Akademie věd České republiky
Ústav teorie informace a automatizace, v.v.i.
Pod Vodárenskou věží 4, 182 08 Praha 8

Doc. RNDr. Martin Kružík, Ph.D.
e-mail: kruzik@utia.cas.cz
web: http://staff.utia.cas.cz/kruzik/
tel.: 00420-266-052-395

July 31, 2017

Referee report on the doctoral thesis by Mr. V. Kulvait

Mathematical Analysis and Computer Simulations of Deformation of Nonlinear Elastic Bodies in the Small Strain Range

The thesis consists of five Chapters and the Appendix. It nicely combines important aspects of mathematical modeling, namely gathering of existing experimental data, building of a model, its mathematical analysis and computational results. Chapter one introduces the notation, describes the problem and achieved analytical and computational results.

Chapter two is devoted to constitutive models. It starts with a historical survey of mathematical modeling in continuum mechanics. Contrary to Cauchy elasticity where the stress is a function of strain, the thesis deals with models where the strain is a function of stress. Various response functions are considered. In particular, the author studies power-law models with different formulas for deviatopric and isochoric part of the strain tensor and strain-limiting models where the strain remains bounded independently of the applied stress.

Chapter three is very interesting because here the author estimates elastic constants needed for his model from experimental data related to tensile tests on beta phase titanium alloys. Many details are provided about used algorithms to obtain the so-called "best fit".

Chapter four establishes the existence of a weak solution to the boundary value problem. First, V. Kulvait regularizes the original problem by introducing a perturbation by a smoothing operator and applies Galerkin's method to establish the existence of a solution. The proof goes rather standardly by first introducing finite-dimensional subspaces, proving the existence of approximate solutions, and finally by a limit passage he finds a solution to the original regularized problem. The next step is to show that solutions of the regularized problems converge to a solution of the original problem. Also this step is performed in the work.

Chapter five is computational and carefully presents obtained numerical results for beta phase titanium alloys in various geometries of the computational domain.

The thesis is equipped with many auxiliary results put together in the appendix so that it is fairly self-content. It contains just a few typos and is written very clearly and precisely.

I have checked at Google Scholar that two paper co-authored by V. Kulvait on the topic of the thesis which are published in impacted international journals (Int. J. Fracture, Int. J. Appl.

Mech Engrg.) have together about 33 citations which certainly shows his increasing international reputation in applied mathematics.

I have a few suggestions and questions for the author:

- I think it is better to call $\mathcal{E}u$ on p. 7 "symmetrized gradient" rather than "symmetric gradient".
- It is better to say "a weak solution to the problem" rather than "a solution to the weak problem" (Sec. 4.2)
- Your computational domain $\Omega = B \times \mathbb{R}$ is unbounded while in Definition 4.5, the set Ω is a bounded Lipschitz domain. Is it a problem?
- Is there some relationship between strain-limiting models and locking materials as introduced by Prager (Prager, W.: On ideal locking materials. *Transactions of The Society of Rheology*, **1**, (1957), 169–175) and further developed by Ciarlet-Nečas (Ciarlet, P.G., Nečas, J.: Unilateral problems in nonlinear, three-dimensional elasticity. *Arch. Rational Mech. Anal.* **87** (1985), 319–338.), or Demengel-Suquet (Demengel, F., Suquet, P.: On locking materials. *Acta Applicandae Mathematicae* **6** (1986), 185–211.)?
- Would that be possible to extend your models to fully nonlinear setting (i.e., Cauchy-Green strain instead of the symmetrized gradient ofthe displacement) How can one ensure that deformations are orientation-preserving?

I have checked at Google Scholar that two paper co-authored by V. Kulvait on the topic of the thesis which are published in impacted international journals (Int. J. Fracture, Int. J. Appl. Mech Engrg.) have together about 33 citations which certainly shows his increasing international reputation in applied mathematics. In my opinion, V. Kulvait has shown that he is able to produce and publish valuable scientific results and I am happy to recommend his thesis for the defence.

Martin Kružík