55 days. In case of BSE we will also not reject this scenario since we have negative average returns and the share of negative observations is 54.84%. On the other hand, Scenario 3 will be rejected for BVB, because we have positive average return, supported by positive median and our average number of days is about almost four days lower than the preestablished threshold of 55 days.

When we will take closer look on what types of pairs we have, we discover that on PSE only one pair showed persistence across our sample periods and occured in six periods. That pair was formed by stocks of Komerční Banka as (KB) and Vienna Insurance Group AG Wiener Versicherung Gruppe (VIG), which share similar industry. KB operates in a banking industry, while VIG is an insurance company, these fields share more characteristics than any other fields that are traded on PSE (only with exception of KB and Erste Group Bank AG, which were paired only in two periods). The situation on another two markets is similar, on BSE one pair stands above all, because it was detected in eleven out of fifteen periods. We are talking about pair formed by Eszak Magyarorszagi Aramszolgaltato Nyrt and Budapesti Elektromos Muvek Nyrt, both companies operate in energetic sector which is in accordance with findings of Gatev, Goetzmann and Rouwenhorst (2006), who had 70% of their portfolio composed by utilities. Two most frequent pairs on BVB are not an exception, one pair is between two construction companies, and another, which might be interesting is between utility company and biotechnology and life sciences company. These insights into pairs supports claim of Do and Faff (2010) that the industry homogeneity is important driver in pairs trading profitability, which is causing our markets to fail in providing positive returns.

The histograms in Figure 2 together with distribution statistics from Table 5 indicate, that the excess returns on PSE and BVB are normally distibuted. We need to note, that their skewness is normal, in case of PSE

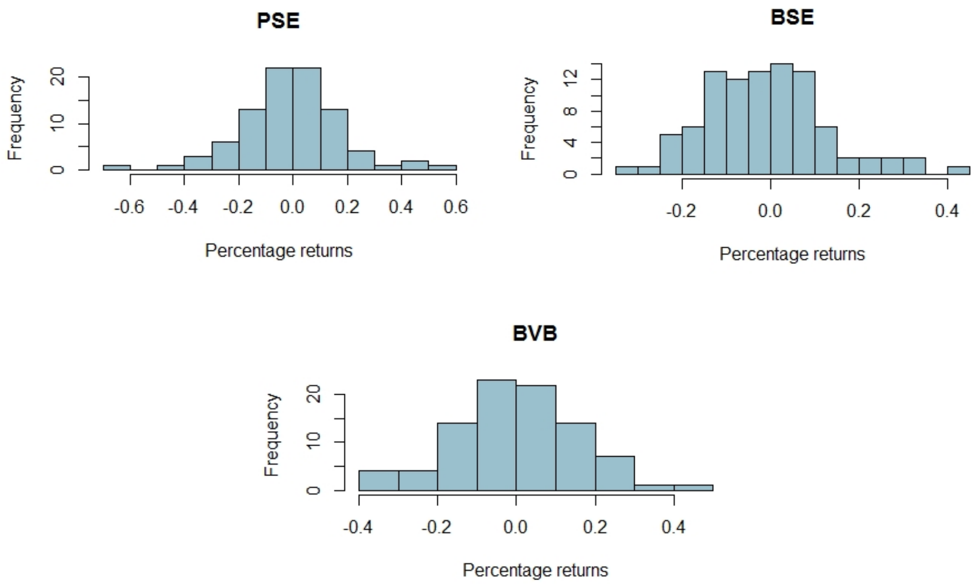

Figure 2: Histograms of Distance Method Trading for PSE, BSE and BVB

it equals even zero. From looking into the statistic of kurtosis and well-known rule of thumb, we can see that our distributions are platykurtic, because kurtosis is lower than three. Platykurtic distributions have lower and wider peak around the mean and thinner tails in comparison with normal distribution, which we can partially spot in our histograms. The excess returns of BSE are right-skewed, or have positive skewness as it is sometimes called, which indicates that we have some ”extreme” scores on the right hand side of the distribution and the curve is pulled in a positive direction. The kurtosis in case of BSE is the same as in the cases of PSE and BVB, it is platykurtic distribution.

When we compare our results to the ones obtained in the study of Gatev, Goetzmann and Rouwenhorst (2006) we must comment that we did not achieve as good performance as they did. The main reason behind this could be that they trade pairs in the S&P 500, where they have much greater number of stocks that could have potential comovement of their stock prices. Also, the higher number of stocks may then form pairs from the same industry, which are thus more robust against industry shocks. It is for sure the reason behind our worse performance. Broussard and Vaihekoski (2012) trade with the different stocks of the same companies and Gatev, Goetzmann and Rouwenhorst (2006) trade with pairs formed from the same sector, mostly utilities, while we are forced to trade with stocks from different industries. This reality is reflected in our statistics, which are all in favor of larger equity markets such as US and Finnish stock markets. Whereas both studies, Gatev, Goetzmann and Rouwenhorst (2006) and Broussard and Vaihekoski (2012), found the pairs trading profitable, we cannot reject zero returns at all. Therefore, according to our results, pairs trading with distance method is not suitable for small equity markets with high diversification of stocks, which supports the conslusions of Jacobs and Weber (2015) that returns are positively correlated with the size of the market.

7.2 Cointegration Method Trading

The descriptive statistics for evaluation of cointegration method that is used for pairs trading in analysed equity markets can be found in Table 6. For using such method we had to make several tests before the formation of potential pairs, like ADF test for testing the unit root, which resulted in dropping several stocks from further analysis. The exact numbers of the dropped stocks and periods in which it has been done can be found in Table 10.

After conducting augmented Engle and Granger tests, we identified only a few pairs in each period, and some periods, exactly three on PSE and one on BSE, were even without a single cointegrated pair. The exact periods and numbers of detected pairs can be found in Table 10. The highest number of detected pairs in a single period is 14, which represents 18% of all possible pairs in that period. Nevertheless, such high numbers are rare, only four periods had number of detected pairs higher than 10%. To be exact, the cointegrated pairs represent only 4.55% of all pairs across all sample periods. Because of this reality, we will reject Scenario 3, which states that none of the cointegrated pairs were found, which is not true. We might as well reject Scenario 1 as it states high percentage of cointegrated pairs, which is not our reality, but we will analyse it in the following text in more detail.

	PSE	BSE	BVB
Total number of trades	43	92	84
Average excess return	-0.0361	-0.0554	-0.0095
Standard error	0.0344	0.0214	0.0205
t-statistic	-1.05	-2.59	-0.46
Average information ratio	0.0286	-0.0023	0.0418
Average number of days the pair is open	61.42	61.6	66.93
Share of round-trips[10]	32.56%	39.13%	28.57%
Excess return distribution			
Median	-0.0246	0.0034	0
Standard deviation	0.2259	0.2057	0.1880
Skewness	-0.71	-1.03	-0.76
Excess kurtosis	2.93	0.85	2.08
Minimum	-0.8552	-0.7242	-0.6667
Maximum	0.5623	0.3776	0.4213
Share of negative observations	51.16%	48.91%	48.81%

Table 6: Trade Returns and Their Distribution for Cointegration Method Trading

From results in Table 6 we can see that cointegration method has somehow similar outcome for all three studied equity markets. Their average excess return is negative for all of them, they have high average of days the pairs are open and they have really similar share of trades that end up in a loss. When we look closely on PSE, we can see quite low total number of trades in comparison with other two markets, which might consequently affect our t-statistic of average return, but it is not quite the case. PSE is with its 61.42 days per trade the market with the lowest average time per trade, which means that we are forced to reject Scenario 1 for all three markets, since neither average number of days criterion or the percentage of all pairs criterion is fullfilled, and to be thorough, nor is the positive average returns criterion. Share of round-trips is comparable to the statistics we obtained in

[10] The percentage of trades that were finished during the period, the rest must have been liquidated on the last day of the period.

the distance method, which again signals, that the average number of days the pair is open might be much higher and our trading period is not long enough for executing the trades. Another explanation could be that the detected cointegration from the formation period did not last until the trading period and the opened pairs would not close even in the future. Share of negative observations supports our claim in rejecting the Scenario 1. On grounds of the discussion above we cannot reject Scenario 2 for PSE, we can even confirm this scenario since all of the three metrics were affirmative.

When we will focus on BSE, we will for sure reject Scenario 1, because we obtained negative average returns which are significant in this case. We can see, that median is positive, however the rest of the descriptive statistics points out, that returns on BSE have negative skewness, which we will confirm in the following text. We are left with Scenario 2, which we will verify, because we have negative average returns, high average number of days the pair is open and the number of cointegrated pairs lies somewhere between four and five percent, it is 4.55% to be specific, for BSE. BVB has again the least significant excess returns, but they are also negative just as the returns on PSE and BSE. So we will reject Scenario 1, it is also supported by the fact, that only in one period we had more than 10% of pairs cointegrated, the rest is much lower and in total it gives 5.04% throughout all periods for BVB. This statistics supports our Scenario 2 in the case of BVB as well.

Further investigation of cointegrated pairs revealed that on PSE no single pair was identified in more than two sample periods, which might signal that the relationships between stocks from different industries are not sustainable for longer periods of time and those pairs therefore end up in the loss. The pair detected most times on BSE was again the one between two utility companies, Eszak Magyarorszagi Aramszolgaltato Nyrt and Budapesti Elektromos Muvek Nyrt, which proved that they have long-lasting relationship. Other than that, we did not find any pair that would show any indications of sustainability. The cointegration method on BVB detected pair, which was not present in any period of distance method, consisting of oil company OMV Petrom SA and financial company SSIF BRK Financial Group SA. Their cointegration was detected in the first three sample periods and looking on the graph[11] of their stock price movements shows big gradual drop in both cases. Our guess is that it is caused by accepting Romania into European Union and the following events triggering drop in both stock prices.

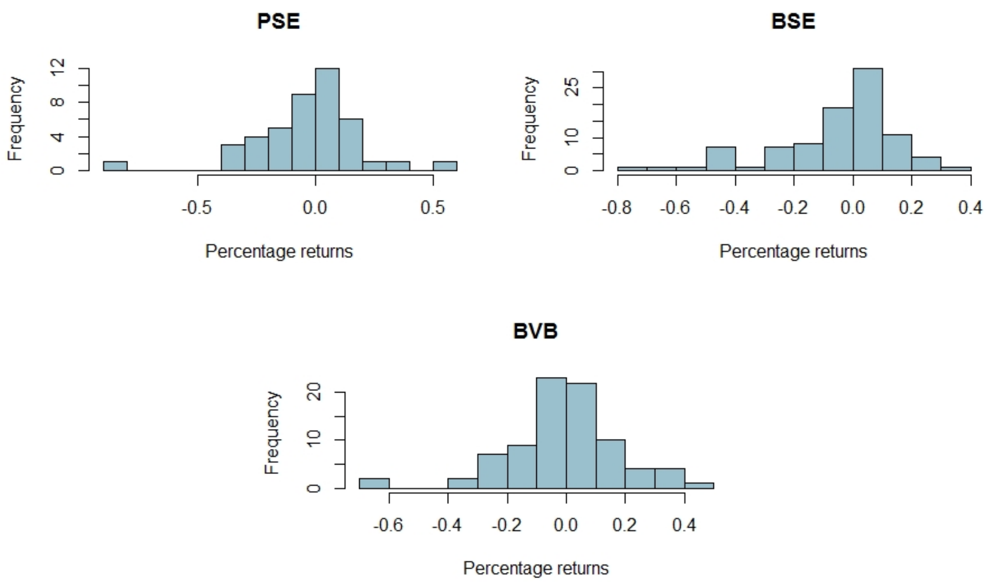

Figure 3: Histograms of Cointegration Method Trading for PSE, BSE and BVB

Our results are confirmed by the histograms of percentage returns per trade shown in Figure 3. In all three cases we can see negative skewness, which is easily visible in our histograms, especially in the histogram of percentage returns on BSE. Negative skewness has a long left tail in negative direction, and it is a rule of thumb, that mean is to the left of the peak, which is confirmed by our descriptive statistics. The histogram of the percentage returns of pairs on PSE has also negative skewness, but from the graph we can see that it is probably caused by an extreme outlier which is also a minimum of our returns, nevertheless it does not change a thing that our results are worse than we anticipated. With regard to excess kurtosis, the percentage returns on PSE are mesokurtic, which signals normal distri-

[11]The graph was ommitted in this thesis.

bution. The percentage returns on BSE and BVB are platykurtic, which indicates flat and wide distribution around the mean, for which we would probably need more observations in order to see those features more clearly in our histograms.

Based on the discussion above, we would expect larger minimum than maximum in absolute value in returns, which is confirmed. The minimum for PSE is -0.8552, which signals that during one trade we lost 85.52% of our initial value of the investment. The maximum is 0.5623, which proves that minimum is larger than maximum in absolute terms. This feature is true for all three markets. Sample standard deviation around 0.2 is an indication of really high volatility of returns, which is for risk-averse investors strong signal not to use this strategy. Our conclusions are finally supported by poor average information ratios that are 0.0286 for PSE, -0.0023 for BSE and 0.0418 for BVB, respectively.

7.3 Comparison of the Distance Method and the Cointegration Method

From the text above and Tables 5 and 6 is visible that the distance method outperforms the cointegration method in almost all statistics on all three markets. The cointegration method has lower average excess return in all three cases, which all alone makes a strong evidence in favor of the distance method. It is then supported by higher average time the pair is open and lower share of round-trips across the whole sample. The last statistics worth mentioning is lower number of all trades, especially on PSE. This would not be an issue if the method chose good pairs and the trades would occur less often, but they would be more profitable. Unfortunately, that is not the case and we have to conclude that the cointegration method is worse on small-sized equity markets with high industry diversification. Our findings are in contradiction with Zeng's and Lee's (2014) and Huck's and Afawubo's (2015) conclusions and empirical evidence from US market that the cointegration method outperforms the distance method even with certain transactions costs. Our obtained results are, however, caused by the characteristics of analysed markets and have not a relevant informative value. What is interesting in both methods is that they were able to identify similar pairs despite the different methodology. The prominent example of such behaviour is the mentioned pair between two Romanian utility companies[12].

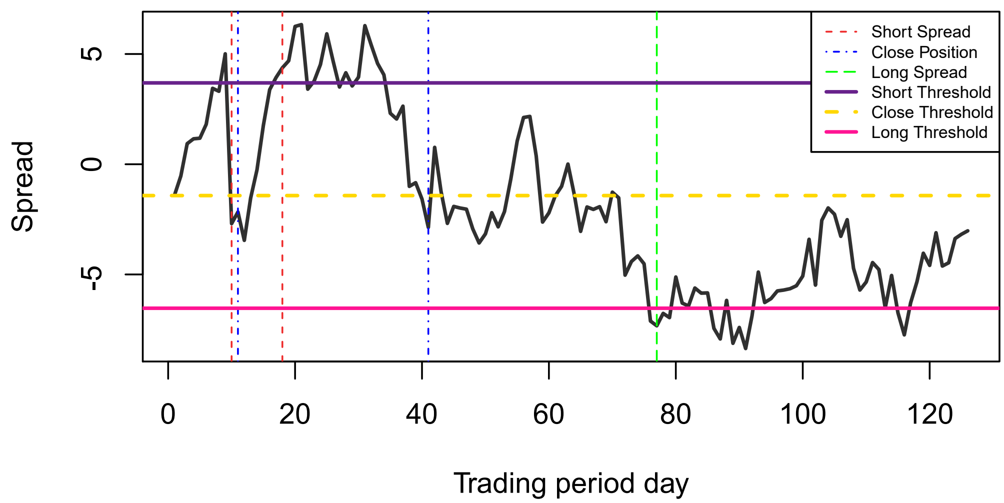

Figure 4: Example of Trading Triggers

Both methodologies achieved execution of trades on all three equity markets, but with no success of providing profits to investors. We will show an example of successful pair and its opening and closing triggers in order to help reader visualize the process in our algorithm. For illustration we selected the most often pair from the distance method on PSE, KB and VIG, from sample period five. Figure 4 shows the pairs trading strategy using the distance method. In the graph we can see the development of the spread between KB and VIG accompanied by several horizontal lines, which represent thresholds for triggering the trade or closing the position. The vertical lines indicate when we should enter or left the trade. We can see that in day 10 we entered the trade and bought the spread, when the spread

[12] Eszak Magyarorszagi Aramszolgaltato Nyrt and Budapesti Elektromos Muvek Nyrt.

crossed the Short Threshold line and the position was closed in the next day which was caused by huge drop in the spread and by crossing the Close Threshold line. The same scenario happens in day 18 and 41, respectively. We entered the trade once again, only with the difference that now we sold the spread, because it crossed the bottom threshold. The last trade was not finished, because the spread did not cross the Close Threshold, but we can see that it was heading towards the long-term equilibrium represented by the Close Threshold line. What is interesting to spot is that we enter the trade with one-day lag after crossing the thresholds. This affect our final returns, especially in the first trade, when we would end up with significant profit, however, with the lag it was not so great. The threshold values were obtained during the formation period according to Equations (20) and (21) or Equations (22) and (23), respectively.

8 Conclusion

In this thesis, we discussed the effectivity of pairs trading in small Central and Eastern European equity markets, more specifically on Prague Stock Exchange, Budapest Stock Exchange and Bucharest Stock Exchange. We focused on the distance method and the cointegration method, which have same theoretical background, but their methodology is quite different. We aimed to examine both methods, how they would perform with comparison to the market and against each other, and especially, if the pairs trading earns similar returns as in the US market.

The pairs trading strategy was tested using a sample period of almost nine years from June 2008 to March 2017. Our findings show that the pairs trading strategy has no significant results on small equity markets. The distance method which followed methodology of Gatev, Goetzmann and Rouwenhorst (2006) had the average return fluctuating around zero, the exact average returns were -0.49%, -1.16% and 0.06%, respectively, but none of them was statistically different from zero. Not so appealing statistics were supported by the fact that more than half of the trades ended up in the loss and the average round-trip lasted between 51 and 61 days. When we compared our results with the ones found by Gatev, Goetzmann and Rouwenhorst (2006), we detected that the method does not yield similar results. The method had negative insignificant results, high volatility and much greater risk. The comparison with Finnish stock market showed in addition to the previous findings two times longer round-trip duration.

The cointegration method, which followed Vidyamurthy (2004) detected several cointegrated pairs but their quantity was on average 4.55% of all possible pairs. These relationships were, nevertheless, quite weak and oftentimes were easily broken by the industry shocks. This method yielded negative returns with enormous volatility and high average of days the pairs were opened. In comparison between the two mentioned methods the first one was better in all measured statistics. Nonetheless, any of the two methods is quite suitable for markets we studied, because it would be much easier to just hold the indices of the Stock Exchanges which is obvious from the small information ratios. We attribute these results to the small size of the equity markets and their high industry diversity. In the final analysis we would not recommend the use of pairs trading strategy in any of the studied markets without extended research of the traded pairs and their possible correlation, since the results are quite straightforward.

The analysis in this thesis shows that robust profitability found among large equity markets cannot be convertible to the markets that are small and supports findings of Jacobs and Weber (2015) who found out that the size of the market matters in the pairs trading strategy. We do not eliminate the possibility of profitability in the small-sized equity markets, but the characteristic of markets in Prague, Budapest and Bucharest does not seem like a right fit. Future research might improve our findings by incorporating industry diversity, risk control measures and extending the number of sample periods. These measurements might bring more significant results, but their indications will probably stay the same.

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Appendix: Companies Used in the Analysis and Period Dates

	Used in periods	Not used in periods
AAA Auto Group NV	1-7	8-15
Central European Media Enterprises Ltd	1-15	None
ČEZ as	1-15	None
ECM Real Estate Investments Ag SA	1-4	5-15
Erste Group Bank AG	1-15	None
Fortuna Entertainment Group NV	6-15	1-5
Kofola Československo as	None	1-15
Komerční Banka as	1-15	None
Moneta Money Bank as	1-15	None
New World Resources Plc	1-13	14-15
O2 Czech Republic as	1-11	12-15
Orco Property Group SA	1-10	11-15
Pegas Nonwovens SA	1-15	None
Philip Morris CR as	1-15	None
Stock Spirits Group Plc	12-15	1-11
Tatry Mountain Resorts as	None	1-15
Unipetrol as	1-15	None
Vienna Insurance Group AG Wiener Versicherung Gruppe	1-15	None

Table 7: Name of the Companies Used in the Analysis - PSE

	Used in periods	Not used in periods
ANY Biztonsagi Nyomda Nyrt	2-15	1
Appeninn Vagyonkezelo Holding Nyrt	6-15	1-5
Budapesti Elektromos Muvek Nyrt	2-15	1
CIG PannoniaEletbiztosito Nyrt	6-15	1-5
ENEFI Energiahatekonysagi Nyrt	1-15	None
Est Media Vagyonkezelo Nyrt	1-15	None
Eszak Magyarorszagi Aramszolgaltato Nyrt	1-15	None
FHB Jelzalogbank Nyrt	1-15	None
Graphisoft Park SE Ingatlanfejleszto Europai Rt	1-15	None
Magyar Telekom Tavkozlesi Nyrt	1-15	None
MOL Plc	1-15	None
OTP Bank Nyrt	1-15	None
PannErgy Nyrt	1-15	None
Plotinus Vagyonkezelo Nyrt	7-15	1-6
Raba Jarmuipari Holding Nyrt	1-15	None
Richter Gedeon Vegyeszeti Gyar Nyrt	1-15	None
Zwack Unicum Likoripari es Kereskedelmi Nyrt	1-15	None

Table 8: Name of the Companies Used in the Analysis - BSE

	Used in periods	Not used in periods
Banca Comerciala Carpatica SA	2-15	1
Banca Transilvania SA	1-15	None
Biofarm SA	1-15	None
BRD Groupe Societe Generale SA	1-15	None
Bursa de Valori Bucuresti SA	6-15	1-5
Compania Nationala de Transport al Energiei Electrice Transelectrica SA	1-15	None
Condmag SA	1-11	12-15
Conpet SA	2-15	1
Dafora SA	1-11	12-15
Electromagnetica SA	1-15	None
Fondul Proprietatea SA	7-15	1-6
Impact Developer & Contractor SA	1-15	None
OMV Petrom SA	1-15	None
Rompetrol Rafinare SA	1-15	None
Societatea Energetica Electrica SA	14-15	1-13
Societatea Nationala de Gaze Naturale Romgaz SA	12-15	1-11
Societatea Nationala de Transport Gaze Naturale Transgaz SA	1-15	None
Societatea Nationala Nuclearelectrica SA	1-15	None
SSIF BRK Financial Group SA	1-15	None

Table 9: Name of the Companies Used in the Analysis - BVB

					PSE			BSE			BVB		
Sample period	Formation period (D.M.Y)	Data points	Trading period (D.M.Y)	Data points	Number of stocks	Stat	Coin	Number of stocks	Stat	Coin	Number of stocks	Stat	Coin
1	2.6.2008 - 30.8.2009	311	31.8.2009 - 28.2.2010	126	13	0	3	12	0	-	12	0	8
2	24.11.2008 - 28.2.2010	315	1.3.2010 - 5.9.2010	131	13	1	4	14	2	12	14	2	8
3	8.6.2009 - 5.9.2010	314	6.9.2010 - 6.3.2011	128	13	0	1	14	2	3	14	2	6
4	7.12.2009 - 6.3.2011	315	7.3.2011 - 4.9.2011	126	13	1	1	14	4	1	14	3	2
5	7.6.2010 - 4.9.2011	317	5.9.2011 - 4.3.2012	126	12	0	2	14	0	5	14	1	4
6	6.12.2010 - 4.3.2012	315	5.3.2012 - 2.9.2012	122	13	0	14	16	1	4	15	3	2
7	6.6.2011 - 2.9.2012	311	3.9.2012 - 3.3.2013	121	13	1	9	17	0	11	16	1	8
8	5.12.2011 - 3.3.2013	305	4.3.2013 - 1.9.2013	123	12	1	-	17	1	5	16	0	2
9	28.5.2012 - 1.9.2013	311	2.9.2013 - 2.3.2014	122	12	1	1	17	2	5	16	2	8
10	26.11.2012 - 2.3.2014	309	3.3.2014 - 31.8.2014	124	13	2	1	17	1	12	16	1	8
11	3.6.2013 - 31.8.2014	308	1.9.2014 - 1.3.2015	122	11	1	1	17	2	1	16	4	7
12	25.11.2013 - 1.3.2015	309	2.3.2015 - 30.8.2015	124	11	1	3	17	2	2	16	3	1
13	2.6.2014 - 30.8.2015	308	31.8.2015 - 28.2.2016	125	11	2	-	17	0	10	16	2	6
14	24.11.2014 - 28.2.2016	312	29.2.2016 - 4.9.2016	130	10	0	-	17	0	9	17	2	4
15	1.6.2015 - 4.9.2016	317	5.9.2016 - 12.3.2017	132	11	1	4	17	1	2	17	4	4

Notes: Stat shows the number of detected stationary stocks in the period, which were then substracted from further analysis; Coin shows the number of cointegrated pairs we detected.

Table 10: Period Dates and Additional Information