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Bankruptcy prediction models in the Czech economy: New specification using Bayesian model averaging and logistic regression on the latest data

Master's Thesis

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Declaration of Authorship

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Prague, July 13, 2017

Signature

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Abstract

The main objective of our research was to develop a new bankruptcy prediction model for the Czech economy. For that purpose we used the logistic regression and 150,000 financial statements collected for the 2002—2016 period. We defined 41 explanatory variables (25 financial ratios and 16 dummy variables) and used Bayesian model averaging to select the best set of explanatory variables. The resulting model has been estimated for three prediction horizons: one, two, and three years before bankruptcy, so that we could assess the changes in the importance of explanatory variables and models' prediction accuracy. To deal with high skew in our dataset due to small number of bankrupt firms, we applied over- and under-sampling methods on the train sample (80% of data). These methods proved to enhance our classifier's accuracy for all specifications and periods. The accuracy of our models has been evaluated by Receiver operating characteristics curves, Sensitivity-Specificity curves, and Precision-Recall curves. In comparison with models examined on similar data, our model performed very well. In addition, we have selected the most powerful predictors for short- and long-term horizons, which is potentially of high relevance for practice.

JEL Classification	C11, C51, C53, G33, M21
Keywords	Bankruptcy prediction, Logistic regression, Bayesian model averaging, Receiver operating characteristics curves

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Abstrakt

Hlavním cílem našeho výzkumu bylo vyvinout nový bankrotní model pro českou ekonomiku. Za tím účelem jsme použili logistickou regresi a vzorek 150 000 finančních výkazů pro období 2002—2016. Pracovali jsme celkově s 41 vysvětlujícími proměnnými, z čehož 25 byly finanční poměrové ukazatele a 16 dummy proměnné, k selekci nejlepších prediktorů bylo využito Bayesovské průměrování modelů. Výsledný model byl odhadnut pro 3 predikční horizonty, jeden, dva a tři roky před bankrotem, abychom mohli vyhodnotit vývoj v signifikanci jednotlivých proměnných a přesnost modelů pro různé horizonty. Protože jsme měli významně méně dat pro bankrotující společnosti, použili jsme metody tzv. over-samplingu a under-samplingu pro data, která byla použita k odhadování našich modelů (80% celého vzorku). Tyto metody se ukázaly být velice efektivní, protože zlepšily predikční schopnosti modelů napříč časovými horizonty. Přesnost predikcí jsme měřili pomocí ROC křivek, Sensitivity-Specificity křivek a Precision-Recall křivek. V porovnání s modely odhadnutými na českých datech náš model dopadl velice dobře. Pomocí analýzy ekonomické a statistické signifikance odhadnutých parametrů jsme navíc vybrali nejlepší proměnné pro predikce v krátkodobém a dlouhodobém horizontu, což má přidanou hodnotu pro praktické použití.

Klasifikace	C11, C51, C53, G33, M21
Klíčová slova	Predikce úpadku, Logistická regrese, Bayesovské průměrování modelů, ROC křivky

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Acronyms

BY Bianco-Yohai (estimator)

CAS Czech Accounting Standards

CF Cash Flow

CPP Cumulative Posterior Probability

EBIT Earnings before Interest and Taxes

EVA Economic Value Added

FPR False Positive Rate

GP Gross Profit

IFRS International Financial Reporting Standards

ISO International Organization for Standardization

LPM Linear Probability Model

MDA Multivariate Discriminant Analysis

MLE Maximum Likelihood Estimation

NACE Nomenclature of Economic Activities

NP Net Profit

PR Precision-Recall (curve)

RE Retained Earnings

ROA Return on Assets

ROC Receiver Operating Characteristics (curve)

ROE Return on Equity

TPR True Positive Rate

Master's Thesis Proposal

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Proposed Topic:

Bankruptcy prediction models in the Czech economy: New specification using latest data

Motivation:

There is a lot of literature regarding bankruptcy prediction modeling and most of it is based on application or modification of three key papers in this area – Altman (1968), Ohlson (1980), and Zmijewski (1984). These authors used different methodologies in estimating their models but the procedure of their structuring is relatively similar. All of them defined key indicators that are supposed to have explanatory power in predicting firm´s failure, then they estimate the model and specify a scale which should serve as an easy predictor if a company is expected to bankrupt in near future or not (i.e. when we plug specific indicators for a particular company into the estimated equation, we get number which is compared with the scale and if it is larger than some frontier we conclude that it is not likely that the company should fail and vice versa).

This quite easy empirical setup has led to expansion of research in this field, especially in re-estimation of accuracy of these models in different countries and suggested improvements in model specification to get higher accuracy - e.g. Thailand examined by Pongsatat, Ramage and Lawrence (2004), Turkey by Canbaş, Önal, Düzakin and Kiliç (2006), Sweden by Yazdanfar (2008) South Korea by Bae (2012), China by Gang and Xiaom (2009) and many other researchers focused on holistic international comparison as Altman, Drozdowska, Laitinen and Suvas (2014). There was also some coverage by Czech researchers – Machek (2014) for example tested the accuracy of the three original models and a couple of other popular models on Czech companies' data but the scope was rather limited. The most prominent researchers in the Czech Republic are probably Inka and Ivan Neumaier who developed a few bankruptcy models based on and adjusted for our economy. In the nineties they introduced 3 bankruptcy models and in 2005 Neumaier and Neumaerová (2005) made public the latest version called IN05. This model has become very popular in Czech settings and it is widely used. Machek (2014), however, says that because of the change of insolvency law in 2008 there is a growing number of bankruptcies which require an update of the models to sustain necessary accuracy of predictions. Also Sušický (2011), although working with Czech data from 1997–2007, rejected a hypothesis that these "national" models perform better than the original ones and their most common variants.

Besides the changes in Czech economy after the crises and European Union expansion, introduction of new laws and other issues, I haven't found any relevant new model which would react on these innovations and would be accepted by broader academic and non-academic circles. It therefore provides space for interesting research and development of new models based on latest empirical evidence.

Hypotheses:

1. Hypothesis #1: There were shifts in Czech economy so bankruptcy has different dynamics.
2. Hypothesis #2: There is a significant difference between industries, firms' sizes and other specifications, so controlling for these improves accuracy of bankruptcy prediction models.
3. Hypothesis #3: Based on new data a better model can be built and improve thus the precision of predictions in the Czech Republic compared to older Czech methods, like IN05, or adjusted original models.

Methodology:

The first step will be broad literature review as there are many methods how to build models for bankruptcy prediction, e.g. which indicators to use, how many control variables to implement, how to structure data etc., and how to estimate them, e.g. use probit or logit. It is crucial to understand all negatives and positives of each method and have robust evidence of their application to be able to develop and structure new model to best fit current situation in the Czech Republic.

The second step will be data. I will use data from Magnus database which contains financial and other data for vast majority of companies operating in the CR that make them public and provides thus extraordinary dataset for purpose of this study. I would like to base my analysis on observations ranging from 2006 to 2015 to obtain robust results. However there are many issues with such large dataset because there can be various disruptions in it and many inconsistencies that could bring about misleading conclusions. For example Sušický (2011) and Neumaier and Neumaerová (2005) disregard in their analyses companies that do not have data for all dates for which they operate. I would like to find a way to use these omissions using quantitative background from the IES as it could improve not just quality of estimators but also robustness of results. Nevertheless, it will be essential to check data thoroughly to detect any suspicious observations (companies) that could lead to various biases. It will be also necessary to define specific industry groups (probably based on NACE codes), specify relevant measures of company features like size, time span of market presence (from establishment), number of employees etc., and generally based on step one to define various control variables that could improve prediction accuracy.

When I have comprehensive dataset and understanding of bankruptcy prediction models´ features (from step one), I will continue with model specification. Here it will be necessary to build multiple models using various ratios (EBIT/assets, Total Liabilities/Total assets etc.) and control variables and test which one gives the best results (apart from evidence from literature). For the purpose of objective comparison of models in next step, it would be appropriate to keep some data for out of sample measurement of predictive power. However, I do not have any concrete idea yet how to proceed here specifically. Here it could be also tested if there were shifts in the Czech economy applying for example IN05 on 2 datasets in different timeframe and compare its accuracy.

In the following step I will compare accuracy of the most commonly used models internationally – Altman´s, Ohlman´s, Zmijewski´s and domestic models – IN05, IN01, on latest Czech data. Having the results for my newly specified model, we could again see if there were significant changes in Czech economy, so that the new model outperforms older ones, or if there are no significant changes at all.

Expected Contribution:

I will conduct update of a methodology for bankruptcy prediction in conditions of the Czech economy. As the most popular models are rather old there is likely a space for improvement to enhance accuracy of predictions using latest data. Having updated model could bring not just possible improvement of ability to predict future distress of companies but could also bring a general discussion about shifts and changes in economies that would require recalibration or complete re-estimation of our prediction models. Besides that I will prepare methodology for adjustment of large datasets from Magnus database and map possible threats so it will be easier for further researchers to work with it.

Outline:

1. Motivation: Introduce why it is important to update current models due to different shifts and changes in the Czech economy.
2. Literature review: Detail overview of features, negatives and positives of various models to be able to use empirical evidence for building of my own model.
3. Data: Working with Magnus database – transparent filtering and structuring of data.
4. Model specification: My own model specification based on extensive literature and tests on latest data.
5. Comparison: Accuracy testing of multiple models to get idea if I achieved some improvement in comparison with this benchmark.

6. Results: I will discuss why it is the case that I got these particular outcomes and what could be potential weaknesses of my approach.
7. Concluding remarks: I will summarize my findings and their implications for policy and future research.

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1 Introduction

We can date the bankruptcy research in the Czech Republic back to 1990s, when the economy was transforming and privatizations brought about substantial changes in the Czech corporate sector. Although during this period a couple of bankruptcy prediction models have been developed and tailored to the conditions of the Czech economy, afterwards the innovation has essentially stopped. So far, the vast majority of bankruptcy research has been limited to the re-estimation of old foreign models like Altman's Z score or ZETA model, and their comparison to the most widely accepted Czech models. Nothing has changed even after the new piece of insolvency legislation came into effect in 2008 and the number of bankruptcies more than doubled between 2008 and 2013, primarily due to adverse economic conditions.

Note that being able to reasonably evaluate company's financial health and future prospects is of crucial importance for many reasons. For instance, managers and shareholders can anticipate potential problems in advance and can act timely to prevent unnecessary losses. Bankers are interested in the creditworthiness of their clients, with reliable scoring models the efficiency of capital allocation could be enhanced. Bankruptcy prediction models can be also used by other related parties, such as suppliers, who can assess the solvency of their customers and protect their interest by the payment schedule's renegotiation when the customer seems to be facing troubles. Efficient classifiers can also be helpful in saving large amounts of money on portfolio screening expenditures, monitoring of customers, or simply timely solution of problems before they become unsolvable.

Having these reasons on mind, the primary objective of this thesis is to specify a new prediction model on the latest data. We want to use the techniques proposed in the foreign literature, which have been never or very rarely employed in the context of the Czech economy. By that we would like to expand the toolbox available to the Czech bankruptcy prediction researchers and contribute to a better understanding of these methods in the context of bankruptcy prediction. Furthermore, we want to provide interested groups with an up-to-date set of predictors, tailored to the specifics of the Czech economy.

We will use logistic regression for the estimation of our bankruptcy model. A set of 25 commonly used financial ratios will be complemented by 16 dummy variables, which should enable to capture the characteristics of individual enterprises more precisely. Out of these 41 variables, just a subset of the most relevant ones will be selected by the Bayesian model averaging method and included in our ultimate model. The resulting model will be estimated for three prediction horizons: one year, two years, and three years in advance. Thus we will be able to evaluate the changes in classifiers' characteristics over time and select the best long- and short-term predictors based on the size parameter estimates and their development across prediction horizons. Our dataset consists of approximately 150,000 financial statements for Czech companies collected for the 2002—2016 period. In addition to the standard MLE estimator, we will employ also the outlier-robust version of the ML estimator proposed by Bianco and Yohai (1996).

The stress will be put on the variables' selection. Czech researchers have been usually using standard F-tests for the assessment of the importance of explanatory variables (see for example (Neumaierová and Neumaier 2005a), (Jakubík and Teplý 2008), (Kalouda and Vaníček 2013)). However, this method is not very effective when the number of explanatory variables to be considered is too large. Therefore, we decided to employ the technique of Bayesian model averaging, which has not been used for bankruptcy prediction in the Czech conditions so far, and could be an extremely useful tool for the Czech academics if proven to work well.

We formulated three main hypotheses to be tested:

1. BMA is an effective method for the selection of powerful predictors in the Czech conditions.
2. Dummy variables such as age, size, and industry provide valuable insights into the characteristics of individual companies and as such should be included in bankruptcy prediction models.

3. It is possible to predict bankruptcies up to two years prediction horizon, but then the accuracy drops significantly. (Altman 1968)

The first hypothesis will be tested by the ability of BMA to select statistically significant variables for our ultimate model. The second hypothesis will be examined by the evaluation of the economic and statistical significance of the dummy variables included in our model. The third hypothesis will be discussed with respect to the changes in the shape of ROC and Sensitivity-Specificity curves for different prediction horizons.

The thesis is structured as follows: In Section 2 we review available literature on bankruptcy prediction in the Czech Republic and internationally. Section 3 describes the development of insolvency legislation and a number of bankruptcies in the Czech Republic. In Section 4 our dataset is described and the methodology is addressed in Section 5. In Section 6 we analyze results of the Bayesian model averaging methodology and interpret the outcomes of resulting logit regressions. In this section we also address the issue of accuracy measurement metrics.

The analysis presented in this work has been conducted using R. Unless stated otherwise, own calculations are sources of data in tables and figures.

2 Literature Review

2.1 World Literature Review

Without any doubts, the history of credit is as old as human civilization. From lenders' perspective, it has always been crucial to evaluate the borrower's creditworthiness to decide if he is able to repay the debt he wants to take on. Although it may sound relatively straightforward, before the era of computers and developed statistical methods, there had been basically no way to approach the issue quantitatively. That is the main reason why bankruptcy prediction and financial scoring methods are quite young field of study. On the following pages, we offer a comprehensive overview of the development in this field of research, and we also discuss in more detail three key papers, that is Altman (1968), Ohlson (1980), and Zmijewski (1984), which have significantly influenced the work of many other researchers. Moreover, these articles nicely summarize the issues that must be tackled in order to build a reliable prediction model.

2.1.1 Beginnings of the bankruptcy prediction

The original interest of academia in the bankruptcy prediction dates back to 1930s, when the USA experienced the Great Depression. The era of severe economic downturn caused some researchers to raise a question if it is possible to estimate which companies are at the risk of future failure and which are not. One of the first studies was conducted by Smith and Winakor (1935) and FitzPatrick (1932), who collected financial data for the set of sound and distressed companies and then simply compared their financials. In 1936, Fisher (1936) introduced a discrimination method, which enabled to detect and test the statistical significance of differences between 2 distinct populations. Although Fisher used it in taxonomy, its application was potentially very wide, extending to the corporate failure research. Beaver (1966) applied the univariate version of Fisher's method to discriminate between 79 sound and 79 bankrupt companies using the list of 30 financial ratios. He found that some ratios have very strong predictive power even 5 years before the company actually files for a bankruptcy. Likewise, Tamari (1966) studied differences between 1610 healthy industrial companies and 28 financially distressed industrial companies. He used a simple comparative analysis to decide which ratios are most important and then used them for the specification of a scoring function. These pioneering papers established a solid base for further research.

2.1.2 Altman's approach

The major breakthrough in the bankruptcy prediction research came with the paper Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy, published in 1968 by Edward I. Altman. Influenced by the studies of Beaver and Tamari, Altman noted that multivariate discriminant analysis (MDA), used for consumer-loan evaluation at that time, could serve the purposes of bankruptcy prediction as well.

His idea was to use the MDA for the estimation of scoring function consisting of financial ratios, which was meant to discriminate between the groups of bankrupt and sound companies. The dataset for the Altman´s analysis consisted of financial ratios for 33 bankrupt and 33 non-bankrupt US companies between the years 1946 and 1965. He used stratified random sampling for the selection of the non-bankrupt companies, so that they had similar characteristics (e.g. size, industry, etc.) as the sample of bankrupt companies. Based on previous literature and analysts´ opinions, he defined 22 financial ratios divided into five main categories: leverage, solvency, liquidity, profitability, and activity ratios. Subsequently, he used F-tests, inter-correlation analysis, and personal judgement to examine which ratios are the best in discriminating bankrupt and non-bankrupt companies. Eventually, he picked five best-performing ratios, that he used to estimate the actual discrimination function's coefficients as

$$Z = 0.012x_1 + 0.014x_2 + 0.033x_3 + 0.006x_4 + 0.999x_5\,,$$

where variables x_1 to x_5 are Working Capital/Total Assets, Retained Earnings/Total Assets, Earnings Before Interest and Taxes/Total Assets, Market Value of Equity/Book Value of Total Debt, and Sales/Total Assets, respectively.

Altman named the discrimination function Z score. Note that all five ratios are intuitively expected to be higher for healthy than distressed companies. As a result, higher scores meant lower probability of failure, and vice versa. Therefore, Altman was able to define a scale which served as a point of reference in deciding whether the firm is or is not likely to go bankrupt. He elucidated it as, "*all firms having a Z score of greater than 2.99 clearly fall into the "non-bankrupt" sector, while those firms having a Z below 1.81 are all bankrupt. The area between 1.81 and 2.99 will be defined as the "zone of ignorance" or "gray area" because of the susceptibility to error classification."* (Altman 1968, p. 19) Applying this scale for one-year horizon on the secondary sample consisting of 66 non-bankrupt and 25 bankrupt companies, Altman obtained truly impressive results. 90% of eventually bankrupt companies were predicted to bankrupt, and 79% of non-bankrupt companies were correctly assigned to the non-bankrupt group. In contrast to Beaver (1966), Altman found that it is possible to reliably predict distress of the company up to two years before the bankruptcy, but the accuracy declines substantially from the third year.

2.1.3 Post-Altman period; Ohlson (1980) and Zmijewsky (1984)

After the paper of Altman (1968) had been published, the field of research experienced a real boom. Many academics applied MDA method on various datasets and studied how different specifications of the discrimination function influence the results of analysis. To name a few important contributors of that decade, we can mention for example Edmister (1972), Deakin (1972), Blum (1974), Libby (1975), Moyer (1977), or Altman et al. (1977). During this period, the discriminant analysis was undoubtedly the most prominent approach. Nonetheless, the method suffers from a few imperfections, which were questioned by some authors. Ohlson (1980) came up with a couple of objections why MDA approach is not statistically appropriate, and proposed a conditional logit function as a solution for these shortcomings.

One of the flaws Ohlson (1980) listed were the requirements on the distributional properties of predictors. For instance, Ohlson mentions the need for variance-covariance matrices of predictors to be the same for both groups (bankrupt and non-bankrupt). Moreover, the use of dummy variables is restricted in case of discrimination analysis, due to the requirement of normally distributed predictors. Although these restrictions do not hamper the specification of discrimination function (since the objective of the MDA is just to discriminate between 2 populations), Ohlson (1980) argues that it imposes significant limits on statistical inference, which cannot be reliably drawn from the MDA.

Besides that, Ohlson (1980) discussed various problems linked to the sample-matching methods. Since two representative samples of the same size are required for the MDA, researchers have to match the minority class of failed companies with the appropriate sub-sample of non-failed companies. Even though measures like total assets, sales, industry, etc., are frequently used for this purpose, Ohlson (1980) opposed to that claiming they are rather arbitrary, and that during the sample-matching process any kind of information can be lost. Therefore, he suggested that everything was directly included in the estimation to avoid the potential loss of information.

To fix the vast majority of MDA shortcomings, Ohlson proposed to use conditional logit analysis. He emphasized the fact that the logit analysis does not require matching-sample procedures and uses available data in full. In other words, other company specifications, such as its size, should be included directly in the logit model, instead of being used just for the selection of matching sample. As a consequence of larger datasets, statistical inference can be drawn on asymptotic sample theory. (Ohlson 1980)

He also argues that only data made public before the firm filed for bankruptcy should be used in order to prevent the look-ahead bias. This means that all financial statements which were made public after the company filed for bankruptcy should be disregarded. Ohlson maintains that many authors use the financial data announced after the company actually filed for bankruptcy, and therefore violate the very logic of prediction.

In the last section of his article, Ohlson discusses the issue of prediction models' accuracy evaluation. Commonly, authors had used the classification matrix with the probability cutoff point of 0.5 for the assessment of prediction´s accuracy. Nevertheless, Ohlson argues that the point of bankruptcy prediction is to predict bankruptcy, and not non-bankruptcy, which means that the loss function is necessarily not symmetric. Hence he suggests weighting type I and type II errors differently. Ohlson realized that shifting of the cutoff thresholds enables to "manipulate" frequencies of both types of errors and thus affect the classifier's accuracy. Although he did not provide any guideline how to select the proper cutoff, he successfully broke the dogma of 0.5 probability cutoff point implicitly assuming a symmetric loss function.

Zmijewski (1984) also examined some of the methodological issues bankruptcy researches frequently face. He pointed especially to the topic of incomplete data, which arises when a part of the dataset is ignored due to the incompleteness of information. This can be seen in multiple bankruptcy prediction studies, since authors tend to disregard companies with missing observations. What they can get as a consequence is a sample selection bias. The bias appears when the bankrupt companies have higher probability of incomplete data. Excluding these firms from the estimation would likely result in an understatement of bankruptcy probabilities. (Zmijewski 1984) While the study showed that the bias is indeed present, the author concluded that it doesn´t seem to have any significant effect on statistical inferences. Furthermore, he did not provide any applicable solutions to these deficiencies.

2.1.4 Latest development

There has been a large number of researchers who have come up with plenty of completely new approaches to the estimation methodology in order to eliminate some of the limitations discussed previously. These methods include for instance semi-parameter models (Klein and Spady 1993), classification trees (Breiman et al. 1984), genetic algorithms (Back et al. 1996, Acosta-González and Fernández-Rodríguez 2014), hazard models (Shumway 2001, Chava and Jarrow 2004, Hillegeist et al. 2004), or very popular neural networks (Tam and Kiang 1992, Sharda and Wilson 1996, Jo et al. 1997, O'leary 1998, Zhang et al. 1999, Alam et al. 2000, Lee et al. 2005).

Along with these new approaches, there have been multiple studies focusing on empirical application of standard models. To name a few, Thailand was examined by Pongsatat et al. (2004), Turkey by Canbaş et al. (2006), Sweden by Yazdanfar and Nilsson (2008), South Korea by Bae (2012), China by Hu and Zhang (2009) and Wang and Campbell (2010), Japan by Xu and Zhang (2009), and the list is far from being exhaustive. There is, however, one article to point out, and that is Altman et al. (2014), which analyzed the prediction accuracy of multiple models with the national data for all EU countries. It was concluded that there is likely not a comprehensive model which could be applied generally in every context. Instead, it is necessary to calibrate models for unique conditions in each country. (Altman et al. 2014) This is the one of the reasons why we believe that our study can specifically contribute to the innovation in the Czech bankruptcy prediction.

2.2 Czech Literature Review

In the late 1990s and early 2000s, there was a clear interest in bankruptcy prediction modelling because of the turbulent developments in the Czech economy. It was the era of privatizations and other significant changes associated with the transformation of economy. As a result, a couple of researchers developed their own bankruptcy prediction models tailored specifically for the Czech Republic. However, there has been only a marginal innovation after this period, and virtually no new models have been introduced. This continued to be true even after the new bankruptcy legislation came into effect in 2008, since researchers have started to focus on different topics linked to corporate failures. We will touch upon that briefly, too.

2.2.1 Czech bankruptcy prediction models

Probably the most prominent Czech researchers in the field of bankruptcy prediction are Inka and Ivan Neumaier, who between 1995 and 2005 introduced four prediction models: IN95, IN99, IN01, and IN05. (Neumaierová and Neumaier 2002, 2005b) They were complemented by Slovak researchers Gurčík and Chrastinová, who developed their own indices: G-index (Gurčík 2002) and CH-index (Chrastinová 1998). Nonetheless, while Gurčík and Chrastinová focused solely on agriculture, Neumaiers tried to build generally applicable models.

Index IN95 was the first model in a row and all of the following three models were derived from it (although there were some modifications; large portion of changes was due to updated data). Since Index IN95 was based on Altman´s original article, it used the discrimination method. The discrimination function proposed by Inka and Ivan Neumaier consisted of six ratios: Assets/Equity, Interest Coverage Ratio, EBIT/Assets, ROA, Current Ratio, and Overdue Payables/Net Income. In addition to the Altman´s basic setup, the authors used different weights for estimated parameters, depending on a sector at which the particular company operates (recall, that dummy variables could not be used in the MDA). As a consequence, authors specified approximately 20 different discrimination functions for each of the main NACE codes (Nomenclature of Economic Activities). However, although Sušický (2011) stressed the necessity of differentiation among industries, he discussed the issue of industry-specific changes in legislation, developments in industrial organization, and effects of other external factors. He concludes that the models must updated regularly to reflect the latest development.

In their second model IN99, Neumaiers approached the problem from a different perspective. Instead of the standard bankruptcy approach, the EVA (Economic Value Added) metric was employed to define the dependent variable. Hence they split their dataset into two groups, one with positive and the second with negative EVA. Subsequently, they used MDA for the estimation of discrimination function as in the case of IN95. Although the model was not a classic bankruptcy prediction model, it represented a handy instrument for the evaluation of companies´ health in general.

The third index in a row was IN01. It was a combination of preceding two indices, IN95 and IN99, where not only the bankruptcy variable was taken into account, but so was the EVA measure. Otherwise, the estimation methodology was identical as in the case of both older indices. The last index, IN05, was virtually just the parameter re-estimation of the IN01 model using the most recent data. (Neumaierová and Neumaier 2005b)

A couple of researchers tried to compare the accuracy of these Czech models with the precision of other internationally accepted models. Machek (2014) compared the prediction accuracy of IN99 and IN05 models with Traffler´s model, Altman´s Z-score, and Kralicek´s Quick test on the sample of more than 8000 Czech companies from 2007–2012. He concluded that IN05 index outperformed Altman´s Z-score, followed by index IN99. On the other hand, Sušický (2011) found that the Czech indices do not perform necessarily better than the foreign ones. He compared predictive abilities of IN99, IN01, and IN05 indices with those of Altman´s Z-score, Altman´s ZETA model, and Traffler´s model. Although there were some variations between the models´ performances conditional on specific industries, overall, the Altman´s ZETA and Z-score models outperformed the Czech IN05 and IN99 indices. Sušický (2011) thus argued that the Czech bankruptcy prediction models do not provide much added value over the Altman´s models. The same was found by Šlégr (2013).

Some researchers tried to apply standard models on specific industries with high occurrence of bankruptcies. Klečka and Scholleová (2010) studied the prediction abilities of Altman's Z-score and index IN05 on glass-making industry, and concluded that both models produce insufficient results. Čámská (2015) and Karas and Režňáková (2017) re-estimated standard models for the construction industry, which was hardly hit by the adverse economic conditions, and were able to improve the accuracy of these models.

To conclude, in the last decade Czech researchers have been so far conducting primarily comparative studies for various internationally and domestically developed models. This caused that there has been a lack of innovation in the bankruptcy prediction field in the Czech Republic. Hence we feel that the application of modern methods for the specification of new prediction models could contribute the academia, and it could also offer some fresh and illuminating insights into the bankruptcy prediction in the Czech context.

2.2.2 Other bankruptcy-related research

Jakubík (2007) and Jakubík and Teplý (2008) analyzed bankruptcies rather from the macroeconomic point of view. Jakubík (2007) studied which macroeconomic indicators are significant for the share of bankruptcies in the economy. He found that the share decreases with the growth of domestic GDP, growth of GDP of main trade partners, and inflation. On the other hand, it increases with higher share of credit on GDP, growth in interest rates, and the appreciation of currency. Jakubík and Teplý (2008) built a financial scoring model for the non-financial sector in the Czech Republic, named JT index, which was designed to assess the aggregate creditworthiness of the Czech corporate sector.

There has been also some research related to the accounting specifics of the Czech economy. Kubíčková (2011) analyzed the impact of different accounting standards on the accuracy of Altman´s Z-score model. She focused on the differences between IFRS and CAS, and found that companies using IFRS seemed to be in worse condition than those using CAS. As a result, in 94% of cases the Z-score was lower for companies using IFRS in comparison with those using CAS. Bokšová et al. (2015) proved that not all Czech companies make their yearly financial statements public as they should according to the current legislation. Although the research was conducted on a quite limited sample of around 200 suppliers of Skoda Auto, it suggested that the incomplete data bias proposed by Zmijewski (1984) could be present in the Czech case, too.

A couple of researchers focused on the evaluation of Act No. 182/2006 Coll. On Bankruptcy and its Settlement Methods, that came into force in 2008. For instance, Smrčka et al. (2014) opposed to the international data of the World Bank, which suggested that the recovery rate in the Czech Republic (i.e. the receivables that creditors get back in the bankruptcy proceeding) increased from 20.9% in 2008 to 65% in 2014. The WB data were in a clear contradiction with their findings, which showed recovery rates of only 24.96% of total claims between 2012 and 2013. Moreover, Smrčka et al. (2014) found out that the cost of the bankruptcy proceedings for creditors is extremely important variable for the efficiency of the system. With regard to this, the new legislation had rather negative impact in the Czech Republic, since the cost of the bankruptcy proceedings as the share of total claims increased from 15% in 2008 to 17% in 2014. (Smrčka et al. 2014)

3 Czech Bankruptcy Legislation and the Latest Development

After the Velvet revolution, new insolvency legislation was enacted and came into force in 1991 as Act. No. 328/1991 Coll.. The main issue was that it was based on fairly old laws. It should be reminded that concept of bankruptcy was completely unknown in planned economies and that is why no progress was made on the insolvency legislation during the communist era. The last relevant piece of legislation was the Act. No. 64/1931 Coll., which was at the time just about 60 years old. Moreover, since the bankruptcy legislation differed widely from state to state, it was not easy to find an appropriate model to follow. (Diblík 2004) As a consequence, after 1991 the bankruptcy legislation had been amended multiple times and the need for updated comprehensive rules has become obvious.

The new law that replaced the original Act. No. 328/1991 Coll. was passed in 2006 and came into force on the January, 1 in 2008 as the Act No. 182/2006 Coll. on Bankruptcy and its Settlement Methods. The main objective of this new legislation was to streamline the bankruptcy proceedings and make it cheaper and more transparent for both creditors and debtors. The aim was to improve the position of creditors to assure that they will be able to collect as much receivables as possible. Debtors, on the other hand, should get motivated to act timely and not when the debts are already too large to be paid back. They should be able to follow clearly defined rules and methods which should not just enhance the transparency of insolvency process for both related parties, but also support debtors in proactively solving the problem of their potentially unpayable debts. (Justice.cz 2006)

Furthermore, the stress in the new legislation was put on keeping the debtor´s business operating if possible. For that purpose, there was a need for capable insolvency administrators who would be able to recognize when the business could be run efficiently to satisfy creditors, and to eliminate at the same time the social impact of company’s bankruptcy on employees and other stakeholders. The increasing importance of the insolvency administrators was reflected in the following piece of legislation, the Act No. 312/2006 Coll., which put further requirements on their training and education.

For example, only a person with Master’s degree could become an administrator, and only after successfully passing the exams prepared by the Ministry of Justice.

Act No. 182/2006 Coll. on Bankruptcy and its Settlement Methods specified two methods for solving the insolvency of legal entities, and it offered a solution for natural persons, too. Businesses could either go bankrupt, meaning that the assets were sold and obtained proceedings distributed to creditors, or they could reorganize the business (e.g. restructure their capital structure) and, by following a very strict repayment plan, gradually satisfy creditors’ claims. Originally, the possibility of reorganization could be given only to companies with turnover of at least CZK 100 m in the last year before the insolvency, and to firms with more than 100 employees. In spite of this, after 2014 these requirements were loosened and limits were lowered to revenues of at least CZK 50 m and more than 50 employees. In addition, natural persons were newly allowed to file for a “personal bankruptcy”. These changes enabled heavily indebted individuals to wipe their debts out by following a repayment plan that should assure the repayment of at least 30% of all lender’s claims. However, since the personal bankruptcies are not the subject of this thesis, we will not elaborate on the details.

In Table 1, there is an overview of the development of declared bankruptcies, insolvency proposals and reorganizations between 2008 and 2016 in the Czech Republic. It is impossible to draw general conclusions form this data, since it has been significantly affected by the economic crises, however, there are a couple of trends worth noticing.

First of all, there was a rapid increase in declared bankruptcies after 2008. In absolute values, the number of declared bankruptcies almost doubled between 2008 and 2013, reaching its peak of 2403 failures in 2014, and then starting to fall. Unfortunately, Creditreform did not provide precise data for the share of tradesmen and companies on the total number of declared bankruptcies before 2012. Hence it is not possible to discuss the development specifically for the segment of firms, but it seems that the peak was reached in 2013, that is one year before the peak in the total number of declared bankruptcies.

Table 1: Development of insolvencies in the Czech Republic after 2008

	2008	2009	2010	2011	2012	2013	2014	2015	2016
Declared bankruptcies	1,141	1,553	1,601	1,778	1,899	2,224	2,403	2,191	1,982
Tradesmen	n/a	n/a	n/a	n/a	555	849	1,110	1,158	1,076
Companies	n/a	n/a	n/a	n/a	1,344	1,375	1,293	1,033	906
Insolvency proposals	5,355	9,493	16,118	24,353	32,228	36,909	35,140	32,353	29,505
Companies[1)]	3,418	5,255	5,559	6,753	8,398	6,021	3,563	3,004	2,438
Consumers[2)]	1,936	4,237	10,559	17,600	23,830	30,888	31,577	29,349	27,067
Reorganizations	6	13	19	17	17	12	31	18	27

[1)] *Since 2013 non-business tradesmen excluded from the Companies segment and included in the Consumers segment*
[2)] *Since 2013 non-business tradesmen included in the Consumers segment*
Data provided by Creditreform

Secondly, there was a significant increase in the number of insolvency proposals that sextupled between 2008 and 2012, and which was fueled mainly by the proposals submitted by natural persons who used the instrument of personal bankruptcy enacted in 2008. To draw any inferences from the development in the segment of companies is again a bit troublesome due to the change in the methodology of Creditreform. It started to exclude non-business tradesmen from the segment of companies since 2013 and did not adjust their time series retrospectively. Nonetheless, it seems that the peak number of insolvency proposals submitted by firms was reached either in 2012 or 2013 (the drop in 2013 was caused by the methodological change discussed above).

Thirdly, it is evident that reorganizations represent just a small number of insolvency cases in the Czech Republic. Although the requirements for companies to be eligible for reorganization were loosened starting from 2014 (see the jump from 17 cases in 2013 to 31 cases in 2014), it continued to be a very rarely used tool. Despite all, it is undoubtedly a very useful instrument, since it is aimed at larger companies with more employees and potentially larger negative social impact in the case of simple bankruptcy.

To sum up, after the year 2008, the Czech Republic has experienced a fairly dynamic development in terms of the number of declared bankruptcies and submitted insolvency proposals. It seems that both variables peaked around the year 2013, when the number of declared bankruptcies and insolvency proposals by companies was twice as large as in 2008. Nonetheless, it is impossible to distinguish between the effects of economic downturn after the 2008 financial crisis and the impact of the new insolvency legislation.

4 Data

4.1 Sources and general characteristics of data

The primary source of data for the thesis was Magnus database. The database consists of information for all registered Czech and Slovak companies gathered from multiple publicly available sources (e.g. Business register, Insolvency register, Courts, etc.), which makes it a very useful source for this type of analysis. Note that we concentrated solely on companies registered in the Czech Republic and the 2002–2016 period.

Moreover, only enterprises with the annual revenue larger than CZK 100 m were included in the dataset. The main reason for setting this lower bound was that the segment of small companies would substantially increase the complexity of our dataset and it would be a rich source of outliers, too. (Altman et al. 2014) Furthermore, relatively larger companies are generally more stable and less likely to change their business orientation and other characteristics.

In addition to this, just 4 main legal business forms were considered: joint-stock companies, limited liability companies, general partnerships, and limited partnerships. It is also important to note that all types of natural persons were excluded as well as other types of legal forms, such as national corporations (e.g. Budvar), all types of public organizations, not for profit organizations, cooperatives, clubs, unions and others were not included in the dataset for obvious reasons. On top of that, only non-financial companies were considered due to the sharp difference in business models, and subsequently also substantially distinctive income statements and balance sheets.

Dependent variable in our model is meant to symbolize if the firm went bankruptcy or not. While there are some nuances in the definition of bankruptcy across various authors, we followed the norm and defined bankrupt company as a company that went bankrupt according to the insolvency court's resolution. Although Magnus database contains some information about bankruptcies and insolvency proceedings, it does not unfortunately identify exact dates when the specific actions and resolutions of insolvency courts took place. Therefore, it was necessary to collect these dates for approximately 1,200 individual companies directly from the Czech insolvency register for the 2008–2016 period and from the Czech evidence of bankrupts for the 2002–2007 period. Since this way of data collection was very time-demanding, we were forced to accept a slight loss on generality of our model and omit closed and liquidated businesses from our dataset. The reason is that there were over 4,000 of such enterprises, and it was not possible to determine if the company was closed or liquidated due to insolvency court resolution or not. Also, companies awaiting the resolution were disregarded due to uncertainty about the outcome of the insolvency proceedings.

Consequently, as we want to test the predictive abilities of models with different time horizons (one, two, and three years in advance), we needed to link the date of bankruptcy to the corresponding financial information. This was achieved simply by linking the year of insolvency court's resolution to the relevant financial statement (one, two, or three years before the resolution). Magnus unfortunately does not contain dates when the financial statements were made public, so we cannot assure that the look-ahead bias is not present (as discussed by Ohlson (1980)). However, this represents a potential issue just for the one-year horizon, since companies make usually their financial statements public in the year following the relevant reporting period.

The most frequently used explanatory variables in the bankruptcy prediction models are financial ratios. We calculate them with data from Magnus for years 2002–2016, so that we cover both pre- and post-2008 periods. Nonetheless, we had to disregard all companies that did not provide information needed for the calculation of these ratios. Although it could bring about the sample selection bias discussed by Zmijewski (1984), it is a standard procedure in literature, and we have not found any remedy. Detailed description of all financial explanatory variables will be provided later.

In addition to the financial data, it was necessary to gather further information about each company in order to enrich classical bankruptcy modelling by "soft" factors, such as age, industry, certifications, etc. Further details will be provided in the section 4.2.2.

Table 2: Number of available financial statements in each period

Year	Non-bankrupt				Bankrupt (2Y)			
	Total	Large	Medium	Small	Total	Large	Medium	Small
2002	1,487	48	251	1,111	-	-	-	-
2003	7,029	259	945	5,439	4	-	-	4
2004	10,359	356	1,251	8,178	9	-	3	6
2005	11,684	373	1,470	9,203	6	-	-	6
2006	12,615	447	1,664	9,826	32	3	5	24
2007	13,657	538	1,939	10,488	46	1	4	41
2008	14,363	598	2,157	10,960	63	3	10	50
2009	14,996	594	2,167	11,643	55	-	8	47
2010	15,393	637	2,299	11,918	73	5	15	53
2011	14,281	414	1,889	11,553	68	-	7	61
2012	15,215	713	2,490	11,686	78	-	9	69
2013	14,064	741	2,465	10,656	57	1	9	47
2014	7,233	499	1,445	5,220	17	1	7	9
Total	**146,530**	**6,217**	**22,432**	**117,881**	**508**	**14**	**77**	**417**

Note: Simplified EU commission's methodology has been used for the size segmentation: small enterprises have total assets less than EUR 10 m, large ones have total assets exceeding EUR 50 m, and medium ones lie somewhere in-between. Average CZK/EUR exchange rate for the whole period equal to 27.7 has been used. Data for bankrupt companies denote the financial statements available for the prediction purposes two years in advance, so the numbers in 2014 row actually refer to 2016 bankruptcies (2 years horizon). Source of data is Magnus Database, segmentation has been done by author.

Also, financials for the most recent years were excluded depending on the prediction horizon. For example, for the three years horizon, the 2014, 2015, and 2016 financials were excluded from the dataset, since at present we do not know if the company will or will not go bankrupt eventually.

All of these specifications resulted in a set of slightly less than 150,000 financial statements for a period from 2002 to 2016. In Table 2 you can find the summary of the data, segmented in terms of size and years. There are 508 bankrupt companies with available financial information two years before bankruptcy, out of which more than 80% belong to the group of small businesses. Similar disproportion is observable in the case of non-bankrupt companies. Note that the number of bankrupt and non-bankrupt companies varies with the time horizons due to the differences in the availability of data. For details inspect Appendix 7. Finally, we have got solid data for the post-2008 period, but just a limited number of observations of distressed enterprises between 2002 and 2007.

It is also good to note that at this point it would be extremely difficult to cope with potential outliers. The sample is fairly complex, and there are many dimensions to take into account in deciding which observation should be treated as an outlier and which not. Instead, we will use the fact that ratios are able to eliminate some of these dimensions and apply the method of winsorization on them to deal with outlying values. Details will be explained in chapter 4.3.

4.2 Definition of explanatory variables

4.2.1 Financial ratios

Virtually in every paper that deals with the bankruptcy prediction modelling, the principal explanatory variables are financial ratios. The chief reason is that they are easily obtainable from financial statements and they provide good picture about company's financial situation. There has also been an extensive research validating the use of ratios in bankruptcy prediction. However, no clear consensus exists about which financial ratios are the best. In the literature it is common to define a set of 20–40 ratios and consequently evaluate their performance in the concrete model. Ohlson (1980) suggests not to include too many of them in the analysis, since "exotic" ratios are often strongly correlated with the standard ones and complicate the analysis with no or small added value. Therefore, instead of defining broad range of new innovative ratios, we decided to focus primarily on the most widely used ratios in the Czech and the most cited international articles. The list of ratios compiled by Jakubík and Teplý (2008) has been a prominent source for this purpose.

Five main groups of ratios were formed for the analysis: liquidity, solvency, leverage, activity, and profitability ratios. Each of these groups evaluates the soundness of company's operations from a different perspective, and thus a full range of company's characteristics is covered. The comprehensive list of financial ratios, including formulas, is drawn up in Table 3.

Table 3: List of financial ratios

Name	Type	Formula[1)]
Current.Ratio	Liquidity	CA / ST payables
Current.Ratio.B	Liquidity	CA / (ST payables + Bank debt)
Quick.Ratio	Liquidity	(CA - Inv) / ST payables
Quick.Ratio.B	Liquidity	(CA - Inv) / (ST payables + Bank debt)
Cash.Ratio	Liquidity	(Cash + MS) / ST payables
Cash.Ratio.B	Liquidity	(Cash + MS) / (ST payables + Bank debt)
Total.Debt.to.Equity	Leverage	Debt / Equity
Debt.to.Equity	Leverage	LT debt / Equity
Financial.leverage	Leverage	Total assets / Equity
R.E.to.Assets	Leverage	R.E. / Total assets
Debt.payback.period	Solvency	Debt / (EBIT + DA + Int. Cost)
Interest.coverage	Solvency	EBIT / (EBIT + Int. Cost)
CashFlow.I	Solvency	(EBIT + DA) / LT debt
CashFlow.II	Solvency	(EBIT + DA) / (LT debt - Reserves)
GP.margin	Profitability	(Sales - COGS) / Sales
Operating.margin	Profitability	EBIT / Sales
NP.margin.I	Profitability	Net Comprehensive Income / Sales
NP.margin.II	Profitability	Net Income / Sales
NI.ROE	Profitability	Net Income / Equity
NI.ROA	Profitability	Net Income / Total assets
EBIT.ROA	Profitability	EBIT / Total assets
AR.days	Activity	365 / AR turnover
Inventory.days	Activity	365 / Inv turnover
AP.days	Activity	365 / AP turnover
Asset.turnover	Activity	Sales / Avg. total assets

[1)]Abbreviations: CA – Current assets, ST– Short-term, LT – Long-term, DA – Depreciation and amortization, MS – Marketable securities, R.E. – Retained earnings, Inv – Inventory, AP – Accounts payable, AR – Accounts receivable

Liquidity ratios evaluate firm’s ability to repay short term liabilities by its current assets. It is reasonable to expect that bankrupt companies will on average report lower liquidity ratios in comparison with sound companies, since liquidity problems are usually the key determinant of bankruptcy. Note that two different ratios were calculated for each liquidity ratio. The reason is that the balance sheet itemization followed by Magnus database (based on Czech Accounting Standards 2003) does not make available precise information about the amount of total current liabilities. It uses two ambiguous items instead: short term payables (mainly trade payables, salaries payable, etc.) and bank debt. Under the bank debt item are hidden not just long term bank debts, but also short term loans and short term borrowings, which makes the definition of current liabilities more complicated. Since it is impossible to set a general rule that would decide which portion of bank debt should be accounted for as a current liability, it was necessary to define two different ratios with both types of denominators (see Table 3). Clearly, it must hold that Current ratio > Quick ratio > Cash ratio for a single company, since only more easily liquidatable assets are included in Quick ratio compared to Current ratio and in Cash ratio compared to Quick ratio.

Solvency and leverage ratios measure, on the other hand, the long term position of a company. Leverage ratios focus primarily on the composition of capital structure (e.g. share of long term debt on total capital). It is widely accepted that companies with larger portion of equity in capital structure are generally less likely to default, and therefore the ratios indicating higher levels of equity should correspond to sounder enterprises. Solvency ratios gauge company's cash flow-generating power and ability to cover expenses arising from long-term liabilities. Obviously, firms with weak cash flows and high fixed costs are likely to be good candidates for future solvency problems.

Profitability ratios are defined to capture the profit-generating power of the company. Intuitively, positive returns are a necessary prerequisite for successful businesses in the long run. Hence, multiple profit margins were defined to cover the whole process of value creation from operations to financing and investing decisions. Moreover, two types of net profit margins were specified, one with standard net income and the second one with net comprehensive income in numerator. The second measure adjusts net income for extraordinary items such as acquisitions, unusual sales of assets, changes in equity, etc. It could be thus useful for the detection of troublesome firms that faced some important extraordinary events in the past. Other profitability ratios measure return on invested capital (debt and equity), which should signal the efficiency of employed resources.

Activity ratios indicate the efficiency with which the company manages its operations. What is problematic about these ratios is that it is not always clear which level is optimal. For example, more days of inventory on hand could indicate poor inventory management; conversely, a small number of days of inventory on hand could mean that the firm loses a part of potential sales, because it does not have enough products in inventory to sell. Similar trade-offs can be identified for other activity ratios, too. Generally, knowing the industry standards is important for the evaluation.

Since not all companies had all information necessary for the calculation of these ratios, firms with incomplete data had to be excluded from the dataset. It is however a standard approach, as this issue is inherent in the bankruptcy prediction analysis. Additionally, the ratios which opposed to reality, such as negative number of account receivable days, were adjusted to correspond to what we observe in real world.

4.2.2 Dummy explanatory variables

To complement the standard approach of purely financial explanatory variables, we decided to define a set of dummy variables to control better for individual companies' specifics. In Table 4 is the overview of these variables accompanied by formal description.

With regard to the specification of size, age, and industry variables, we got inspiration from the study of Altman et al. (2014), who used them in the logit regression with promising results. Note that we used only three industry dummies, although there are 20 NACE codes in total. The reason is that we were able to encompass more than two thirds of all observations just with these three variables, and the rest was relatively fragmented across remaining NACE segments. We are mainly interested in the effect of construction dummy variable, since it has recently attracted attention of Czech researchers, see for example Karas and Režňáková (2017), Karas and Režňáková (2015), or Čámská (2015), who discuss the negative impact of adverse economic conditions on this industry.

Remaining dummy variables were defined using the non-financial information for individual companies extracted from Magnus database. There were no specific expectations, nor theory behind their selection, but they could serve as potentially interesting predictors for further research if proven to

Table 4: List of dummy explanatory variables

Variable's Name	Description
Size.Small	1 if total assets < EUR 10 m; 0 otherwise
Size.Medium	1 if EUR 10 m < total assets < EUR 50 m; 0 otherwise
Size.Large	1 if total assets > EUR 50 m; 0 otherwise
FirmAge.New	1 if time from establishment < 5 years; 0 otherwise
FirmAge.Medium	1 if time from establishment between 5-10 years; 0 otherwise
FirmAge.Established	1 if time from establishment > 10 years; 0 otherwise
Wholesale.Retail	1 if the company operates in W&R (NACE code G); 0 otherwise
Processing	1 if the company operates in processing (NACE code C); 0 otherwise
Construction	1 if the company operates in construction (NACE code F); 0 otherwise
Joint_Stock	1 for joint stock companies; 0 otherwise
Ltd.Company	1 for limited liability companies; 0 otherwise
Incor.pages.Few	1 if number of pages in certificate of incorporation < 10; 0 otherwise
Incor.pages.Moderate	1 if number of pages in certificate of incorporation between 10-20
Incor.pages.Many	1 if number of pages in certificate of incorporation > 20; 0 otherwise
ISO.9001	1 if the company received the ISO 9001 certification; 0 otherwise
PRG	1 if the company resides in Prague; 0 otherwise

Note: Average CZK/EUR exchange rate for the 2002–2016 period equal to 27.7 has been used for the inclusion in the groups with respect to company's size

play some role in the predictions of bankruptcies. The only issue with these variables isthat they hold as of the date of extraction from the database (1/2017), because Magnus does not provide any information about changes in these non-financial indicators retrospectively. Therefore these variables should be treated cautiously, rather as proxies than hard data.

4.3 Data adjustments

4.3.1 Coping with outliers - Winsorization

Treatment of outlying values is indissociably one of the essential steps in the estimation of bankruptcy prediction model. However, similarly to the selection of appropriate financial ratios, there is no clear consensus about this issue in the literature, and researchers approach it differently. Some use advanced methods to detect multivariate outliers and subsequently delete them from the dataset (see for example the study of Hauser and Booth (2011), who used the deviance residuals from maximum likelihood regression model to detect deviating values); others prefer to trim outlying values so that they do not skew the estimation (but keep the observation in the dataset). For example, Neumaierová and Neumaier (2005b) suggested to cap the values of interest coverage ratios by 9, since knowing that the actual ratio is 300 does not offer much added value for the analysis. Instead, a problem with extremely large or small values distorting the estimation could arise. In this thesis we adopt the commonly used method of winsorization, which is frequently applied in the bankruptcy prediction context.

Winsorization is a data censoring method. Mulry et al. (2016) describe the procedure as one that "replaces extreme values with less extreme values, effectively moving the original extreme values toward the center of the distribution. Winsorization therefore both detects and treats influential values." (Mulry et al. 2016, p. 1) The basic idea is the following: observations are ordered from the smallest to the largest value, then the k^{th} and $100\text{-}k^{th}$ percentile values (e.g. 5^{th} and 95^{th} percentiles) are found, and all values above or below this threshold are replaced by the k^{th} and $100\text{-}k^{th}$ values, respectively. Extreme values are thus censored rather than entirely dismissed, so that the information contained in these observations is preserved. Subsequently, the resultant dataset allows for better estimation of central tendency and other statistics. More complete discussion can be found for example in Clark (1995) or Martinoz et al. (2015).

The question is at what level the outliers should be winsorized. Literature does not offer any guidance how to proceed here, however, virtually only 2 threshold values are widely used in practice: 1^{st} and 5^{th} percentiles. 1% levels were chosen for example by Giordani et al. (2011), González-Aguado and Moral-Benito (2012), or Timmermans (2014); on the other hand, 5^{th} and 95^{th} percentiles were applied for instance by Campbell et al. (2011), Dewaelheyns and Van Hulle (2004), or Duda and Schmidt (2009). Since our dataset is fairly sizeable and complex, 5% threshold has been chosen over the 1% one.

As the data for bankrupt companies are of the major significance for the analysis, and they could be censored inappropriately by winsorization, two different datasets were made to overcome this

Table 5: Descriptive statistics: Non-bankrupt vs. Bankrupt companies

Financial ratios	Non-bankrupt	Bankrupt (3Y)	Bankrupt (2Y)	Bankrupt (1Y)
Current.Ratio	2.347	1.424	1.398	1.179
Quick.Ratio	1.615	0.943	0.942	0.798
Cash.Ratio	0.446	0.098	0.084	0.065
Total.Debt.to.Equity	1.581	4.697	3.872	1.855
Financial.leverage	3.108	7.182	6.09	3.584
Debt.payback.period	4.867	8.445	10.316	7.458
CashFlow.I	0.297	0.056	0.044	0.006
GP.margin	0.217	0.14	0.139	0.122
Operating.margin	0.044	-0.0003	-0.011	-0.047
NP.margin.I	0.027	-0.015	-0.026	-0.063
NI.ROA	0.05	-0.013	-0.029	-0.079
AR.days	80.537	95.072	101.125	107.979

Note: 3Y refers to data corresponding to information available three years before bankruptcy, 2Y to two years and 1Y to one year before bankruptcy.

potential flaw. One consisted of financials for bankrupt companies and the second one of the remaining observations. The same 5% threshold has been applied for both datasets. RobustHD package in R by Alfons (2016) has been used for the application of winsorization method.

In Table 5 you can find resulting means for a selection of financial ratios for non-bankrupt companies, and companies one, two, and three years before bankruptcy. Comprehensive descriptive statistics for all financial ratios, including standard deviation and percentile values, can be found in Appendix 1, Appendix 2, Appendix 3, and Appendix 4. You can notice interesting differences among distressed and sound companies which could be used for their classification. Regarding the liquidity, bankrupt companies show significantly worse ratios on average; moreover, a clear deterioration is observable the closer to the actual bankruptcy the enterprises are. Leverage ratios are generally higher for bankrupt companies, suggesting the higher risk of failure. Also, the profitability is significantly lower for bankrupt companies. Average net profit margin is less than -6% one year prior the bankruptcy for distressed companies, compared to almost 3% in the case of healthy firms. Other profit measures exhibit similar pattern. Obviously, failing companies also struggle with the collection of accounts receivable.

4.3.2 Test sample for out-of-sample accuracy assessment

Lastly, we split the original dataset into two smaller ones: test and train samples. Train sample will be used for the estimation of the parameters in our model and models accuracy will be subsequently tested on the test sample. Train sample consists of 80% of the original data and has been selected by random sampling so that the proportion of bankrupt and non-bankrupt companies stays the same in both datasets. Caret package in R by Kuhn (2017) has been used for this purpose.

5 Methodology

5.1 Model

Corporate defaults are frequently modelled using binomial probability models with the dependent variable equal to 1 if the firm went bankrupt in the studied period, and 0 otherwise. This relationship is then best captured by the simple Bernoulli distribution model as

$$P\{Y_i = y_i\} = \pi_i^{y_i}(1-\pi_i)^{1-y_i}, \tag{1}$$

where Y_i is a random variable, which can take on values of 1 and 0 with probability π_i and $1 - \pi_i$, respectively. However, the outcome probability of company's bankruptcy is unobserved in the majority of cases and needs to be estimated. The simplest way to model probability would be to estimate it as a linear function of covariates as

$$\pi_i = x_i'\beta, \tag{2}$$

where x_i' is a vector of explanatory variables, and β is a vector of regression parameters. This model is called *linear probability model.* However, one of the shortcomings of the LPM is that the function $x_i'\beta$ can yield values outside the 0 to 1 range. This in effect contradicts the whole concept of probability, and indeed is one of the major reasons why the LPM is not used in practice very often. To address these shortcomings, probit and logit models are more commonly used for binary dependent variables. These use the advantages of standard normal and logistic functions respectively, in order to eliminate the problem of yielding values outside the range 0 to 1. We have decided the use the logit model in our study as it is extensively used in the literature that this work is based on and seems to be a reliable and proven method for our purposes.

In the logistic regression the probability is modelled in the form of odds, so that

$$log\frac{\pi_i}{1-\pi_i} = x_i'\beta, \tag{3}$$

which can be rearranged to

$$\pi_i = \frac{e^{x_i'\beta}}{1+e^{x_i'\beta}}\,. \tag{4}$$

In this form, the $x_i'\beta$ term can range from negative infinity to positive infinity, and the probability will always be within the 0 and 1 boundaries.

Standard approach to estimate the parameters in equation (4) is *Maximum Likelihood Estimation.* This method takes a likelihood function defined as a product of densities of Bernoulli distribution from equation (1), giving

$$L_\beta = \prod_{i=1}^{n} \pi_i^{y_i}(1-\pi_i)^{1-y_i}, \tag{5}$$

which is subsequently transformed into the Log-likelihood function (for computational purposes)

$$\log(L_\beta) = \sum_{i=1}^{n} y_i \log(\pi_i) + (1-y_i)\, log(1-\pi_i), \tag{6}$$

where π_i is defined as a function of parameters β and data, as in the equation (4). The last step is to maximize the Log-likelihood function by the selection of appropriate regression parameters, so that

$$\hat{\beta}_n^{ML} = \; arg \max_{\beta} \sum_{i=1}^{n} y_i \log(\pi_i) + (1-y_i)\, log(1-\pi_i), \tag{7}$$

which is equivalent to

$$\hat{\beta}_n^{ML} = \; arg \min_{\beta} \sum_{i=1}^{n} -y_i \log(\pi_i) - (1-y_i)\, log(1-\pi_i). \tag{8}$$

We will reformulate equation (8) for easier work to

$$\hat{\beta}_n^{ML} = \; \arg\min_{\beta} - \sum_{i=1}^{n} \log\bigl(f(x_i'\beta, y_i)\bigr)\,. \tag{9}$$

The estimation itself is then done by taking the first derivative of equation (9) with respect to the parameters and finding a point at which the function equal 0.

One of the main issues of the MLE method is that it is sensitive to the presence of outliers in the data at hand. (Šimečková 2005) Although the method of winsorisation has been used to address this issue, we have further decided to employ a robust estimation method that should serve as a robustness check for our model. Indeed, there are multiple robust estimators with various characteristics, so we have decided to use the one proposed by Bianco and Yohai (1996) as it has been already used by Hauser and Booth (2011) in the bankruptcy prediction context and proved to be a good method in their analysis.

Bianco and Yohai (1996) introduced a consistent and robust estimator, from now on referred to as BY estimator, by adding a bounded function ρ to the log-likelihood function and defining a bias correction term. The BY estimator is then defined as

$$\hat{\beta}_n^{BY} = \arg\min_{\beta} \sum_{i=1}^{n} \{\rho_k\big(d(x_i'\beta, y_i)\big) + C(x_i'\beta, y_i)\}, \tag{10}$$

where *d* is defined as the log function in equation (9), including the negative sign, ρ_k is the bounded function and C is the correction term. The correction term is defined as

$$C(x_i'\beta, y_i) = \mathrm{G}\big(\mathrm{F}(x_i'\beta)\big) + G(1 - F(s)), \tag{11}$$

where F is a standard logistic function defined in equation (4) and G is given by

$$G(x) = \int_0^x \rho' \,(-\ln(u))\, du. \tag{12}$$

Authors of the BY estimator suggested to employ the bounded function in the form of

$$\rho_k(k) = \begin{cases} x - \dfrac{x^2}{2k} & if\ x\ \le k \\ \dfrac{k}{2} & otherwise \end{cases} \tag{13}$$

but also noted, that it could be replaced by any other suitable bounded function. For further details and discussion refer to Bianco and Yohai (1996) or Croux and Haesbroeck (2003).

Having these two estimators should give us a solid base for conclusions about the robustness of our results, and partly also help to evaluate the efficiency of the winsorization method.

For the estimation of regression parameters the R package robustbase by Maechler et al. (2005) has been used.

5.2 Rebalancing of the unbalanced dataset

The next issue when using the ML estimator that needs to be addressed is that the estimates it yields are sensitive to any imbalances in the data. Olson (2005) for example tested the performance of multiple estimators on a set of three unbalanced datasets. He found that the results tend to be biased as the MLE disregards the minority class, focusing in the most common values instead. This feature of the ML estimators is clearly very important for our analysis as there is significantly less bankrupt than prosperous companies in the sample, making the dataset highly unbalanced. The underlying idea of the rebalancing approach is that the minority class in the training sample (in our case the bankrupt companies) is brought to the same level as the original majority class. Subsequently, the adjusted dataset should help to estimate the model parameters more effectively and improves the rate of correct classifications in the test sample. (Elhassan et al. 2016)

Out of the numerous techniques to treat class imbalances, the standard approach seems to be to inflate the number of the minority class observations by repeating the minority class data. Alternatively, the same effect can be achieved by decreasing the number of the majority class observations such that the size of the majority class sample is approximately equal to the minority one (which is frequently used for the MDA discussed in the literature review). These methods have an important advantage that they are proven to work well and are relatively easy to implement. However, recently there has been an upsurge in the use of synthetic methods like SMOTE (Synthetic Minority Over-sampling Technique) proposed by Chawla et al. (2002) or ROSE (Random Over-Sampling Examples) developed by Lunardon et al. (2014). The general idea of these synthetic approaches is that the replicated data are not identical to the original observations but are artificially created with respect to the similarities across the data instead. Although there may presently be some evidence that the synthetic methods may outperform the classical data rebalancing techniques, we have decided not to adopt them in our study. This is due to the relative ease of implementation of the classical methods. We also want to fully understand what is happening to our dataset during the rebalancing process, and the synthetic techniques are not very transparent. Moreover, a number of researchers have shown that the classical methods work very well in the logistic regression framework such as Oommen et al. (2011), or more recently Elhassan et al. (2016). A comprehensive overview of the frequently used resampling methods with their applications can further be found in Ali et al. (2015) or Elhassan et al. (2016).

ROSE package in R by Lunardon et al. (2014) has been used for rebalancing the dataset.

5.3 Selection of explanatory variables: Bayesian model averaging

Many researchers construct the models of their research topic based primarily on their prior beliefs of what are the relevant factors for the specification of the true model. However, without sufficient guidance from the theory, there is the inherent issue with model uncertainty. In other words, it may be unclear, which explanatory variables should be included in the model, and which should be omitted. This issue becomes even more burning with increasing number of potential explanatory variables. However, model averaging represents an effective tool for the resolution of this uncertainty as it evaluates a large number of models. Subsequently, model are ranked according to a specific criterion from the most suitable to the least. By estimating and contrasting the whole universe of the models, the model uncertainty is considerably lowered.

Bayesian model averaging (BMA henceforth) is one of the model averaging methods that has already been extensively used in the corporate failure research literature. For example, Hayden et al. (2014) compared the stepwise and the BMA methods for logistic regressions in credit risk applications, Figini and Giudici (2013) evaluated multiple specifications of BMA approach on the performance of a bankruptcy prediction models, and González-Aguado and Moral-Benito (2012) tested the importance of the extensive group of financial ratios among other variables for the corporate failure predictions using the BMA technique. Research conducted by these academics gave a decent base for the use of the BMA method in our analysis. The logic of the BMA method will now be briefly explained.

As mentioned above, the central idea of BMA is to estimate whole universe of models and evaluate their quality with respect to the other alternatives. Simply, let say we have K explanatory variables; consequently, these K variables could be used to specify 2^K different models. These models can be labelled as M_j, where j = 1,….,2^K , and they are dependent upon parameters β^j. The posterior distribution of the parameters for various models given data y is then described as

$$g(\beta^j|y, M_j) = \frac{f(y|\beta^j, M_j)g(\beta^j|M_j)}{f(y|M_j)}. \tag{14}$$

In the next step we will use the fact that the Bayesian rule can also be used to derive the probability of an unknown event (probability of model being the true model) using a known event (our data, y). Specifying the prior model probability as $P(M_j)$, using Bayesian rule, the posterior probability is then given as

$$P(M_j|y) = \frac{f(y|M_j)P(M_j)}{f(y)}, \tag{15}$$

where $P(M_j)$ is a prior probability representing the belief about probability that the model M_j is the correct one. $f(y|M_j)$ is called the marginal likelihood, and is calculated from equation (14). $f(y)$ is called the integrated likelihood and is constant for all models M_j as it depends on the data only. Subsequently, the posterior probability can be employed to rank the models and assess, which of them are the most likely to be the true ones.

If we were interested in the point estimates, we could calculate them as

$$E(\beta^j|y) = \sum_{j=1}^{2K} P(M_j|y)\, E(\beta^j|y, M_j), \tag{16}$$

where posterior probabilities are used as weights for 2^K estimates of β^j. Formula for standard deviation can be derived in a similar way.

It is noteworthy how important the prior probability $P(M_j)$ is in the estimation process. Indeed, it is the key term for the posterior probability estimation in (15). This is then the main factor in raking the models compared. As the prior probability has to be specified in advance and it influences the outcomes of the BMA analysis, it has already been investigated by numerous researchers. In our analysis we decided to follow advice of Figini and Giudici (2013), who tested various prior specifications for credit risk predictions on 2 sets of data; one with small number of explanatory variables (5 in total), and the second with more than 20 variables. They have found that although the non-uniform priors can yield more stable results when the dataset is small, the non-informative uniform priors are better when the datasets are large. This indeed is the case of our dataset, which is comparatively rather large. Note that the uniform prior is said to be uninformative, as it assigns the same prior probability to all 2^K models and does not give any preference to any of them.

Another important consideration is, that the BMA method is conditional on data (see equations (14) to (16)). As we have decided to take rebalanced datasets into account, it will be necessary to apply BMA repeatedly for more datasets.

Lastly, we are not primarily interested in the point estimates from the BMA, but we want to use the approach mainly for the selection of the most important explanatory variables. Therefore, the posterior probabilities are the main factor that we look for. However, point estimates can still be used for the evaluation of expected impact of each explanatory variable.

For further details about the BMA technique see for example Hoeting et al. (1999). For the analysis the BMA package in R by Raftery et al. (2017) has been utilized.

5.4 Model evaluation – ROC and PR curves

Confusion matrix, depicted in Figure 1, has been a standard measure of the classifiers' quality in the bankruptcy prediction research for a long time. It was taken as a principal measure of models' accuracy by many researchers (Altman 1968, Zmijewski 1984, Pongsatat et al. 2004) and has been used extensively by the Czech academics as well (Neumaierová and Neumaier 2005b, Sušický 2011, Machek 2014). However, some researchers have been arguing that it may often be a poor measure of the predictive ability of the respective models as it evaluates the model just at a specific cut-off point. (Fawcett and Provost 1997) Instead, a new measurement method of Receiver operating characteristics (ROC) curves has been proposed and has recently become widely accepted among the academic community. The main benefits of this approach compared to the classifier matrix will now be briefly outlined and discussed.

Firstly, it is necessary to define a few terms we will be working with. Following the notation from Figure 1, we define

$$\text{True Positive Rate (Sensitivity)} = \frac{TP}{TP + FN} \ , \tag{17}$$

$$\text{True Negative Rate (Specificity)} = \frac{TN}{TN + FP} \ , \tag{18}$$

$$\text{False Positive Rate } (1 - \text{Specificity}) = \frac{TN}{TN + FN} \ . \tag{19}$$

The ROC graph is then constructed with the *false positive rate* on horizontal axis and the *true positive rate* on vertical axis. The example of ROC curves can be viewed in Figure 2. The curve essentially shows the classifier´s performance for a multiple cut-off threshold in terms of sensitivity and the false positive rate. It is therefore much more informative than the conventional confusion matrix. Also, there usually is a trade-off between the x and y axis measures, so that the ROC curve is upward-sloping. In Figure 2 you can see 4 different ROC curves. Curve A, which goes from (0,1) to (1,1), represents the optimal test, when all positives can be selected with 100% accuracy and no false positives are incurred.

Figure 1: Potential outcomes of a bankruptcy test (classification matrix)

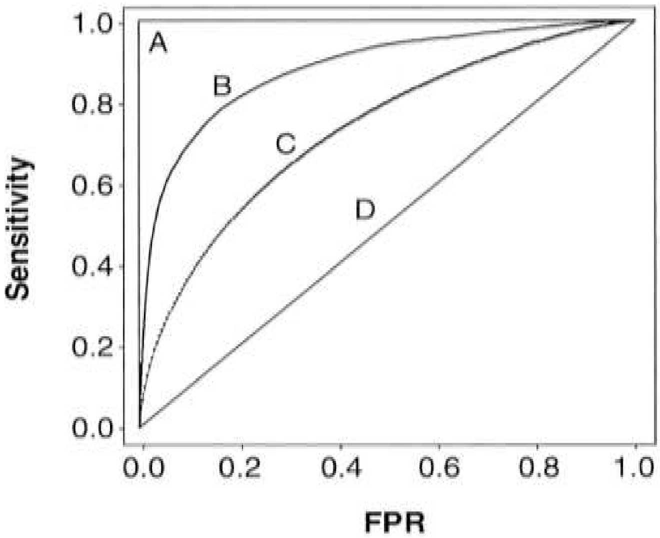
		Predicted condition	
		Positive (bankrupt)	**Negative (prosperous)**
True condition	**Positive (bankrupt)**	True positive (TP)	False negative (FN)
	Negative (prosperous)	False positive (FP)	True negative (TN)

Conversely, curve D shows a situation of random selection. Generally, the closer the classifier's ROC curve gets to the curve A, the more accurate the classifier is. To complement the common ROC analysis, it is possible to study the behavior of sensitivity and specificity curves for different cut-offs separately. This analysis can provide a useful piece of information for the practical use of the estimation model. In Appendix 8 and Appendix 9 you can find descriptive plots of sensitivity and specificity curves, respectively. The most important thig to note is that they are directly linked to the classification matrix and hence they can be used for the management of the type I and type II errors for various cutoffs.

The key advantage of the ROC curves is that they are not sensitive to any imbalances in the data. (Fawcett, 2006) It essentially means that the tested sample´s size does not affect the ROC´s curvature. As the researchers frequently encounter datasets with large class imbalances, this property of the ROC curve is indeed quite useful in practice, especially for making comparisons of results across articles.

An important measure for the comparison of classifiers' quality is the area under the ROC curve (AUC hence forth). (Bradley 1997) In practice the AUC values range between 0.5, for a random test, and 1, for a perfect test (see Figure 2). AUC can be also interpreted as a probability that the classifier will rank a randomly chosen positive observation higher than a randomly chosen negative observation. (Fawcett 2006) Furthermore, it is related to the GINI coefficient, namely GINI + 1 = 2AUC. (Breiman et al. 1984)

Figure 2: ROC curves (from Park et al. (2004))



All of the above-mentioned features make the AUC simple and very effective measure of classifiers' predictive ability. Therefore, it has become a widely used tool also in bankruptcy prediction setting, where it is frequently used for the comparison of models' performance (see for instance Duda and Schmidt (2009), Kalouda and Vaníček (2013), Altman et al. 2014 (2014), Timmermans (2014), or Affes and Hentati-Kaffel (2017).

However, in spite of being insensitive to class imbalances, the ROC curves can sometimes be overly optimistic about the classifier´s ability, especially when the skew in the data is substantial. In such a case, other measures such as the Precision-Recall (PR) curves should be considered. (David and Goadrich 2006) Precision is defined as

$$\text{Precision} = \frac{TP}{TP + FP}, \tag{20}$$

and Recall is in effect just a different name for sensitivity. Although the PR curves as a measurement of classifiers´ predictive ability have up to date been virtually not used in the corporate bankruptcy context, we believe that they have the ability to shed some light on the true model´s predictive ability. Moreover, Precision clearly is of a substantial importance for the real life application of the classifier.

For plotting ROC and PR curves the ROCR package in R by Sing et al. (2005) has been utilized.

6 Results

The empirical findings resulting from using the methodology outlined above on my data shall now be presented and analyzed. All models are estimated on a randomly selected train sample that represents 80% of the whole original dataset. The classifiers are then evaluated using an out-of-sample test dataset that consists the remaining 20% of our data.

Moreover, three different rebalancing methods described in the methodology have been employed on the train sample: over-sampling, under-sampling, and the combination of these two sampling methods. Comprehensive overview of a number of observations used for each estimation of the regression parameters is shown in Appendix 7.

This chapter is divided into three sections dealing with BMA, estimation of regression parameters, and evaluation of predictive ability using ROC and PR curve analysis. For the presentation of regression results the stargazer package in R by Hlavac (2015) has been employed.

6.1 Bayesian model averaging

At this point we would like to remind reader of a couple of considerations that could be useful to bear in mind before we move to the analysis of BMA results.

Firstly, we are not primarily interested in the point estimates obtained by the model averaging approach. The reason why we employ BMA is to select the most relevant explanatory variables for our logistic model.

Secondly, posterior probability estimates are conditional on the data. Therefore, it is necessary to differentiate between the original and the rebalanced datasets. For simplicity and to ensure a clear presentation of the results, we apply BMA just on the original dataset and the dataset constructed by the combination of under- and over-sampling methods. This should to some extent be representative of the both (although under-sampling is more distinct due to the small number of bankrupt firms).

Table 6: Correlation matrix – selected variables with high level of correlation

Correlation matrix	Total.D/Equity	Debt.to.Equity	Fin.leverage	CashFlow.I	CashFlow.II	Oper.margin	NP.margin.I	NP.margin.II	NI.ROA	EBIT.ROA
Total.D/Equity	1.00									
Debt.to.Equity	**0.97**	1.00								
Fin.leverage	**0.96**	**0.99**	1.00							
CashFlow.I	-0.15	-0.17	-0.17	1.00						
CashFlow.II	-0.16	-0.17	-0.17	**0.99**	1.00					
Oper.margin	-0.04	-0.05	-0.04	0.54	0.55	1.00				
NP.margin.I	-0.03	-0.04	-0.04	0.53	0.53	**0.95**	1.00			
NP.margin.II	-0.03	-0.04	-0.04	0.53	0.53	**0.95**	**0.99**	1.00		
NI.ROA	-0.02	-0.04	-0.04	0.63	0.63	0.67	0.70	0.71	1.00	
EBIT.ROA	-0.02	-0.04	-0.04	0.65	0.65	0.68	0.66	0.66	**0.96**	1.00

Note: Bolded variables have correlation higher than 0.9

Furthermore, just a model with the dependent variable lagged by two years is considered for higher clarity. As a consequence, two blocks of BMA estimates will be discussed.

Third, stemming from the very definitions of some of the financial ratios presented in Table 3, there are very high levels of correlation between a few variables. In Table 6, there are the most correlated variables, where very high correlation scores above 0.9 are to be found (see Appendix 5 and Appendix 6 for complete correlation matrix). This present challenges to our analysis, as it caused some of the highly correlated variables to have counter-intuitive parameter estimates, when included in the same model (e.g. companies with larger D/E ratio would be less likely to bankrupt, or firms with higher operating profit would be more likely to bankrupt). Parameters also become more sensitive to the model specifications. The issue has been resolved by evaluating the performance of multiple correlated variables separately and selecting the top performing predictors. In general, variables with the B suffix (see Table 3) were found to be better in explaining the separation of the bankrupt and non-bankrupt companies. This may be due to the bank debt term included in liquidity ratios' denominators.

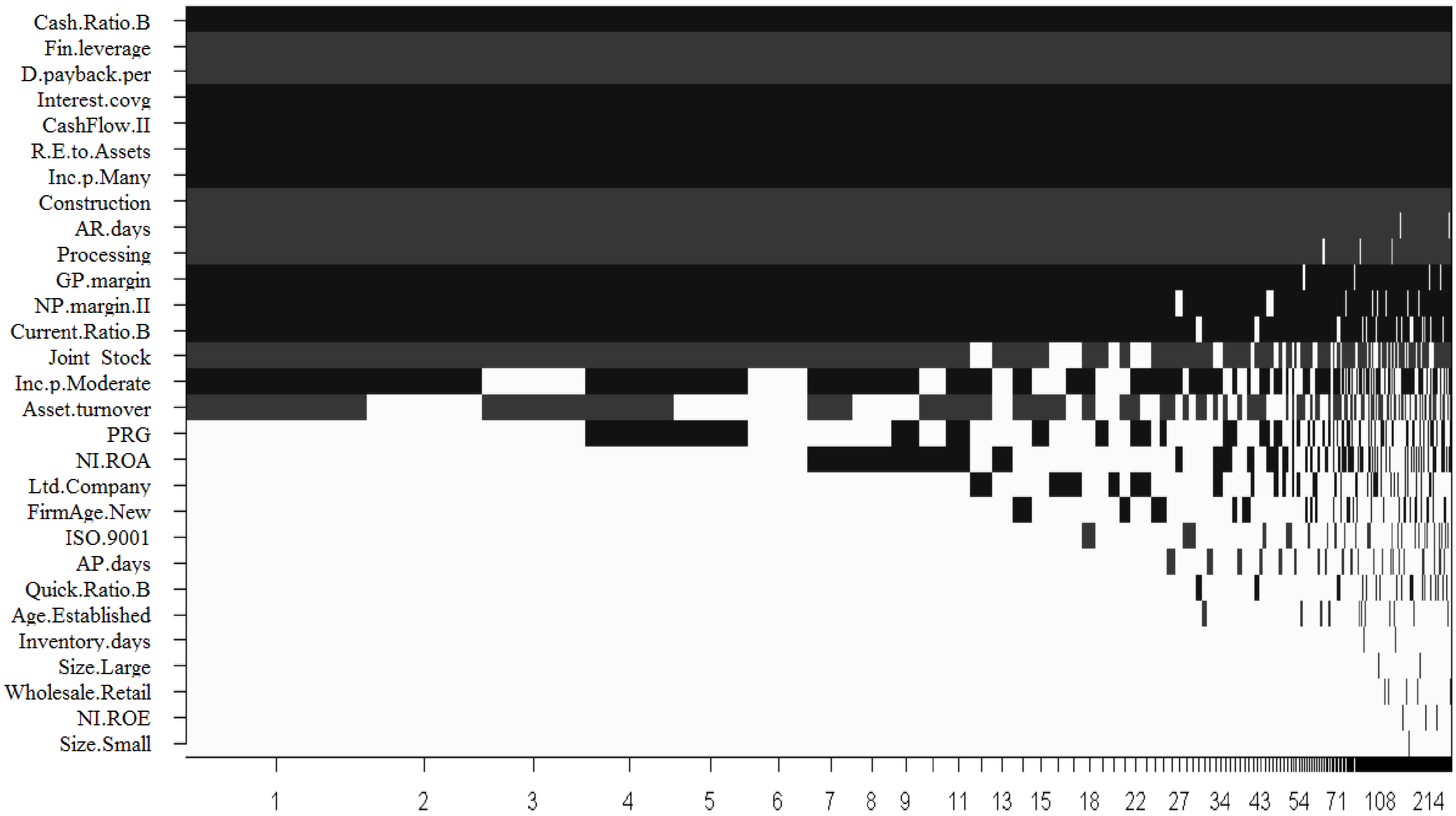
Financial leverage performed slightly better than other solvency measures, and variables with net income in the numerator proved to be superior to ratios based on EBIT (one of the potential reasons is, that the size of interest matters, and therefore net income provides more information than EBIT).

6.1.1 BMA for the original unbalanced dataset

In Table 7 you can find summary of the BMA results on the original unbalanced dataset. Figure 3 shows a graphical representation of these results. The horizontal axis represents various model specifications, where the width of each segment stands for the marginal posterior probabilities of various models; and the vertical axis is a list of relevant explanatory variables. The figure comprises of more than 200 best models selected based on their posterior probabilities and the method of Occam's window, which uses Bayesian information criterion (BIC) to eliminate models with significantly lower probability of being true model than the best model (fuller discussion of this matter can be found in Madigan and Raftery (1994)).

We are interested in the variables that were included in as many models with the sufficient cumulative posterior probabilities as possible. For example, the Cash.ratio.B was one of the explanatory variables in all of the analyzed models (i.e. there is no blank spaces in the associated row), which very likely makes it a solid predictor. On the other hand, there were only two models with a very low posterior probability, where the Inventory.days variable was included. This means, that days of inventory on hand do not play a significant role in our prediction. With regard to the importance of the single explanatory variables, we will follow the suggestions of Eicher et al. (2011), who consider the variable's importance to be substantial if the cumulative posterior probability (CPP henceforth) lies between 0.75 and 0.95, strong when between 0.95 and 0.99, and decisive when it is above 0.99. Therefore, we have included all variables with CPP larger than 0.75 in our ultimate model (see Table 7).

Figure 3: Variable selection with BMA for the original unbalanced dataset



Note: x-axis = model specifications (width is a marginal posterior probability of that model), y-axis = list of potential explanatory variables; black shading = negative sign of parameter, grey shading = positive sign of parameter. Completely shaded row symbolizes cumulative posterior probability equal to 1, blank row means zero CPP for that variable

Table 7: Detailed BMA results for the original unbalanced dataset (2Y lag)

	Cumulative Post. Prob	EV	SD	model 1	model 2	model 3
Intercept	100	-5.485	2.832e-01	-5.662	-5.345	-5.744
Cash.Ratio.B	100	-1.296	2.186e-01	-1.291	-1.293	-1.296
Financial.leverage	100	1.067e-01	8.187e-03	1.061e-01	1.062e-01	1.060e-01
Debt.payback.period	100	3.240e-02	3.317e-03	3.247e-02	3.190e-02	3.265e-02
Interest.coverage	100	-3.529e-03	5.351e-04	-3.631e-03	-3.622e-03	-3.622e-03
CashFlow.II	100	-1.557	2.518e-01	-1.617	-1.610	-1.621
R.E.to.Assets	100	-7.244e-01	1.156e-01	-7.260e-01	-7.424e-01	-7.330e-01
Incor.pages.Many	100	-9.279e-01	1.756e-01	-9.458e-01	-9.642e-01	-8.402e-01
Construction	100	8.957e-01	1.302e-01	9.305e-01	8.774e-01	9.252e-01
AR.days	99.8	2.551e-03	4.902e-04	2.637e-03	2.237e-03	2.626e-03
Processing	99.5	5.726e-01	1.275e-01	6.342e-01	5.638e-01	6.291e-01
GP.margin	99.4	-1.297	2.591e-01	-1.255	-1.359	-1.255
NP.margin.II	98.2	-5.069	1.229	-5.623	-5.297	-5.645
Current.Ratio.B	98.0	-4.676e-01	1.054e-01	-4.826e-01	-4.656e-01	-4.797e-01
Joint_Stock	87.8	4.314e-01	1.874e-01	5.129e-01	4.749e-01	4.623e-01
Incor.pages.Moderate	69.6	-2.980e-01	2.212e-01	-4.159e-01	-4.188e-01	.
Asset.turnover	58.4	6.041e-02	5.481e-02	1.033e-01	.	1.041e-01
PRG	28.9	-9.910e-02	1.663e-01	.	.	.
NI.ROA	23.9	-5.334e-01	1.035	.	.	.
Ltd.Company	11.8	-5.250e-02	1.475e-01	.	.	.
FirmAge.New	7.2	-2.161e-02	8.372e-02	.	.	.
ISO.9001	5.0	1.242e-02	5.856e-02	.	.	.
AP.days	4.3	1.063e-05	5.386e-05	.	.	.
Quick.Ratio.B	2.2	-1.188e-02	8.353e-02	.	.	.
FirmAge.Established	1.8	3.451e-03	2.838e-02	.	.	.
Inventory.days	0.4	1.032e-06	2.210e-05	.	.	.
Size.Large	0.3	-8.555e-04	2.173e-02	.	.	.
Wholesale.Retail	0.3	5.062e-04	1.202e-02	.	.	.
NI.ROE	0.2	-4.066e-05	4.088e-03	.	.	.
Size.Small	0.2	1.336e-04	6.164e-03	.	.	.
nVar				16	15	15
BIC				-1.743e+06	-1.743e+06	-1.743e+06
post prob				0.143	0.091	0.082

Also, two types of shadings are to be noticed in Figure 3: black and grey. Black shading symbolizes that the parameter has a negative sign in the corresponding model; grey shading symbolizes the opposite. It is obvious, that the parameter estimates for all the relevant explanatory variables are consistent across our models under consideration (i.e. all of them have the same shading for all models). Moreover, other estimates using the BMA method besides the CPP are found in Table 7 including the weighted point estimates for all models and specific parameter estimates for top three models. We have included this as a further robustness check for our logit regression.

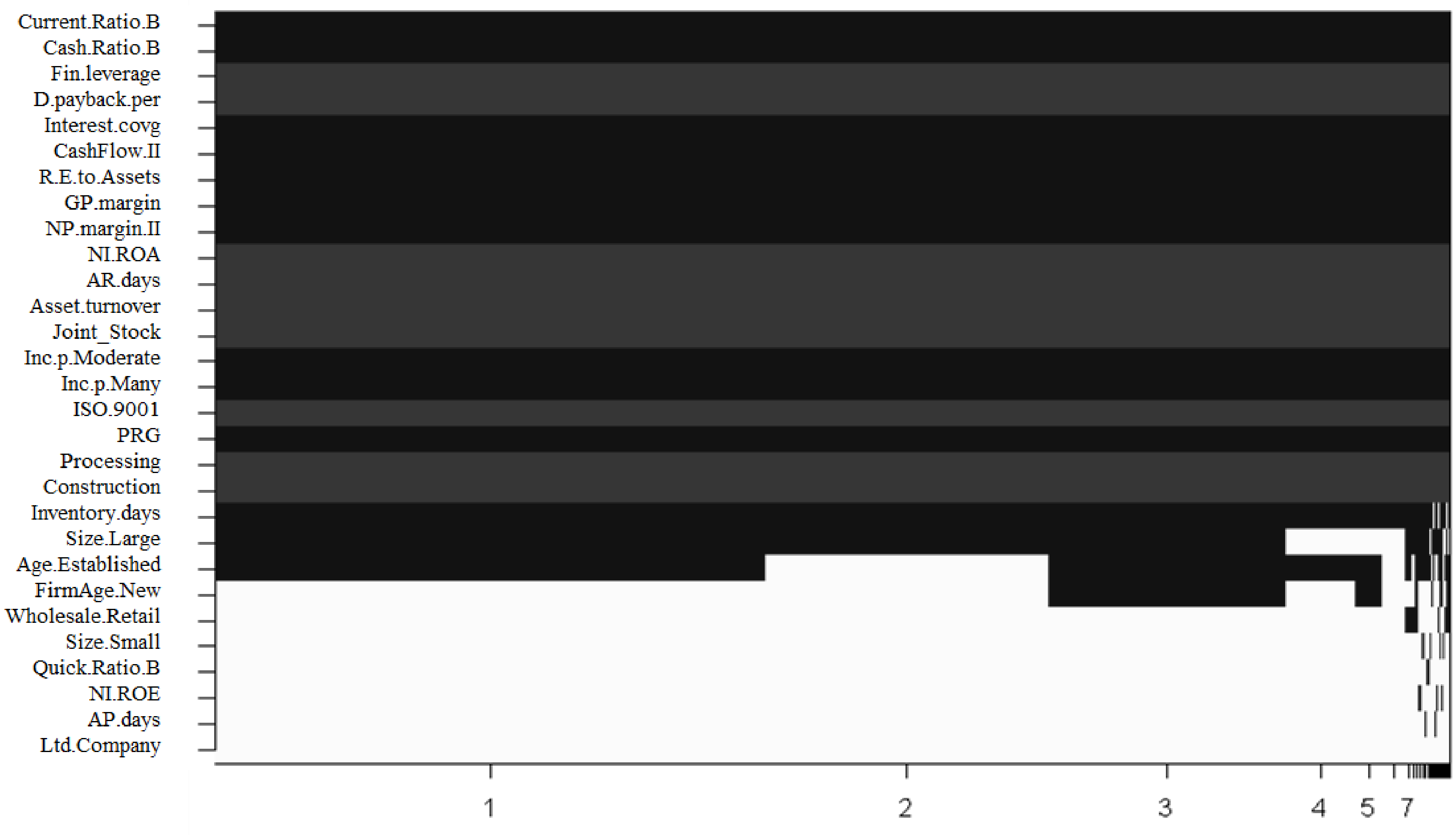
6.1.2 BMA for the rebalanced dataset

Results of the BMA for the rebalanced dataset are summarized in Table 8 and the graphical representation can be found in Figure 4. Although the analysis is analogous to the one for unbalanced dataset, there are significant differences in our results.

For example, substantially lower number of models is included in Figure 4 than in Figure 3; as just the top three models together gain more than 0.85 posterior probability. Occam´s window is therefore the reason why so few models are depicted in Figure 4, since the top model dominates the inclusion probability. Nevertheless, this does not pose any serious challenge to our analysis. On the contrary, we can be confident to a very large extent about the validity of our chosen explanatory variables, because the top three models seem to be clearly superior over the remaining ones. Just as above, we will apply the cutoff rate of 0.75 CPP.

Note that the number of variables with CPP above 0.75 increased from 14 to 21 when compared to the original unbalanced dataset. Moreover, 19 variables have CPP equal to unity. This inflation in the models´ size can be explained by the rebalancing of the minority class in the dataset. Using Table 5 it can be argued that there are noticeable differences between the bankrupt and the non-bankrupt companies. However, this difference is to a large extent neglected in the unbalanced dataset. This is because the MLE estimator favors the majority over the minority class. As a result of this, rebalancing

Figure 4: Variable selection with BMA for rebalanced dataset (2Y lag)



Note: x-axis = model specifications (width is a marginal posterior probability of that model), y-axis = list of potential explanatory variables; black shading = negative sign of parameter, grey shading = positive sign of parameter. Completely shaded row symbolizes cumulative posterior probability equal to 1, blank row means zero CPP for that variable

Table 8: Detailed BMA results for the rebalanced dataset (2Y lag)

	Cumulative Post. Prob	EV	SD	model 1	model 2	model 3
Intercept	100	1.838	6.168e-02	1.835	1.807	1.884
Current.Ratio.B	100	-8.879e-01	3.334e-02	-8.856e-01	-8.904e-01	-8.897e-01
Cash.Ratio.B	100	-3.558	1.145e-01	-3.560	-3.555	-3.549
Financial.leverage	100	2.451e-02	1.331e-03	2.444e-02	2.454e-02	2.454e-02
Debt.payback.period	100	2.093e-02	1.188e-03	2.097e-02	2.077e-02	2.096e-02
Interest.coverage	100	-4.644e-03	1.339e-04	-4.657e-03	-4.623e-03	-4.644e-03
CashFlow.II	100	-4.606	1.465e-01	-4.606	-4.601	-4.602
R.E.to.Assets	100	-1.054	5.014e-02	-1.051	-1.055	-1.058
GP.margin	100	-2.326	1.070e-01	-2.316	-2.340	-2.334
NP.margin.II	100	-3.555	2.737e-01	-3.550	-3.559	-3.536
NI.ROA	100	1.875	2.492e-01	1.859	1.887	1.876
AR.days	100	2.839e-03	1.550e-04	2.844e-03	2.839e-03	2.841e-03
Asset.turnover	100	1.331e-01	7.451e-03	1.326e-01	1.328e-01	1.337e-01
Joint_Stock	100	5.670e-01	2.596e-02	5.678e-01	5.682e-01	5.690e-01
Incor.pages.Moderate	100	-6.130e-01	2.999e-02	-6.118e-01	-6.193e-01	-6.105e-01
Incor.pages.Many	100	-1.360	3.879e-02	-1.360	-1.360	-1.354
ISO.9001	100	1.456e-01	2.584e-02	1.493e-01	1.377e-01	1.447e-01
PRG	100	-2.710e-01	2.687e-02	-2.733e-01	-2.658e-01	-2.713e-01
Processing	100	2.890e-01	2.928e-02	2.907e-01	2.889e-01	2.892e-01
Construction	100	8.888e-01	3.709e-02	8.905e-01	8.819e-01	8.916e-01
Inventory.days	99.6	-2.544e-04	5.534e-05	-2.527e-04	-2.607e-04	-2.595e-04
Size.Large	89.9	-2.170e-01	9.340e-02	-2.393e-01	-2.458e-01	-2.414e-01
FirmAge.Established	74.3	-7.108e-02	4.997e-02	-8.246e-02	.	-1.258e-01
FirmAge.New	22.3	-2.229e-02	4.447e-02	.	.	-1.004e-01
Wholesale.Retail	1.4	-8.318e-04	8.461e-03	.	.	.
Size.Small	0.7	2.493e-04	4.326e-03	.	.	.
Quick.Ratio.B	0.4	-8.836e-05	4.431e-03	.	.	.
NI.ROE	0.4	-3.566e-05	1.289e-03	.	.	.
AP.days	0.4	-4.265e-08	1.809e-06	.	.	.
nVar				22	21	23
BIC				-6.117e+05	-6.117e+05	-6.117e+05
post prob				0.446	0.229	0.192

improved the model´s ability to discriminate between good and bad predictors (i.e. explanatory variables).

Lastly, all of the selected explanatory variables have consistent parameter estimates across all our models, as observed in the case of the unbalanced data.

At this point, we have all necessary information to specify our prediction models, and shortly we should be also able to evaluate the ability of BMA methodology to select a solid set of predictors.

6.2 Estimation of the model

6.2.1 General comments and findings

Firstly, for the sake of clarity it is important to explain the structure of our results. We have defined 3 different dependent variables, for one, two, and three years before bankruptcy. We have used this method to see whether the parameter estimates are changing over different time horizons. Also, we have defined three basic types of datasets for the periods 2002–2016, 2008–2016, and 2002–2007. This should help us to detect potential developments after the new legislation came into force in 2008. Moreover, we used 3 rebalancing methods denoted by Under (for under-sampling), Over (for over-sampling), and Both (for the combination of previous methods); original unbalanced dataset is denoted by Full data. We have also used two different estimators in regressions, standard MLE and BY estimator as described in the methodology section.

As a result, we have run 72 regressions overall. Table 9 shows point estimates for the bankruptcy model with the one-year time horizon. Table 10 shows estimates for the model predicting 2 years in advance, and Table 11 does the same for three years horizon. Note that results presented in these three tables are for the full period of 2002–2016. Estimates for the remaining periods are presented in Appendix 10 to Appendix 14 (with the exception of the results for one year between 2002 and 2007, due to the insufficient number of observations and the inability to estimate the model by the BY estimator). Explanatory variables are those selected by BMA using 0.75 CPP threshold.

Before we move to the general analysis of our results, it is worth noting some general relations that we observe. With regard to the dependent variables, bankrupt firms are denoted by 1 and financially sound companies by 0. This implies, that if the sign of the estimated parameter is positive, higher values of the underlying variable increase the likelihood of a company's failure, and vice versa. A typical example of the first case could be financial leverage, which increases the probability of bankruptcy when raised and is therefore positively related to the likelihood of a firm´s bankruptcy. Conversely, gross profit margin, which decreases the probability of bankruptcy the higher it is, is then negatively related to the aforementioned probability. However, the numerical values of the parameters estimated using the logit model are indeed not as easy to interpret as those estimated using a simple linear model due to the very nature of the logistic function itself. We will explain this in more detail in the following sub-section.

Analysing the tables on pages 49, 50, and 51, we recognize a few common patterns worth of noting.

First, BY and standard MLE estimates are very similar. Signs are identical across the datasets and variables and the size of point estimates does not vary too much. The only exception is variable AR.days in Table 9, where the point estimate by BY markedly exceeds that of the standard MLE. Nevertheless, overall it seems that the employed method of winsorizing has coped with the issue of large sample outliers fairly well.

Second, signs of the estimates stay consistent across all three tables for the vast majority of variables. This is an important finding, as it suggests that the relations under analysis persist over time. There are two exceptions, NP.margin.II and NI.ROA. The expected sign of these profitability ratios is negative, since the higher the profitability measures, the less likely the probability of bankruptcy should be. This holds for the predictions one year in advance (Table 9), but does not hold for the three years predictions (Table 11). Even when we look closer at descriptive statistics in the appendices, the

Table 9: Logit model estimation for full dataset (1Y)

	Dependent variable: Bankrupt (1Y)							
	Full data MLE	Over MLE	Under MLE	Both MLE	Full data BY	Over BY	Under BY	Both BY
Constant	-5.7***	2.6***	2.3**	2.7***	-5.6***	3.2***	2.4*	3.2***
	(0.2)	(0.03)	(0.9)	(0.1)	(0.6)	(0.1)	(1.2)	(0.1)
Current.Ratio.B	-0.6***	-1.5***	-1.3**	-1.4***	-0.6***	-1.8***	-1.3	-1.7***
	(0.1)	(0.02)	(0.6)	(0.04)	(0.1)	(0.04)	(1.1)	(0.1)
Cash.Ratio.B	-1.1***	-6.6***	-6.6***	-6.9***	-1.1***	-7.9***	-6.9	-8.3***
	(0.3)	(0.1)	(2.2)	(0.2)	(0.2)	(0.2)	(4.2)	(0.3)
Financial.leverage	0.04**	0.02***	0.01	0.01***	0.04	0.02***	0.01	0.02***
	(0.02)	(0.001)	(0.02)	(0.002)	(0.1)	(0.001)	(0.02)	(0.002)
R.E.to.Assets	-0.8***	-1.4***	-2.0***	-1.4***	-0.8***	-1.6***	-2.1	-1.7***
	(0.1)	(0.03)	(0.7)	(0.1)	(0.1)	(0.1)	(7.0)	(0.2)
Debt.payback.period	0.02***	0.004***	0.01	0.003**	0.02	0.005***	0.01	0.004**
	(0.01)	(0.001)	(0.02)	(0.001)	(0.02)	(0.001)	(0.03)	(0.002)
Interest.coverage	-0.004***	-0.004***	-0.002	-0.004***	-0.004***	-0.005***	-0.002	-0.005***
	(0.001)	(0.0001)	(0.002)	(0.0002)	(0.0005)	(0.0001)	(0.002)	(0.0002)
CashFlow.II	-1.5***	-3.7***	-4.2**	-4.1***	-1.5***	-4.4***	-4.4	-5.0***
	(0.3)	(0.1)	(1.9)	(0.2)	(0.2)	(0.4)	(15.2)	(0.7)
GP.margin	-1.4***	-3.3***	-4.2**	-3.4***	-1.4***	-3.9***	-4.4	-4.1***
	(0.3)	(0.1)	(1.9)	(0.1)	(0.2)	(0.2)	(6.6)	(0.3)
NP.margin.II	-7.2***	-4.4***	-12.3***	-3.8***	-7.2***	-5.3***	-12.9	-4.6***
	(0.8)	(0.1)	(4.0)	(0.3)	(1.2)	(0.2)	(12.7)	(0.4)
NI.ROA		-2.6***	2.6	-2.5***		-3.1***	2.8	-3.0***
		(0.1)	(2.2)	(0.2)		(0.2)	(3.3)	(0.4)
Inventory.days		-0.001***	-0.001	-0.001***		-0.001***	-0.001	-0.001***
		(0.0000)	(0.001)	(0.0001)		(0.0000)	(0.001)	(0.0001)
AR.days	0.003***	0.004***	0.003**	0.004***	0.003***	0.01***	0.004	0.01***
	(0.001)	(0.0001)	(0.002)	(0.0002)	(0.001)	(0.0002)	(0.002)	(0.0004)
Asset.turnover		0.1***	0.2	0.1***		0.1***	0.2	0.1***
		(0.005)	(0.1)	(0.01)		(0.01)	(0.2)	(0.02)
Size.Large		-0.5***	-0.2	-0.4***		-0.6***	-0.2	-0.5***
		(0.03)	(0.9)	(0.1)		(0.03)	(0.7)	(0.1)
Construction	1.0***	0.7***	0.3	0.7***	1.0***	0.8***	0.3	0.8***
	(0.2)	(0.02)	(0.5)	(0.04)	(0.3)	(0.02)	(0.6)	(0.05)
Processing	0.7***	0.2***	0.3	0.2***	0.7***	0.3***	0.3	0.3***
	(0.2)	(0.02)	(0.4)	(0.03)	(0.2)	(0.02)	(0.5)	(0.04)
ISO.9001		0.1***	0.4	0.1*		0.1***	0.4	0.1*
		(0.01)	(0.4)	(0.03)		(0.02)	(0.6)	(0.03)
PRG		-0.2***	-0.3	-0.2***		-0.2***	-0.4	-0.3***
		(0.02)	(0.4)	(0.03)		(0.02)	(0.5)	(0.04)
Joint_Stock		0.4***	0.5	0.4***		0.5***	0.5	0.5***
		(0.01)	(0.4)	(0.03)		(0.01)	(0.4)	(0.03)
Incor.pages.Many	-0.4*	-0.8***	-0.8*	-0.8***	-0.4*	-0.9***	-0.9	-0.9***
	(0.2)	(0.02)	(0.5)	(0.04)	(0.2)	(0.02)	(0.5)	(0.04)
Incor.pages.Moderate		-0.5***	-0.3	-0.5***		-0.6***	-0.3	-0.6***
		(0.02)	(0.4)	(0.03)		(0.02)	(0.6)	(0.03)
Observations	*117,400*	*234,585*	*406*	*60,000*	*117,400*	*234,585*	*406*	*60,000*
Log Likelihood	*-1,254.7*	*-76,622.3*	*-134.5*	*-19,469.7*				
Akaike Inf. Crit.	*2,537.3*	*153,288.6*	*313.1*	*38,983.3*				
Note:								*p<0.1; **p<0.05; ***p<0.01

Table 10: Logit model estimation for full dataset (2Y)

	Dependent variable: Bankrupt (2Y)							
	Full data MLE	Over MLE	Under MLE	Both MLE	Full data BY	Over BY	Under BY	Both BY
Constant	-5.2***	1.8***	1.9***	1.8***	-5.2***	2.0***	1.9***	2.0***
	(0.1)	(0.03)	(0.5)	(0.1)	(0.2)	(0.03)	(0.6)	(0.1)
Current.Ratio.B	-0.5***	-0.9***	-1.0***	-0.9***	-0.5***	-1.0***	-1.0***	-1.0***
	(0.1)	(0.02)	(0.3)	(0.03)	(0.1)	(0.02)	(0.4)	(0.04)
Cash.Ratio.B	-1.2***	-3.7***	-4.7***	-3.6***	-1.2***	-4.0***	-4.9**	-3.8***
	(0.2)	(0.1)	(1.1)	(0.1)	(0.1)	(0.1)	(2.1)	(0.2)
Financial.leverage	0.1***	0.02***	0.02**	0.02***	0.1***	0.03***	0.03***	0.03***
	(0.01)	(0.001)	(0.01)	(0.001)	(0.03)	(0.001)	(0.01)	(0.001)
R.E.to.Assets	-0.7***	-1.0***	-0.5	-1.1***	-0.7***	-1.1***	-0.5	-1.1***
	(0.1)	(0.03)	(0.4)	(0.1)	(0.1)	(0.04)	(0.6)	(0.1)
Debt.payback.period	0.03***	0.02***	0.02**	0.02***	0.03***	0.02***	0.02**	0.02***
	(0.004)	(0.001)	(0.01)	(0.001)	(0.01)	(0.001)	(0.01)	(0.001)
Interest.coverage	-0.004***	-0.005***	-0.003**	-0.005***	-0.004***	-0.005***	-0.003***	-0.01***
	(0.001)	(0.0001)	(0.001)	(0.0001)	(0.0003)	(0.0001)	(0.001)	(0.0001)
CashFlow.II	-1.6***	-4.5***	-4.1***	-4.6***	-1.6***	-4.9***	-4.2	-5.0***
	(0.2)	(0.1)	(1.4)	(0.1)	(0.1)	(0.1)	(3.3)	(0.3)
GP.margin	-1.2***	-2.2***	-3.1***	-2.3***	-1.2***	-2.4***	-3.3**	-2.5***
	(0.3)	(0.1)	(1.0)	(0.1)	(0.1)	(0.1)	(1.4)	(0.1)
NP.margin.II	-5.7***	-3.8***	-10.0***	-3.6***	-5.7***	-4.2***	-10.4**	-3.9***
	(0.7)	(0.1)	(2.9)	(0.3)	(0.7)	(0.2)	(4.1)	(0.3)
NI.ROA		2.2***	4.4*	1.9***		2.4***	4.6*	2.0***
		(0.1)	(2.3)	(0.2)		(0.2)	(2.5)	(0.3)
Inventory.days		-0.0003***	0.0001	-0.0003***		-0.0003***	0.0001	-0.0003***
		(0.0000)	(0.0005)	(0.0001)		(0.0000)	(0.0004)	(0.0000)
AR.days	0.002***	0.003***	0.005***	0.003***	0.002***	0.003***	0.005*	0.003***
	(0.0005)	(0.0001)	(0.002)	(0.0002)	(0.0005)	(0.0001)	(0.003)	(0.0002)
Asset.turnover		0.1***	0.1*	0.1***		0.2***	0.1	0.1***
		(0.004)	(0.1)	(0.01)		(0.005)	(0.1)	(0.01)
Size.Large		-0.2***	0.1	-0.2***		-0.2***	0.1	-0.3***
		(0.03)	(0.6)	(0.1)		(0.04)	(0.8)	(0.1)
Construction	0.9***	0.9***	1.0***	0.9***	0.9***	1.0***	1.1***	1.0***
	(0.1)	(0.02)	(0.3)	(0.04)	(0.2)	(0.02)	(0.3)	(0.04)
Processing	0.5***	0.3***	0.4	0.3***	0.5***	0.3***	0.4	0.3***
	(0.1)	(0.01)	(0.2)	(0.03)	(0.1)	(0.02)	(0.3)	(0.03)
ISO.9001		0.1***	0.3	0.1***		0.1***	0.3	0.1***
		(0.01)	(0.2)	(0.03)		(0.01)	(0.2)	(0.03)
PRG		-0.3***	-0.4	-0.3***		-0.3***	-0.4*	-0.3***
		(0.01)	(0.2)	(0.03)		(0.01)	(0.2)	(0.03)
Joint_Stock		0.6***	0.5**	0.6***		0.6***	0.6*	0.6***
		(0.01)	(0.2)	(0.03)		(0.01)	(0.3)	(0.03)
Incor.pages.Many	-1.0***	-1.4***	-1.8***	-1.4***	-1.0***	-1.5***	-1.9***	-1.5***
	(0.2)	(0.02)	(0.3)	(0.04)	(0.2)	(0.02)	(0.4)	(0.04)
Incor.pages.Moderate		-0.7***	-0.7***	-0.6***		-0.7***	-0.8***	-0.7***
		(0.01)	(0.2)	(0.03)		(0.01)	(0.3)	(0.03)
Observations	*117,604*	*234,549*	*848*	*60,000*	*117,604*	*234,549*	*848*	*60,000*
Log Likelihood	*-2,335.7*	*-94,748.8*	*-336.5*	*-24,095.4*				
Akaike Inf. Crit.	*4,699.4*	*189,541.5*	*717.1*	*48,234.8*				
Note:								*p<0.1; **p<0.05; ***p<0.01

Table 11: Logit model estimation for full dataset (3Y)

	Dependent variable: Bankrupt (3Y)							
	Full data MLE	Over MLE	Under MLE	Both MLE	Full data BY	Over BY	Under BY	Both BY
Constant	-4.9***	2.1***	2.4***	2.2***	-4.9***	2.3***	2.5***	2.3***
	(0.1)	(0.03)	(0.4)	(0.1)	(0.1)	(0.04)	(0.6)	(0.1)
Current.Ratio.B	-0.4***	-0.8***	-1.2***	-0.8***	-0.4***	-0.9***	-1.3***	-0.9***
	(0.1)	(0.02)	(0.2)	(0.03)	(0.1)	(0.02)	(0.3)	(0.04)
Cash.Ratio.B	-1.1***	-3.3***	-2.6***	-2.8***	-1.1***	-3.6***	-2.7	-3.0***
	(0.2)	(0.1)	(0.7)	(0.1)	(0.1)	(0.1)	(1.7)	(0.2)
Financial.leverage	0.1***	0.04***	0.04***	0.04***	0.1***	0.04***	0.04***	0.04***
	(0.01)	(0.001)	(0.01)	(0.001)	(0.01)	(0.001)	(0.01)	(0.001)
R.E.to.Assets	-0.7***	-0.7***	-0.8**	-0.7***	-0.7***	-0.7***	-0.8	-0.7***
	(0.1)	(0.02)	(0.4)	(0.05)	(0.1)	(0.04)	(0.7)	(0.1)
Debt.payback.period	0.02***	0.005***	0.01	0.004***	0.02***	0.01***	0.01	0.004***
	(0.004)	(0.001)	(0.01)	(0.001)	(0.01)	(0.001)	(0.01)	(0.001)
Interest.coverage	-0.003***	-0.004***	-0.004***	-0.004***	-0.003***	-0.004***	-0.004***	-0.004***
	(0.0005)	(0.0001)	(0.001)	(0.0001)	(0.0003)	(0.0000)	(0.001)	(0.0001)
CashFlow.II	-1.8***	-5.4***	-7.0***	-5.8***	-1.8***	-5.7***	-7.3***	-6.2***
	(0.2)	(0.1)	(1.1)	(0.1)	(0.1)	(0.1)	(1.9)	(0.2)
GP.margin	-1.4***	-2.5***	-1.8**	-2.8***	-1.4***	-2.7***	-1.9	-3.0***
	(0.2)	(0.1)	(0.8)	(0.1)	(0.1)	(0.1)	(1.2)	(0.2)
NP.margin.II	-3.7***	0.1	1.7	1.0***	-3.7***	0.1	1.7	1.0***
	(0.7)	(0.1)	(2.1)	(0.3)	(0.5)	(0.1)	(2.2)	(0.3)
NI.ROA		1.7***	3.7**	1.4***		1.8***	3.9	1.5***
		(0.1)	(1.8)	(0.2)		(0.2)	(2.6)	(0.3)
Inventory.days		-0.001***	0.0000	-0.001***		-0.001***	0.0000	-0.001***
		(0.0000)	(0.0004)	(0.0001)		(0.0000)	(0.0005)	(0.0000)
AR.days	0.002***	0.001***	0.003***	0.002***	0.002***	0.002***	0.003	0.002***
	(0.0004)	(0.0001)	(0.001)	(0.0002)	(0.0003)	(0.0001)	(0.003)	(0.0002)
Asset.turnover		0.1***	0.2***	0.1***		0.1***	0.2***	0.1***
		(0.004)	(0.1)	(0.01)		(0.01)	(0.1)	(0.01)
Size.Large		-0.2***	-0.01	-0.2***		-0.2***	-0.01	-0.2***
		(0.03)	(0.5)	(0.1)		(0.03)	(0.5)	(0.1)
Construction	0.8***	0.8***	0.7***	0.8***	0.8***	0.9***	0.7***	0.9***
	(0.1)	(0.02)	(0.3)	(0.04)	(0.1)	(0.02)	(0.3)	(0.04)
Processing	0.5***	0.3***	0.1	0.4***	0.5***	0.3***	0.1	0.4***
	(0.1)	(0.01)	(0.2)	(0.03)	(0.1)	(0.01)	(0.2)	(0.03)
ISO.9001		0.1***	-0.01	0.1***		0.1***	-0.01	0.1***
		(0.01)	(0.2)	(0.02)		(0.01)	(0.2)	(0.03)
PRG		-0.1***	-0.3*	-0.1***		-0.1***	-0.4*	-0.1***
		(0.01)	(0.2)	(0.03)		(0.01)	(0.2)	(0.03)
Joint_Stock		0.5***	0.4**	0.5***		0.5***	0.4*	0.5***
		(0.01)	(0.2)	(0.03)		(0.01)	(0.2)	(0.03)
Incor.pages.Many	-0.8***	-1.1***	-1.1***	-1.0***	-0.8***	-1.2***	-1.1***	-1.1***
	(0.2)	(0.02)	(0.3)	(0.04)	(0.2)	(0.02)	(0.3)	(0.03)
Incor.pages.Moderate		-0.5***	-0.5**	-0.5***		-0.6***	-0.6***	-0.6***
		(0.01)	(0.2)	(0.03)		(0.01)	(0.2)	(0.03)
Observations	*117,724*	*234,542*	*1,084*	*60,000*	*117,724*	*234,542*	*1,084*	*60,000*
Log Likelihood	*-2,925.4*	*-100,432.5*	*-442.7*	*-25,758.0*				
Akaike Inf. Crit.	*5,878.8*	*200,909.0*	*929.4*	*51,559.9*				
Note:								*p<0.1; **p<0.05; ***p<0.01

situation is not clearer. This is because the bankrupt companies have generally negative probability, and vice versa (see Appendix 1 to Appendix 4). However, when we take into account the average sizes of these two parameters, which are generally less than 1 and seems to be most commonly close to zero, we believe that their impact is economically not very significant. Therefore we concluded, that this anomaly does not represent a serious flaw and shall not undermine our inferences.

Third, overwhelming majority of independent variables is significant at 5% level. That confirms the usefulness of the BMA approach, which helped us to select the set of best predictors. The only deviation is the significance levels for the under-sampled dataset, which cannot be reached primarily due to the insufficient number of observations.

Finally, same patterns can be found also in the datasets for 2002–2007 and 2008–2016. Reader can inspect detailed summaries for these two periods in appendices, but due to the relative resemblance, we will not comment these results separately. Instead, will put the models' predictive abilities into perspective in the next section.

6.2.2 Role and significance of independent variables

It is important to keep in mind that the parameters of logit model are not easily interpretable. Recall the equation (3) from methodology, which can be fitted, so that

$$lo\,g \frac{\hat{\pi}_i}{1-\hat{\pi}_i} = x_i'\hat{\beta}\ , \tag{21}$$

where $\hat{\beta}$ is the vector of estimated parameters, and the left hand side of the equation represents odds of going bankrupt. As a consequence, the interpretation of dummy variables is very straightforward, since the dummy of value 1 has higher odds of bankruptcy by $e^{\hat{\beta}}$, keeping all other variables fixed. The interpretation of continuous variables is more complex, because the size of the coefficients has to be assessed with respect to the mean values of the underlying variables. To evaluate and compare the importance of predictors (we will refer to it as to economic significance), the difference in mean values for non-bankrupt and bankrupt companies will be used. Consequently, the higher the $e^{\widehat{\beta}(\bar{X}_{sound} - \bar{X}_{bankrupt})}$ term, all else being equal, the greater is the economic significance attached to the predictor, and vice versa. We essentially compare the effects of average bankrupt and non-bankrupt companies on the odds of getting bankrupt. For the sake of simplicity, we will use just means for one-year horizon to be found in Appendix 2, and parameter estimates for Both MLE model.

In Tables 9, 10, and 11 you can see, that variables have been grouped by their specialization (e.g. liquidity, solvency, activity, etc.). This enables an easy assessment of predictor's importance within each group, which is very useful for the selection of the top measures to be used for bankruptcy prediction. Furthermore, it shows, that all groups from Table 3 are represented in our model. In following paragraphs each group will be analyzed separately and also the comparison with the results of other researchers will be provided.

Liquidity ratios are generally accepted predictors in the bankruptcy models. The most widely used ratio is the Current ratio, which has been selected as an important variable for instance by Zmijewski (1984), Ohlson (1980), or Altman (2000), and in the Czech context by Neumaierová and Neumaier (2005b) and Kalouda and Vaníček (2013). Cash ratio usually does not attract much attention, but for example Beaver (1966), Jakubík and Teplý (2008), or Kalouda and Vaníček (2013), found an evidence that it can be a very solid predictor. In our logit model the Cash ratio seems to be more important with the coefficient of -6.8 in the one-year horizon, as Current ratio's estimate is just -1.4. However, when we compare the measures for economic significance, we get almost equal values of $e^{-2.1}$ for Cash ratio and $e^{-2.2}$ for Current ratio. This suggests, that both ratios are similarly important predictors in one-year horizon. Nonetheless, the size of the Cash ratio's parameter decreases to -2.8 for three years horizon compared to less significant drop to -0.8 for Current ratio. From this we conclude, that both ratios are very strong predictors one year before bankruptcy, but the relevance of Cash ratio diminishes with longer periods. The last point is in a slight contradiction with findings of Kalouda and Vaníček (2013), who concluded that Cash ratio is actually even better predictor for long-term than short-term horizons, but similarly to us they arrived to conclusion that the Current ratio is generally stronger predictor than Cash ratio.

Financial leverage and Retained Earnings to Assets are central leverage ratios used in the literature. However, although both ratios are statistically significant in our model, Financial Leverage is of remarkably lower economic significance compared to RE ratio (our measure of the economic significance is $e^{0.004}$ for Financial Leverage and $e^{-0.28}$ for Retained Earnings to Assets). However, the importance of Financial Leverage increases with longer prediction horizons to $e^{0.16}$ for three years horizon, suggesting that it is of some relevance for long-term predictions. The opposite is true for the RE ratio where the importance decreases with longer horizons.

Higher importance of RE to Assets can be partly explained by the fact, that the ratio is perceived to contain more information and hence is often used as a proxy for other specifications, such as age, reinvestment policy, or profitability. (Altman 2000) One of the possible reasons why Financial Leverage seems not to be very significant could be, that we could not use market values of equity and debt as did for example Altman (1968) and Altman (2000).

Out of three solvency ratios chosen by BMA, Cash Flow ratio is the most significant predictor followed by Interest Coverage ratio, with $e^{-1.1}$ and $e^{-0.5}$, respectively. Debt payback period is of low economic significance. The size of parameter estimates remains on similar levels across horizons, so these two ratios are good candidates for being the effective long-term predictors, too.

Interest Coverage ratio has been frequently used for bankruptcy prediction. Moreover, there has been also some evidence that it is a useful predictor in the Czech economy, see Kalouda and Vaníček (2013), Jakubík and Teplý (2008), and Neumaierová and Neumaier (2005b). On the other hand, Cash Flow ratios have not attracted much attention so far. For instance, Ohlson (1980) used it in his prediction model, but in the literature EBITDA is usually compared to total assets instead of sales or long-term debts, as we did.

Albeit all activity ratios are statistically significant at very low significance levels, none of them is economically significant. Hence we can see, why they are rarely used in bankruptcy prediction models. The main problem with activity ratios is, that they are very individual and their interpretation can thus be ambiguous. Although Jakubík and Teplý (2008) and Kalouda and Vaníček (2013) proposed the Days of Inventory to be strong variable especially for long-term predictions, we have not found supportive evidence in our data.

In the case of profitability ratios we get conflicting estimates across horizons for Net Profit margin and Return on Assets, which disables any reasonable inference. However, Gross Profit margin is of reasonable economic significance, with $e^{-0.31}$, and the size of coefficients persists with increasing prediction horizon. Thus we come to the same conclusion as Jakubík and Teplý (2008), who proposed the GP margin as the profitability ratio with the highest predictive power.

The most interesting finding from the group of dummy variables is, that construction companies have more than 2 times higher odds of bankruptcy than other companies (non-processing and non-construction). It stems from the fact that $e^{0.7} = 2.014$, where 0.7 is the point estimate for one-year horizon. This is consistent with findings of Karas and Režňáková (2017) and Čámská (2015), who focused specifically on the construction industry due to higher rate of corporate failures. Moreover, joint-stock companies have 1.5 times higher odds of bankruptcy than other legal forms and enterprises with total assets above EUR 50 m have 1.5 times lower odds of distress for one-year horizon. The size effect in our estimation supports the results of Ohlson (1980), Altman (2000), and Altman et al. (2014), who came to similar conclusions. Additionally, companies operating in Prague seem to have lower probability of bankruptcy and the number of pages in the incorporation documentation seems to be a solid proxy for some unobservable variables. Interestingly, ISO certification is estimated to have negative impact on the odds of bankruptcy. It could be caused by the limitations the certification imposes on the company, which then potentially becomes less flexible. However, the interpretation of the last dummies has to be taken cautiously, due to the limitations mentioned in the chapter describing our data.

6.2.3 The most efficient predictors for practical use

One of the goals of our research was to provide shareholders, managers, analysts, suppliers, and other interested groups, with the updated list of the best predictors of future financial distress in the context of the Czech economy. Figure 5 shows the outcomes of our analysis.

We divided the top preforming financial ratios into two tiers, based on the argumentation presented in the previous section. Apparently, Current and Cash Flow II ratios are strong predictors for both long- and short-term horizons. Cash ratio performs very well in shorter term, but loses part of its predictive abilities the longer the prediction horizon is. However, it still retains a solid predictive power also in the long-term horizon. Generally, Tier 2 predictors are of somehow smaller economic significance than the Tier 1 ratios, but they can be used to efficiently complement the Tier 1 ratios. Both Interest Coverage and Gross Profit margin keep their predictive power even for longer prediction periods, which is consistent with the findings of Jakubík and Teplý (2008) and Kalouda and Vaníček (2013). Share

Figure 5: Top financial predictors for corporate failures

	Short-term horizon	Long-term horizon
Tier 1 predictors	Cash ratio, Current ratio, Cash Flow II	Current ratio, Cash Flow II
Tier 2 predictors	Retained Earnings to Assets, Interest Coverage, Gross Profit margin	Cash ratio, Financial Leverage, Interest Coverage, Gross Profit margin

Note: Tier 1 and Tier 2 ratios represent financial ratios selected upon the analysis in the previous section. Short-term horizon is intended for one to two years predictions, long-term horizon is intended for longer than two years horizons.

of Retained Earnings on total Assets is a good predictor for short term, but its ability deteriorates with longer prediction periods. The opposite is true for Financial Leverage, which does not perform well in short term, but can be employed for longer-term predictions.

As a result of our analysis we thus provide all interested groups essentially with 2 sets of financial ratios, one to be used for long-term and the second for short-term prediction horizons. Since these ratios have been evaluated on the updated data up to the year 2016, they should reflect the latest developments in the Czech economy.

6.3 ROC and PR curves

6.3.1 ROC curves for various models and datasets

In this part, the predictive ability of the models outlined in chapter 6.2 will be evaluated. As a main measure, we have selected the method of ROC curves that have recently gained prominence in the relevant academic literature. We assess the models using the out-of-sample observations that were put aside before estimating the regression parameters (corresponding to 20 per cent of the data).

In Figure 6 to Figure 8 the visualized results for 68 models are shown. We split them into three sets of diagrams with respect to the prediction horizons (4 modes for BY estimator in 2002–2007 could not be estimated due to the small number of observations).

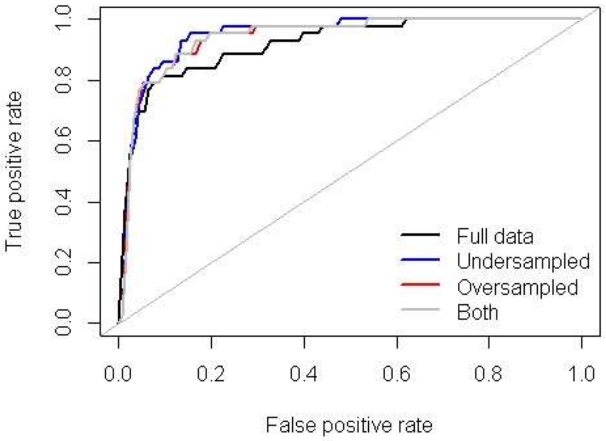
Although all diagrams look quite similarly, a couple of important patterns and tendencies are noticeable.

First of all, ROC curves for standard MLE and BY models are almost identical, which stems from the marginal nuances in the parameter estimates debated in previous section.

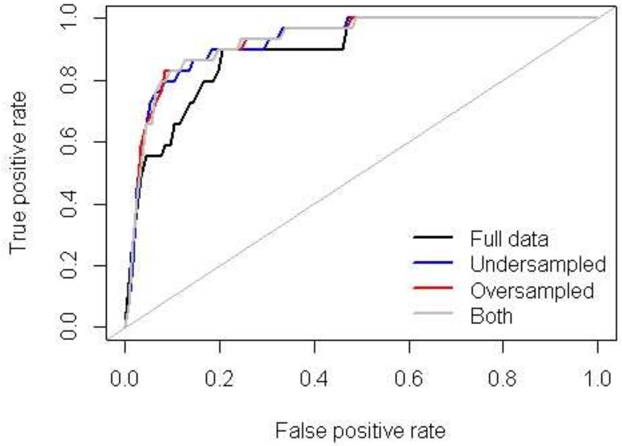
It is also clear that the models estimated on the original unbalanced datasets (represented by the black line on the diagrams) perform comparatively worse to those estimated by the rebalanced datasets. This rule holds throughout all our datasets and time horizons studied. On the other hand,

Figure 6: ROC curves for standard and robust estimates one year in advance

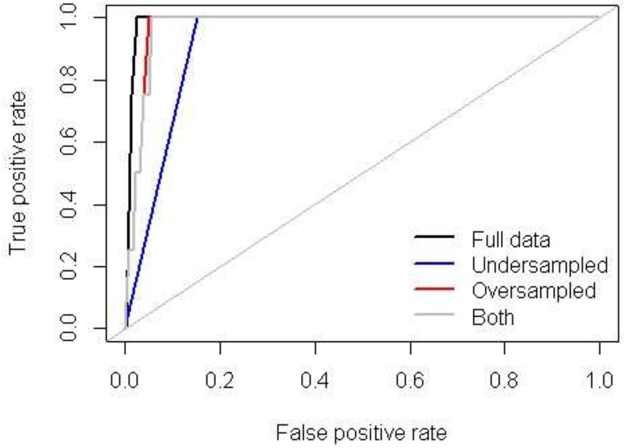
2002-2016, standard MLE



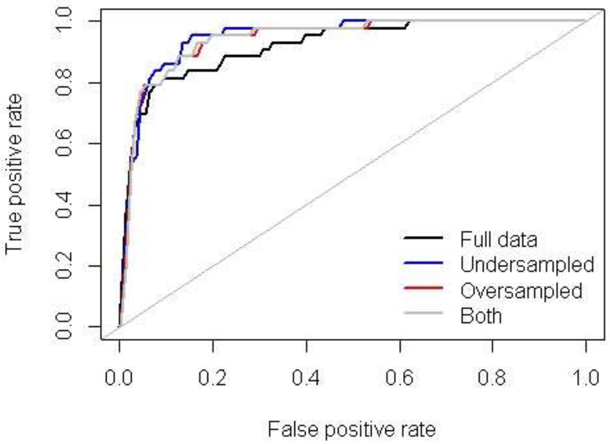
2008-2016, standard MLE



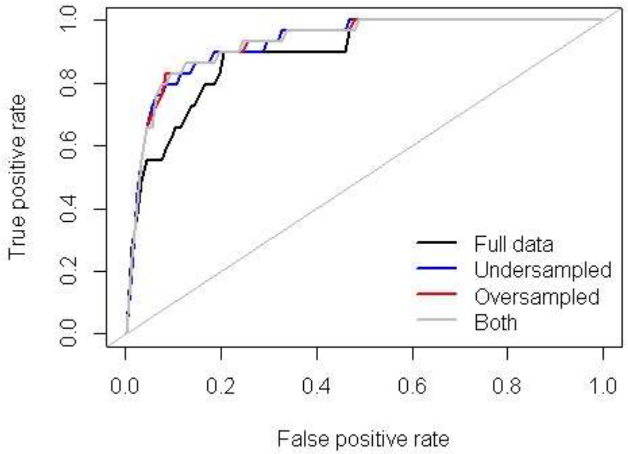
2002-2006, standard MLE



2002-2016, BY



2008-2016, BY



2002-2007, BY estimate not available

Figure 7: ROC curves for standard and robust estimates two years in advance

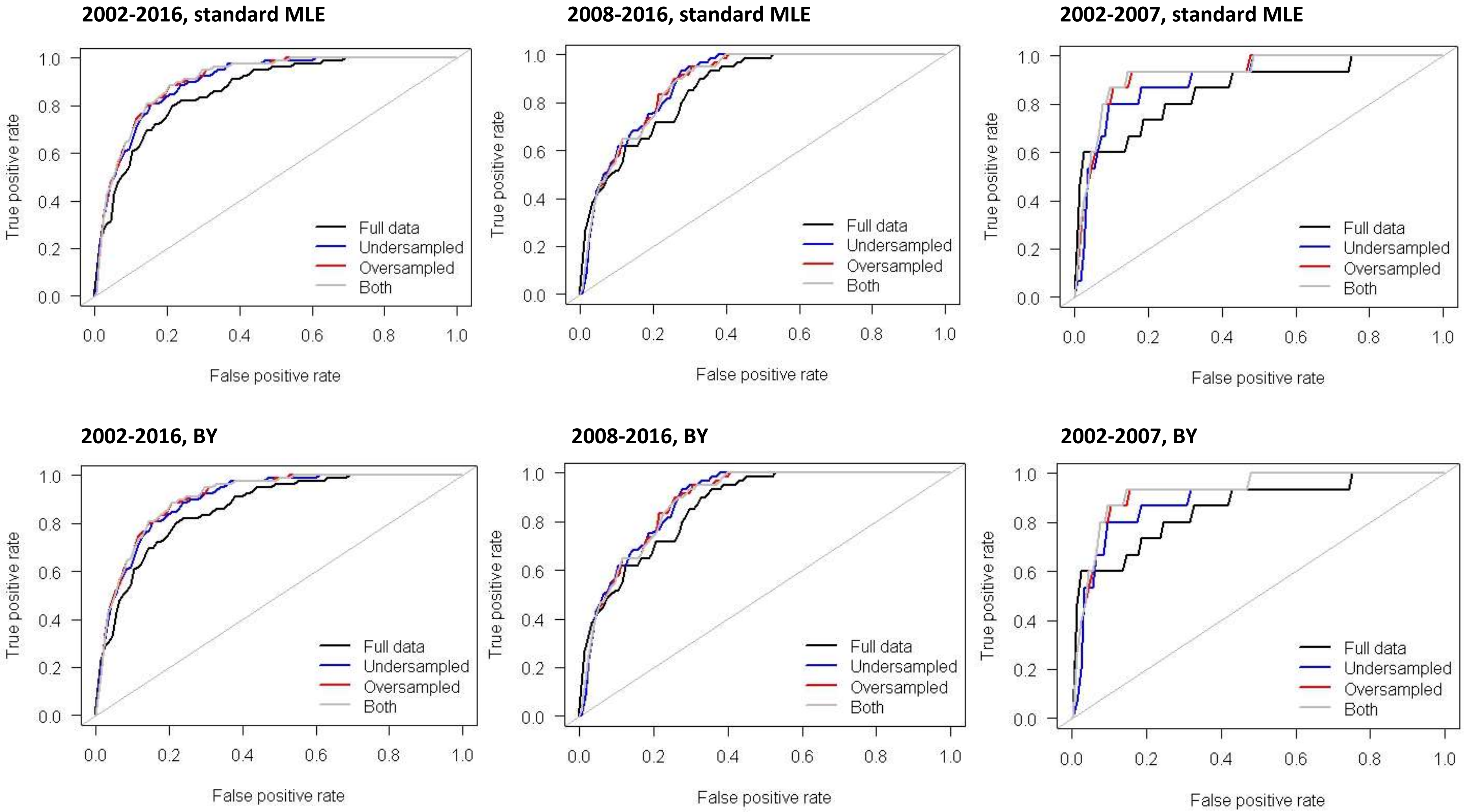
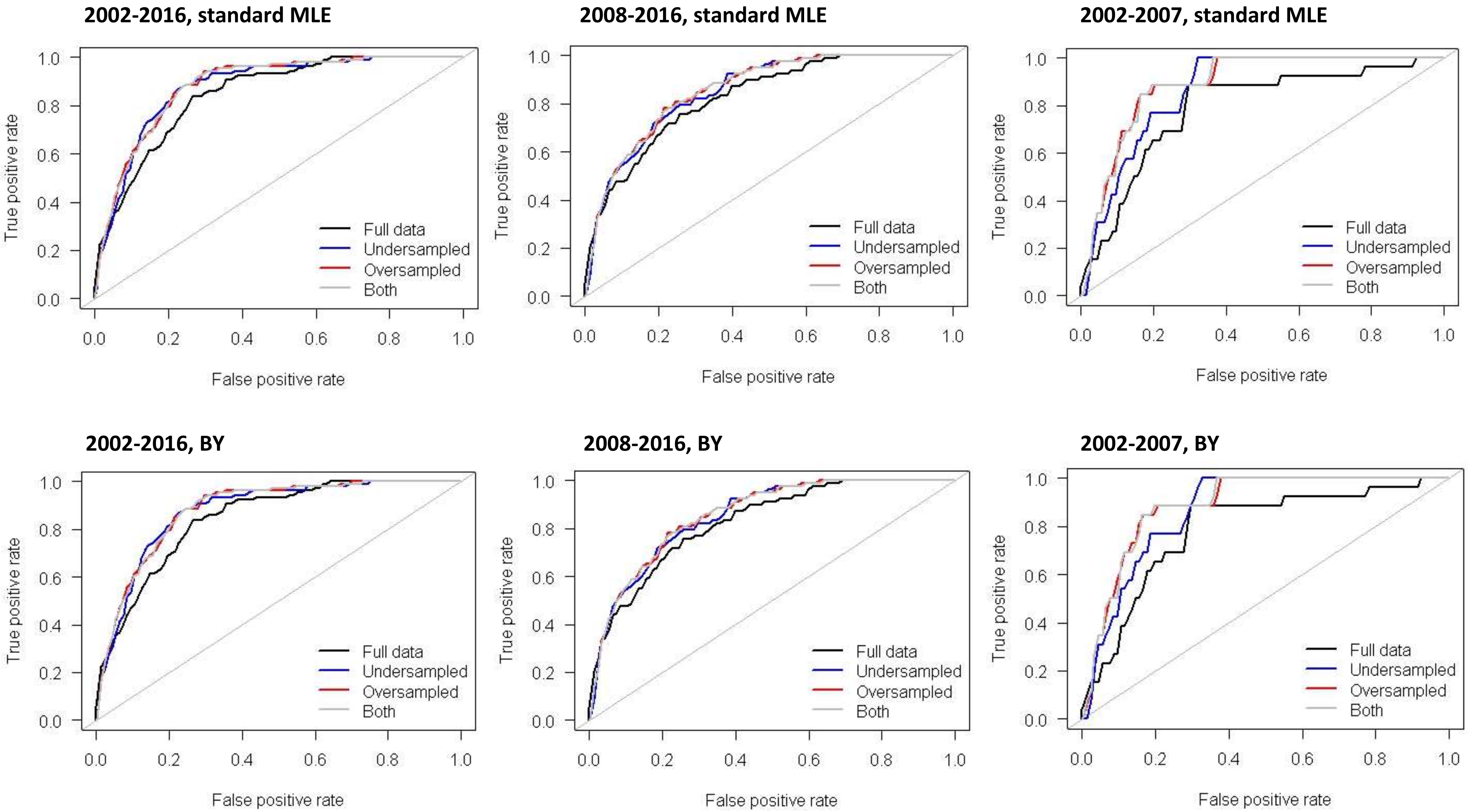


Figure 8: ROC curves for standard and robust estimates three years in advance



over-sampling and combination of over- and under-sampling methods work very well. AUC metric shows that too (see Table 12).

Furthermore, the predictive ability deteriorates with expanding time horizon. While one year in advance the true positive rate can get to 80 per cent while showing just a very small increase in the false positive rates, longer time horizons require higher false positive rate to reach the same level of true positives. In other words, in shorter horizons the classifier is able to predict company's bankruptcy with lower number of tries. It can be nicely observable on the 2008–2016 example, where the ROC curves get more and more rounded with every year. It is also obvious from AUCs presented in Table 12, which decline with longer horizons. Nonetheless, it is obvious that our models retain predictive abilities even in the three year horizon, which opposes to the conclusion of Altman (1968), who concluded that the accuracy of predictions deteriorates substantially with longer than 2 years horizons.

Lastly, it is rather difficult to compare the results for the 2002–2007 and 2008–2016 periods. This is because we do not have an extensive dataset for the year from 2002 to 2007. Therefore, the test sample consist of limited number of observations (with only 4 bankrupt companies for one-year horizon for example). Due to this challenge to our evidence, we have decided to not make any explicit conclusions in regard to inter-period dissimilarities. However, reader can inspect detailed information provided in appendices and draw conclusion on his own.

In order to better see the characteristics of our classifier, we will apply it on a fictitious sample of 1000 sound and 100 bankrupt companies. Also, for the sake of clarity we will focus only on the cases of 2002—2016 period and standard MLE estimates for resampled data.

Let's assume we are interested in discrimination between companies that will and will not go bankrupt in one year. From the top-left diagram in Figure 6 we can infer that 80% true positive rate (tpr) can be reached with about 5% false positive rate (fpr). This implies that our classifier would assign 80 out of 100 bankrupt and 950 out of 1000 non-bankrupt companies correctly. Additionally, 20 bankrupt and 50 non-bankrupt firms would be incorrectly assigned to the group of healthy and failing firms, respectively. If we were targeting higher tpr, we would have to accept higher fpr. Intuitively, the

Table 12: Summary of AUCs for all models and data

1 Year	Full MLE	Under MLE	Over MLE	Both MLE	Full BY	Under BY	Over BY	Both BY
Complete	0.92	0.95	0.94	0.94	0.92	0.95	0.94	0.94
2008–2016	0.89	0.93	0.93	0.93	0.89	0.93	0.93	0.93
2002–2007	0.99	0.92	0.97	0.97	n/a	n/a	n/a	n/a
Average	0.93	0.93	0.95	0.95	0.91	0.94	0.94	0.94

2 Years	Full MLE	Under MLE	Over MLE	Both MLE	Full BY	Under BY	Over BY	Both BY
Complete	0.87	0.90	0.91	0.91	0.87	0.90	0.91	0.91
2008–2016	0.86	0.89	0.89	0.89	0.86	0.89	0.89	0.89
2002–2007	0.86	0.90	0.93	0.93	0.86	0.90	0.90	0.93
Average	0.86	0.90	0.91	0.91	0.86	0.90	0.90	0.91

3 Years	Full MLE	Under MLE	Over MLE	Both MLE	Full BY	Under BY	Over BY	Both BY
Complete	0.85	0.88	0.88	0.88	0.85	0.88	0.88	0.88
2008–2016	0.83	0.85	0.86	0.86	0.83	0.85	0.86	0.86
2002–2007	0.79	0.87	0.89	0.89	0.79	0.87	0.89	0.89
Average	0.82	0.87	0.88	0.88	0.82	0.87	0.88	0.88

classifier would require more attempts to select larger portion of distressed enterprises. In consequence, to reach 95% tpr we would have to tolerate the increase in fpr to approximately 20%. Hence, in order to select additional 15 bankrupt companies to get to 95 out of 100 success rate, total of 200 healthy companies would have to be assigned incorrectly to the bankrupt group. You can notice that the ROC curve becomes flat and equal to unity after the fpr reaches approximately 50%. This is the point at which all bankrupt companies are correctly assigned. Nevertheless, about 500 incorrectly assigned healthy companies are needed to attain this level of tpr. Noteworthy, after reaching the 100% tpr it does not make any sense to further sacrifice any increase in fpr as the maximum tpr has been achieved.

The logic is analogous also for further time horizons. Generally, as the ROC curves get more rounded with the expanding prediction horizons, higher rate of false positives is required to reach the same level of true positives. For instance, to reach the 80% tpr in three years horizon requires approximately

Table 13: AUCs of top models from relevant studies (Czech companies only)

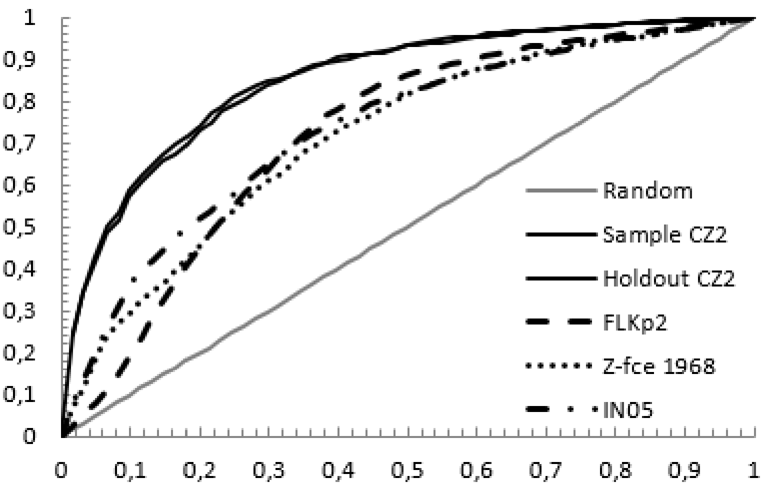
Article	Horizon	Years of interest	No. of firms	Method	M1	M2	M3	M4
Altman et al. (2014)	1 year	2007-2010	100,000	Logit	0.84	0.83	0.82	0.82
Kalouda and Vaníček (2013)	2 years	2008-2012	140,000	MDA	0.85	0.73	0.72	0.71
Karas and Režňáková (2017)	1 year	2011-2014	700	CART	0.86	0.78	0.55	0.54

Note: Both Altman et al. (2014) and Kalouda and Vaníček (2013) used comprehensive datasets for estimation, Karas and Režňáková (2017) focused on construction industry only. M2 in Kalouda and Vaníček (2013) is index IN05

20% fpr compared to only 5% in one-year horizon. For our fictitious dataset it would mean, that to select 80 out of 100 bankrupt companies correctly, 200 incorrect picks would be needed. Furthermore, the ROC curve gets flat and equal to unity at higher fpr rates with longer prediction horizons.

To put our results into perspective, we compare them to the results of Altman et al. (2014), Kalouda and Vaníček (2013), and Karas and Režňáková (2017), who estimated their bankruptcy models on the Czech data. Table 13 shows a summary of the 4 best models in terms of AUC from the respective papers. Remarkably, all of the mentioned studies use a different set of data, different time horizons and different methodology among other things. However, they can still be quite representative of what levels of accuracy could reasonably be expected. Probably the closest model to ours is the M1 model by Altman et al. (2014), which represents a logit regression on a dataset of a comparable size, with set of dummy variables including industry, age, and size. We can see, that our models perform fairly well in comparison. For the one year time horizon our models reach AUC of 0.95, for two years horizon AUC of 0.91, and for three years horizon 0.88, which is still above the top model's value from Table 13. Figure 9 shows resulting ROC curves from Kalouda and Vaníček (2013) for 5 models they evaluated. Note that the curves are for two years horizon and all of them are closer to random classification than ours. In addition, model IN05 performs fairly poorly.

Figure 9: ROC curves for comparison, 2Y horizon (from Kalouda and Vaníček (2013))



However, claiming the superiority of our model was by no means the goal of the comparison. It should primarily provide the reader with benchmark, to which the models' performance could be compared.

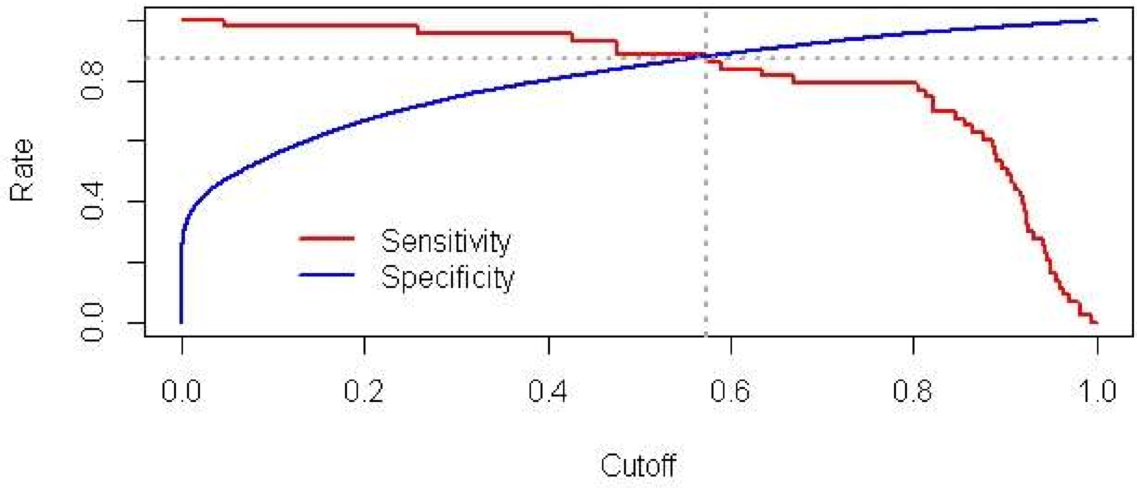
6.3.2 Sensitivity and Specificity curves

In this section, we shall take a closer look at the various features of our model in order to analyze the main strengths and the main weaknesses of the classifier. We will analyze the behavior of sensitivity and specificity curves for various cut-off thresholds to allow us to tailor the classifiers to special objectives (e.g. are we interested in correct discrimination of non-bankrupt companies, or do we target the highest possible total accuracy rate?). A similar approach is taken for example in medical testing, where different levels of type I and type II errors can potentially save millions of dollars.

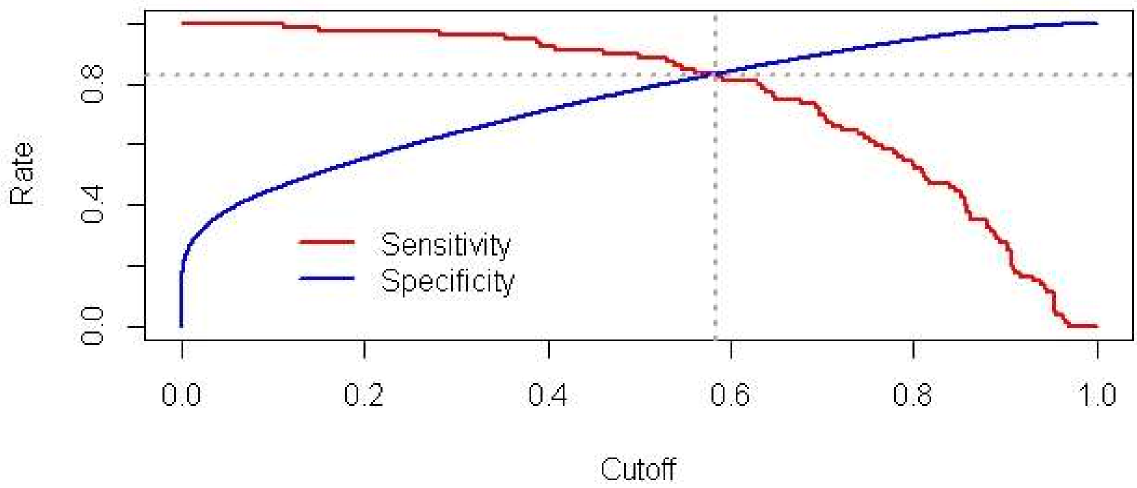
Figure 10 shows the plotted sensitivity and specificity curves for standard MLE estimates on Both data (this model was selected mainly due to solid performance in previous section). The x-axis represents the cut-off rate and the y-axis is a standard rate ranging from 0 to 1. Recall that sensitivity is defined as a rate of correctly predicted positive cases (out of all positives) and specificity corresponds to

Figure 10: Sensitivity-Specificity curves for standard MLE estimates (Both data)

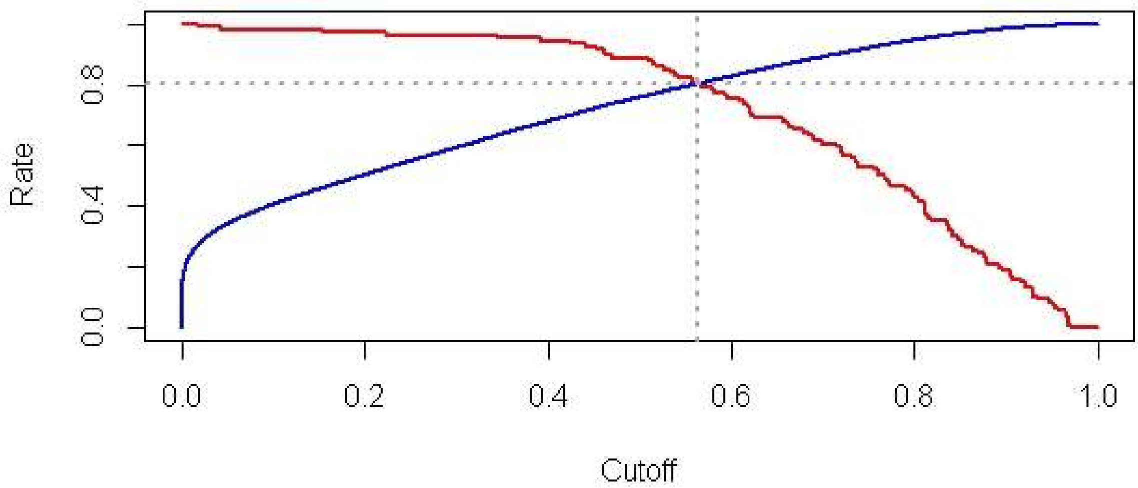
A. One year prediction model



B. Two years prediction model



C. Three years prediction model



correctly predicted negative cases (out of all negatives). Curves in Figure 10 could be then viewed as the dynamic representation of confusion matrix from Figure 1, including both types of error.

We will begin with the analysis of specificity curve. It is evident from the first graph (one-year horizon), that we are able to accurately select about 40% companies that will not go bankrupt within 1 year by setting the cutoff threshold very low. This stems from the fact, that low cutoff brings about almost 40% true negative rate, and at the same time all positives are correctly predicted (no false negatives are made). See, that although this low-cutoff success rate diminishes with increasing time horizon, it persists also in the three years prediction horizon. This can potentially be an extremely useful piece of information. For example, when there is just a limited amount of resources for testing the whole population with a precise but expensive method (like in the case of cancer for example), the technique of inexpensive pre-selection could lower the total costs significantly.

On the other hand, sensitivity deals with the positive observations. With low cut-off, the rate of correctly predicted positive cases is high. Furthermore, it is negatively related to the cut-off threshold yielding a downward-slopping curve. Also, in the one-year horizon, the sensitivity curve is quite flat, and therefore a relatively high accuracy can be retained even at higher cut-off rates. Specifically, 80% true positive rate can be sustained with approximately 0.8 cut-off. For longer horizons this property deteriorates, since the sensitivity curves start to decrease at lower cut-offs. Clearly, the understanding of the sensitivity curve´s behavior at various cut-off levels brings a very important insight into the classifier´s features.

The previous analysis has shown how the knowledge of the sensitivity and specificity curves´ curvature could be used for the classifier´s optimization. As the sensitivity and specificity curves can be viewed as the dynamic representation of type I and type II errors, the specificity and sensitivity plots can be used to manipulate the test in order to achieve some specific objectives (such as to use the classifier for a low-cost preselection). Although some authors discuss the need for more comprehensive measurement and evaluation of classifiers' characteristics (see for example Ohlson (1980) or Klepáč and Hampel (2016)), many authors stick to the ROC metric and neglect other evaluation methods. We believe that more attention should be paid to the analysis of the classifiers´ characteristics as they are so important for their practical use.

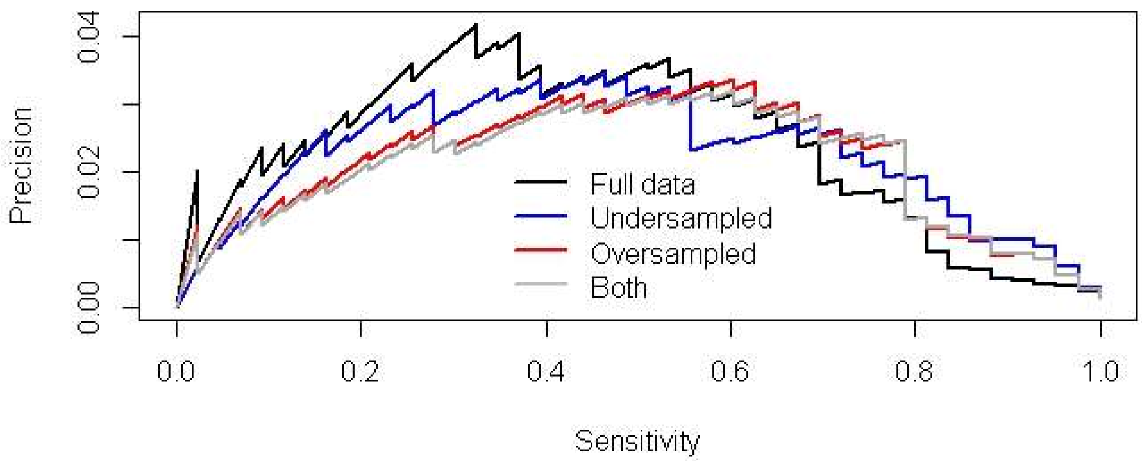
6.3.3 Precision-Recall curves

The ROC methodology is used by many authors as the principal and often the only measure to assess their model´s quality. However, as discussed in the previous section, there are other dimensions of the accuracy assessment that are often neglected, but are very important for putting the model into practice. One of these dimensions is precision, which quantifies how many tries were necessary to get one hit. Clearly, this information is vitally important for a real life application. Also, even though the ROC curves are not sensitive to class imbalances, they can be relatively misguiding in the case of seriously unbalanced datasets, which is frequently the case in bankruptcy prediction models. (David and Goadrich 2006)

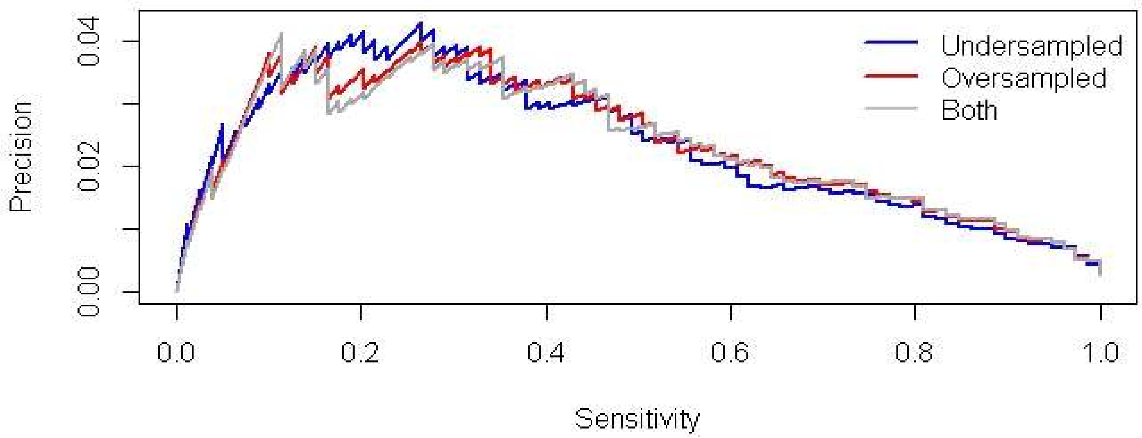
Figure 11 shows the plotted precision-recall curves for the standard ML estimates for all three prediction time horizons. The rate of prevision is on the y-axis and recall (another name for sensitivity) is on the x-axis. It can be seen that the precision rate rarely exceeds 4%, corresponding to the point where 25 shots are needed for a hit. This is a rather low precision rate indeed. However, although almost nobody uses explicitly the precision metric, those who do, get usually relatively comparable results. Taavi (2014) evaluated models precision using 12 months horizon, and 1 year in advance the precision was far below 10%. Kalouda and Vaníček (2013) did not use precision metric directly, but provided data from which it could be calculated. The precision of any of their four models did not exceed 3%. Therefore, we think that the low precision could potentially be a common characteristic of the bankruptcy prediction models in general.

Figure 11: Precision-Sensitivity curves for standard MLE estimates

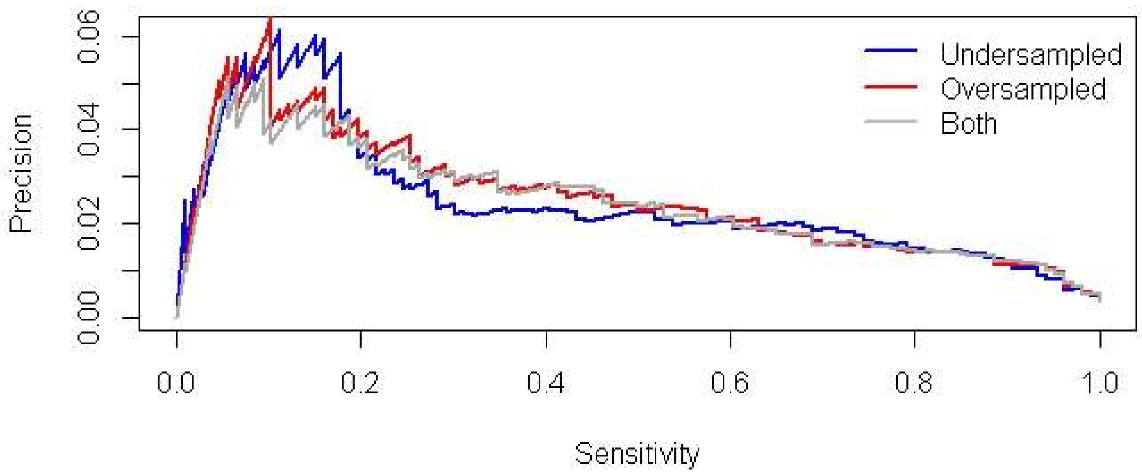
A. One year prediction model



B. Two years prediction model



C. Three years prediction model



Note: "Full data" model is not included in the last two graphs, since the curve had very different trajectory

Importantly, this by no means implies that these models are useless. It is just their feature, which should be definitely taken into account, when applying these classifiers in practice. For example, these models could be employed to select a rough sample of companies in potential risk of bankruptcy (for some predetermined level of sensitivity). The resulting sample would be of significantly lower size and some more accurate discrimination method (likely requiring new data, inputs, time etc.) could be used on this new dataset afterwards. In other words, likewise with the cancer tests, more time demanding and resources intensive methods could be utilized after the initial low-cost screening is done.

7 Conclusion

The main goal of our analysis was to specify a new bankruptcy model for the Czech economy using up-to-date dataset. We wanted to use new methods and test their viability in the context of the Czech economy, so that they could be potentially employed by other researchers, too. Moreover, we wanted to examine which financial ratios are of the highest relevancy for the purposes of bankruptcy prediction.

Our classifier attains a very solid accuracy in comparison with other classifiers estimated on the data for Czech companies. The accuracy has been tested on a secondary sample consisting of 20% of our original data, and our model reached AUC of 0.95, 0.91, and 0.88, for one, two, and three years prediction horizons in 2002—2016 period, respectively. In one-year horizon we were able to achieve 80% true positive rate with only 5% false positive rate. 95% true positive rate required approximately 20% false positive rate. Furthermore, at the 50% false positive rate all bankrupt companies were correctly assigned to bankrupt group. The accuracy of our model deteriorated with the expansion of the prediction horizons, so to reach 80% true positive rate in three years horizon, 20% false positive rate was needed. However, it seems that the model is able to discriminate between bankrupt and non-bankrupt companies even for longer prediction horizons.

Bayesian model averaging has proved to work well in our analysis. We were able to select 20 explanatory variables out of the set of 45 variables. Moreover, these 20 variables were, with marginal exceptions, statistically significant at very low significance levels.

For all three prediction horizons, estimation periods, and estimators, rebalanced train samples improved model's predictive accuracy. Hence we conclude that rebalancing can be an effective technique for the treatment of highly skewed datasets. Additionally, standard MLE and BY estimators provided very similar estimates of regression parameters. Since BY is an outliers-robust method and we employed winsorization to deal with outliers, we arrived to the conclusion that winsorization is an effective way to deal with outlying values.

Furthermore, we assessed the statistical and economic significance of individual variables and defined two sets of top-performing financial ratios, one for a short-term (up to two years) and second for a long-term prediction horizons. The Tier 1 predictors for short-term horizons are Cash ratio, Current ratio, and Cash Flow II ratio. RE to Assets, Interest Coverage, and Gross Profit margin were put in the group of Tier 2 predictors. In the long term, Current ratio and Cash Flow II ratio preserve their predictive abilities, but Cash ratio's significance deteriorates. Other Tier 2 ratios for long-term predictions are Financial Leverage, Interest Coverage, and Gross Profit margin.

In addition to the financial ratios, we have got some interesting results for some dummy variables, too. For example, companies operating in the construction have more than 2 times higher odds of bankruptcy, and large companies are less susceptible to bankruptcies. On the contrary, joint-stock companies are more likely to bankrupt compared to other legal forms. Interestingly, the larger the number of pages of the incorporation documentation, the less likely the firm's bankruptcy is. This variable is hard to interpret, but it might be a proxy for unobservable characteristics, like sophistication of owners, management, complexity of organization structure, or company's size. We have not, on the other hand, found any supportive evidence for the positive effect of company's age on the odds of going bankrupt.

With regard to our hypotheses, we were not able to reject the hypothesis that BMA is an effective instrument for the variable selection in the Czech context. Moreover, it seems that dummy variables are indeed useful for capturing firm-specific characteristics, and thus should be used in bankruptcy prediction models. Lastly, it seems that the predictive ability of our model persists also for longer prediction horizons than two years, which contradicts the findings of Altman (1968).

To conclude, we aware of the fact that our dataset was far from perfect. For instance, we were forced to omit closed and liquidated businesses. Furthermore, we were unable to collect sufficient number of bankrupt companies for the 2002—2007 period to compare pre- and post-2008 periods. Therefore, in order to enhance the generality of our model, it would be desirable to re-apply the proposed methodology on a more complete dataset.

Moreover, BMA offers much broader opportunities than we could use in our research. It could be interesting to use BMA for the selection of explanatory variables in each prediction horizon separately, to track the changes in variables' predictive power in time. Also, a larger number of variables could be included in the original dataset. Besides the financial ratios, there is definitely room for more dummies and other variables, such as macroeconomic conditions, industry specific variables, etc.

Furthermore, it could be as much intriguing to analyze the impact of synthetic rebalancing methods like SMOTE or ROSE on the classifiers' accuracy, and compare it with the standard ones.

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Appendices

Appendix 1: Financial ratios - descriptive statistics for non-bankrupt firms

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
Current.Ratio	146,530	2.347	1.545	-9.357	1.439	1.806	2.773	17.204
Current.Ratio.B	146,530	1.730	1.196	-6.809	1.077	1.316	1.995	10.569
Quick.Ratio	146,530	1.615	1.111	-8.412	0.981	1.270	1.932	10.883
Quick.Ratio.B	146,530	1.240	0.953	-6.265	0.689	0.961	1.471	9.584
Cash.Ratio	146,530	0.446	0.569	-2.947	0.117	0.225	0.533	4.453
Cash.Ratio.B	146,530	0.358	0.500	-2.304	0.079	0.157	0.411	2.915
Total.Debt.to.Equity	146,530	1.581	2.954	-12.024	0.483	0.812	1.957	13.909
Debt.to.Equity	146,530	2.073	3.449	-17.139	0.691	1.173	2.636	19.911
Financial.leverage	146,530	3.108	3.474	-16.599	1.710	2.206	3.693	21.491
Debt.payback.period	146,530	4.867	7.270	0.000	1.632	2.514	5.007	59.542
Interest.coverage	146,530	138.258	201.068	-566.883	7.000	19.093	251.569	604.968
CashFlow.I	146,530	0.297	0.359	-1.928	0.113	0.175	0.367	2.828
CashFlow.II	146,530	0.311	0.371	-1.987	0.116	0.180	0.393	2.987
R.E.to.Assets	146,530	0.134	0.269	-2.889	0.042	0.101	0.259	2.806
GP.margin	146,530	0.217	0.147	-2.191	0.131	0.191	0.283	1.000
Operating.margin	146,530	0.044	0.072	-0.539	0.017	0.034	0.069	0.639
NP.margin.I	146,530	0.027	0.063	-0.423	0.007	0.021	0.050	0.501
NP.margin.II	146,530	0.027	0.063	-0.427	0.007	0.021	0.050	0.500
NI.ROE	146,530	0.154	0.314	-2.775	0.051	0.123	0.243	2.935
NI.ROA	146,530	0.050	0.086	-0.638	0.016	0.036	0.082	0.730
EBIT.ROA	146,530	0.077	0.101	-0.661	0.033	0.058	0.114	0.946
AR.days	146,530	80.537	76.870	0.000	46.560	63.275	87.706	776.802
Inventory.days	146,530	148.607	230.259	0.000	27.178	51.226	154.357	1,343.635
AP.days	146,530	346.196	464.210	0.000	109.628	170.639	356.015	2,913.256
Asset.turnover	146,530	2.126	1.599	-8.657	1.335	1.702	2.446	21.569

Appendix 2: Financial ratios - descriptive statistics for bankrupt firms 1Y in advance

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
Current.Ratio	253	1.179	0.484	0.163	0.944	1.155	1.273	4.753
Current.Ratio.B	253	0.883	0.282	0.162	0.739	0.920	0.986	2.781
Quick.Ratio	253	0.798	0.322	0.046	0.618	0.807	0.897	2.831
Quick.Ratio.B	253	0.591	0.243	0.039	0.451	0.597	0.680	1.909
Cash.Ratio	253	0.065	0.066	-0.022	0.030	0.058	0.070	0.493
Cash.Ratio.B	253	0.048	0.055	-0.011	0.022	0.042	0.050	0.447
Total.Debt.to.Equity	253	1.855	8.017	-33.786	-0.022	1.720	2.637	39.988
Debt.to.Equity	253	2.562	9.489	-39.572	0.200	2.350	3.773	42.080
Financial.leverage	253	3.584	9.547	-38.900	1.040	3.388	4.810	43.093
Debt.payback.period	253	7.458	13.111	0.000	2.156	4.083	6.434	96.057
Interest.coverage	253	6.152	56.035	-174.945	-4.629	1.989	5.188	184.033
CashFlow.I	253	0.006	0.121	-0.393	-0.059	0.022	0.057	0.780
CashFlow.II	253	0.005	0.123	-0.426	-0.059	0.022	0.058	0.780
R.E.to.Assets	253	-0.074	0.259	-1.239	-0.104	0.007	0.046	0.369
GP.margin	253	0.122	0.098	-0.183	0.073	0.121	0.155	0.590
Operating.margin	253	-0.047	0.081	-0.285	-0.099	-0.024	0.009	0.197
NP.margin.I	253	-0.063	0.081	-0.279	-0.120	-0.038	0.001	0.189
NP.margin.II	253	-0.063	0.082	-0.279	-0.121	-0.036	0.001	0.189
NI.ROE	253	0.230	0.957	-3.530	-0.088	0.114	0.409	3.557
NI.ROA	253	-0.079	0.130	-0.497	-0.132	-0.031	0.005	0.243
EBIT.ROA	253	-0.054	0.129	-0.461	-0.113	-0.012	0.025	0.257
AR.days	253	107.979	93.649	6.226	62.604	76.148	117.603	602.735
Inventory.days	253	102.579	154.576	0.000	35.890	52.211	85.329	926.775
AP.days	253	421.591	487.570	36.240	189.019	233.443	395.868	2,607.345
Asset.turnover	253	2.050	1.400	0.360	1.452	1.750	2.160	11.455

Appendix 3: Financial ratios - descriptive statistics for bankrupt firms 2Y in advance

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
Current.Ratio	508	1.398	0.757	0.119	1.052	1.210	1.511	6.196
Current.Ratio.B	508	0.974	0.318	0.082	0.820	0.969	1.076	2.650
Quick.Ratio	508	0.942	0.594	0.009	0.677	0.842	1.026	4.824
Quick.Ratio.B	508	0.647	0.279	0.007	0.496	0.630	0.779	1.916
Cash.Ratio	508	0.084	0.114	-0.333	0.030	0.061	0.095	1.056
Cash.Ratio.B	508	0.061	0.091	-0.304	0.020	0.043	0.065	0.849
Total.Debt.to.Equity	508	3.872	9.360	-35.658	1.054	2.096	5.141	45.856
Debt.to.Equity	508	5.045	11.061	-50.820	1.693	2.934	6.998	53.179
Financial.leverage	508	6.090	11.123	-50.705	2.732	4.000	8.012	54.867
Debt.payback.period	508	10.316	15.872	0.000	3.167	4.623	10.758	94.779
Interest.coverage	508	13.718	54.691	-174.225	0.126	3.573	6.837	182.763
CashFlow.I	508	0.044	0.130	-0.599	0.006	0.049	0.074	1.023
CashFlow.II	508	0.045	0.131	-0.604	0.007	0.049	0.076	1.024
R.E.to.Assets	508	-0.031	0.277	-1.877	-0.034	0.010	0.069	0.608
GP.margin	508	0.139	0.108	-0.467	0.079	0.126	0.179	0.693
Operating.margin	508	-0.011	0.069	-0.277	-0.028	0.007	0.020	0.342
NP.margin.I	508	-0.026	0.072	-0.288	-0.046	0.001	0.007	0.330
NP.margin.II	508	-0.026	0.072	-0.288	-0.046	0.0002	0.007	0.328
NI.ROE	508	0.001	0.902	-3.679	-0.048	0.087	0.232	3.562
NI.ROA	508	-0.029	0.111	-0.536	-0.049	0.002	0.014	0.405
EBIT.ROA	508	-0.004	0.113	-0.529	-0.026	0.019	0.037	0.474
AR.days	508	101.125	87.583	6.505	57.989	75.441	114.614	607.118
Inventory.days	508	137.325	191.986	0.000	37.071	59.033	146.469	927.375
AP.days	508	408.020	463.620	18.822	174.174	218.608	407.011	2,605.504
Asset.turnover	508	2.217	1.784	0.258	1.395	1.769	2.373	15.272

Appendix 4: Financial ratios - descriptive statistics for bankrupt firms 3Y in advance

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
Current.Ratio	659	1.424	0.704	0.023	1.084	1.228	1.586	6.453
Current.Ratio.B	659	1.005	0.348	0.071	0.865	0.981	1.122	3.392
Quick.Ratio	659	0.943	0.520	-0.017	0.698	0.863	1.070	3.991
Quick.Ratio.B	659	0.663	0.309	-0.013	0.491	0.642	0.786	3.377
Cash.Ratio	659	0.098	0.134	-0.349	0.029	0.063	0.111	1.214
Cash.Ratio.B	659	0.069	0.097	-0.319	0.021	0.045	0.080	0.710
Total.Debt.to.Equity	659	4.697	9.358	-35.948	1.109	2.358	5.326	45.651
Debt.to.Equity	659	6.123	11.155	-38.593	1.786	3.289	7.296	55.086
Financial.leverage	659	7.182	11.208	-37.555	2.767	4.319	8.439	56.587
Debt.payback.period	659	8.445	11.096	0.000	3.127	4.932	9.323	96.388
Interest.coverage	659	19.937	58.438	-172.705	1.372	3.994	9.997	188.893
CashFlow.I	659	0.056	0.121	-0.655	0.019	0.055	0.092	0.803
CashFlow.II	659	0.057	0.123	-0.668	0.019	0.056	0.093	0.804
R.E.to.Assets	659	-0.003	0.250	-1.838	-0.016	0.012	0.075	0.583
GP.margin	659	0.140	0.107	-0.358	0.081	0.128	0.183	0.721
Operating.margin	659	-0.0003	0.065	-0.386	-0.007	0.012	0.024	0.364
NP.margin.I	659	-0.015	0.064	-0.329	-0.019	0.003	0.009	0.278
NP.margin.II	659	-0.015	0.064	-0.329	-0.019	0.002	0.010	0.278
NI.ROE	659	0.082	0.736	-3.471	0.0005	0.097	0.239	3.592
NI.ROA	659	-0.013	0.087	-0.511	-0.027	0.005	0.019	0.411
EBIT.ROA	659	0.012	0.092	-0.471	-0.008	0.025	0.047	0.461
AR.days	659	95.072	77.358	0.000	57.525	75.226	109.103	644.569
Inventory.days	659	138.808	196.477	0.000	35.265	58.734	145.365	932.576
AP.days	659	381.312	452.571	18.672	155.457	212.969	348.355	2,655.174
Asset.turnover	659	2.249	1.757	0.358	1.415	1.786	2.443	15.486

Appendix 5: Correlation matrix for financial ratios (Part 1/2)

Correlation matrix	Current.Ratio	Current.Ratio.B	Quick.Ratio	Quick.Ratio.B	Cash.Ratio	Cash.Ratio.B	Total.D/Equity	Debt.to.Equity	Fin.leverage	Debt.payback.period	Interest.coverage
Current.Ratio	1.00	0.76	0.84	0.62	0.56	0.45	-0.19	-0.17	-0.17	-0.20	0.03
Current.Ratio.B	0.76	1.00	0.74	0.88	0.58	0.66	-0.15	-0.19	-0.19	-0.15	0.23
Quick.Ratio	0.84	0.74	1.00	0.83	0.67	0.59	-0.17	-0.17	-0.17	-0.19	0.14
Quick.Ratio.B	0.62	0.88	0.83	1.00	0.64	0.73	-0.14	-0.17	-0.17	-0.14	0.30
Cash.Ratio	0.56	0.58	0.67	0.64	1.00	0.92	-0.15	-0.16	-0.16	-0.17	0.19
Cash.Ratio.B	0.45	0.66	0.59	0.73	0.92	1.00	-0.13	-0.16	-0.16	-0.14	0.27
Total.D/Equity	-0.19	-0.15	-0.17	-0.14	-0.15	-0.13	1.00	0.97	0.96	0.19	-0.09
Debt.to.Equity	-0.17	-0.19	-0.17	-0.17	-0.16	-0.16	0.97	1.00	1.00	0.17	-0.13
Fin.leverage	-0.17	-0.19	-0.17	-0.17	-0.16	-0.16	0.96	1.00	1.00	0.17	-0.12
Debt.pay.per	-0.20	-0.15	-0.19	-0.14	-0.17	-0.14	0.19	0.17	0.17	1.00	-0.11
Int.coverage	0.03	0.23	0.14	0.30	0.19	0.27	-0.09	-0.13	-0.12	-0.11	1.00
CashFlow.I	0.34	0.44	0.39	0.44	0.38	0.40	-0.15	-0.17	-0.17	-0.25	0.29
CashFlow.II	0.34	0.44	0.39	0.45	0.38	0.41	-0.16	-0.17	-0.17	-0.26	0.29
R.E.to.Assets	0.26	0.28	0.26	0.26	0.21	0.21	-0.07	-0.08	-0.08	-0.10	0.12
GP.margin	0.12	0.09	0.16	0.13	0.18	0.14	-0.11	-0.11	-0.11	-0.15	0.06
Oper.margin	0.15	0.13	0.18	0.15	0.18	0.15	-0.04	-0.05	-0.04	-0.16	0.17
NP.margin.I	0.16	0.16	0.19	0.18	0.19	0.17	-0.03	-0.04	-0.04	-0.13	0.20
NP.margin.II	0.16	0.16	0.19	0.18	0.19	0.17	-0.03	-0.04	-0.04	-0.13	0.20
NI.ROE	-0.05	-0.02	-0.01	0.01	0.01	0.01	-0.05	-0.08	-0.08	-0.09	0.11
NI.ROA	0.11	0.16	0.16	0.19	0.16	0.17	-0.02	-0.04	-0.04	-0.16	0.27
EBIT.ROA	0.09	0.14	0.14	0.17	0.14	0.15	-0.02	-0.04	-0.04	-0.19	0.26

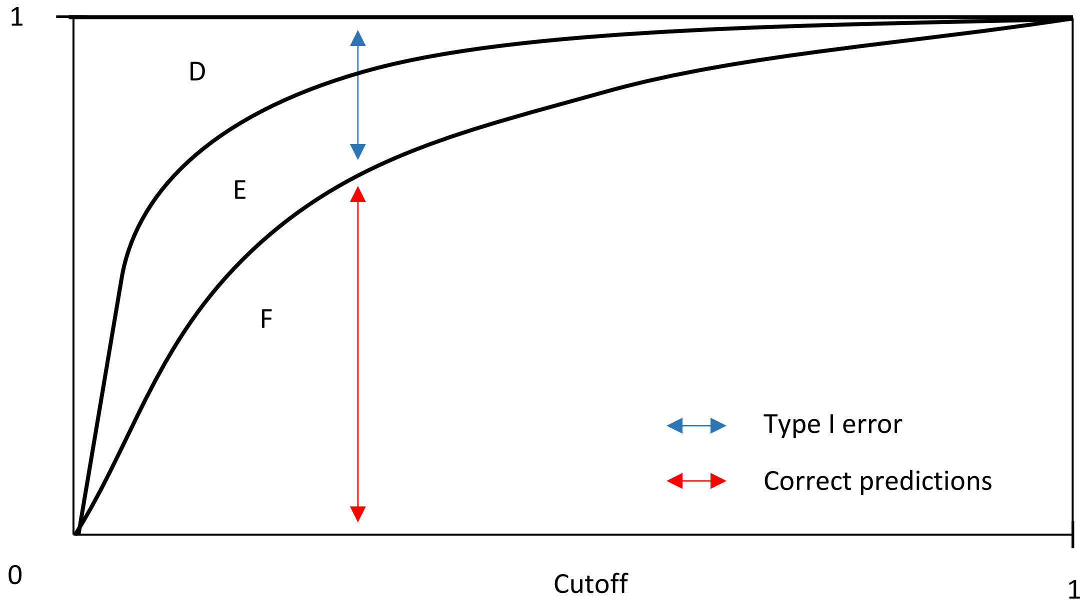
Appendix 6: Correlation matrix for financial ratios (Part 2/2)

Correlation matrix	CashFlow.I	CashFlow.II	R.E.to.Assets	GP.margin	Oper.margin	NP.margin.I	NP.margin.II	NI.ROE	NI.ROA	EBIT.ROA
Current.Ratio	0.34	0.34	0.26	0.12	0.15	0.16	0.16	-0.05	0.11	0.09
Current.Ratio.B	0.44	0.44	0.28	0.09	0.13	0.16	0.16	-0.02	0.16	0.14
Quick.Ratio	0.39	0.39	0.26	0.16	0.18	0.19	0.19	-0.01	0.16	0.14
Quick.Ratio.B	0.44	0.45	0.26	0.13	0.15	0.18	0.18	0.01	0.19	0.17
Cash.Ratio	0.38	0.38	0.21	0.18	0.18	0.19	0.19	0.01	0.16	0.14
Cash.Ratio.B	0.40	0.41	0.21	0.14	0.15	0.17	0.17	0.01	0.17	0.15
Total.D/Equity	-0.15	-0.16	-0.07	-0.11	-0.04	-0.03	-0.03	-0.05	-0.02	-0.02
Debt.to.Equity	-0.17	-0.17	-0.08	-0.11	-0.05	-0.04	-0.04	-0.08	-0.04	-0.04
Fin.leverage	-0.17	-0.17	-0.08	-0.11	-0.04	-0.04	-0.04	-0.08	-0.04	-0.04
Debt.pay.per	-0.25	-0.26	-0.10	-0.15	-0.16	-0.13	-0.13	-0.09	-0.16	-0.19
Int.coverage	0.29	0.29	0.12	0.06	0.17	0.20	0.20	0.11	0.27	0.26
CashFlow.I	1.00	0.99	0.22	0.30	0.54	0.53	0.53	0.20	0.63	0.65
CashFlow.II	0.99	1.00	0.22	0.30	0.55	0.53	0.53	0.20	0.63	0.65
R.E.to.Assets	0.22	0.22	1.00	0.07	0.14	0.16	0.16	-0.03	0.17	0.15
GP.margin	0.30	0.30	0.07	1.00	0.37	0.33	0.32	0.03	0.21	0.21
Oper.margin	0.54	0.55	0.14	0.37	1.00	0.95	0.95	0.26	0.67	0.68
NP.margin.I	0.53	0.53	0.16	0.33	0.95	1.00	1.00	0.27	0.70	0.66
NP.margin.II	0.53	0.53	0.16	0.32	0.95	1.00	1.00	0.27	0.71	0.66
NI.ROE	0.20	0.20	-0.03	0.03	0.26	0.27	0.27	1.00	0.39	0.40
NI.ROA	0.63	0.63	0.17	0.21	0.67	0.70	0.71	0.39	1.00	0.96
EBIT.ROA	0.65	0.65	0.15	0.21	0.68	0.66	0.66	0.40	0.96	1.00

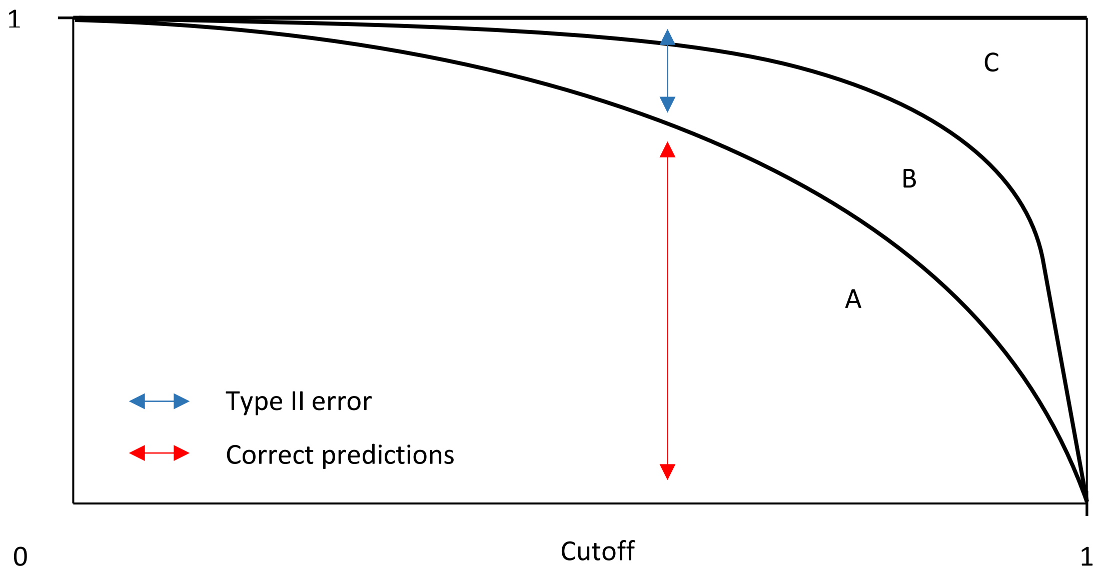
Appendix 7: No. of observations in original and rebalanced datasets

	2Y in advance		3Y in advance		1Y in advance	
Full dataset	0	1	0	1	0	1
Full	146,497	510	146,497	658	146,497	253
Over	117,175	117,374	117,172	117,370	117,190	117,395
Under	419	429	532	552	196	210
Both	29,951	30,049	29,951	30,049	29,951	30,049
2002–2007	0	1	0	1	0	1
Full	53,753	97	53,753	183	53,753	33
Over	42,998	43,057	42,991	43,054	42,999	43,058
Under	80	82	145	157	29	29
Both	29,951	30,049	29,951	30,049	29,951	30,049
2008–2016	0	1	0	1	0	1
Full	92,744	411	92,744	475	92,744	220
Over	74,173	74,160	74,178	74,165	74,180	74,167
Under	342	351	385	397	175	191
Both	29,951	30,049	29,951	30,049	29,951	30,049

Appendix 8: Examples of sensitivity curves



Appendix 9: Examples of specificity curves



Appendix 10: Logit model estimation for 2008–2015 (1Y)

	Dependent variable: Bankrupt (1Y)							
	Full data MLE	Over MLE	Under MLE	Both MLE	Full data BY	Over BY	Under BY	Both BY
Constant	-5.4***	2.9***	4.4***	2.8***	-5.4***	3.6***	5.2***	3.5***
	(0.2)	(0.04)	(0.9)	(0.1)	(1.4)	(0.1)	(1.1)	(0.1)
Cash.Ratio.B	-1.1***	-5.6***	-8.4***	-5.7***	-1.1***	-6.9***	-10.0**	-6.9***
	(0.3)	(0.1)	(2.6)	(0.2)	(0.3)	(0.2)	(4.1)	(0.3)
Financial.leverage	0.1***	0.02***	0.02	0.02***	0.1	0.02***	0.02	0.02***
	(0.02)	(0.001)	(0.02)	(0.002)	(0.3)	(0.001)	(0.02)	(0.002)
Debt.payback.period	0.02***	0.004***	0.01	0.002*	0.02	0.005***	0.01	0.003*
	(0.01)	(0.001)	(0.02)	(0.001)	(0.02)	(0.001)	(0.03)	(0.001)
Interest.coverage	-0.003***	-0.003***	-0.004	-0.003***	-0.003***	-0.004***	-0.004	-0.004***
	(0.001)	(0.0001)	(0.002)	(0.0001)	(0.001)	(0.0001)	(0.003)	(0.0001)
CashFlow.II	-1.4***	-5.3***	-4.3***	-5.6***	-1.4***	-6.5***	-5.1***	-6.9***
	(0.3)	(0.1)	(1.6)	(0.2)	(0.2)	(0.2)	(1.8)	(0.4)
R.E.to.Assets	-0.7***	-1.3***	-0.5	-1.3***	-0.7***	-1.6***	-0.6	-1.5***
	(0.2)	(0.04)	(0.7)	(0.1)	(0.3)	(0.1)	(1.1)	(0.2)
Incor.pages.Many	-0.6**	-1.1***	-0.4	-1.1***	-0.6**	-1.3***	-0.5	-1.3***
	(0.2)	(0.03)	(0.6)	(0.04)	(0.2)	(0.03)	(0.4)	(0.04)
ISO.9001		0.1***	-0.3	0.1***		0.2***	-0.3	0.2***
		(0.02)	(0.4)	(0.03)		(0.02)	(0.6)	(0.03)
PRG		-0.3***	-0.2	-0.3***		-0.3***	-0.2	-0.3***
		(0.02)	(0.4)	(0.03)		(0.02)	(0.6)	(0.04)
Construction	1.0***	0.6***	0.9*	0.7***	1.0***	0.8***	1.1*	0.8***
	(0.2)	(0.03)	(0.5)	(0.04)	(0.3)	(0.03)	(0.6)	(0.05)
Inventory.days		-0.001***	-0.002**	-0.001***		-0.002***	-0.002**	-0.002***
		(0.0000)	(0.001)	(0.0001)		(0.0000)	(0.001)	(0.0001)
Size.Large		-0.2***	-0.2	-0.1**		-0.3***	-0.2	-0.2***
		(0.04)	(0.9)	(0.1)		(0.04)	(0.6)	(0.1)
AR.days	0.003***	0.003***	0.002	0.003***	0.003*	0.004***	0.003	0.004***
	(0.001)	(0.0001)	(0.002)	(0.0002)	(0.001)	(0.0002)	(0.002)	(0.0003)
Asset.turnover		0.1***	-0.1	0.1***		0.1***	-0.1	0.1***
		(0.01)	(0.1)	(0.01)		(0.01)	(0.2)	(0.01)
Joint_Stock		0.5***	0.2	0.5***		0.6***	0.2	0.6***
		(0.02)	(0.4)	(0.03)		(0.02)	(0.4)	(0.03)
Incor.pages.Moderate		-0.5***	-0.7	-0.5***		-0.6***	-0.8	-0.6***
		(0.02)	(0.4)	(0.03)		(0.02)	(0.7)	(0.04)
Processing	0.7***	0.1***	0.1	0.1***	0.7***	0.1***	0.1	0.1***
	(0.2)	(0.02)	(0.5)	(0.03)	(0.2)	(0.02)	(0.5)	(0.04)
GP.margin	-1.5***	-3.5***	-5.2***	-3.1***	-1.5***	-4.2***	-6.1**	-3.8***
	(0.3)	(0.1)	(2.0)	(0.1)	(0.3)	(0.2)	(3.1)	(0.3)
NP.margin.II	-8.1***	-5.4***	-0.7	-5.4***	-8.1***	-6.5***	-0.8	-6.6***
	(0.8)	(0.2)	(3.0)	(0.2)	(3.0)	(0.2)	(2.9)	(0.3)
NI.ROA		0.5***	-3.5	0.5**		0.6***	-4.2	0.6**
		(0.1)	(2.9)	(0.2)		(0.2)	(3.8)	(0.3)
Current.Ratio.B	-0.6***	-1.4***	-1.8***	-1.4***	-0.6***	-1.8***	-2.1**	-1.7***
	(0.1)	(0.03)	(0.6)	(0.04)	(0.1)	(0.03)	(0.9)	(0.1)
Observations	*74,371*	*148,347*	*366*	*60,000*	*74,371*	*148,347*	*366*	*60,000*
Log Likelihood	*-1,073.5*	*-50,148.4*	*-119.3*	*-20,238.2*				
Akaike Inf. Crit.	*2,175.1*	*100,340.8*	*282.6*	*40,520.5*				
Note:								*p<0.1; **p<0.05; ***p<0.01

Appendix 11: Logit model estimation for 2008–2016 (2Y)

	Dependent variable: Bankrupt (2Y)							
	Full data MLE	Over MLE	Under MLE	Both MLE	Full data BY	Over BY	Under BY	Both BY
Constant	-4.9***	1.8***	2.2***	1.8***	-4.9***	2.0***	2.4**	2.0***
	(0.1)	(0.03)	(0.5)	(0.1)	(0.2)	(0.04)	(0.9)	(0.1)
Cash.Ratio.B	-1.2***	-3.8***	-3.6***	-3.7***	-1.2***	-4.1***	-3.9**	-4.0***
	(0.3)	(0.1)	(1.1)	(0.1)	(0.1)	(0.1)	(1.8)	(0.2)
Financial.leverage	0.1***	0.02***	0.01	0.02***	0.1***	0.02***	0.01	0.02***
	(0.01)	(0.001)	(0.01)	(0.001)	(0.03)	(0.001)	(0.01)	(0.001)
Debt.payback.period	0.03***	0.02***	0.02**	0.03***	0.03***	0.03***	0.02*	0.03***
	(0.004)	(0.001)	(0.01)	(0.001)	(0.01)	(0.001)	(0.01)	(0.001)
Interest.coverage	-0.003***	-0.004***	-0.005***	-0.004***	-0.003***	-0.004***	-0.01***	-0.004***
	(0.001)	(0.0001)	(0.001)	(0.0001)	(0.0004)	(0.0001)	(0.001)	(0.0001)
CashFlow.II	-1.4***	-3.3***	-4.8***	-3.2***	-1.4***	-3.6***	-5.3	-3.5***
	(0.3)	(0.1)	(1.5)	(0.1)	(0.2)	(0.2)	(3.9)	(0.3)
R.E.to.Assets	-0.7***	-1.1***	-1.9***	-1.2***	-0.7***	-1.2***	-2.1***	-1.4***
	(0.1)	(0.03)	(0.5)	(0.1)	(0.1)	(0.1)	(0.6)	(0.1)
Incor.pages.Many	-0.8***	-1.0***	-0.9**	-1.0***	-0.8***	-1.1***	-0.9***	-1.1***
	(0.2)	(0.02)	(0.4)	(0.04)	(0.2)	(0.02)	(0.4)	(0.04)
ISO.9001		0.3***	0.2	0.3***		0.3***	0.3	0.3***
		(0.02)	(0.2)	(0.03)		(0.02)	(0.3)	(0.03)
PRG		-0.3***	-0.4	-0.3***		-0.4***	-0.4	-0.3***
		(0.02)	(0.2)	(0.03)		(0.02)	(0.3)	(0.03)
Construction	1.0***	1.1***	1.0***	1.1***	1.0***	1.2***	1.1***	1.3***
	(0.1)	(0.02)	(0.3)	(0.04)	(0.2)	(0.03)	(0.4)	(0.04)
Inventory.days		-0.0005***	-0.0004	-0.0004***		-0.001***	-0.0004	-0.0005***
		(0.0000)	(0.001)	(0.0001)		(0.0000)	(0.0005)	(0.0000)
Size.Large		-0.4***	-0.3	-0.3***		-0.5***	-0.4	-0.3***
		(0.04)	(0.6)	(0.1)		(0.05)	(0.7)	(0.1)
AR.days	0.002***	0.002***	0.002	0.002***	0.002***	0.002***	0.002	0.002***
	(0.001)	(0.0001)	(0.001)	(0.0001)	(0.001)	(0.0001)	(0.002)	(0.0002)
Asset.turnover		0.2***	0.2**	0.2***		0.2***	0.2	0.2***
		(0.005)	(0.1)	(0.01)		(0.01)	(0.2)	(0.01)
Joint_Stock		0.5***	0.5*	0.4***		0.5***	0.5*	0.5***
		(0.02)	(0.2)	(0.03)		(0.02)	(0.3)	(0.03)
Incor.pages.Moderate		-0.6***	-0.5	-0.5***		-0.6***	-0.5*	-0.6***
		(0.02)	(0.3)	(0.03)		(0.02)	(0.3)	(0.03)
Processing	0.4***	0.1***	0.1	0.1***	0.4***	0.1***	0.1	0.1***
	(0.1)	(0.02)	(0.3)	(0.03)	(0.1)	(0.02)	(0.3)	(0.03)
GP.margin	-1.3***	-2.6***	-2.9***	-2.6***	-1.3***	-2.9***	-3.2	-2.8***
	(0.3)	(0.1)	(1.0)	(0.1)	(0.1)	(0.1)	(2.1)	(0.2)
NP.margin.II	-5.5***	-4.2***	-5.0*	-4.2***	-5.5***	-4.6***	-5.5*	-4.6***
	(0.8)	(0.2)	(2.6)	(0.3)	(0.8)	(0.2)	(3.2)	(0.3)
NI.ROA		2.2***	4.8**	2.2***		2.4***	5.3	2.4***
		(0.2)	(2.4)	(0.2)		(0.2)	(4.1)	(0.4)
Current.Ratio.B	-0.5***	-1.0***	-1.0***	-1.0***	-0.5***	-1.1***	-1.1***	-1.1***
	(0.1)	(0.02)	(0.3)	(0.03)	(0.1)	(0.03)	(0.4)	(0.04)
Observations	*74,524*	*148,333*	*693*	*60,000*	*74,524*	*148,333*	*693*	*60,000*
Log Likelihood	*-1,836.9*	*-60,289.3*	*-272.3*	*-24,490.4*				
Akaike Inf. Crit.	*3,701.9*	*120,622.6*	*588.6*	*49,024.7*				
Note:								*p<0.1; **p<0.05; ***p<0.01

Appendix 12: Logit model estimation for 2002–2007 (2Y)

	Dependent variable: Bankrupt (2Y)							
	Full data MLE	Over MLE	Under MLE	Both MLE	Full data BY	Over BY	Under BY	Both BY
Constant	-6.1***	2.9***	4.0*	3.0***	-6.1***	3.1***	3.7	3.2***
	(0.4)	(0.1)	(2.3)	(0.1)	(0.5)	(0.1)	(2.9)	(0.1)
Cash.Ratio.B	-1.7***	-8.5***	-8.3	-8.9***	-1.7***	-9.2***	-7.7	-9.5***
	(0.5)	(0.1)	(5.1)	(0.2)	(0.3)	(0.2)	(7.5)	(0.2)
Financial.leverage	0.1***	0.03***	0.1	0.03***	0.1	0.03***	0.05	0.03***
	(0.02)	(0.001)	(0.04)	(0.001)	(0.1)	(0.001)	(0.1)	(0.001)
Debt.payback.period	0.03***	0.02***	0.02	0.02***	0.03	0.02***	0.01	0.02***
	(0.01)	(0.001)	(0.04)	(0.001)	(0.03)	(0.002)	(0.1)	(0.002)
Interest.coverage	-0.002**	-0.002***	-0.003	-0.002***	-0.002***	-0.002***	-0.003	-0.002***
	(0.001)	(0.0001)	(0.004)	(0.0001)	(0.001)	(0.0001)	(0.01)	(0.0001)
CashFlow.II	-2.2***	-7.4***	-13.8**	-7.9***	-2.2***	-8.0***	-12.8	-8.4***
	(0.5)	(0.2)	(6.9)	(0.2)	(0.2)	(0.3)	(21.5)	(0.4)
R.E.to.Assets	-1.1***	-1.7***	-11.2***	-1.8***	-1.1***	-1.8***	-10.4	-1.9***
	(0.3)	(0.1)	(4.2)	(0.1)	(0.2)	(0.2)	(6.9)	(0.2)
Incor.pages.Many	-0.9**	-1.5***	-1.6	-1.4***	-0.9*	-1.6***	-1.5	-1.5***
	(0.4)	(0.03)	(1.1)	(0.04)	(0.5)	(0.04)	(1.8)	(0.04)
ISO.9001		-0.04*	-0.7	-0.04		-0.05*	-0.7	-0.05
		(0.02)	(0.8)	(0.03)		(0.03)	(1.4)	(0.03)
PRG		-0.02	0.6	-0.005		-0.03	0.5	-0.005
		(0.03)	(0.8)	(0.03)		(0.03)	(1.4)	(0.03)
Construction	0.7*	0.4***	3.0*	0.4***	0.7	0.4***	2.8	0.4***
	(0.4)	(0.04)	(1.8)	(0.05)	(0.5)	(0.04)	(2.2)	(0.04)
Inventory.days		0.0002***	-0.001	0.0001		0.0002***	-0.001	0.0001
		(0.0001)	(0.001)	(0.0001)		(0.0000)	(0.001)	(0.0001)
Size.Large		0.4***	-0.03	0.4***		0.4***	-0.03	0.5***
		(0.1)	(1.2)	(0.1)		(0.1)	(3.1)	(0.1)
AR.days	0.001	0.0002	-0.0000	0.0003	0.001	0.0003	-0.0000	0.0003
	(0.002)	(0.0002)	(0.01)	(0.0002)	(0.001)	(0.0002)	(0.01)	(0.0002)
Asset.turnover		-0.2***	-0.1	-0.2***		-0.2***	-0.1	-0.2***
		(0.01)	(0.4)	(0.01)		(0.01)	(0.6)	(0.02)
Joint_Stock		0.6***	0.2	0.6***		0.7***	0.2	0.6***
		(0.02)	(0.7)	(0.03)		(0.03)	(1.6)	(0.04)
Incor.pages.Moderate		-1.2***	-0.2	-1.2***		-1.3***	-0.2	-1.3***
		(0.03)	(0.9)	(0.04)		(0.04)	(1.5)	(0.1)
Processing	1.2***	1.4***	2.3**	1.4***	1.2***	1.5***	2.1	1.5***
	(0.3)	(0.03)	(0.9)	(0.03)	(0.3)	(0.04)	(2.1)	(0.05)
GP.margin	-1.8***	-5.1***	-3.9	-5.3***	-1.8***	-5.4***	-3.6	-5.6***
	(0.6)	(0.1)	(5.9)	(0.2)	(0.3)	(0.2)	(15.4)	(0.2)
NP.margin.II	-4.9***	2.9***	-2.9	3.7***	-4.9***	3.2***	-2.7	3.9***
	(1.6)	(0.3)	(18.3)	(0.4)	(1.3)	(0.3)	(18.5)	(0.4)
NI.ROA		-0.1	5.4	0.2		-0.1	5.0	0.2
		(0.3)	(11.3)	(0.3)		(0.4)	(14.8)	(0.4)
Current.Ratio.B	-0.4	-0.5***	-0.6	-0.4***	-0.4***	-0.5***	-0.6	-0.4***
	(0.2)	(0.03)	(1.6)	(0.04)	(0.1)	(0.05)	(3.0)	(0.1)
Observations	*43,080*	*86,055*	*162*	*60,000*	*43,080*	*86,055*	*162*	*60,000*
Log Likelihood	*-488.4*	*-29,370.1*	*-40.0*	*-20,303.3*				
Akaike Inf. Crit.	*1,004.8*	*58,784.2*	*124.0*	*40,650.6*				
Note:								*p<0.1; **p<0.05; ***p<0.01

Appendix 13: Logit model estimation for 2008–2016(3Y)

	Dependent variable: Bankrupt (3Y)							
	Full data MLE	Over MLE	Under MLE	Both MLE	Full data BY	Over BY	Under BY	Both BY
Constant	-4.8^{***}	2.0^{***}	2.6^{***}	2.0^{***}	-4.8^{***}	2.1^{***}	2.7^{***}	2.1^{***}
	(0.1)	(0.03)	(0.5)	(0.1)	(0.1)	(0.05)	(0.6)	(0.1)
Cash.Ratio.B	-1.0^{***}	-2.3^{***}	-3.0^{***}	-2.3^{***}	-1.0^{***}	-2.5^{***}	-3.1^{***}	-2.4^{***}
	(0.3)	(0.1)	(0.9)	(0.1)	(0.1)	(0.1)	(1.2)	(0.1)
Financial.leverage	0.1^{***}	0.1^{***}	0.05^{***}	0.05^{***}	0.1^{***}	0.1^{***}	0.05^{***}	0.1^{***}
	(0.01)	(0.001)	(0.01)	(0.002)	(0.01)	(0.001)	(0.01)	(0.002)
Debt.payback.period	0.01^{***}	0.004^{***}	0.01	0.004^{***}	0.01^{**}	0.004^{***}	0.01	0.004^{***}
	(0.005)	(0.001)	(0.01)	(0.001)	(0.01)	(0.001)	(0.01)	(0.001)
Interest.coverage	-0.003^{***}	-0.004^{***}	-0.004^{***}	-0.004^{***}	-0.003^{***}	-0.004^{***}	-0.004^{***}	-0.004^{***}
	(0.001)	(0.0001)	(0.001)	(0.0001)	(0.0004)	(0.0001)	(0.001)	(0.0001)
CashFlow.II	-1.6^{***}	-3.8^{***}	-2.9^{***}	-4.0^{***}	-1.6^{***}	-4.1^{***}	-3.1	-4.2^{***}
	(0.3)	(0.1)	(1.0)	(0.1)	(0.1)	(0.2)	(2.2)	(0.3)
R.E.to.Assets	-0.7^{***}	-1.1^{***}	-1.6^{***}	-1.1^{***}	-0.7^{***}	-1.1^{***}	-1.7^{***}	-1.1^{***}
	(0.1)	(0.03)	(0.4)	(0.05)	(0.1)	(0.1)	(0.5)	(0.1)
Incor.pages.Many	-1.0^{***}	-1.2^{***}	-1.6^{***}	-1.3^{***}	-1.0^{***}	-1.3^{***}	-1.7^{***}	-1.4^{***}
	(0.2)	(0.02)	(0.3)	(0.04)	(0.3)	(0.02)	(0.3)	(0.04)
ISO.9001		0.2^{***}	0.4^{*}	0.2^{***}		0.2^{***}	0.4^{*}	0.2^{***}
		(0.02)	(0.2)	(0.03)		(0.02)	(0.2)	(0.03)
PRG		-0.03	-0.5^{**}	-0.1^{**}		-0.03^{*}	-0.5^{**}	-0.1^{**}
		(0.02)	(0.2)	(0.03)		(0.02)	(0.2)	(0.03)
Construction	0.9^{***}	1.0^{***}	0.7^{**}	1.0^{***}	0.9^{***}	1.0^{***}	0.7^{*}	1.0^{***}
	(0.1)	(0.02)	(0.3)	(0.03)	(0.1)	(0.03)	(0.4)	(0.04)
Inventory.days		-0.001^{***}	-0.001^{*}	-0.001^{***}		-0.001^{***}	-0.001^{*}	-0.001^{***}
		(0.0000)	(0.0005)	(0.0001)		(0.0000)	(0.0005)	(0.0000)
Size.Large		-0.9^{***}	-0.6	-0.9^{***}		-0.9^{***}	-0.7	-1.0^{***}
		(0.04)	(0.6)	(0.1)		(0.04)	(0.6)	(0.1)
AR.days	0.001^{***}	0.001^{***}	0.001	0.001^{***}	0.001^{***}	0.001^{***}	0.001	0.001^{***}
	(0.001)	(0.0001)	(0.001)	(0.0001)	(0.0004)	(0.0001)	(0.001)	(0.0002)
Asset.turnover		0.1^{***}	0.1	0.1^{***}		0.1^{***}	0.1	0.1^{***}
		(0.004)	(0.1)	(0.01)		(0.01)	(0.1)	(0.01)
Joint_Stock		0.4^{***}	0.5^{**}	0.4^{***}		0.4^{***}	0.6^{**}	0.4^{***}
		(0.02)	(0.2)	(0.03)		(0.02)	(0.2)	(0.03)
Incor.pages.Moderate		-0.4^{***}	-0.4^{*}	-0.4^{***}		-0.4^{***}	-0.4	-0.4^{***}
		(0.02)	(0.2)	(0.03)		(0.02)	(0.3)	(0.03)
Processing	0.4^{***}	0.3^{***}	-0.04	0.4^{***}	0.4^{***}	0.3^{***}	-0.04	0.4^{***}
	(0.1)	(0.02)	(0.2)	(0.03)	(0.1)	(0.02)	(0.3)	(0.03)
GP.margin	-1.4^{***}	-3.3^{***}	-4.9^{***}	-3.4^{***}	-1.4^{***}	-3.5^{***}	-5.1^{***}	-3.6^{***}
	(0.2)	(0.1)	(1.0)	(0.1)	(0.1)	(0.1)	(1.2)	(0.2)
NP.margin.II	-3.9^{***}	-0.6^{***}	-3.1	-0.3	-3.9^{***}	-0.6^{***}	-3.3	-0.4
	(0.8)	(0.2)	(2.4)	(0.3)	(0.7)	(0.2)	(2.4)	(0.3)
NI.ROA		1.1^{***}	2.2	1.1^{***}		1.1^{***}	2.3	1.2^{***}
		(0.1)	(2.1)	(0.2)		(0.2)	(2.5)	(0.3)
Current.Ratio.B	-0.4^{***}	-0.8^{***}	-0.9^{***}	-0.9^{***}	-0.4^{***}	-0.9^{***}	-0.9^{***}	-0.9^{***}
	(0.1)	(0.02)	(0.3)	(0.03)	(0.1)	(0.02)	(0.3)	(0.03)
Observations	*74,575*	*148,343*	*782*	*60,000*	*74,575*	*148,343*	*782*	*60,000*
Log Likelihood	*-2,039.5*	*-63,113.5*	*-336.1*	*-25,419.2*				
Akaike Inf. Crit.	*4,107.0*	*126,271.0*	*716.2*	*50,882.5*				
Note:								*p<0.1; **p<0.05; ***p<0.01

Appendix 14: Logit model estimation for 2002–2007 (3Y)

	Dependent variable: Bankrupt (3Y)							
	Full data MLE	Over MLE	Under MLE	Both MLE	Full data BY	Over BY	Under BY	Both BY
Constant	-5.4^{***}	3.0^{***}	3.1^{***}	3.0^{***}	-5.4^{***}	3.3^{***}	3.3^{***}	3.3^{***}
	(0.3)	(0.1)	(1.0)	(0.1)	(0.3)	(0.1)	(1.2)	(0.1)
Cash.Ratio.B	-1.6^{***}	-7.8^{***}	-6.9^{***}	-7.6^{***}	-1.6^{***}	-8.6^{***}	-7.3^{**}	-8.3^{***}
	(0.4)	(0.1)	(2.3)	(0.2)	(0.2)	(0.2)	(3.2)	(0.2)
Financial.leverage	0.1^{***}	0.1^{***}	0.1^{**}	0.05^{***}	0.1^{***}	0.1^{***}	0.1^{**}	0.1^{***}
	(0.01)	(0.001)	(0.02)	(0.002)	(0.04)	(0.002)	(0.02)	(0.002)
Debt.payback.period	0.02^{**}	-0.002	0.003	-0.002^{*}	0.02	-0.002	0.003	-0.003
	(0.01)	(0.001)	(0.03)	(0.001)	(0.01)	(0.001)	(0.03)	(0.002)
Interest.coverage	-0.003^{***}	-0.003^{***}	-0.005^{**}	-0.003^{***}	-0.003^{***}	-0.003^{***}	-0.005^{***}	-0.003^{***}
	(0.001)	(0.0001)	(0.002)	(0.0001)	(0.001)	(0.0001)	(0.002)	(0.0001)
CashFlow.II	-2.3^{***}	-10.7^{***}	-13.6^{***}	-10.2^{***}	-2.3^{***}	-11.7^{***}	-14.3^{***}	-11.2^{***}
	(0.4)	(0.2)	(3.8)	(0.2)	(0.2)	(0.3)	(5.5)	(0.3)
R.E.to.Assets	-0.9^{***}	-0.8^{***}	-3.6^{**}	-0.8^{***}	-0.9^{***}	-0.9^{***}	-3.8	-0.9^{***}
	(0.2)	(0.05)	(1.7)	(0.1)	(0.1)	(0.1)	(2.6)	(0.1)
Incor.pages.Many	-0.3	-0.7^{***}	-1.6^{***}	-0.8^{***}	-0.3	-0.8^{***}	-1.7^{***}	-0.8^{***}
	(0.2)	(0.03)	(0.5)	(0.04)	(0.3)	(0.03)	(0.6)	(0.04)
ISO.9001		-0.002	0.6	0.01		-0.002	0.6	0.01
		(0.02)	(0.4)	(0.03)		(0.02)	(0.5)	(0.03)
PRG		-0.3^{***}	-0.4	-0.3^{***}		-0.3^{***}	-0.4	-0.3^{***}
		(0.02)	(0.5)	(0.03)		(0.03)	(0.6)	(0.03)
Construction	0.4^{*}	-0.1^{**}	0.01	-0.04	0.4	-0.1^{**}	0.01	-0.04
	(0.3)	(0.04)	(0.7)	(0.04)	(0.4)	(0.03)	(0.8)	(0.04)
Inventory.days		-0.001^{***}	-0.001	-0.001^{***}		-0.001^{***}	-0.001	-0.001^{***}
		(0.0000)	(0.001)	(0.0001)		(0.0000)	(0.001)	(0.0001)
Size.Large		0.3^{***}	0.3	0.3^{***}		0.3^{***}	0.3	0.3^{***}
		(0.05)	(0.9)	(0.1)		(0.1)	(1.3)	(0.1)
AR.days	0.003^{***}	0.001^{***}	0.001	0.001^{***}	0.003^{***}	0.001^{***}	0.001	0.001^{***}
	(0.001)	(0.0002)	(0.003)	(0.0002)	(0.001)	(0.0002)	(0.002)	(0.0002)
Asset.turnover		-0.1^{***}	-0.2	-0.1^{***}		-0.1^{***}	-0.2	-0.2^{***}
		(0.01)	(0.2)	(0.01)		(0.01)	(0.2)	(0.01)
Joint_Stock		1.0^{***}	1.0^{**}	1.0^{***}		1.1^{***}	1.0^{*}	1.1^{***}
		(0.02)	(0.4)	(0.03)		(0.03)	(0.6)	(0.03)
Incor.pages.Moderate		-0.7^{***}	-1.2^{**}	-0.7^{***}		-0.7^{***}	-1.2^{**}	-0.8^{***}
		(0.03)	(0.5)	(0.03)		(0.03)	(0.6)	(0.04)
Processing	0.8^{***}	0.5^{***}	0.5	0.5^{***}	0.8^{***}	0.5^{***}	0.5	0.5^{***}
	(0.2)	(0.03)	(0.5)	(0.03)	(0.2)	(0.03)	(0.4)	(0.03)
GP.margin	-1.5^{***}	-2.2^{***}	-1.6	-2.3^{***}	-1.5^{***}	-2.5^{***}	-1.6	-2.6^{***}
	(0.5)	(0.1)	(2.3)	(0.1)	(0.2)	(0.2)	(2.7)	(0.2)
NP.margin.II	-3.4^{***}	0.7^{**}	-12.5	0.6	-3.4^{***}	0.7^{**}	-13.3^{*}	0.6
	(1.3)	(0.3)	(8.0)	(0.4)	(0.9)	(0.3)	(7.1)	(0.4)
NI.ROA		4.0^{***}	12.0^{**}	3.7^{***}		4.4^{***}	12.6	4.1^{***}
		(0.3)	(5.9)	(0.3)		(0.4)	(8.1)	(0.5)
Current.Ratio.B	-0.4^{**}	-0.5^{***}	0.04	-0.5^{***}	-0.4^{***}	-0.5^{***}	0.04	-0.6^{***}
	(0.2)	(0.03)	(0.5)	(0.04)	(0.1)	(0.04)	(0.6)	(0.05)
Observations	*43,148*	*86,045*	*302*	*60,000*	*43,148*	*86,045*	*302*	*60,000*
Log Likelihood	*-848.7*	*-31,889.8*	*-103.1*	*-22,369.1*				
Akaike Inf. Crit.	*1,725.3*	*63,823.6*	*250.3*	*44,782.2*				

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$