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**Application of Premiums and Discounts to
the Company Valuation**

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Prohlášení

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V Praze dne 10.9.2009

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Poděkování

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Abstract

In the light of the current market downturn, the need of the most accurate valuation appears to be more crucial than ever before. This thesis provides the reader with both the theoretical and practical background of the use of valuation premiums and discounts which apply directly to the value of the company reached by conventional separate valuation techniques. The most important premiums and discounts we focus our attention on are control premium/minority interest discount and lack of liquidity discount. The thesis presents an overview of the basic methodology of the theoretical concepts related to the valuation premiums and discounts. Moreover, based on a sample of 202 mergers and acquisitions transactions of the companies listed in the Central and Eastern Europe (“CEE”), we examine the size and key determinants of the control premium applicable within the CEE region.

Abstrakt

Potřeba přesného ocenění se dnes, ve světle ochladnutí světových trhů, zdá snad ještě zásadnější než kdykoliv předtím. Tato práce svému čtenáři předkládá jak teoretické, tak praktické aspekty týkající se prémie a diskontů, které se přímo aplikují na hodnotu společnosti získanou skrze konvenční cesty oceňování. Nejdůležitější premie a diskonty, na které se v naší práci zaměřujeme, jsou kontrolní premie/diskont za minoritní podíl a diskont za nedostatečnou likviditu. Naše práce mimo jiné představuje základní metodologii, která se váže k teoretickým konceptům prémie a diskontů užívaných při oceňování společností. Na vzorku 202 transakcí z oblasti fúzí a akvizic společností kótovaných na burzách střední a východní Evropy zkoumáme jak velikost, tak hlavní determinanty kontrolní premie v daném regionu.

Table of Contents

INTRODUCTION	1
1. BASIC ASPECTS OF COMPANY VALUATION	3
1.1 THE INCOME APPROACH	3
1.2 THE MARKET APPROACH	4
1.3 THE ASSET-BASED APPROACH	5
2. BASIC REVIEW OF PREMIUMS AND DISCOUNTS AND BASES TO WHICH THEY APPLY	6
2.1 LEVELS OF VALUE	7
2.1.1 <i>Company-Level Discounts</i>	8
2.1.2 <i>Shareholder-Level Discounts</i>	8
2.1.3 <i>Degree of Control/Minority and Degree of Marketability</i>	11
2.1.4 <i>Valuation Principles and Levels of Value They Arrive At</i>	11
2.2 STANDARDS OF VALUE	12
2.2.1 <i>Fair Market Value</i>	13
2.2.2 <i>Investment Value</i>	14
2.2.3 <i>Fair Value</i>	14
2.3 INTEGRATED THEORY OF BUSINESS VALUATION	15
2.3.1 <i>Gordon Model</i>	15
2.3.2 <i>Marketable Minority Interest Level of Value</i>	17
2.3.3 <i>Financial Control Level of Value</i>	18
2.3.4 <i>Strategic Control Level of Value</i>	20
2.3.5 <i>Non-Marketable Minority Interest Level of Value</i>	21
2.3.6 <i>Theory Integration</i>	24
3. CONTROL PREMIUM / MINORITY INTEREST DISCOUNT.....	26
3.1 BASIC DEFINITIONS	26
3.2 PREROGATIVES OF CONTROL	28
3.3 PRIVATE BENEFITS OF CONTROL.....	29
3.4 FACTORS INFLUENCING THE CONTROL PREMIUM	31
3.5 VALUATION METHODOLOGY EFFECT ON THE MINORITY INTEREST DISCOUNT AND CONTROL PREMIUM	34
3.5.1 <i>Income Approach and Control Premium Application</i>	34
3.5.2 <i>Market Approach and Control Premium Application</i>	35
3.5.3 <i>Asset-Based Approach and Control Premium Application</i>	36

3.6 PUBLICLY TRADED MINORITY STOCKS AND CONTROL PREMIUM.....	36
3.7 EMPIRICAL EVIDENCE OF CONTROL PREMIUMS AND MINORITY DISCOUNTS.....	39
4. LIQUIDITY/MARKETABILITY PREMIUM.....	45
4.1 BASIC DEFINITIONS.....	45
4.2 FACTORS INFLUENCING DISCOUNT FOR LACK OF MARKETABILITY.....	46
4.3 EMPIRICAL EVIDENCE ON LACK OF MARKETABILITY DISCOUNT.....	50
4.3.1 <i>Restricted Stock Studies</i>	51
4.3.2 <i>Studies of Private Transactions before Initial Public Offerings</i>	57
4.3.3 <i>Option Pricing Method</i>	59
4.4 LIMITATIONS TO THE LACK OF MARKETABILITY STUDIES AND THE STUDIES' PROPER APPLICATION.....	62
5. EMPIRICAL EXAMINATION OF CONTROL PREMIUMS APPLICABLE TO THE CEE M&A TRANSACTIONS.....	64
5.1 RESEARCH HYPOTHESES.....	64
5.2 METHODOLOGY.....	67
5.3 DATA SAMPLE.....	69
5.3.1 <i>Initial Selection Criteria</i>	69
5.3.2 <i>Databases Used</i>	70
5.3.3 <i>Final Sample</i>	71
5.4 DESCRIPTIVE STATISTICS.....	73
5.5 EMPIRICAL RESULTS.....	78
5.6 SUMMARY OF THE RESULTS.....	81
CONCLUSION.....	83
REFERENCES.....	85
APPENDICES.....	92
APPENDIX 1 – BLACK-SCHOLES OPTION PRICING MODEL APPLIED BY CHAFFE.....	92
APPENDIX 2 – REGRESSION OUTPUTS RELATED TO THE HYPOTHESIS 3.....	93

List of Abbreviations

ASA	American Society of Appraisers
CAPM	Capital Asset Pricing Model
CEE	Central and Eastern Europe
DCF	Discounted Cash Flow
DLOC	Discount for Lack of Control
DLOM	Discount for Lack of Marketability
GLS	Generalized Least Squares
IPO	Initial Public Offering
M&A	Mergers & Acquisitions
NAV	Net Asset Valuation
OLS	Ordinary Least Squares
R	R Software
SEC	Security and Exchange Commission
UK	United Kingdom
US	United States/United States of America
WMA	Willamette Management Associates

List of Figures

Figure 1 - Levels of Value Chart	9
Figure 2 - Put Prices According to Chaffe's Black-Scholes Model Application	61
Figure 3 - Development of the Number of Transactions in Time	74
Figure 4 - Transactions' Matrix According to Target's and Acquirer's Domiciles	75
Figure 5 - Control Premiums Distribution	76

List of Tables

Table 1 - Relationship between Marketable Minority Interest Level of Value and Financial Control Level of Value	19
Table 2 - Relationship among Marketable Minority Interest, Financial Control, and Strategic Control Levels of Value	21
Table 3 - Relationship between the Marketable Minority and Non-Marketable Minority Levels of Value.....	23
Table 4 - Relationship between the Strategic Control, Financial Control, Marketable Minority and Non-Marketable Minority Levels of Value	24
Table 5 - Nath's Research Output	37
Table 6 - Bellinger Study Output	41
Table 7 - Houlihan, Lokey, Howard and Zukin Study Output.....	42
Table 8 - Kasper Study Output	42
Table 9 - Mercer Study Output	43
Table 10 - Pratt Study Output	44
Table 11 - SEC Study Output	53
Table 12 - Gelman Study Output	53
Table 13 - Columbia Financial Advisors Study Output.....	56
Table 14 - Overview of Restricted Stock Studies	57
Table 15 - Emory Studies' Output	58
Table 16 - WMA Studies' Output.....	59
Table 17 - Nature of the Transactions in the Sample	73
Table 18 - Newly Acquired Stake and Final Stake of the Acquirer in the Company.....	75
Table 19 - Control Premium Characteristics according to the Target's Country	77
Table 20 - Control Premium Characteristics according to the Target's Industry.....	77
Table 21 - Output of the OLS Regression Regarding the Pure Control Dummy	78
Table 22 - Output of the OLS Regression Regarding the Termination Year Variable	78
Table 23 - Output of the OLS Regression Regarding the Poland Dummy	79
Table 24 - Output of the OLS Regression Regarding the Turkey Dummy.....	79
Table 25 - Output of the OLS Regression Regarding the Cross-Boarder Dummy	80
Table 26 - Output of the OLS Regression Regarding the Non-CEE Acquirer Dummy.....	80
Table 27 - Output of the OLS Regression Regarding the Strategic Acquirer Dummy	80
Table 28 - Output of the OLS Regression Regarding the Cash Payment Dummy	81
Table 29 - Summary of the Results.....	82
Table 30 - Output of the OLS Regression Regarding the Bulgaria Dummy.....	93
Table 31 - Output of the OLS Regression Regarding the Croatia Dummy.....	93
Table 32 - Output of the OLS Regression Regarding the Czech Dummy	93
Table 33 - Output of the OLS Regression Regarding the Estonia Dummy	93
Table 34 - Output of the OLS Regression Regarding the Hungary Dummy	93

Table 35 - Output of the OLS Regression Regarding the Latvia Dummy	93
Table 36 - Output of the OLS Regression Regarding the Lithuania Dummy	94
Table 37 - Output of the OLS Regression Regarding the Poland Dummy	94
Table 38 - Output of the OLS Regression Regarding the Romania Dummy	94
Table 39 - Output of the OLS Regression Regarding the Russia Dummy.....	94
Table 40 - Output of the OLS Regression Regarding the Serbia Dummy	94
Table 41 - Output of the OLS Regression Regarding the Slovakia Dummy	94
Table 42 - Output of the OLS Regression Regarding the Slovenia Dummy	94
Table 43 - Output of the OLS Regression Regarding the Turkey Dummy	95
Table 44 - Output of the OLS Regression Regarding the Ukraine Dummy.....	95

Introduction

“There is often more money in dispute in determining the discounts and premiums in a business valuation than in arriving at the pre-discount valuation itself. Discounts and premiums affect not only the value of the company but also play a crucial role in determining the risk involved, control issues, marketability contingent liability, and a host of other factors that can make or break a deal.”¹

Given the current lame market conditions, investors have become extremely cautious and perceptive to any market signals. As a result, the previous buoyant years in the mergers and acquisitions (hereinafter also “M&A”) market have been replaced by a gloomy reality when every penny matters. In such state of the world the most accurate valuation is apparently more crucial than ever before.

One of the most challenging parts of the valuation for M&A purposes is the application of discounts and premiums. The main task in M&A, equity underwritings, private equity investments, and generally all the situations when firms or their stakes are to be valued, is how to adjust the obtained values for liquidity/marketability and control characteristics. Numerous studies² provided evidence that liquidity/marketability and control have their own intrinsic value. Consequently, they found that the presence of these attributes or their lack is worth some additional compensation. This compensation can take the form of either a premium or a discount. Whereas premium means additional compensation above the underlying asset’s value, discount works the other way around.

In addition, in the company valuation there exist two main groups of premiums and discounts. One is distinguished from the other in terms of its application. Premiums

¹ Citation of Pratt in Bruner, R.F. and Palacios M. (2004), "Valuing Control and Marketability", Darden Graduate Business School University of Virginia, Batten Institute, May 2004.

² For more detail see Mercer, Z.C. (2002), "Money for Nothing, Application of Control Premiums Under the Fair Value Standard", Mercer Capital, April 2002; Nath, E.W. (1990), "Control Premiums and Minority Interest Discounts in Private Companies", Business Valuation Review, Vol. 9, No. 2, June 1990; Damodaran, A. (2005), "Marketability and Value: Measuring the Illiquidity Discount", Stern School of Business, July 2005; Emory, J.D. Sr., Dengel, F.R. and Emory, J.D. Jr. (2002), "Discounts for Lack of Marketability: Emory Pre-IPO Discount Studies 1980-2000, As Adjusted October 10, 2002", Emory

and discounts belonging to the first group are applied directly to the value of the company, which results from the conventional separate valuation procedure. On the contrary, the second group of premiums and discounts is usually reflected in the discount rate employed in the basic valuation model. In our analysis we focus solely on the first group of premiums and discounts. The main discounts and premiums of this group are those reflecting the attributes of marketability/liquidity and control.

The aim of the thesis is to provide the reader with sufficient theoretical, as well as practical background for the application of discounts and premiums. In practice, the correct application of discounts and premiums requires an a priori analysis of levels of value to which the premium/discount applies, standards of value according to which the premium/discount materializes, etc. Therefore, the thesis presents an overview of the basic methodology of these theoretical concepts. Furthermore, we examine the size and key determinants of the control premiums in the region of the Central and Eastern Europe (hereinafter also “CEE”). In this respect, our study is distinguished from the previous research which focuses mainly on the developed markets of Western Europe and the United States. Given the lack of relevant regional data for lack of marketability discount’s assessment, we leave the discussion on marketability issues on the theoretical level only.

The thesis is structured as follows. Chapter 1 handles the topic of basic aspects of company valuation. Levels and standards of value together with the Integrated Theory of Business Valuation are the main topics covered by Chapter 2. The key aspects of control premium/minority interest discount and discount for lack of marketability are presented in Chapter 3 and Chapter 4, respectively. The pivotal part of the thesis is the empirical section described in Chapter 5. It uses M&A data obtained from three sources – Bloomberg, Mergermarket and Zephyr databases. The main goal of Chapter 5 is to closely address the issues of control premiums and their application in the CEE region. We focus on the size of the historically applicable control premiums in the region and its key determinants. Findings of our analysis are summarized in the last section of the thesis.

1. Basic Aspects of Company Valuation

Before we discuss premiums and discounts applicable to a value of companies, we would like to briefly comment on the basic business valuation concepts and methodologies. In the following section we will provide the reader with the basic aspects of various approaches to business valuation. Understanding of these approaches is essential for the correct application of discounts and premiums. Generally, to determine the value of net business assets, whole enterprises, their partial shareholdings, or individual shares (as an investment instrument representing an entitlement to a part of net business assets), three fundamental traditional approaches are usually considered:³

- (i) **Income approach;**
- (ii) **Market approach;** and
- (iii) **Asset-based approach.**

1.1 The Income Approach

The income approach is a general way of determining the value indication of a business, business ownership interest, intangible asset, or security by using one or more methods through which the anticipated benefits are converted into value.⁴ Basic methods acceptable while using this approach include the method of capitalization of benefits and the method of discounted future benefits. Whereas, in the method of discounted future benefits⁵ the benefits are estimated for each of the selected numbers of future periods, in the method of capitalization of benefits, there is only one representative benefit level which is multiplied or divided by an appropriate capitalization factor in order to convert the benefit into value.

The DCF method, the most frequently used example of this approach, values future “cash flows” while both the risks of attaining such cash flows and their current value are the two attributes taken into account at the same time. The probability of

on Stock Prices", Financial Analyst Journal, July-August 1991.

³ In Lockwood, W.A. (2003), "Valuation of Closely Held Business Interests", Empire Valuation Consultants, Inc., ABA's 14th Annual Real Property, Estate Planning Symposia, 2003.

⁴ In American Society of Appraisers (2008), "ASA Business Valuation Standards", July 2008 Updated Version.

⁵ One can also include the discounted cash flow (hereinafter also “DCF”) method into the conceptual framework of the method of discounted future benefits.

attaining certain future income is given by internal, as well as external factors specific to each company or project.

A sub-category of the DCF method is the Discounted Dividend Revenue method. The biggest advantages of this method are its orientation to future income and its ability to more precisely determine the discount rate. On the other hand its biggest disadvantage is the inability to fulfil the prognoses upon which the valuation method employed is based. A second method, which is relatively rarely used, is known under the name of the capitalization of profit. It is primarily based on the historical profit-to-capitalization ratios. Valuation professionals usually consider this method to yield an overly approximate result, however, under certain circumstances it may be a useful indicator. The revenue approach is always predicated upon the availability of selected economic indicators within a certain time framework, e.g., future prognosis (DCF) or past results (profit capitalization method).

1.2 The Market Approach

This approach emphasizes comparison of the characteristics of the assets under valuation and is useful especially in cases where sufficient amount of data regarding the market for comparable goods is available.⁶ This assumes the existence of a developed and long-standing market economy (stock exchange data, similar transactions' history, licensing analogues, etc).

For example, taken into account the current conditions in our domestic (Czech) capital market, as well as in majority of other CEE countries' capital markets, the comparison approach may generally be applicable only by deriving relevant data from foreign capital markets. The foreign capital markets have been experiencing markedly higher standards of liquidity, transparency and pricing policies. Moreover, Czech capital market, as a representative of the CEE region, suffers from insufficiently long time series to be used to derive any market approach conclusions. On the other hand, developed capital markets that can be found e.g., in the US, UK and other countries can heavily benefit from using this straight forward model.

⁶ As stated in Cimasi, R.J. (2007), "Valuation Discounts for Lack of Marketability", Physician's News Digest, August 2007, this method is premised on the foundation that actual transactions of similar entities provide guidance to properly state the value.

1.3 The Asset-Based Approach

The asset-based approach⁷ is based on an indicative valuation of assets net of liabilities and may be analogous to the cost approach which can be found in other appraisal disciplines.⁸ The final result is given by deduction of the value of total liabilities from the value of total assets. Therefore, it is a so-called static valuation method, predicated on accounting values adjusted to their market values under the going concern assumptions.

In special cases the variation method called the liquidation value is used, i.e., the value of a company which does not continue as a going concern any more, but is liquidated⁹ instead. The use of this method is warranted shall the liquidation yield a greater profit than the continuance of the company's business; hence, the liquidation value outweighs the income value.

⁷ It is also sometimes referred to as the net asset valuation (hereinafter also "NAV") principle or the substantive method.

⁸ In American Society of Appraisers (2008), "ASA Business Valuation Standards", July 2008 Updated Version.

⁹ This act means that company's activities will terminate and its individual asset components will be sold.

2. Basic Review of Premiums and Discounts and Bases to Which They Apply

The purpose of this chapter is to make the reader familiar with the basic concepts of business valuation premiums and discounts and clarify the relationship between the premiums and discounts and the levels of value. As already indicated in the introductory part of this thesis, the concept of premiums and discounts has been prevalingly developed in the Anglo-Saxon countries with the rest of the world only implementing the main methodology with some marginal adjustments.¹⁰ Below see the abstract from the generally accepted key methodological steps specified by the American Society of Appraisers¹¹ in their publication of Business Valuation Standards on Valuation Discounts and Premiums¹²:

- (i) A discount or premium has no meaning until the conceptual basis underlying the base value to which it is applied is defined;
- (ii) A discount or premium is warranted when characteristics affecting the value of the subject interest differ sufficiently from those inherent in the base value to which it is applied;
- (iii) A discount or a premium quantifies an adjustment to account for differences in characteristics affecting the value of the subject interest relative to the base value to which it is compared;
- (iv) The purpose, applicable standard of value, or other circumstances of an appraisal may indicate the need to account for differences between the base value and the value of the subject interest. If so, appropriate discount should be applied;
- (v) The base value to which the discount or premium is applied must be specified and defined;

¹⁰ This statement is only the author's subjective perception of the reality based upon his own experience and as such does not has to be the general true.

¹¹ The American Society of Appraisers (hereinafter also "ASA") is an international organization of appraisal professionals and others interested in the appraisal profession. ASA is the oldest and only major appraisal organization representing all of the disciplines of appraisal specialists. The society originated in 1936 and incorporated in 1952. ASA among others helps find an ASA accredited appraiser and is the only professional valuation organization that accredits members in every appraisal discipline.

¹² In American Society of Appraisers (2008), "ASA Business Valuation Standards", July 2008 Updated Version.

- (vi) Each discount or premium to be applied to the base value must be defined;
- (vii) The primary reasons why each selected discount or premium applies to the appraised interest must be stated;
- (viii) The evidence considered in deriving the discount or premium must be specified; and
- (ix) The appraiser's reasoning in arriving at the conclusion regarding the size of any discount or premium applied must be explained.

To put it more clear, the main if not the only aim of the premiums and discounts is to reach a reasonable adjustment from a base value¹³. Such an adjustment shall reflect different characteristics of the subject interests compared to those of the base group upon which the value indication is based. As a result, the characteristic differences usually create changes in risk perception, in most cases as a consequence of lack of marketability, lack of control, and other likely factors. Since premiums increase value and discounts work the other way around, if the combination of these measures is applied correctly, the resulting difference in value should correspond to a new level of expected rate of return required by an investor to be compensated for differences in investment characteristics.

The process of determining the proper discount or premium into the subject interest valuation is a demanding procedure, not only because there is often more money involved in determining the applicable level of discount or premium than it is in arriving to the base value. The comprehensive understanding of the fundamental valuation of discounts and premiums, as well as situations in which each of them is and is not applicable, and their quantification shall be a standard equipment of every person involved in any kind of business valuation procedures.

2.1 Levels of Value

Some categories of discounts and premiums are applicable to the company as a whole¹⁴ and others reflect the ownership characteristics¹⁵. In other words, discounts and

¹³ Base value is a term representing the value of a company or its given shareholding prior to any discount or premium application.

¹⁴ Discounts belonging into this group could be e.g., environmental liability discount or key person discount.

premiums can be applied to a value either at company level or at shareholder level – we can distinguish between “company-level” or “entity-level” discounts or premiums and “shareholder-level” discounts or premiums which we apply to specific block of stocks.¹⁶

2.1.1 Company-Level Discounts

Selected discounts or premiums apply to the company as a whole or to all of its shareholders, either as a group or individually, regardless of any individual shareholder’s characteristics or attributes. These company-level discounts which are usually applied before the so-called shareholder discounts and are normally applied to a control-level of value, include, for example:¹⁷

- (i) Discount for trapped-in capital gains;
- (ii) Key person discount;
- (iii) Discount for known or potential environmental liability;
- (iv) “Portfolio”, “conglomerate”, or “non-homogenous assets” discount;
- (v) Discount for pending litigation; or
- (vi) Discount for risk of loss or non-renewal of significant customers or suppliers due to extreme concentration.

In some cases, this group of discounts can be implemented directly into discount or capitalization rates while using the income valuation approach or valuation multiples in the market approach in order to reflect the additional level of risk they carry. With respect to the methodological guideline of the American Society of Appraisers this step should be clearly explained.

2.1.2 Shareholder-Level Discounts

The fact that prior to any discount or premium implementation it is necessary to have a well-defined base to which such discount or premium is applied holds even more strictly in the case of the shareholder-level discounts. The levels of value chart as a cornerstone of discounts concerning the characteristics of ownership appears in works

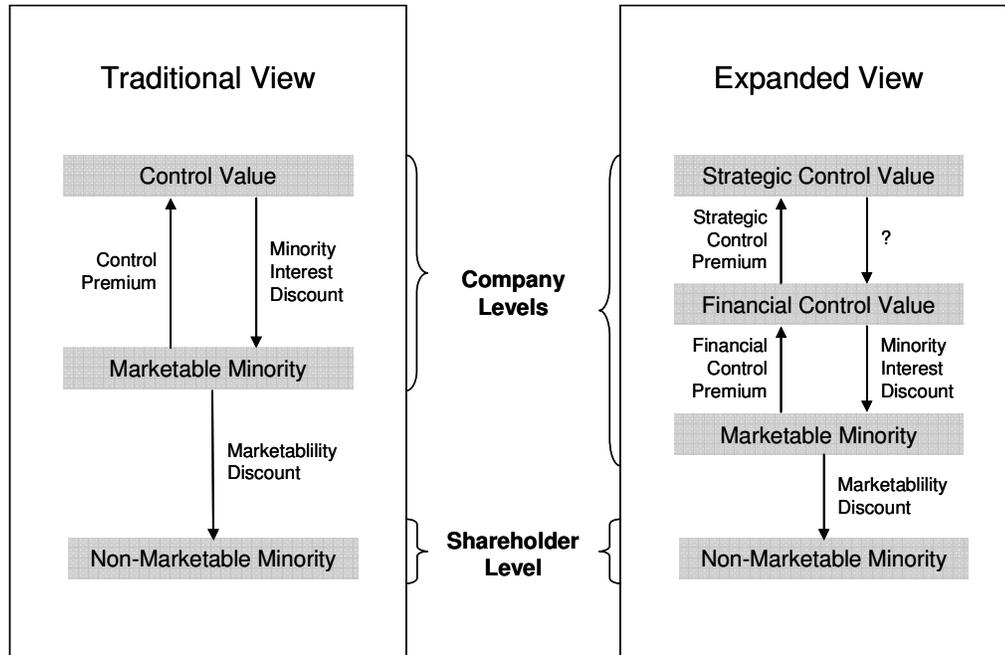
¹⁵ Discounts and premiums which belong into this group are e.g., control premium vs. the minority interest discount or lack of marketability discount.

¹⁶ In Pratt, S.P. (2001), "Business Valuation Discounts and Premiums", John Wiley & Sons, Inc., New York, 2001.

¹⁷ Ibid.

of e.g., Mercer (1997, 1999, 2002a, 2002b), Hyde (2000), or Nath (2003). For illustration see the following Figure 1.

Figure 1 - Levels of Value Chart



Source: Mercer¹⁸ with author's adjustments

The above mentioned scheme depicts the traditional view (left-hand side) of the level of value chart which has recently been further elaborated¹⁹ to the expanded version (right-hand side). Whereas the financial control stands for the level of value a financial investor is able and willing to pay for a control over the business, the strategic, sometimes called synergistic, control represents the control level a strategic buyer is prepared to pay for a controlling stake in the company.²⁰ From the traditional point of view,²¹ the starting point for any discussions concerning discounts or premiums related to the characteristics of ownership could be either control value or marketable minority

¹⁸ In Mercer, Z.C. (2002b), "Money for Nothing, Application of Control Premiums Under the Fair Value Standard", Mercer Capital, April 2002.

¹⁹ Pratt (2001), Hyde (2000) and other economists engaged in the application of premiums and discounts indicate Christopher Z. Mercer as an inventor of this scheme. The same person is awarded a credit for its further expansion.

²⁰ Financial investor usually acquires companies based on their current potential to generate future profits. On the other hand, strategic investor acquires a company where the investor is aware of any future additional gains he or she would otherwise not be able to extract, the so-called synergies. For such synergies is the strategic investor prepared to pay additional strategic control premium.

²¹ Compared to the traditional view, the control value has been only split into two subsets – strategic and financial control values – in the expanded version.

value²². The discounts and premiums which represent characteristics of ownership and will be further discussed in the following chapters fall broadly into two major categories:²³

- (i) Control premium or lack of control discount;²⁴ and
- (ii) Lack of marketability discount.²⁵

Marketability and control are distinct issues, but the degree of control or lack of control has an influence on the size of the discount for lack of marketability, as well as on the selection of the appropriate method to quantify the discount for lack of marketability (hereinafter also “DLOM”). It is the main reason of the preferential consideration of the degree of control prior to degree of marketability in the valuation analysis. There is one practical reason why the non-marketable minority level of value is not used as a starting point of discounts and premiums application. The reason is that there does not exist any unified database recording information on the arm’s length of minority non-marketable transactions²⁶ and even no other empirical data lead to this level of value. Whereas marketability or its lack has usually not been solved at the controlling interest level²⁷ it has been distinguished as an important issue at the minority interest level.

On the marketable minority level, the term “liquid” or “marketable” refers to an actively traded stock that can be easily liquidated and the cash proceeds from the sale transaction can be received within three working days.²⁸ Controlling interests are usually much less liquid compared to the usually traded securities instead; however, more liquid than minority shares in privately held companies.

²² Depending on the author the marketable minority value is also sometimes called the stock market value, publicly traded equivalent value, or more loosely “as-if-freely tradable value”. All these terms have the equivalent meaning and will be used interchangeably in this thesis.

²³ In Pratt, S.P. (2001), "Business Valuation Discounts and Premiums", John Wiley & Sons, Inc., New York, 2001.

²⁴ The topic of voting vs. nonvoting shares may be considered as a separate issue or as a subcategory of control.

²⁵ Some sources use the term liquidity instead of marketability. This phenomenon will be discussed further in more detail.

²⁶ Arm’s length transactions generally refer to the sales of shareholdings in the privately owned companies.

²⁷ There are countless disputes among valuation professionals regarding the issue of marketability on the controlling level of value. Nevertheless, the current mainstream promotes not to apply marketability discount to the controlling interest.

²⁸ In Mercer, Z.C. (1997), "Quantifying Marketability Discounts", Peabody Publishing, 1997.

2.1.3 Degree of Control/Minority and Degree of Marketability

The degree of ownership can range from 100% shareholding to the minority ownership represented just by a few or even a single share of a respective company. Besides quantity, also quality of the control present directly influences the magnitude of the discount for lack of control (hereinafter also “DLOC”).

As in the case of the degree of ownership control, also the marketability degree covers a full spectrum. It can range from a 100% liquidity represented by the above mentioned example of the sale and obtaining the sale price within three working days²⁹ to a liquidity which is very limited³⁰. As already indicated, there can sometimes be a distinction between liquidity and marketability. Nonetheless, as we see only a marginal difference between these two features we will join the current trend of the business appraisers and use them interchangeably within the thesis.

2.1.4 Valuation Principles and Levels of Value They Arrive At

As different valuation methods stand upon different assumptions and principles, their outcomes may result in different levels of value. It is fundamental to understand what base value was carried out by the valuation method employed and consequently be aware how the selected valuation method impacts the applicable discounts or premiums.

In the chapter 1.5 called General Valuation Methodologies we have identified three valuation approaches and each of them arrives to the base value according to the following pattern:³¹

- (i) **Income approach:**
 - DCF method – minority or control value,
 - Capitalization of income – minority or control value;
- (ii) **Market approach:**

²⁹ Such a procedure is common for the majority of trades carried out on the New York Stock Exchange (hereinafter also “NYSE”) or the American NASDAQ.

³⁰ Such a case of disabled liquidity can arise when for example the listed company besides its ordinary shares decides from different reasons to issue also the so called restricted shares. The owners of these shares have different rights and obligations compared to ordinary shareholders. E.g., according to the Security Exchange Commission’s (hereinafter also “SEC”) Rule 144 the owners of such shares are not allowed to trade these shares within one year period after their issuance. This topic will be discussed in more detail further in the thesis.

- Guideline companies – minority value,
 - M&A transactions – control value; and
- (iii) **Asset-based approach:**
- Adjusted net assets – control value.

Regardless which method (either DCF or CAPM) is used to estimate the discount rate in the income approach, the rate is gained based on the public market data and as such it reflects the assumption of full marketability. Therefore, if control cash flows are used, the result should be the control value, whereas if minority interest cash flows are processed, the result should be the marketable minority level value.³²

As the guideline public companies actively trade their minority interests, the guideline companies' method has traditionally been assumed to produce marketable minority value. On the other hand, if using M&A guideline companies in order to derive market multiples, the transactions generally represent controlling interests and this method is therefore assumed to reflect control value.³³

In case of the adjusted net assets' value, as a representative of the asset-based approach, the general assumption says that it reflects control over the assets; hence, the control value with respect to the levels of value chart is calculated.³⁴

2.2 Standards of Value

Eventhough, the standard of value which has been used by majority of professionals (e.g., Cavendish and Kammerer, 2008; Emory, 1999; Hall and Polacek, 1994; Mercer, 1999; or Sansing, 1999) is called fair market value, there have been two more standards of value described by the literature (e.g., Pratt, 2001). These two additional approaches are called investment value and fair value.³⁵

The reason why it is extremely important for all parties involved in the valuation process prior to its initiation to agree on the current standard of value is that it

³¹ In Schlueter, R.A. (2004), "Control vs. Minority Interests and Marketability Discounts for ESOPs", Ohio Employee Ownership Center, April 2004.

³² In Pratt, S.P. (2001), "Business Valuation Discounts and Premiums", John Wiley & Sons, Inc., New York, 2001.

³³ Ibid.

³⁴ Ibid.

³⁵ All of these concepts are quite strictly defined, especially by the American courts, but are much more loosely used in practise.

may have considerable impact on the applicability of appropriate premiums and discounts, particularly, in the case of control / minority interest and marketability.

2.2.1 Fair Market Value

The most frequently used definition of fair market value says that the fair market value of the ownership interest is the price at which such property would change hands between a willing buyer and a willing seller, neither being under any compulsion to buy or to sell, and both having reasonable knowledge of relevant facts.³⁶ This definition intentionally does not take into consideration a specific buyer or seller in order to eliminate the influence of their potentially specific motivations. Instead, the fair market value concept tries to be objective. It intends to find a price at which a hypothetical investor³⁷ would be indifferent between buying or selling his or her ownership interest in the company, excluding any potential synergies with a particular buyer.

According to John Emory³⁸, the process of valuing interests in closely held companies is much art as a science and numerous factors are to be considered within the valuation itself. The most important factors influencing the fair market value include among others the history and nature of the business, general economic and specific industrial outlooks, company's economic condition, company's goodwill and other intangible assets' quantitative and qualitative features, or the company's unique earning capacity. The appraiser is then left with large portion of autonomy to weigh each of these factors. How big the autonomy actually is, it has already been documented many times when two different appraisers having the same sources of information arrived at two diametrically different valuation conclusions of the same ownership interest.

Under the fair market value standard, the appraiser has to be focused on the specific ownership interest which is about to be valued "as it is" involving its minority interest/control and marketability characteristics. When considering a minority ownership interest in a closely held company, it is fairly standard in a fair market value

³⁶ In Sansing, R. (1999), "Economic Foundations of Valuation Discounts", Dartmouth College Paper, January 1999.

³⁷ Such a hypothetical investor represents a group of competing buyers and sellers with a common set of motivations.

³⁸ In Emory, J.D. (1999), "Discounts and Premiums in Business Valuation: What Business Lawyers Should Know", Business Laws, Inc., November 1999.

analysis to substantially decrease the total price for these shares to reflect both lack of marketability and control.³⁹

2.2.2 Investment Value

Investment value substantially differs from the above mentioned fair market value, because the investment value stands for the value to some particular buyer or seller rather than to a hypothetical buyer or seller.⁴⁰ As a result the price computed under the investment value approach may incorporate the value of synergies that a particular investor may be willing to count into the strategic premium paid above just a pure control premium.⁴¹

It is the best option for an appraiser to thoroughly study all the relevant cases, because it is widely spread phenomenon that one can easily find “fair market value” written in the text, but after closer identification gets to know that in fact the accepted valuation methodology corresponds to the investment value principle. The investment value is also sometimes referred to as an “intrinsic value”.

2.2.3 Fair Value

As stated by Pratt (2001), until 1999 the fair value was defined by the Model Business Corporation Act as the value of the shares immediately before effectuation of the corporate action to which the dissenter objects, excluding any appreciation or depreciation in anticipation of the corporate action unless exclusion would be inequitable.⁴² Although, this definition does not touch the issue of lack of marketability or lack of control, it clearly eliminates any premium that would encompass synergistic value with a potential acquirer above the firm’s financial control value on a stand-alone basis.

³⁹ In Cavendish, R.C. and Kammerer, C.W. (2008), "Determining the Fair Value of Minority Ownership Interests in Closely Held Corporations: Are Discounts for Lack of Control and Lack of Marketability Applicable?", *The Florida Bar Journal*, Vol. 82, No. 2, February 2008.

⁴⁰ In Pratt, S.P. (2001), "Business Valuation Discounts and Premiums", John Wiley & Sons, Inc., New York, 2001.

⁴¹ In the US there can often be found the investment value principle application in legal precedents, prevailing in the family law courts, where judges seek the “value to the owner“. For example, if there is a family-owned company, there may be improper to apply the minority interest discount to a minority owner, because through family attribution, the minority owner is perceived as a part of the “controlling group“.

Even in these days there is no clear and generally accepted position regarding the fair value with respect to control/minority and marketability issues. On the other hand, in the United States, there is the fair value concept usually interpreted by the courts in dissenters' and minority oppression cases to mean proportion of the enterprise value without any lack of control discount, even in the case when the valuation subject corresponds to a minority shareholding.

2.3 Integrated Theory of Business Valuation

The integrated theory of business valuation represents a straightforward and easy-to-understand analysis of discounts and premiums implementation in the context of conceptual levels of value. This mathematical-based theory has been developed and published by Christopher Z. Mercer in 2002.⁴³ As we find this approach unique and essential for a comprehensive understanding of discounts' and premiums' application, we will briefly describe and discuss its main ideas and implications.

2.3.1 Gordon Model

The Gordon model which is frequently used within the valuation procedures is in fact the starting point of the integrated theory of business valuation. The basic Gordon model is a single-period income capitalization model and can be described by the following Equation (1):⁴⁴

$$V_0 = \frac{CF_1}{r - g} = \frac{CF_0(1 + g)}{r - g}, \text{ where} \quad (1)$$

- (i) V_0 stands for the value of the underlying asset at time 0;
- (ii) CF_1 represents the cash flow in the next period of time;
- (iii) r means the company's discount rate; and
- (iv) g stands for the expected growth rate of the company's dividends.

The basic interpretation of the formula above states that the value of the underlying asset today equals the value of cash flows attainable in the next period

⁴² In Pratt, S.P. (2001), "Business Valuation Discounts and Premiums", John Wiley & Sons, Inc., New York, 2001.

⁴³ Further details on this issue could be found in Mercer, Z.C. (2002a), "An Integrated Theory of Business Valuation", Mercer Capital, October 2002.

⁴⁴ In CFA Institute (2009), "CFA Program Curriculum – Equity", Pearson, Custom Publishing; 2009

divided or capitalized by the company's discount rate less the expected growth rate of company's dividends. This formula is a summary of the basic DCF valuation method under the following assumptions:

- (i) The attainable cash flows grow over the period at the constant rate g ; and
- (ii) All the cash flows are reinvested (or otherwise distributed and available for reinvestment) within the firm at the discount rate r .

The development of the discount rates belongs to the fundamentals of the whole Gordon model. The discount rates used within the Gordon model are prevailingly developed by two primarily means. The first one is the so-called direct method which is based on the pricing of publicly traded securities and the second one is the so-called indirect method which makes a use of the findings of the CAPM or the adjusted CAPM.⁴⁵

To meet the main aim of the integrated theory of business valuation, which is the provision of theoretical integration of the Gordon model, appraisers valuing firms have to accomplish the following:⁴⁶

- (i) Adapt and shortly explain the Gordon model to represent all conceptual levels of value;
- (ii) Understand the definition of the conceptual adjustments between the levels of value⁴⁷ while using the components of the Gordon model;
- (iii) Explain the differences in the levels of value in terms of the components of the Gordon model; and
- (iv) Explain why the integrated theory of business valuation is illustrative of pricing behaviour in the market for entire companies (controlling interest levels of value), in the marketplace for public securities (marketable minority levels of value), and the "market" for illiquid minority interests of closely-held companies (non-marketable minority levels of value).

⁴⁵ In Mercer, Z. C. (1992), "Adjusted Capitalization Rates for the Differences between Net Income and Net Free Cash Flow", *Business Valuation Review*, December 1994.

⁴⁶ In Mercer, Z.C. (2002a), "An Integrated Theory of Business Valuation", Mercer Capital, October 2002.

⁴⁷ E.g., the adjustments between the control premium/minority interest discount and the marketability discount.

2.3.2 Marketable Minority Interest Level of Value

It is generally accepted fact that the Gordon model provides a representation of the value of publicly traded securities at the marketable minority interest level of value. The minority interest level is perceived as a conceptual level of value which helps to derive other levels of value. For closely held companies, it provides an indication of the comparable level of value which is sometimes known under the term “as-if-freely-traded” value.⁴⁸ The marketable minority interest level of value could be defined in the Gordon model’s context in the way as described in Equation (2):⁴⁹

$$V_{mm} = \frac{CF_{e(mm)}}{r_{mm} - g_{mm}}, \text{ where} \quad (2)$$

- (i) V_{mm} stands for equity value of a publicly traded security at the marketable minority level of value and the value of the closely held security at the same, i.e., as-if-freely-traded value;
- (ii) $CF_{e(mm)}$ equals to the cash flow of the enterprise at the marketable minority level of value for the next period;⁵⁰
- (iii) r_{mm} is the discount rate applicable for a public company or the discount rate at the marketability level of value;⁵¹ and
- (iv) g_{mm} represents the expected growth rate of earnings belonging to the publicly traded security under the assumption that all the company’s earnings are distributed to its shareholders.

At this stage of the analysis Mercer (2002a) defines the marketable minority level of value as an enterprise level of value.⁵² The importance of this definition becomes evident in the moment the remaining relationships of the mathematical nature are discovered.

⁴⁸ This issue has already been discussed above.

⁴⁹ The formula is taken from Mercer (2002a) and restated according to our needs.

⁵⁰ The marketable minority level of cash flows is assumed to be normalized for non-recurring or unusual events.

⁵¹ For the closely-held companies, r_{mm} is usually estimated by using the adjusted CAPM as mentioned above.

⁵² It is only a Mercer’s definition taken from Mercer, Z.C. (2002a), "An Integrated Theory of Business Valuation", Mercer Capital, October 2002.

2.3.3 Financial Control Level of Value

As already discussed in one of the previous sections of the thesis, the financial control level which does not include any synergistic value is only one out of two substances⁵³ which together, according to the traditional approach, form the control level of value. According to Mercer's study (2002a) the careful review of the real-life data available to appraisers, brings the results, which usually reflect transactions driven by the strategic or synergistic motivations. If his presumptions and findings are correct, then the available data on control market premium correspond to the sum of the financial and strategic control premiums.⁵⁴

Keeping the above mentioned facts in mind, we can introduce the conceptual math behind the financial control level of value as depicted in Equation (3):⁵⁵

$$V_{e(c,f)} = \frac{CF_{e(c,f)}}{[r_f - (g_{mm} + g_f)]}, \text{ where} \quad (3)$$

- (i) $V_{e(c,f)}$ stands for equity value of an enterprise from the point of view of a typical buyer of the entire enterprise who does not experience any synergistic or strategic benefits that could further increase value relative to the marketable minority value;
- (ii) $CF_{e(c,f)}$ equals to the cash flow of the enterprise from the point of view of a financial control buyer;⁵⁶
- (iii) r_f is the discount rate applicable for the universe of financial buyers;⁵⁷
and

⁵³ The second fundament is the so-called strategic or synergistic level of value and will be discussed later.

⁵⁴ If this finding is true, the use of available control premium studies as a basis for derivation of minority interest discount within the fair market value concept is not methodologically correct unless strategic buyers are a generally accepted norm. The use of such studies would result in overstating the discount for the minority interest and applying this discount to the financial control values would not bring the appraiser to the marketable minority level of value but something lower than that level of value. Moreover, this newly arrived level of value would not be conceptually defined.

⁵⁵ The formula is taken from Mercer (2002a) and restated according to our needs.

⁵⁶ $CF_{e(c,f)}$ is derived from $CF_{e(mm)}$ at first, normalizing the earnings stream, and second, by implementation of judgements regarding the ability of a new control buyer to upgrade the stream of earnings beyond the process of normalization. Especially the second step could include the ability of a specific buyer to improve the current operations, which might not be applicable under the fair market value concept, where the buyers, as well as the sellers, are only hypothetical ones.

⁵⁷ r_f can be identical to r_{mm} in the real world as mentioned by Mercer (2002a), but in our case the r_f is designated specifically to allow for the fact that the leverage considered by the financial buyer could increase the discount rate slightly above the r_{mm} .

(iv) $(g_{mm} + g_f)$ represents the expected growth rate of earnings for the financial control buyer.⁵⁸

The relationship between the two levels of value we discussed so far, i.e., the marketable minority interest level of value and financial control level of value, is illustrated in the Table 1.

Table 1 - Relationship between Marketable Minority Interest Level of Value and Financial Control Level of Value

Level of Value	Conceptual Math	Relationships	Value Implications
Financial Control Value	$\frac{CF_{e(c,f)}}{r_f - (g_{mm} + g_f)}$	$CF_{e(c,f)} \geq CF_{mm}$ $g_f \geq 0$ $r_f \geq r_{mm}$	$V_{e(c,f)} \geq V_{mm}$
Marketable Minority Interest	$\frac{CF_{e(mm)}}{r_{mm} - g_{mm}}$	$CF_{e(mm)} = CF_{mm}$	V_{mm}

Source: Mercer⁵⁹ with author's adjustments

$CF_{e(c,f)}$ would outweigh $CF_{e(mm)}$ if and only if the typical buyer of the company would be expecting to improve its operations and would share the idea of the expected benefits with the hypothetical seller other things being equal. However, this is hardly to materialize in reality. g_f would be larger than zero if the typical financial control buyer believes he or she could improve the future cash flows growth rates and shares this expected benefit with the hypothetical seller. First then $V_{e(c,f)}$ could outweigh $V_{(mm)}$, ceteris paribus. The notation further allows for r_f being greater than r_{mm} which could be true for a specific buyer, but is highly improbable, as the market forces are expected to push all the buyers to accept r_{mm} as the basis discount rate.

⁵⁸ The g_{mm} is the same variable as in the previous formula, but the g_f represents the increment in the earnings growth rate that a financial control buyer may expect to generate. Even in here, the second factor can hardly be reasonable under the fair market value concept out of two reasons. First, the whole universe of the hypothetical buyers may not expect such an increment in earnings growth rate; and second, a specific buyer who would be able to reach the accelerated growth can hardly be expected to share this attribute with the selling party within the acquisition negotiations.

⁵⁹ In Mercer, Z.C. (2002a), "An Integrated Theory of Business Valuation", Mercer Capital, October 2002.

2.3.4 Strategic Control Level of Value

Adding an attribute of potential synergetic effects we can easily arrive to a slightly adjusted conceptual math formula, compared to Equation 4 (the previous one) in the case of the financial control level of value. The basic formula is valid for the strategic control level of value and could be depicted as follows:⁶⁰

$$V_{e(c,s)} = \frac{CF_{e(c,s)}}{[r_{ss} - (g_{mm} + g_s)]}, \text{ where} \quad (4)$$

- (i) $V_{e(c,s)}$ stands for equity value of an enterprise from the point of view of a buyer of the entire enterprise who may expect to experience some synergistic or strategic benefits that could further increase value relative to the financial control level of value;⁶¹
- (ii) $CF_{e(c,s)}$ equals to the cash flow of the enterprise from the point of view of a strategic control buyer;⁶²
- (iii) r_{ss} is the discount rate applicable for the universe of potential financial buyers;⁶³ and
- (iv) $(g_{mm} + g_s)$ represents the expected growth rate of earnings for the strategic control buyer.⁶⁴

The relationship among the three company levels of value we discussed so far, i.e., the marketable minority interest level of value, financial control level of value, and strategic control level of value, is illustrated in the Table 2.

⁶⁰ The formula was taken from Mercer (2002a) and restated according to our needs.

⁶¹ As already mentioned above, this new increased level of value is sometimes called strategic control level of value (e.g., Mercer, 2002a) or synergistic control level of value (e.g., Pratt, 2001).

⁶² $CF_{e(c,s)}$ is derived in the same way as in the case of $CF_{e(c,f)}$. First, the earnings are normalized to derive $CF_{e(mm)}$, and second, additional adjustments are to be implemented afterwards. These adjustments differ a bit from the previous case and shall reflect the nature and scope of potential improvements that a typical financial buyer might expect to make by operating the firm in a more profitable way, and compose of concrete expectations concerning synergies or strategic benefits. To put it in a different way, besides any expectations of operating the firm better, the strategic control buyer takes into consideration expected potential benefits from operating the company differently.

⁶³ r_{ss} may be lower than r_{mm} for two different reasons. First, majority of strategic buyers are several times larger than companies they usually desire to acquire and as such their cost of capital is considerably lower compared to their potential targets, and second, a strategic control buyer of a comparable size may expect reduced risk as a result of a strategic combination, hence considers a lower discount rate.

⁶⁴ The g_{mm} is always the same and the g_s represents the increment in the earnings growth rate that a strategic control buyer may expect to generate.

Table 2 - Relationship among Marketable Minority Interest, Financial Control, and Strategic Control Levels of Value

Level of Value	Conceptual Math	Relationships	Value Implications
Strategic/Synergistic Control Value	$\frac{CF_{e(c,s)}}{r_{ss} - (g_{mm} + g_s)}$	$CF_{e(c,s)} \geq CF_{e(c,f)}$ $g_s \geq 0$ $r_{ss} \leq r_{mm}$	$V_{e(c,s)} \geq V_{e(c,f)}$
Financial Control Value	$\frac{CF_{e(c,f)}}{r_f - (g_{mm} + g_f)}$	$CF_{e(c,f)} \geq CF_{mm}$ $g_f \geq 0$ $r_f \geq r_{mm}$	$V_{e(c,f)} \geq V_{mm}$
Marketable Minority Interest	$\frac{CF_{e(mm)}}{r_{mm} - g_{mm}}$	$CF_{e(mm)} = CF_{mm}$	V_{mm}

Source: Mercer⁶⁵ with author's adjustments

$CF_{e(c,s)}$ might outweigh both $CF_{e(c,f)}$ and $CF_{e(mm)}$ if and only if the strategic control buyer of the company would be expecting to reach some synergistic and strategic benefits which are not available to the financial control investor. g_s would be larger than zero if the strategic control buyer believes he or she could improve the future cash flows growth rates and shares this expected benefit with the hypothetical seller. In such a case $V_{e(c,s)}$ could outweigh both $V_{e(c,f)}$ and $V_{(mm)}$, ceteris paribus. The notation further allows for r_{ss} being lower compared to r_f or r_{mm} , as has already been mentioned above. If this is true, the strategic control level of value can be higher than the other company levels of value, ceteris paribus.

2.3.5 Non-Marketable Minority Interest Level of Value

So far we have been discussing and comparing different company levels of value. However, in the integrated theory of business valuation it is important to analyse also the relationship between the marketable minority level of value and the only representative of shareholder level of value, the non-marketable minority level of value.

The conceptual math formula corresponding to non-marketable level of value could be characterized as in the Equation (5):⁶⁶

$$V_{sh} = \frac{CF_{sh}}{r_{hp} - g_v}, \text{ where} \quad (5)$$

- (i) V_{sh} stands for equity value of an enterprise that does not have an active market for its shares from the point of view of a shareholder of such an interest;⁶⁷
- (ii) CF_{sh} equals to the portion of cash flow of the enterprise from the point of view and expected to be received by the company's shareholders;
- (iii) r_{hp} is the discount rate of the minority shareholder who invested into a non-marketable equity security for an expected duration of the holding period;⁶⁸ and
- (iv) g_v represents the expected growth rate in value of the enterprise's equity yielding the terminal value of the company at the end of the holding period.⁶⁹

The relationship between the marketable minority interest level of value and the non-marketable minority interest level of value is illustrated in the Table 3.

⁶⁵ In Mercer, Z.C. (2002a), "An Integrated Theory of Business Valuation", Mercer Capital, October 2002.

⁶⁶ The formula was taken from Mercer (2002a) and restated according to our needs.

⁶⁷ To obtain the V_{sh} in the case of asset holding entities, appraisers in practise usually begin with the net asset value (assuming it to be the financial control level of value) and consequently subtract the minority interest and marketability discounts, respectively.

⁶⁸ r_{sh} can be equal or larger than r_{mm} . r_{sh} can be interpreted as a sum of r_{mm} and the so-called holding period premium. The holding period premium can equal to zero if and only if, there is no holding period risk connected to the respective equity security. Such a zero holding period risk is assumed in the case of publicly traded securities where it is possible to liquidate one's positions within three business days, as mentioned in the thesis earlier.

⁶⁹ The expected growth rate of the enterprise's equity value equals to r_{mm} for a publicly traded security without paying any dividends. However, due to the leakage of the cash flows from the unlisted enterprise, the g_v will be lower than r_{mm} .

Table 3 - Relationship between the Marketable Minority and Non-Marketable Minority Levels of Value

Level of Value	Conceptual Math	Relationships	Value Implications
Marketable Minority Interest	$\frac{CF_{e(mm)}}{r_{mm} - g_{mm}}$	$CF_{e(mm)} = CF_{mm}$	V_{mm}
Non-Marketable Minority Interest	$\frac{CF_{sh}}{r_{hp} - g_v}$	$CF_{sh} \leq CF_{mm}$ $g_v \leq r_{mm}$ $r_{hp} \geq r_{mm}$	$V_{sh} \leq V_{mm}$

Source: Mercer⁷⁰ with author's adjustments

Following the above mentioned Table 3, we can easily deduce that V_{sh} , the non-marketable minority interest value, will be lower compared to V_{mm} , the marketable minority interest value, if at least one of the following conditions holds. First, CF_{sh} is less than CF_{mm} which is true when the cash flows expected by the shareholders are lower than the expected cash flows of the company, which holds in cases where not all of the company's cash flows are distributed among its shareholders. Such cash flows can be paid out to firm's controlling shareholders or be reinvested in the enterprise. Second, g_v is lower in comparison to r_{mm} when the company cash flows are reinvested sub-optimally⁷¹ or not fully reinvested in the enterprise.⁷² Finally, r_{hp} is greater than r_{mm} as a result of a greater risk which is beard by the holder of an illiquid asset compared to an otherwise similar asset with an active public market.⁷³ which could be true for a specific buyer, but is highly improbable, as the market forces are expected to push all the buyers to accept r_{mm} as the basis discount rate.

⁷⁰ In Mercer, Z.C. (2002a), "An Integrated Theory of Business Valuation", Mercer Capital, October 2002.

⁷¹ Sub-optimal reinvestment is simply characterized by reinvestment of cash flows at rates lower than the discount rates.

⁷² g_v lower than r_{mm} means a lower terminal value at the end of the holding period, hence lower value.

⁷³ This incremental risk stems from several reasons, the most fundamental of which is connected to a potentially long and unidentified holding period of such non-marketable asset.

2.3.6 Theory Integration

This section of the thesis generally intends to link all the findings discovered in the four previous subchapters and stress the benefits of the integrated theory of business valuation. For illustration we included Table 4, which incorporates all the useful formulas crucial for understanding the relationships among the strategic control, financial control, marketable minority interest, and non-marketable minority interest levels of value.

Table 4 - Relationship between the Strategic Control, Financial Control, Marketable Minority and Non-Marketable Minority Levels of Value

Level of Value	Conceptual Math	Relationships	Value Implications
Strategic/Synergistic Control Value	$\frac{CF_{e(c,s)}}{r_{ss} - (g_{mm} + g_s)}$	$CF_{e(c,s)} \geq CF_{e(c,f)}$ $g_s \geq 0$ $r_{ss} \leq r_{mm}$	$V_{e(c,s)} \geq V_{e(c,f)}$
Financial Control Value	$\frac{CF_{e(c,f)}}{r_f - (g_{mm} + g_f)}$	$CF_{e(c,f)} \geq CF_{mm}$ $g_f \geq 0$ $r_f \geq r_{mm}$	$V_{e(c,f)} \geq V_{mm}$
Marketable Minority Interest	$\frac{CF_{e(mm)}}{r_{mm} - g_{mm}}$	$CF_{e(mm)} = CF_{mm}$	V_{mm}
Non-Marketable Minority Interest	$\frac{CF_{sh}}{r_{hp} - g_v}$	$CF_{sh} \leq CF_{mm}$ $g_v \leq r_{mm}$ $r_{hp} \geq r_{mm}$	$V_{sh} \leq V_{mm}$

Source: Mercer⁷⁴ with author's adjustments

The major benefits of the integrated theory of the business valuation can be summarized in the following bullet points:⁷⁵

⁷⁴ In Mercer, Z.C. (2002a), "An Integrated Theory of Business Valuation", Mercer Capital, October 2002.

⁷⁵ Ibid.

- (i) It helps to understand each level of value in the context of valuation and financial theory and explains why value differs from level to level in financial and economic terms;
- (ii) Explains why the integrated model is illustrative of pricing behaviour observed in both non-public and public markets for equity interests;
- (iii) Defines the conceptual adjustments related to various levels of value in terms of DCF analysis summarized by the Gordon model; and
- (iv) Gains an increased level of understanding of the value of control and minority interest value.

3. Control Premium / Minority Interest Discount

The main idea of the contemporary finance states that majority of the listed companies have dispersed ownership where the benefits are distributed among the shareholders on their pro rata shareholdings in the respective company. To the contrary, recently it has been more and more often mentioned⁷⁶ that certain shareholders can exercise control over crucial corporate decisions⁷⁷ that is disproportionate to their shareholdings.⁷⁸ This feature has an intrinsic value and causes that such shareholders can easily obtain the so called private benefits of control.⁷⁹ There cannot be any doubt that such a control is highly valued and many investors are prepared to pay for it considerable control premiums beyond the sole enterprise value of the targeted company. This phenomenon has been documented by an extensive research of e.g., Dyck and Zingales (2004a, 2004b); Hanouna, Sarin and Shapiro (2001); Finnerty and Emery (2004); or Bruner and Palacios (2004); and is the leitmotiv of the whole upcoming chapter.

3.1 Basic Definitions

Until recently the professionals engaged in control premiums and their application thought that the only factor influencing the scope of the control premium is the legal power. Nevertheless, according to the concept first presented by Wayne Jankowske at the 1995 American Appraisers International Conference⁸⁰, the value of control has besides the legal powers and rights also its economic dimension. This economic dimension is represented by the economic potential of the company being under investigation. Hence, unprofitable business with no reasonable prospects for profitability brings no additional value of control compared to the “historical approach” when just the legal powers and rights mattered. On the other side, when there are

⁷⁶ For more detail refer to Hanouna, P., Sarin, A. and Shapiro, A.C. (2001), "Value of Corporate Control: Some International Evidence", Marshall School of Business, Working Paper, 2001.

⁷⁷ Such shareholders dispose of the so called prerogatives of control and these will be discussed in more detail later in this chapter.

⁷⁸ Also considerable empirical evidence has been carried out to monitor this issue.

⁷⁹ Private benefits of control are advantages available not to all of the company's shareholders but to their selected part only.

⁸⁰ For more detail refer to Pratt, S.P. (2001), "Business Valuation Discounts and Premiums", John Wiley & Sons, Inc., New York, 2001.

significant benefits that could be extracted from exercising some prerogatives of control⁸¹, the applicable control premium can rise substantially.

In order to make the reader of the thesis more familiar with the terminology linked to the control premiums and discounts we include the definitions of the most often used terms. As none of the authors explicitly states the full scope of related definitions, we decided to provide the reader with the definitions stated in the American Society of Appraisers' Business Valuation Standards:⁸²

- (i) **Control** – the power to direct the management and policies of a business enterprise;
- (ii) **Control Premium** – an amount or a percentage by which the pro rata value of a controlling interest exceeds the pro rata value of a non-controlling interest in a business enterprise, to reflect the power of control;
- (iii) **Discount for Lack of Control** – an amount or percentage deducted from the pro rata share of value of 100% of an equity interest in a business to reflect the absence of some or all of the powers of control;
- (iv) **Majority Control** – the degree of control provided by a majority position;
- (v) **Majority Interest** – an ownership interest greater than 50% of the voting interest in a business enterprise;
- (vi) **Minority Discount** – a discount for lack of control applicable to a minority interest and can be calculated as follows:⁸³

$$MinorityDiscount = 1 - \left(\frac{1}{1 + Control\ Premium} \right) \quad (6); \text{ and}$$

- (vii) **Minority Interest** – an ownership interest less than 50% of the voting interest in a business enterprise.

⁸¹ Prerogatives of control are a large topic closely related to the control premiums and will be discussed in the next subchapter in more detail.

⁸² In American Society of Appraisers (2008), "ASA Business Valuation Standards", July 2008 Updated Version.

⁸³ Taken from Prodělal, F. (2004), "The Minority Discount and Control Premium within the Company Valuation", A&CE Consulting, 2004; this duality issue of control premium and minority interest discount has been touched by many professionals and their articles, e.g., Emory, J.D. (1999), "Discounts and Premiums in Business Valuation: What Business Lawyers Should Know", Business Laws, Inc., November 1999.

3.2 Prerogatives of Control

There cannot be any doubt that there is a prevailing perception of less risk in an investment, such as merger or acquisition, where the investor concerned has the privilege to control the business' future course of action. Pro rata controlling interest in a company usually encompasses a higher price consideration compared to a pro rata minority interest in the same business. On the other hand, the adjustment for lack of control or a minority interest⁸⁴ is appropriate, providing the business was initially valued as a whole but now just its part is being offered to the market. That is, when a company or a partnership interest is valued as a single entity (e.g., as if 100% of the stock were going to be sold) the value of a portion of the ownership interest is usually not pro rata; an adjustment to pro rata value is generally observed in the marketplace when dealing with an interest that provides less than full and comprehensive control.⁸⁵

But what is that valuable on the controlling interest compared to the ordinary minority shareholdings? The answer is relatively simple and has been touched even in the previous subchapter. The holder of the controlling stake has the power to control and dictate a company's management and policies. This competence is usually referred to as prerogatives of control and their list can be found in every basic work⁸⁶ on the value of control. The basic list of prerogatives of control includes:⁸⁷

- (i) Appoint or change operational management;
- (ii) Appoint or change members of the board of directors;
- (iii) Determine management compensation and perquisites;
- (iv) Set operational and strategic policies and change the course of the business;

⁸⁴ In the literature, in addition to the notion of an interest suffering from "lack of control" there has been used another term dealing with the same phenomenon which is a "minority interest". Both of these terms are used interchangeably in this thesis.

⁸⁵ In Weaver, W.C. (1998), "Discounts and Other Adjustments to Fair Market Value Estimates", University of Central Florida, College of Business Orlando, 1998.

⁸⁶ In e.g., Booth, R.A. (2000), "Minority Discounts and Control Premiums in Appraisal Proceedings", Working Paper, 2000; Emory, J.D. (1999), "Discounts and Premiums in Business Valuation: What Business Lawyers Should Know", Business Laws, Inc., November 1999; Mard, M.J. (2001), "Financial Factors - Valuation Premiums and Discounts", The Licensing Journal, April 2001; Mercer, Z.C. (2002), "An Integrated Theory of Business Valuation", Mercer Capital, October 2002; or Weaver, W.C. (1998), "Discounts and Other Adjustments to Fair Market Value Estimates", University of Central Florida, College of Business Orlando, 1998.

⁸⁷ Ibid.

- (v) Decide what products and services to offer, what markets and locations to serve, and which customers categories to market and which not;
- (vi) Acquire, lease, or liquidate business assets, including plants, property, and equipment;
- (vii) Select suppliers, vendors, and subcontractors and award contracts;
- (viii) Negotiate and consummate mergers and acquisitions;
- (ix) Liquidate, dissolve, sell out, or recapitalize the company;
- (x) Register the company's equity or debt securities for an initial or secondary public offerings;
- (xi) Decide on the company's dividend policy;
- (xii) Sell a controlling interest in the company with or without participation of minority shareholders;
- (xiii) Change the capital structure;
- (xiv) Change the articles of incorporation or bylaws; or
- (xv) Block any or all of the above actions.

To sum it up, the prerogatives of control illustrate that the controlling shareholder is entitled to use all his or her rights and responsibilities to run the business for the benefit of the controlling shareholder.

3.3 Private Benefits of Control

The control may be valuable for two reasons. First, it grants the control owner the right to determine the course of the business, as described by the prerogatives of control. Second, control may provide the option to gain the so called private benefits through expropriation of wealth of non-controlling shareholders.⁸⁸ The theoretical literature often identifies private benefits of control as the "psychic" value some shareholders attribute simply to being in control. Although this may certainly be an

⁸⁸ As stated by Sansing, R. (1999), "Economic Foundations of Valuation Discounts", Dartmouth College Paper, January 1999, the corporation cannot pay dividends to the majority (controlling) shareholder without paying an appropriate portion to the minority shareholders. However, in the corporation there exist numerous ways to choose policies that provide personal benefits to the controlling shareholder without violating any laws.

important factor in some cases, the historically paid multimillion premiums can only hardly be justified by a pure pleasure to command.⁸⁹

Although, the private benefits of control by definition are difficult to be measured, there have been proposed two main methods in the literature to quantify their size.⁹⁰ The first method focuses on firms with dual class shares and derives the value of control from the applicable voting premium.⁹¹ The second method observes the pricing in the sales of control block shareholdings. It measures the difference between the market price of the company's shares and the price per share offered in the transactions where the control block of shares was of the key interest. The second method is represented by the attitudes of e.g., Dyck and Zingales (2004a), or Hanouna, Sarin and Shapiro (2001).

As mentioned also by the authors of the scientific works, both of these methods have several shortcomings. For example, the control block premium includes also the basic control premium and can reflect many other aspects, such as the expected synergy effects or the relative bargaining power between the seller and the buyer. In the case of the voting premium, the problem might be that it is calculated from the prices defined by the minority shareholders, not by those benefiting from having the control. Additionally, there is a sample selection bias problem when the estimates of the private benefits are derived from dual class companies. It is simply because the voting rights are separated from the cash flow rights. Hence, private benefits of control are higher in firms with dual class shares compared to companies where there is applicable the one share-one vote rule.⁹²

The voting premiums estimates vary considerably across countries. To depict how great these differences can be we selected the extreme example presented by Zingales⁹³ who estimated the average voting premium applicable in the Italian environment of 82% on one hand, and 10.5% applicable in the United States on the

⁸⁹ In Dyck A. and Zingales L. (2004), "Private Benefits of Control: An International Comparison", *The Journal of Finance*, Vol.59, No. 2, April 2004, p. 537-600.

⁹⁰ In Doidge, C. (2003), "U.S. Cross-listings and the Private Benefits of Control: Evidence From Dual Class Firms", University of Toronto, January 2003.

⁹¹ The voting premium can easily be derived as the difference in market prices between the two groups of shares with similar or identical dividend rights, when the first group has and the second group lacks the voting competence, *ceteris paribus*.

⁹² In Doidge, C. (2003), "U.S. Cross-listings and the Private Benefits of Control: Evidence From Dual Class Firms", University of Toronto, January 2003.

other hand. As it is relatively difficult to argue that the reason for the huge difference between the voting premiums in Italy and the United States stems from more remarkable takeover activity in Italy than in the United States⁹⁴, it simply has to reflect significant differences of the private benefits between these two countries. This only means that in Italy, compared to the United States, the controlling shareholders weaken the property rights of the minority shareholders.

To generalize this finding, in most of the countries outside the Anglo-Saxon world (i.e., the US and the UK), the probability of the control contest is not high. Therefore, the control premium can be used as a good proxy for the private benefits of control and differences in voting premiums across countries can be roughly interpreted as differences in the size of the private benefits of control. Further, the differences and gaps in the private benefits of control across countries can be explained by different levels of minority shareholders' protection.⁹⁵

3.4 Factors Influencing the Control Premium

Prior to any specific calculations of control premiums it is extremely important to learn that there are numerous factors that can influence the degree of control in the respective company. Consequently, these affect the magnitude of the discount for the minority interest (lack of control) if starting from the control level of value, or the control premium if starting from the minority interest level of value. In the following section of the thesis we would like to mediate the results of Pratt (2001) related to this important issue. Hence, the key factors affecting the degree of control are as follows:⁹⁶

- (i) **Anything less than 100%** - Any ownership shareholding lower than 100% enables minority shareholders to attack some prerogatives of control.⁹⁷ It heavily depends on the parameters of the articles of

⁹³ For more detail refer to Zingales, L. (1995), "What Determines the Value of Corporate Votes?", Quarterly Journal of Economics, Vol.110, November 1995.

⁹⁴ In fact, not only in the times when Zingales' work was published, but also today the opposite is most likely true.

⁹⁵ In Doidge, C. (2003), "U.S. Cross-listings and the Private Benefits of Control: Evidence From Dual Class Firms", University of Toronto, January 2003.

⁹⁶ For more detail refer to Pratt, S.P. (2001), "Business Valuation Discounts and Premiums", John Wiley & Sons, Inc., New York, 2001.

⁹⁷ For example, if the company intends to sell out or take any other corporate action, any minority shareholder might be able to exercise disapproving stockholder rights. Such a step may considerably raise the costs of the whole corporate action or even disable it.

association valid for each respective company, but taking into account the number of ways a minority shareholder can cause inconvenience to a majority shareholder, the reduction to a control premium or the discount for lack of control can reach substantial amounts in companies where the controlling shareholder holds less than pure 100% stake;

- (ii) **Supermajority requirements** – It is relatively common among companies and organizations that more than just 50% plus one share is needed in case any fundamental corporate action is to be approved.⁹⁸ Such amount of votes which have to in favour of the fundamental corporate action in order to materialize is called the supermajority of the votes. If a certain block of shares establishes right to decide on certain actions, but is not big enough to control other corporate actions, it falls in between pure minority interest value and control value.⁹⁹ Nevertheless, if the size of the block falls into such neutral area, an analyst has to be extremely cautious about all the available characteristics, he or she has to take into consideration in order to properly value the respective discount or premium;
- (iii) **Shareholder oppression statutes** – In some countries¹⁰⁰ under specified circumstances, minority shareholders can initiate a lawsuit to dissolve the partnership or corporation and be paid their pro rata share of the proceeds from the liquidation. Where exists such a possibility for the minority shareholders, the controlling shareholders have the right to prevent such oppression by paying the minority shareholders a fair value for their shares. If there is a prospect for such a dissolution, the presence of such a statute may slightly reduce the applicable control premium or minority discount;

⁹⁸ Such a fundamental corporate action could be represented by the approval of a merger on one hand, or sell out on the other hand.

⁹⁹ For example, if one starts with a control value, lack of absolute control has to be awarded by some discount for minority interest. We have not find any empirical studies related to the quantification of such a discount but as stated by Pratt (2001), such discounts usually fall into the range of 5 to 15%. On the other hand, if a minority interest block of shares has the power to prevent certain corporate action, such a block of shares is usually awarded some premium, the so called blockage premium, beyond the pure minority interest level of value. As in the previous case, this premium falls into the range of 5 to 15%.

¹⁰⁰ Such possibility exists e.g., in several states of the United States.

- (iv) **Interest of 50%** - the interest of 50% has a specific status as it is neither pure control nor pure minority. The 50% interest usually has enough power to prevent certain corporate actions to happen, but does not necessarily have to have enough power to cause them to happen.¹⁰¹ In case there are two 50% interests in one company and only one of them can exercise some prerogatives of control under a contractual agreement, the two 50% interests do not have to suffer from the same lack of control;
- (v) **Legal and regulatory constraints** – Legal and/or regulatory limitations can simply prevent the control owner from exercising the prerogatives of control to the complete extent. From the previous statement it is clear that these limitations narrow the gap between minority and control levels of value. Hence, the potential applicable control premium or minority interest discount are reduced compared to those without these legal limitations;
- (vi) **Minority shareholder ability to elect directors** – Some minority interest blocks of shares can encompass the right to elect one or more of the company's directors. This attribute stems from either of two circumstances. First, cumulative voting¹⁰², and second, contractual agreement¹⁰³. Regardless, which of the two above mentioned circumstances allows to the holder of the minority interest to exercise this additional right, it generally tends to reduce the minority interest discount.

¹⁰¹ We have not find any empirical data for guidance in quantification of the 50% interest percentage premiums over minority interest discount or discounts from control value. However, as stated in Pratt (2001) the 50% interest sometimes is discounted by 15% from the control value to reflect the lack of control.

¹⁰² It is a common phenomenon in vast majority of contemporary companies that a majority of the company's shares can elect all its directors; however, there still exist some companies which allow for cumulative voting and through that enable to minority shareholders to elect one or more directors. The cumulative voting works the following way – if there are say five directors to be elected, every 20% of the company's shares have the right to elect one director, no matter whether the majority shareholder agrees or not.

¹⁰³ Regarding the contractual agreement, there can be and in the real life there sometimes are various reasons why certain blocks of stock can be granted a contractual competence to elect for one or more directors in the company. Such institute can usually be found in companies which engaged in venture capital financing.

3.5 Valuation Methodology Effect on the Minority Interest Discount and Control Premium

Let us now recall the most important idea of the second chapter, which states that no premium or discount is meaningful unless the valuation basis is well-defined. Moreover, it does not have to be sufficient to define the so called valuation basis, but it is crucial to understand the relationship of the premium or discount to the basis. This subchapter addresses the issue of the major valuation approaches and the consequences they imply regarding the appropriateness of the control premiums and minority interest discounts. Whereas in some cases the application of control premium or lack of control discount is relatively straightforward, in other cases there may be identified some controversy issues regarding their application.

As we have discussed the general aspects and attributes of the basic valuation methodologies in the first chapter of this thesis. In the following section of the thesis we would like to stress the main consequences and implications each of these valuation principles has from the premiums and discounts application point of view.¹⁰⁴

3.5.1 Income Approach and Control Premium Application

As mentioned in the first chapter, the income approach can arrive at either control or minority interest level of value. As stated by Pratt (2001), there is a general agreement among analysts on the extent, to which the income approach produces a minority interest or control value. It primarily depends on the level of the cash flows or revenues being discounted. I.e., if the projected cash flows are those, the controlling shareholder could expect to obtain, a control premium would already be incorporated in the value of the shareholding.¹⁰⁵

However, some analysts still believe that the income approach always arrives at the publicly traded minority interest level of value, since both the CAPM and build-up method produce discount and capitalization rates from minority interest transaction

¹⁰⁴ For more detail refer to Pratt, S.P. (2001), "Business Valuation Discounts and Premiums", John Wiley & Sons, Inc., New York, 2001.

¹⁰⁵ According to this logic the same would be true in the case of cash flows from the minority shareholding owner's perspective and the consequent incorporation of minority interest discount.

data in the public markets. We agree with the quote of Roger Ibbotson¹⁰⁶ stating: “When you are purchasing a company you are acquiring the ability to potentially control future cash flows. To acquire this option to exercise control, you must pay a premium. Holding all else constant, it should not impact the discount rate.”¹⁰⁷

In order to precisely estimate the control value when the minority level of value is known, two approaches are mostly used. First, adjustment of the cash flows upward to the level a controlling owner could expect¹⁰⁸; second and most common adjustment is the introduction of some amount of a long-term debt which causes substitution of the purely equity capital structure. Thus it lowers the overall cost of capital and raises the present value of the projected cash flows.

3.5.2 Market Approach and Control Premium Application

Within the framework of the guideline of the merged and acquired company method, if control transactions are used to value some interest lower than the controlling one, generally some discount for lack of control shall be granted. As a result, when using the available empirical data, an analyst is obliged to determine whether the consideration paid was the price for the common equity only or a full “deal price”¹⁰⁹.

In case the consideration paid was formed by the “deal price”, the value of the debt and/or preferred stock must be subtracted prior to the application of the discount for lack of control, since it applies only to the common equity. On the other hand, within the framework of the guideline publicly traded company method, which is usually based on the application of valuation multiples observable in the daily public stock trading, it would be inappropriate to apply any minority interest discounts, since the transactions forming the plot of this approach are minority interest operations.¹¹⁰

¹⁰⁶ As stated in Pratt, S.P. (2001), "Business Valuation Discounts and Premiums", John Wiley & Sons, Inc., New York, 2001.

¹⁰⁷ For controlling investors it is rational to pay premium if they expect to increase their future cash flows from the project, not to pay control premium because they are ready to accept a lower expected rate of return.

¹⁰⁸ This method is preferred to the application of a percentage control premium to a minority value.

¹⁰⁹ Such “deal price” reflects both the entire capital structure, including the assumed debt, and very likely the preferred stock.

¹¹⁰ To the contrary, it is reasonable to apply a control premium when the method of the guideline publicly traded companies is used as a starting point for valuation of a controlling interest, following the same logic.

3.5.3 Asset-Based Approach and Control Premium Application

Both of the most often used methods within this approach, which are the net asset value method¹¹¹ and the excess earnings method¹¹², arrive at the control value. As a result, if valuing a controlling interest, no additional control premium is desirable, whereas if valuing anything less than the controlling interest it is usually appropriate to apply the discount for lack of control.

3.6 Publicly Traded Minority Stocks and Control Premium

Since 1990 there has been a debate whether publicly traded minority shareholdings reflect control premium or minority interest discount. This debate was triggered by the Eric Nath's paper published in the Business Valuation Review.¹¹³ Nath in short stated a hypothesis arguing that most public companies tend to trade at or close to their takeover, or controlling interest values. If this can be proved, any valuation approaches based on the public companies' data analyses should yield value that represents a controlling interest value rather than minority interest one. Moreover, discount rates, which are used for discounting company's future cash flows, derived from the public market are controlling interest discount rates which when applied yield controlling interest value.

Nath stated four main reasons how the investor can justify paying sometimes enormous premiums:¹¹⁴

- (i) The target company's shares may be undervalued if the company is not well managed or otherwise underutilized. Such a situation can allow an investor to make a considerable profit if the investor is able to make the target company run more effectively, even after paying a premium;

¹¹¹ Also called the asset accumulation method adjusts all the tangible and intangible assets to their current values and subtracts the liabilities at the same time.

¹¹² As stated by Pratt (2001), in this method all the tangible assets are adjusted to their current values and these values are consequently multiplied by a reasonable rate of return on these assets. If the company's return is higher than this number, the whole difference is called the excess earnings. The excess earnings are capitalized at the rate that takes into consideration the riskiness of those earnings, and the result of these calculations represents the collective value of all intangible assets. The sum of the tangible and intangible assets values equals the value of the company.

¹¹³ For more detail refer to Nath, E.W. (1990), "Control Premiums and Minority Interest Discounts in Private Companies", Business Valuation Review, Vol. 9, No. 2, June 1990.

¹¹⁴ Ibid.

- (ii) Even if the company is well managed and the shareholder's value is maximized, the fact that this is not effectively communicated to the market by the company's management may cause that company's shares are significantly undervalued by the market;
- (iii) If the company is well managed and it has been effectively communicated to the public, there still may be enough space for a strategic investor to pay the premium while taking over the company – mostly due to synergistic effect; and
- (iv) From time to time, there appears someone in the market who simply overestimates the target's value.

If a company does not meet any of the first three above mentioned criteria, it can hardly be suspected to become a takeover target in a short run. This implies that either majority of the publicly traded companies is formed by permanent takeover targets or a substantial control premium can only hardly be applicable and payable by any rational investor. As the first part of the previous statement can hardly be perceived as a truthful viewpoint, the second part very much likely better reflects the reality. To his study, Nath also attached a table supporting the statement that only a minor part of the public equity dollars traded on the three major US stock exchanges were purchased in the M&A transactions. As depicted in the following Table 5, in 1987 and 1988¹¹⁵, only 3.2% and 4% respectively of total publicly traded equities materialized through takeover transactions.

Table 5 - Nath's Research Output

Stock Exchange	Year End Total Market Values [\$ bn]			Takeover Values [\$ bn]			Percent of Total Exchange Traded Value [%]		
	1987	1988	1989	1987	1988	1989	1987	1988	1989
NYSE	2,200.0	2,460.0	3,030.0	63.0	79.7	98.5	2.9%	3.2%	3.3%
AMEX	99.2	112.2	130.8	2.2	7.8	6.4	2.2%	7.0%	4.9%
NASDAQ	325.5	338.7	386.3	17.6	27.4	18.3	5.4%	8.1%	4.7%
<i>Total</i>	2,624.7	2,910.9	3,547.1	82.8	115.1	123.2	3.2%	4.0%	3.5%

Source: Nath¹¹⁶

¹¹⁵ The year 1988 was the most active year for takeovers in the Nath's sample.

¹¹⁶ In Nath, E.W. (1990), "Control Premiums and Minority Interest Discounts in Private Companies", Business Valuation Review, Vol. 9, No. 2, June 1990.

Another issue we feel crucial to stress here once again is the fact that most takeovers are at least partially motivated by some strategic considerations.¹¹⁷ The so-called synergistic premiums paid in the strategic takeover transactions represent sometimes a big impediment for an appraiser trying to find the correct fair market value. Another problem on the way to ascertain the most appropriate price for the target company is that the control premium statistics can be very misleading. Some statistics for example exclude all the transactions where a negative control premium was detected, some on the other hand include them, some exclude negative premium transactions only after exceeding certain level of the negative premium (e.g., 15%), etc. We have not found any single case, where each transaction in the sample would be separately examined and tested whether the negative premium is reasonable or not.¹¹⁸ The same holds in the case of excessive positive control premiums paid (e.g., 100% and more). Last but not least, the problem an analyst has to face is that the takeover market does not necessarily have to be a reliable indicator of the true value compared to other approaches. Generally, it is true that each takeover process is unique and time-specific, and some parties involved in the takeover transaction overpay for an acquisition.¹¹⁹

Michael Bolotsky in his paper¹²⁰ supports Nath's hypothesis and recognizes that shares of certain public companies are in fact traded at prices roughly equal to levels a buyer of the entire company would be willing to pay. At the same time he states that this fact does not in any respect influence the status of the individual shares which still is a representative of minority and lacks prerogatives of control. Therefore he does not see any incentive to deduct a minority interest discount from publicly traded minority interest values.

To develop this viewpoint a little bit more, let us raise a sequence of rhetorical questions. What happens to minority dissenting shareholders who simply have to sell

¹¹⁷ As mentioned by Mercer, Z.C. (2002a), "An Integrated Theory of Business Valuation", Mercer Capital, October 2002; or Nath, E.W. (1990), "Control Premiums and Minority Interest Discounts in Private Companies", Business Valuation Review, Vol. 9, No. 2, June 1990.

¹¹⁸ E.g., due to some objective reasons, such as market decline, deterioration of the targets value in during the takeover process.

¹¹⁹ The question always is, or should be, whether if a "fool" is willing to pay twice as much a company is really worth, does it make a reliable measure of a subject company's value.

¹²⁰ For more detail refer to Bolotsky, M.J. (1991), "Adjustments for Differences in Ownership Rights, Liquidity, Information Access, and Information Reliability: An Assessment of Prevailing Wisdom Versus the Nath Hypothesis", Business Valuation Review, September 1991.

their shares to the currently controlling shareholder?¹²¹ Is the minority discount applicable then? How does the real life experience handle a topic like this? All of these questions have been tackled by numerous theoreticians. Let us name e.g., Booth (2000) or Cavendish and Kammerer (2008). Both of the studies mention the mainstream point of view of the US courts saying that minority shareholders are still shareholders and the shares they hold are legally equal to every other share of the company of the same class. Moreover, the fact that these shares cannot exercise control gives no reason to assign minority shares a lesser value than other comparable shares. Thus, a dissenting shareholder should receive a pro rata share of the company's value without application of any discounts simply because a minority lacks control. The main rationale for the inapplicability of the discount is that the sale to the majority shareholder is different from the sale to a third party, because the sale to the former increases the interest of the majority shareholder who already retains control.

3.7 Empirical Evidence of Control Premiums and Minority Discounts

As we have discussed the most important aspects concerning the theoretical background of the control premiums and minority discounts, we perceive it is the proper time to include the overview of the empirical works¹²² coping with the issue of the premiums and discounts application. All of the empirical studies we have for guidance while quantifying the control premiums and minority interest discounts have one thing in common – all of them are developed from the public markets for stocks or partnership interests. The available empirical data fall broadly into three main categories:¹²³

- (i) **Premiums paid for acquiring controlling interests in firms** – definitely belongs to the most extensively developed procedure of the control premium assessment. This method generally uses the following Equation (7) for control premium calculation:

¹²¹ Right now we have in mind e.g., the case of a squeeze-out of minority shareholders from the company.

¹²² The main aim of this overview is to present all the relevant key findings and results of the empirical studies.

¹²³ In Hanouna, P., Sarin, A. and Shapiro, A.C. (2001), "Value of Corporate Control: Some International Evidence", Marshall School of Business, Working Paper, 2001.

$$\text{Control Premium} = \left(\frac{\text{Share Purchase Price} - \text{Share Unaffected Price}}{\text{Share Unaffected Price}} \right)^{124} \quad (7)$$

However, besides control the premium paid in takeover attempts may represent a compensation for variety of other factors.¹²⁵ As a result, premiums paid within the takeover transactions can ideally serve as the upper bound for control premiums;

- (ii) **Dual class shares studies** – for the proper application of this method it is vitally important to find statistically significant sample of companies which are dual-listed.¹²⁶ The control premium developed by this method is relatively small in comparison to the previous approach and varies among countries¹²⁷; and
- (iii) **Examination of the pricing of block trades** – one of the pioneering works developing this approach is that one published by Barclay and Holderness (1989). It argues that the premiums at which it arrives reflect private benefits that accrue only to a block-holder because of their voting power.

In the following part of the thesis we will address the empirical studies on the scope of control premiums and discounts, which have been developed by the world notable economists. The studies were executed in different time periods, but as can be seen further, there are other factors considerably influencing the size of the final control premium or minority interest discount. One of the most influential factors is the method employed to develop the premium or discount in each respective case.

¹²⁴ We arrived to this equation by making some marginal adjustments to the formula mentioned by Officer (2005). The “Control Premium” was explained in the initial section of this chapter, “Share Purchase Price” is the pro rata price per share which was paid for company’s stake in the underlying transaction, and “Share Unaffected Price” stands for the market price of the share prior to the transaction long enough not to be affected by the market rumours regarding the underlying transaction.

¹²⁵ This topic has partially been discussed in the previous subchapter within the Nath’s understanding of the investors’ justification of paying sometimes enormous takeover premiums.

¹²⁶ The dual listing means that one company within the scope of its existence has issued two classes of shares. These two kinds of shares are identical, except that the former or the later does not dispose of any voting rights. Shares without voting rights are perceived as shares lacking control, thus are usually traded at a lower price which compared to the second group of shares gives the value of the control premium.

¹²⁷ For example, Horner (1988) found a 20% premium for Switzerland, Levy (1983) found an average voting premium of 45.4% for Israel, and Rydqvist (1996) found a 6.5% voting control premium in Sweden.

- (i) **Barclay and Holderness Study**¹²⁸ - this study analyzed the pricing of 63 block trades between 1978 and 1982 involving at least 5% of the common stock of the companies listed at NYSE or AMEX. Barclay and Holderness found that these blocks of shares were typically priced at an average premium of 20% compared to the post-announcement exchange price.
- (ii) **Bellinger Study**¹²⁹ - this study is one of those which tried to derive the minority interest discount through the scope of voting rights connected to the acquired stake of company's shares. It differentiates among six groups of transactions based on two characteristics – voting right on one side and the fact whether the rest of the company's shares are owned by a majority owner or not on the other side. For more detail refer to the following Table 6.

Table 6 - Bellinger Study Output

Share of Total Company's Stock in the Transaction	Minority Interest Discount per Share	
	No Majority Shareholder	Majority Shareholder Exists
0 - 9.9%	30%	50%
10 - 24.9%	25%	35%
25 - 49.9%	15%	-
50 - 74.9%	10%	-
75 - 94.9%	5%	-
95 - 100%	0%	-

Source: Mařík¹³⁰

- (iii) **Houlihan, Lokey, Howard and Zukin Study**¹³¹ - this study provides analysis of premiums based upon actual mergers and acquisitions of one company by another. Its gives quarterly results from beginning of 1995 to the first quarter of 1997. For more detail refer to the following Table 7.

¹²⁸ In Barclay, M.J. and Holderness, C.G. (1989), "Private Benefits from Control of Public Corporations", *Journal of Financial Economics*, Vol. 25, 1989.

¹²⁹ In Mařík, M. a kolektiv (2003), "Metody oceňování podniku, Proces ocenění - základní metody a postupy", *Ekopress*, 2003.

¹³⁰ Ibid.

¹³¹ In Weaver, W.C. (1998), "Discounts and Other Adjustments to Fair Market Value Estimates", University of Central Florida, College of Business Orlando, 1998.

Table 7 - Houlihan, Lokey, Howard and Zukin Study Output

Quarter	1995		1996		1997	
	Minority Interest Discount	No. of Transactions	Minority Interest Discount	No. of Transactions	Minority Interest Discount	No. of Transactions
1	26%	70	24%	114	23%	111
2	27%	62	19%	90	N/A	N/A
3	25%	74	19%	130	N/A	N/A
4	21%	127	22%	124	N/A	N/A

Note: N/A – not available; Source: Weaver¹³²

- (iv) **Kasper Study**¹³³ - this study recorded means and medians of minority interest discounts and control premiums based upon market transactions that materialized between 1980 and 1994. Kasper study results are summarized in the following Table 8.

Table 8 - Kasper Study Output

Year	No. of Transactions	Control Premium (Mean)	Control Premium (Median)	Minority Interest Discount (Mean)	Minority Interest Discount (Median)
1980	169	49.9%	44.6%	33.3%	30.8%
1981	166	48.0%	41.9%	32.4%	29.5%
1982	176	47.4%	43.5%	32.2%	30.3%
1983	168	37.7%	34.0%	27.4%	25.4%
1984	199	37.9%	34.4%	27.5%	25.6%
1985	331	37.1%	27.7%	27.1%	21.7%
1986	333	38.2%	29.9%	27.6%	23.0%
1987	237	38.3%	30.8%	27.7%	23.5%
1988	410	41.9%	30.9%	29.5%	23.6%
1989	303	41.0%	29.0%	29.1%	22.5%
1990	175	42.0%	32.0%	29.6%	24.2%
1991	137	35.1%	29.4%	26.0%	22.7%
1992	142	41.0%	34.7%	29.1%	25.8%
1993	173	38.7%	33.0%	27.9%	24.8%
1994	260	41.9%	35.0%	29.5%	25.9%
Average		41.4%	34.5%	29.8%	26.3%

Source: Prodělal¹³⁴

¹³² Ibid.

¹³³ In Prodělal, F. (2004), "The Minority Discount and Control Premium within the Company Valuation", A&CE Consulting, 2004.

¹³⁴ Ibid.

- (v) **Lyons and Wilczynski Study**¹³⁵ - this study interprets data from the 20 year period from 1968 to 1987 published by Mergerstat Review. The authors derived the minority interest discount from premiums paid in the M&A transactions and found that the most typical discount was between 27 and 28%.¹³⁶
- (vi) **Mercer Study**¹³⁷ - this study is, as well as the previous one, based upon the Mergerstat data. Mercer points out that while implied minority discounts can be easily estimated from Mergerstat data on one hand, there are usually wide distributions around the mean estimates on the other hand. He further comments on the error of bias connected to the Mergerstat related statistics, since they exclude all transactions with for example negative premiums. Mercer adjusted his analysis to reflect all the weaknesses he perceived the Mergerstat data suffered from and returned to results which can be summarized by the following Table 9.

Table 9 - Mercer Study Output

Statistics Description	Mean Minority Interest Discount	Median Minority Interest Discount
All transactions included	31%	23%
Excluded transactions with premiums < 0% and > 150%	26%	22%
Excluded transactions with premiums > 150% and included transactions with premiums < 0%	22%	20%
Excluded transactions with premiums > 100% and included transactions with premiums < 0%	19%	19%

Source: Mercer¹³⁸

- (vii) **Pratt Study**¹³⁹ - this is a study reporting discounts for lack of control which are taken from the market data. It assumes that in the transactions where the acquirer taking control over the target is often willing to pay a premium above the market price for the respective amount of stocks

¹³⁵ In Lyons, R.P. and Wilczynski, M.J. (1989), "Discounting Intrinsic Value", Trusts & Estates, February 1989.

¹³⁶ Since the Mergerstat data are based upon the shares' price only five business days prior to the public announcement of the intended merger or acquisition and the market rumours seem to start influencing the market behaviour much earlier, these data are likely to be biased downward.

¹³⁷ In Mercer, Z.C. (1997), "Quantifying Marketability Discounts", Peabody Publishing, 1997.

¹³⁸ Ibid.

¹³⁹ In Pratt, S.P. (1993), "Valuing Small Businesses and Professional Practices", Second Edition, Irwin Professional Publishing, January 1993.

ensuring control. Pratt in his study presents the minority interest discount as a percentage discount from the buyout price at which target's stocks were selling shortly before the acquisition announcement from 1980 to 1991. The study's results are summarized in the following Table 10.

Table 10 - Pratt Study Output

Year	Minority Interest Discount (Mean)	Year	Minority Interest Discount (Mean)	Year	Minority Interest Discount (Mean)
<i>1980</i>	33.3%	<i>1984</i>	27.5%	<i>1988</i>	29.5%
<i>1981</i>	32.4%	<i>1985</i>	27.1%	<i>1989</i>	29.1%
<i>1982</i>	32.2%	<i>1986</i>	27.6%	<i>1990</i>	29.6%
<i>1983</i>	27.4%	<i>1987</i>	27.7%	<i>1991</i>	26.0%

Source: Pratt¹⁴⁰

(viii) **Schild Study**¹⁴¹ - This study is in fact only a review and analysis of several previous studies. Schild refers to the study carried out by Munrow, Park and Johnson which found, in the sample of 32 non-distributing equity partnerships, an average minority interest discount of 64%. In the sample of 87 equity partnerships regularly distributing funds they found an average discount of 41%. Schild further stresses a study by Kam, Schroeder and Smith, which in the case of non-distributing partnerships identified discounts exceeding 80%.

¹⁴⁰ Ibid.

¹⁴¹ In Schild, J. (1996), "Discounts for Minority Interests", Business Valuation Review, Vol. 15, No. 4, December 1996.

4. Liquidity/Marketability Premium

The concept of marketability/liquidity premium¹⁴² is relatively straightforward. To put it briefly, any asset that is liquid, compared to the same asset with the only difference hidden in the fact that the later does not have the attribute of liquidity, is more valuable. Hence, if the underlying asset lacks liquidity¹⁴³, such an asset has to be of a lower value. The difference between the values of a liquid and less liquid (illiquid) asset is usually called a liquidity premium or lack of liquidity discount, if we move the other way around.

The effect of liquidity has been widely discussed in numerous financial articles.¹⁴⁴ The authors of vast majority if not all of them concluded that the asset's pricing improves with greater marketability, *ceteris paribus*. As it is relatively simple to decide whether any lack of liquidity discount is applicable or not, the problem arises when we have to quantify the extent of such a discount. It is the reason why in this chapter we will try to provide the reader with all the key aspects needed to be taken into consideration while establishing the proper lack of liquidity discount. Moreover, in the last part of this chapter we will introduce the findings of an extensive research connected to the assessment of the lack of liquidity discount.

4.1 Basic Definitions

As will be clearer after reading the definitions of the two seemingly different terms, which are “marketability” and “liquidity”, there is a broad consensus¹⁴⁵ regarding their interchangeability. This holds at least in cases when selected assets are concerned and these are subject to valuation principles. Marketability usually represents the relative ease and promptness with which the underlying asset may be sold at a representative current price without material concession in price caused by the need to

¹⁴² The ambiguity of the marketability and liquidity will be described further in this chapter. Nevertheless, even now we can say that these terms will be used interchangeably.

¹⁴³ Such a fact can easily be discovered if we compare the underlying asset to its freely marketable counterparty.

¹⁴⁴ E.g., Ben-Rephael, A., Kadan, O. and Wohl, A. (2008); Bajaj, M., Denis, D.J., Ferris, S.P. and Sarin, A. (2001); Bolotsky, M.J. (1991); Bowers, H. and Stephenson, T. (2004); Bruner, R.F. and Palacios M. (2004); Cimasi, R.J. (2007); Damodaran, A. (2005); Davis, J.D. (2005); Feldman, S.J. (2002); Chafkin, M.J. (2007); Paschall, M.A. (1994); Robak, E. (2007a, 2007b); Schlueter, R.A. (2004); and many others.

¹⁴⁵ Our statement can be proved e.g. by studies carried out by Pratt, S.P. and Niculita, A.V. (2008) or Williams, P. and Linder, J. (2001).

sell. Liquidity generally stands for the amount of time required to convert the underlying asset into cash or pay a liability. Moreover, for non-current assets, liquidity refers to marketability.¹⁴⁶ As a result of all the above mentioned aspects, we decided to use both of the terms, i.e. liquidity and marketability, as equivalents.

As in the case of the terminology connected to the control premiums and discounts we decided to provide the reader with the definitions of the most often used terms related to the topic of liquidity discounts' application. As none of the authors treating the liquidity discounts explicitly states the full scope of the interrelated definitions, we decided to mention the definitions stated in the American Society of Appraisers' Business Valuation Standards:¹⁴⁷

- (i) **Liquidity** – the ability to readily convert an asset, business, business ownership interest, security or intangible asset into cash without significant loss of a principle;
- (ii) **Marketability** – the capability and ease of transfer or saleability of an asset, business, business ownership interest, security or intangible asset;
- (iii) **Discount for Lack of Liquidity** – an amount or a percentage deduced from the value of an ownership interest to reflect the relative inability to quickly convert property into cash; and
- (iv) **Discount for Lack of Marketability** – an amount or percentage deduced from the value of an ownership interest to reflect the relative absence of marketability.

4.2 Factors Influencing Discount for Lack of Marketability

We have already explained the basic ideas behind the application of the discounts for lack of marketability and that marketability has an inherent value to investors. The inherent value of marketability lies in the fact that its lack increases potential opportunity costs for the holder of an asset. Consequently, assets' values are positively correlated with the assets' marketability, ceteris paribus. Putting it the other way around, expected returns from holding the underlying asset are negatively

¹⁴⁶ For further detail see Pratt, S.P. and Niculita, A.V. (2008), "Valuing a Business: The Analysis and Appraisal of Closely Held Companies", Fifth Edition, The McGraw-Hill Companies, Inc., 2008.

¹⁴⁷ In American Society of Appraisers (2008), "ASA Business Valuation Standards", July 2008 Updated Version.

correlated with the asset's marketability. Hence, the higher expected return for less marketable asset reflects a compensation to an investor for bearing the additional risk in the form of opportunity costs which is caused by the inability to quickly convert the asset into cash, with a minimal price impact.¹⁴⁸

On the other hand, so far we have not stressed that the whole environment surrounding the marketability is not a game with a black or white result only. Taking a company ownership interest as an example of the underlying asset, since there are countless degrees of marketability, one can hardly say that such a stake is purely "marketable"¹⁴⁹ or "non-marketable". The degree of marketability depends on all the circumstances present and applicable to each of the companies whose ownership interest is being valued at the time being.

In the period when the lack of marketability started becoming a phenomenon a fixed illiquidity discount¹⁵⁰ with the appraiser's subjective judgement determining its range used to be applied.¹⁵¹ This situation is evident in the first studies related to the determination of the lack of marketability discount and will be discussed in a more detailed way in the upcoming subchapter. Compared to the early stages of the discount assessment, today another approach has been employed. This could be called a firm-specific discount evaluation method. As stated by Damodaran (2005a) much of the theoretical and empirical discussions support the view that lack of marketability discounts should vary across businesses.

Below we are trying to name and briefly characterize the most often mentioned factors influencing the degree of marketability. Despite the lack of any officially accepted formula assessing the impact of these factors, we are convinced that the comprehensive assessment of these factors will provide the appraiser with the guidance on a reasonable range of discount for lack of marketability. The most often mentioned factors are as follows:

¹⁴⁸ In Bajaj, M., Denis, D.J., Ferris, S.P. and Sarin, A. (2001), "Firm Value and Marketability Discounts", *Journal of Corporate Law*, Vol. 27, No. 1, 2001.

¹⁴⁹ The term "freely tradable" would be more appropriate in the case of a listed company.

¹⁵⁰ In a better case a range of the lack of marketability discount has been indicated.

¹⁵¹ In Damodaran, A. (2005a), "Marketability and Value: Measuring the Illiquidity Discount", *Stern School of Business*, July 2005.

- (i) **Liquidity of assets owned by the company**¹⁵² - the fact that a private company itself might be difficult to sell may be moderated if the firm's assets are highly liquid¹⁵³;
- (ii) **"Put" rights**¹⁵⁴ - their existence generally belongs to the most powerful factors reducing the discount for lack of marketability. The contractual right to sell the ownership interest under the pre-agreed circumstances (incl. the selling price) provides the holder of the ownership interest with such strong guarantees which may even cause a full elimination of any discount for lack of marketability at the end of the day;
- (iii) **Potential buyers' community**¹⁵⁵ - the existence of a substantial number of potential investors could dampen the lack of marketability discount (sometimes one strong potential investor is sufficient to have the same effect);
- (iv) **Financial stability of the firm and volatility of earnings**¹⁵⁶ - generally, a financially health company with a strong flow of revenues and positive cash flow with low volatilities should be subject to a lower illiquidity discounts compared to companies suffering from losses and a negative cash flow generation power;
- (v) **Restrictions on the transfer**¹⁵⁷ - numerous closely held companies are subject to provisions that restrict the rights of the holder of the ownership interest to transfer the stock. Any limitation of the owner's rights connected to the transferability of his or her interest in the company negatively influences marketability;

¹⁵² Ibid.

¹⁵³ Such liquid assets could be represented e.g., by cash or marketable securities.

¹⁵⁴ In Pratt, S.P. and Niculita, A.V. (2008), "Valuing a Business: The Analysis and Appraisal of Closely Held Companies", Fifth Edition, The McGraw-Hill Companies, Inc., 2008.

¹⁵⁵ In Damodaran, A. (2005a), "Marketability and Value: Measuring the Illiquidity Discount", Stern School of Business, July 2005; and Pratt, S.P. and Niculita, A.V. (2008), "Valuing a Business: The Analysis and Appraisal of Closely Held Companies", Fifth Edition, The McGraw-Hill Companies, Inc., 2008.

¹⁵⁶ In DiMattia, R.D. (2008), "Controlling Interests - Discount for Lack of Marketability: The Empirical Evidence", CPA Expert, Summer 2008; and Damodaran, A. (2005a), "Marketability and Value: Measuring the Illiquidity Discount", Stern School of Business, July 2005.

¹⁵⁷ In Pratt, S.P. and Niculita, A.V. (2008), "Valuing a Business: The Analysis and Appraisal of Closely Held Companies", Fifth Edition, The McGraw-Hill Companies, Inc., 2008; and DiMattia, R.D. (2008), "Controlling Interests - Discount for Lack of Marketability: The Empirical Evidence", CPA Expert, Summer 2008

- (vi) **Interest's size**¹⁵⁸ - the empirical evidence usually suggests that larger blocks tend to have larger illiquidity discounts compared to the smaller stakes of the same company. There could be several reasons behind, but the most prominent one is likely connected with the (iii) item in this list. As the ownership interest grows, the potential buyer's community shrinks what makes the discount for lack of liquidity grow.
- (vii) **Size of the company**¹⁵⁹ - if the illiquidity discount is set as a percentage of the company's value, it should become smaller as the company's size increases. As in the case of the previous item, also this one could have many motives but most likely is caused by the psychological effect which generally makes investors to perceive a big company as a more stable and less risk compared to its smaller peers¹⁶⁰;
- (viii) **Dividend distribution policy**¹⁶¹ - it is very reasonable to expect that stocks with a low or no dividend stream suffer from higher lack of liquidity discounts. There is one major reason for that which states that the owner of the interest in the company which does not pay any dividends faces a bigger risk and cannot diversify it over the period of holding the stock compared to the shares paying out sufficient dividends. The investors into the non-dividend-paying stock have to be compensated for this additional risk, hence the illiquidity discount has to be larger;
- (ix) **Control component**¹⁶² - generally it holds that while investing into a closely held company, it is more attractive to acquire a controlling interest of e.g., 51%, as this grants the investor bigger competences compared to for example 49% stake. As a result, the bigger the acquired stake in the company the lower the discount for lack of marketability;

¹⁵⁸ Ibid.

¹⁵⁹ In Pratt, S.P. and Niculita, A.V. (2008), "Valuing a Business: The Analysis and Appraisal of Closely Held Companies", Fifth Edition, The McGraw-Hill Companies, Inc., 2008; and Damodaran, A. (2005a), "Marketability and Value: Measuring the Illiquidity Discount", Stern School of Business, July 2005.

¹⁶⁰ For us it is a question whether this statement is valid across the investors' community also in these days when there can hardly be found someone who would doubt the influence of the largest financial institutions on the current markets' bad shape.

¹⁶¹ In Pratt, S.P. and Niculita, A.V. (2008), "Valuing a Business: The Analysis and Appraisal of Closely Held Companies", Fifth Edition, The McGraw-Hill Companies, Inc., 2008; and Damodaran, A. (2005a).

- (x) **Prospects of future public offering or sale of the business**¹⁶³ - the more are the investors convinced of an imminent public offering or sale of the business, the lower the illiquidity discount could be. However, as the literature says, such prospects are almost never certain, and the degree of offset to the discount of the lack of marketability is problematic since majority of the empirical evidence illustrating the scope of the discount is taken from companies that consequently went public; and
- (xi) **Information access and reliability**¹⁶⁴ - the empirical works suggest that the degree to which the information on the company is made available, also to the non-controlling equity owners, as well as the reliability of that information significantly affect the discount for lack of liquidity. The more information is available and the more such information is reliable, the lower the applicable discount for lack of marketability.

4.3 Empirical Evidence on Lack of Marketability Discount

Until now we have been discussing the most important aspects and factors regarding and generally influencing the scope of the lack of marketability discount. In this moment we would like to turn into more empirical area and focus on the works which for long decades tried hard to discover the answer to a very complex and comprehensive question: “Which discount for lack of liquidity is applicable in each particular case?”

The existing empirical methodologies used to quantify the lack of marketability discount related to the closely held securities or shares of the company fall broadly into three main categories and each of them will be discussed in a more detail in the following subchapters:

- (i) **Restricted stock studies;**

¹⁶² In Damodaran, A. (2005a), "Marketability and Value: Measuring the Illiquidity Discount", Stern School of Business, July 2005.

¹⁶³ In Pratt, S.P. and Niculita, A.V. (2008), "Valuing a Business: The Analysis and Appraisal of Closely Held Companies", Fifth Edition, The McGraw-Hill Companies, Inc., 2008; and Damodaran, A. (2005a), "Marketability and Value: Measuring the Illiquidity Discount", Stern School of Business, July 2005.

¹⁶⁴ In Pratt, S.P. and Niculita, A.V. (2008), "Valuing a Business: The Analysis and Appraisal of Closely Held Companies", Fifth Edition, The McGraw-Hill Companies, Inc., 2008.

- (ii) **Pre-IPO studies;** and
- (iii) **Chaffe's option pricing method.**

4.3.1 Restricted Stock Studies

This kind of empirical research trying to discover the applicable discount for lack of marketability belongs to the most traditional ones and has been developed since the 1970s in the US.¹⁶⁵ All of the studies employing the method of restricted stocks have one attribute in common – they isolate the pricing implications of liquidity from all other valuation-related tasks and focus their attention towards letter stocks. A letter stock is a stock issued by a company which is identical in all respects to its freely traded stock except for the fact that it is restricted from trading¹⁶⁶ on the open market for a certain period.¹⁶⁷ Since the degree of marketability is the only difference between the restricted stock and its freely traded counterpart, simple comparison of prices of these two securities in the moment of the letter stock issuance may provide the analyst with an estimate of the lack of marketability discount.

As stated before, the restricted securities are securities issued by a company, but not registered with the Security Exchange Commission (herein after also “SEC”). It means these can be sold via private placement to investors, but are disabled from resell in the open market. The period over which such restricted shares cannot be openly traded has been regulated by the SEC Rule 144. Prior to February 1997, investors had to wait for two years for their shares to become marketable.¹⁶⁸ Nevertheless, the currently valid version of the Rule 144, which came into force in 1997, applies the general rule stating:¹⁶⁹

“A minimum of one year must elapse between the later of the date of the acquisition of the securities from the issuer or from an affiliate of an issuer, and any resale of such securities in reliance on this section for the account of either the acquirer or any subsequent holder of those securities.”

¹⁶⁵ Among the original studies belong e.g., SEC Institutional Investor Study, Moroney Study, Gelman Study, or Trout Study.

¹⁶⁶ From this feature stems the name “restricted stock”.

¹⁶⁷ In Pratt, S.P. and Niculita, A.V. (2008), "Valuing a Business: The Analysis and Appraisal of Closely Held Companies", Fifth Edition, The McGraw-Hill Companies, Inc., 2008.

¹⁶⁸ In Bajaj, M. (2002), "Response to Shannon Pratt's Critique of Mukesh Bajaj Work on Marketability Discounts", Business Valuation Resources, 2002.

Generally speaking, there might be several reasons for companies to issue restricted stocks instead of regular shares, but the most probable ones are the time and cost factors of these two transactions. Registering the new share issue at SEC is relatively time-demanding process compared to an issuance of restricted stocks. As a result, it is very impractical in case a company needs immediate financial sources to finish a contemplated acquisition or quickly raise private capital for other reasons.

So far we have been talking only on the publicly traded companies which are used as a reference group. On the other hand there are plenty of closely-held companies which mostly never experience the benefits of public markets. As such, these are expected to require a higher lack of marketability discount compared to restricted stocks of ordinary listed companies. In fact, the market demands a significantly higher discount for closely-held minority interest ownerships than in case of restricted stocks of a publicly-traded company.¹⁷⁰ Nonetheless, the restricted stock studies still provide a good approximation of discounts, even for the minority interest closely-held transactions.

In the following section of this subchapter we would like to provide the readers with the outcomes of the studies we came through while studying the questions regarding the lack of marketability discounts. The most crucial studies are according to our opinion the following ones:

- (i) **SEC Study**¹⁷¹ - perhaps the oldest and definitely one of the most respected studies including the actual market data. It is based on 398 private transactions in total (in only 278 of the transactions there were available complete data sets) which stretch over the period from January 1966 to June 1969. Almost 73% of all the transactions included in the study recorded a discount falling into range from 0% to 50%. The overall mean of the price discount was close to 26% and the median price discount was about the same value. Moreover, clear inverse relationship between the size of the company (measured by the sales volumes) and

¹⁶⁹ In United States Securities and Exchange Commission (2006), "Rule 144: Persons Deemed Not to Be Engaged in a Distribution and therefore Not Underwriters", SEC, Washington, December 2006.

¹⁷⁰ In Pratt, S.P. and Niculita, A.V. (2008), "Valuing a Business: The Analysis and Appraisal of Closely Held Companies", Fifth Edition, The McGraw-Hill Companies, Inc., 2008.

¹⁷¹ In The United States 92nd Congress (1971), "Discounts Involved in Purchases of Common Stock", 1st Session, Institutional Investor Study Report of the Securities and Exchange Commission, March 1971.

the amount of the applicable discount has been discovered. For more detail refer to the following Table 11.

Table 11 - SEC Study Output

Size of the Lack of Marketability Discount	Transaction Statistics	
	No. of Transactions	Value of the Transactions [% of the total value]
-15.0 – 0.0%	26	7.3%
0.1 – 10.0%	67	10.9%
10.1 – 20.0%	78	21.4%
20.1 – 30.0%	77	23.1%
30.1 – 40.0%	67	25.7%
40.1 – 50.0%	35	4.6%
50.1 – 80.0%	48	7.0%
Total	398	100%

Source: Institutional Investor Study Report of the SEC¹⁷²

- (ii) **Gelman Study**¹⁷³ - also one of the pioneering works which was elaborated at about the same time as the SEC Study is based upon 89 transactions between 1968 and 1970. Gelman found that the mean, as well as median price discounts were close to 33% and almost two thirds of the purchases recorded a discount of at least 30%. For more detail refer to the following Table 12.

Table 12 - Gelman Study Output

Size of the Lack of Marketability Discount	Transaction Statistics	
	No. of Transactions	% of Total
Less than 15.0%	5	6%
15.0 – 19.9%	9	10%
20.0 – 24.9%	13	15%
25.0 – 29.9%	9	10%
30.0 – 34.9%	12	13%
35.0 – 39.9%	9	10%
40.0% and more	32	36%
Total	89	100%

Source: Gelman¹⁷⁴

¹⁷² Ibid.

¹⁷³ In Gelman, M.(1972), "An Economist-Financial Analyst's Approach to Valuing Stock of a Closely Held Companies", Journal of Taxation, June 1972, p. 353-354.

¹⁷⁴ Ibid.

- (iii) **Trout Study**¹⁷⁵ - Trout analyzed 60 restricted stocks purchased by mutual funds between 1968 and 1972. He constructed a multiple regression model which provided him with a result of a mean price discount of over 33% for restricted stock compared to a freely traded stock;¹⁷⁶
- (iv) **Moroney Study**¹⁷⁷ - in his study Moroney identified 10 registered investment companies that held in total 146 purchases of restricted equity securities. The average price discount which was applied to these transactions amounted almost 36% with the median accounting for about 33%. There was a considerable variation in his results since the standard deviation about Moroney's mean reached almost 18%;
- (v) **Maher Study**¹⁷⁸ - Maher closely studied the period of 1969 to 1973. He recognized an average discount slightly above 35% with the standard deviation close to 18%, as in the case of Moroney's research. After obtaining the first set of results Maher eliminated the bottom and top 10 percent of purchases in order to remove especially low- and high-risk situations. Nevertheless, his result was comparable to the initial one – the mean price discount accounted for slightly less than 35%;
- (vi) **Pittock and Stryker Study**¹⁷⁹ - this study examined 28 private placements of restricted stocks which materialized between October 1978 and June 1982. The range of the discounts which were present in the above described transactions was from 7 to 91% with a median of about 45%;

¹⁷⁵ In Trout, R.R. (1977), "Estimation of the Discount Associated with the Transfer of Restricted Securities", Taxes, June 1977, p. 381-385.

¹⁷⁶ Trout further found out that companies whose ordinary stocks are traded on national exchanges suffer from lower discounts on their restricted stock transactions compared to companies with stocks traded on OTC only.

¹⁷⁷ In Moroney, R.E. (1973), "Most Courts Overvalue Closely Held Companies", Taxes, March 1973, p. 144-154.

¹⁷⁸ In Maher, J.M. (1976), "Discounts for Lack of Marketability for Closely Held Business Interests", Taxes, September 1976, p.562-571.

¹⁷⁹ In Pittock, W.F. and Stryker, C.H. (1983), "Revenue Ruling 77-287 Revisited", SCR Quarterly Reports, Spring 1983, p. 1-3.

- (vii) **Silber Study**¹⁸⁰ - in his study Silber discusses 310 private placements between 1981 and 1988. After eliminating issues that had warrants or other special provisions, he identified 69 private placements of common stock of publicly traded companies. In his sample he found a mean discount of slightly below 34% with a standard deviation of 23.7%. Another important findings of Silber's research were that generally larger and more profitable companies trade at significantly lower discounts compared to their smaller, less well capitalized counterparties, and that size of the price discount tends to be higher for private placements that were larger as a percentage of the shares outstanding;
- (viii) **Hall and Polacek Study**¹⁸¹ - this study reported on analysis of more than 100 restricted stock transactions from 1971 to April 1992. The result of this research was the mean discount of about 23% with a decrease over the period of 1991 to 1992 to almost 21%. Hall and Polacek further concluded that the lack of liquidity discount negatively correlates with the total volume of company's revenues and earnings, and is influenced by the exchange, where the stock is traded;
- (ix) **Management Planning, Inc. Study**¹⁸² - this study covers all the transactions which occurred between 1980 and 1996. The overall result of the study identifies the mean discount of almost 28% with a clear size effect meaning smaller companies tend to have larger discounts and as their revenues grow, the applicable discount diminishes; and
- (x) **Columbia Financial Advisors Study**¹⁸³ - this study examines the transactions from 1996 to 1998. It is the only study we found which at least partially reflects the change in SEC Rule 144 which occurred in May 1997. As such its author divided it internally into two sections – the first one covers the period from January 1996 to April 1997, the second

¹⁸⁰ In Silber, W.L. (1991), "Discounts on Restricted Stocks: The Impact of Illiquidity on Stock Prices", *The Financial Analyst Journal*, July/August 1991, p. 60-64.

¹⁸¹ In Hall, L.S. and Polacek, T.C. (1994), "Strategies for Obtaining the Largest Valuation Discounts", *Estate Planning*, January/February 1994.

¹⁸² In Oliver, R. and Meyers, R., "Discounts Seen in Private Placements of Restricted Stocks", *The Management Planning Inc., Long-term Study (1980-1996)*.

¹⁸³ In Aschwald, K.F. (2000), "Restricted Stock Discounts Decline as Result of One-Year Holding Period", *Shannon Pratt's Business Valuation Update*, May 2000, p. 1-5.

part discusses the period starting from May 1997 and ending December 1998. In the first section, 23 transactions were identified with a discount ranging from 0.8 to 67.5%, mean value of 23% and median equal to 14%. Whereas in the second section, there were mentioned 15 relevant transactions with a discount ranging from 0 to 30%, mean value equal to 13% and median 9%. For a review of the results refer to the following Table 13.

Table 13 - Columbia Financial Advisors Study Output

Respective Period for the Transactions Involved [No. of Transactions]	Transaction Statistics		
	Discount Range in the Sample [%]	Mean Discount [%]	Median Discount [%]
January 1996 – April 1997 (23 transactions)	0.8 – 67.5%	23%	13%
April 1997 – December 1998 (15 transactions)	0.0 – 30.0%	14%	9%

Source: Aschwald¹⁸⁴

In the previous paragraphs we presented the history of the restricted stock studies by pointing ten of the most important ones. Taking the results of all of these studies together we would get hundreds of restricted stock transactions spread over the period from 1966 to 1998. Although the covered period encompasses more than 30 years in a row, the results are remarkably consistent with seemingly the only deviation caused by the SEC Rule 144 change in year 1997 what is documented by the Columbia Financial Advisors Study. We tried to summarize the key results of the above mentioned studies in the following Table 14.

¹⁸⁴ Ibid.

Table 14 - Overview of Restricted Stock Studies

Name of the Study	Studies' Statistics			
	Period Covered	Discount Range [%]	Mean Discount [%]	Median Discount [%]
SEC Study	1966 – 1969	-15% to 80%	26%	-
Gelman Study	1968 – 1970	-15% to over 40%	33%	33%
Trout Study	1968 – 1972	-	33%	-
Moroney Study	-	-	36%	33%
Maher Study	1969 – 1973	-	35%	-
Pittock and Stryker Study	1978 – 1982	7% to 91%	-	45%
Silber Study	1981 – 1988	-	34%	-
Hall and Polacek Study	1971 – 1990	-	23%	-
Management Planning Study	1980 – 1996	-	28%	-
Columbia Fin. Adv. Study I	1996 – Apr 1997	0.8% to 67.5%	23%	14%
Columbia Fin. Adv. Study II	Apr 1997 - 1998	0% to 30%	13%	9%

Source: Author

4.3.2 Studies of Private Transactions before Initial Public Offerings

Before the 1980s almost all of the empirical research, focused on the quantification of lack of marketability discount, employed the method of restricted stocks which was described in the previous chapter. Since approximately the mid-1980s a new method started to address this issue. This newly developing method tries to reflect the above mentioned consensus that the discount for lack of liquidity for ownership interests of closely held firms should be significantly higher than those applicable to restricted shares of publicly traded companies.

This new method attempts to quantify the lack of marketability discount based on comparison of the post-initial public offering (hereinafter also “IPO”) price of the company’s shares with transaction prices of the shares of the same company which realized prior to the IPO. Generally, there exist two most comprehensive and long-term series of studies using the so called pre-IPO approach. These are John Emory’s Studies and Willamette Management Associates (hereinafter also “WMA”) Studies:

- (i) **Emory Studies**¹⁸⁵ - so far nine studies of John Emory were conducted and they in total cover the period from 1980 to 2000. Over the 20-years

¹⁸⁵ In Emory, J.D. Sr., Dengel, F.R. and Emory, J.D. Jr. (2002), "Discounts for Lack of Marketability: Emory Pre-IPO Discount Studies 1980-2000, As Adjusted October 10, 2002", Emory Valuation, 2002.

long period John Emory and his colleagues analyzed more than 4,000 prospectuses of the companies undergoing the IPO. They tried to identify the relationship between the price at which the stock was offered at the IPO, and the price at which the private transactions of the company occurred not later than five months prior to the IPO. Following these guidelines and after elimination of the development-stage companies¹⁸⁶ and firms with no recorded transactions within the five months prior to the IPO, 543 qualifying transactions remained in the aggregate sample of the nine studies. The mean and median price discount for all of these qualifying studies was 46% and 47%, respectively. The fact that Emory's results are more than 10 percentage points higher compared to the restricted stock studies is something what one could reasonably expect regarding the above mentioned consensus. On the other hand there might be found some analysts who perceive the lack of marketability discount of 46% as excessive and unrealistic one.¹⁸⁷ For more detail on the nine studies result refer to the following Table 15.

Table 15 - Emory Studies' Output

Period Covered	No. of Reviewed IPO Prospectuses	No. of Qualifying Transactions	Mean Discount [%]	Median Discount [%]
2000 – 1997	1,847	266	50%	52%
1997 – 1995	732	84	43%	41%
1995 – 1994	318	45	45%	47%
1993 – 1992	443	49	45%	40%
1992 – 1990	266	30	34%	33%
1990 – 1989	157	17	46%	40%
1989 – 1987	98	21	38%	43%
1986 – 1985	130	19	43%	43%
1981 – 1980	97	12	59%	68%
All nine Studies	4,088	543	46%	47%

Source: Emory¹⁸⁸

¹⁸⁶ Under the term “development-stage companies” one can understand companies with history of operating losses.

¹⁸⁷ One of the opponents of such large discount for lack of marketability could be Bajaj, M., Denis, D.J., Ferris, S.P. and Sarin, A. with their work "Firm Value and Marketability Discounts", Journal of Corporate Law, Vol. 27, No. 1, 2001.

¹⁸⁸ In Emory, J.D. Sr., Dengel, F.R. and Emory, J.D. Jr. (2002), "Discounts for Lack of Marketability: Emory Pre-IPO Discount Studies 1980-2000, As Adjusted October 10, 2002", Emory Valuation, October 2002.

- (ii) **WMA Studies**¹⁸⁹ - Willamette Management Associates have been conducting pre-IPO studies for relatively a long period of time. With the most recent one they cover years from 1975 through 2002. Compared to Emory who has been using IPO prospectuses in his analyses, WMA have been using complete SEC registration statements. These require full disclosure of all private transactions in the stock within the three-year period preceding the IPO.¹⁹⁰ For each transaction where meaningful revenue data were available, as well as both the private transaction and IPO dates, the price/earnings multiple the transaction was compared with the subsequent IPO multiple. Firms with no meaningful data at any stage were eliminated from the sample. The results of the WMA studies are illustrated in the following Table 16. The 2001 and 2002 results can hardly be perceived as representative ones as only two statistically meaningful private market transactions were identified in 2001 and only seven transactions qualified a year after.

Table 16 - WMA Studies' Output

Period Covered	Mean Discount [%]	Median Discount [%]
1975 – 1997	from 28.9% (in 1991) to 56.8% (in 1979)	from 31.8% (in 1991) to 73.1% (in 1984)
1998	35.0%	49.4%
1999 – 2000	50.0%	52.0%
2001	-195.8%	-195.8%
2002	55.8%	N/A

Note: N/A – not available

Source: WMA¹⁹¹

4.3.3 Option Pricing Method

Taken it from the chronological perspective, comparing it to the restricted stock or pre-IPO approaches we have to qualify the option pricing model as the most recent method of quantification of the lack of marketability discount. The pioneering article of David Chaffe appeared first in December 1993 in the Business Valuation

¹⁸⁹ In Pratt, S.P. and Niculita, A.V. (2008), "Valuing a Business: The Analysis and Appraisal of Closely Held Companies", Fifth Edition, The McGraw-Hill Companies, Inc., 2008.

¹⁹⁰ Whereas the prospectuses which are required to disclose transactions with affiliated parties only.

Review.¹⁹² According to our findings, David Chaffe is the only theoretician who seriously tried to develop the method of option pricing with respect to the discount for the lack of liquidity.

The idea behind Chaffe's approach is relatively simple and straightforward stating that when provided with a put option, otherwise non-marketable share is granted marketability. Following his logic, the price of the put option represents all (or at least the major portion) of the discount which is to be taken from the marketable price in order to arrive to the price of a non-marketable share. To sum up, if one holds the restricted stocks as described above and at the same time buys the option to sell these shares at the free market price, the purchaser in fact acquired the marketability for the shares.

In his study, Chaffe used two simplifying adjustments in order to examine the option pricing theory with respect to valuation of private company shares. These adjustments are as follows:¹⁹³

- (i) Use of the European option¹⁹⁴ – as this kind of option is exercisable only at the end of the period, it brings to the Chaffe's model an attribute comparable to Rule 144's implications in the restricted stock studies; and
- (ii) The strike price of the option was stated as the freely traded (marketable) price at time of purchase (valuation date); hence, marketability of the shares is assumed at the end of the holding period which is the same date as the exercise date of the option.

Considering that the price difference between the marketable and otherwise identical non-marketable security is the cost of the put option, the application of the put pricing formula forms Chaffe's new approach defining the lack of marketability discount. As the Black-Scholes simplified option pricing model was adopted by Chaffe (for more detail refer to Appendix 1), all of its key variables had to be examined. One of

¹⁹¹ In Pratt, S.P. and Niculita, A.V. (2008), "Valuing a Business: The Analysis and Appraisal of Closely Held Companies", Fifth Edition, The McGraw-Hill Companies, Inc., 2008.

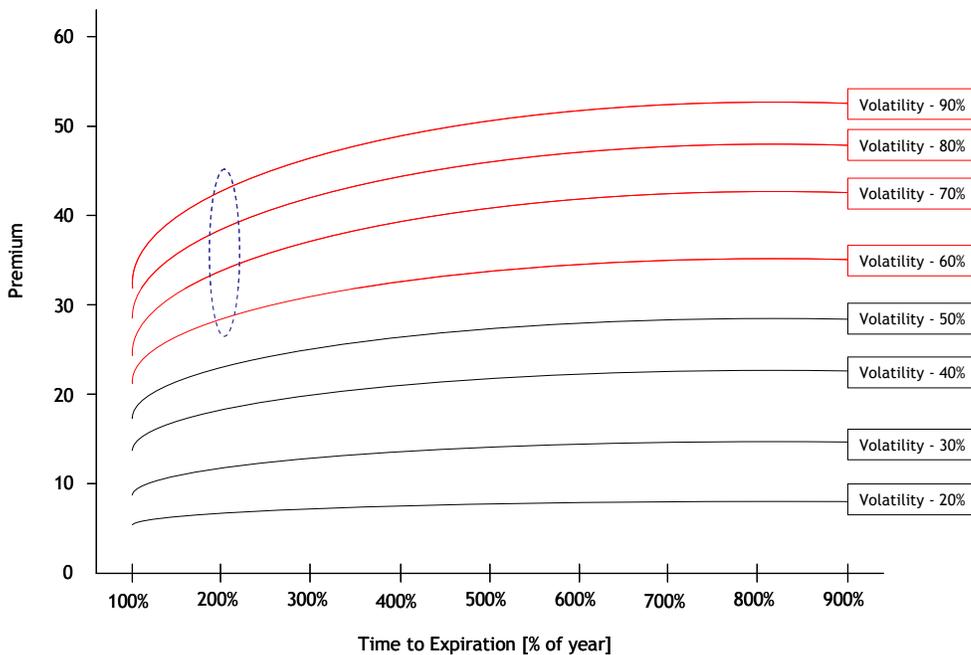
¹⁹² In Chaffe, D.B.H. (1993), "Option Pricing as a Proxy for Discount for Lack of Marketability in Private Company Valuations", *Business Valuation Review*, December 1993, p. 182-188.

¹⁹³ *Ibid.*

¹⁹⁴ The use of the European option results in lower option prices than if the American option was used or in case variations are considered to give the effect of the perpetual put adjusted constantly to the then marketable price.

the trickiest variables included in Chaffe's pricing formula was volatility¹⁹⁵, which showed a considerable influence on the overall pricing formula. However, to get some relevant volatility estimates for privately held shares, it was necessary to confront the private company's financial and operating data with those of the comparable public companies for which volatility can be determined. The result of the Black-Scholes formula applied by Chaffe over the range of time, with an interest rate fixed at 5% and varying volatility is depicted in the Figure 2.

Figure 2 - Put Prices According to Chaffe's Black-Scholes Model Application



Source: Chaffe¹⁹⁶ with author's adjustments

As can be seen in Figure 2 the put price raises sharply to year two or three for all the levels of volatility and flattens as time further proceeds. Moreover, assuming that the volatility of shares of majority of smaller, privately held companies fits the volatility curves of 60 to 90%, a range of put prices of approximately 28% to 41% of the marketable price is implied by the two year intercept. The put prices expressed as a percentage of the strike price imply the discount for lack of marketability close to the

¹⁹⁵ The volatility is in fact an expression of the total risk of a stock and is most often derived using the historical fluctuations of price of the underlying stock.

¹⁹⁶ In Chaffe, D.B.H. (1993), "Option Pricing as a Proxy for Discount for Lack of Marketability in Private Company Valuations", *Business Valuation Review*, December 1993, p. 182-188.

range of 28% to 49%. Such a result is very much comparable to either to the restricted stocks or pre-IPO studies.

4.4 Limitations to the Lack of Marketability Studies and the Studies'

Proper Application

So far we have mentioned and briefly described all the approaches and studies dealing with determination of the lack of marketability discount we got into contact during our research. In order to show that they have to be used with caution, we would like to spend few lines by discussing both, their general shortcomings and the proper way, how the studies shall be used. The most serious criticism of the existing studies covers the following areas:¹⁹⁷

- (i) **Lack of current market data** – as majority of the available studies stretches over the period from the 1960s to the 1980s, these entries can only hardly be perceived as data fully reflecting the current market dynamics. This is true especially today in the turbulent times of the global economic downturn;
- (ii) **Change in holding period for restricted stocks** – this issue has already been discussed but to keep the list of the studies' shortcomings complete, let us once again stress it. Due to the legal change in Rule 144, which materialized in 1997, investors' expectation regarding the length of time that is needed to convert an illiquid security into cash has changed. As mentioned in the previous bullet point as the vast majority of studies is dated before 1997, this change has not been fully reflected in the empirical works carried out so far; and
- (iii) **Bias** – as the studies use the data from the transactions of the public companies, there is a clear bias towards the transactions within the scope of privately held companies or blocks of stocks sold in private placements to selected investors.

As a result, all the studies' limitations, either the key ones mentioned above or those picked up in the previous chapters, according to our opinion the studies should not

be used as the only reference standing above all other indices and analyses. Without any doubt all the lack of marketability discount studies provide a helpful tool for all the valuers since they help to employ the proper methodology and offer some range for the final discount applicable in the observed transaction. However, their outcomes should not be adopted without a prior detailed examination of the respective transaction and all of its aspects.

¹⁹⁷ In Bowers, H. and Stephenson, T. (2004), "Determinants of the Discount for Lack of Marketability", Woodward Group, May 2004.

5. Empirical Examination of Control Premiums Applicable to the CEE M&A Transactions

As indicated in the introductory part of the thesis, the pivotal part of our work consists of the empirical section which uses the M&A transactions in order to examine the premiums applicable in the market. If both of the key premiums and discounts, which have been discussed throughout the thesis, should be empirically tested, it would allow only for a superficial analysis. To avoid this we decided to focus in this chapter only on the aspects of the control premium. Moreover, we took into consideration the fact that the relevant capital market evidence regarding quantification of the value of control premium in the region of the Central and Eastern Europe (hereinafter also “CEE”) remains the virgin territory in the business valuation. As a result, we decided to concentrate our attention towards the examination of the control premiums applicable to the CEE M&A transactions.

5.1 Research Hypotheses

Based predominantly upon the previous research dealing with the issue of the control premiums, we formulated the following set of research hypotheses. All of these hypotheses intend to discover the control premiums applicable in the CEE region and will be tested further in this chapter.

According to our research and our economic rational, it would make more sense if an acquirer of the pure control in the company pays statistically significant premium over the acquirer generally involved in a control transaction only. While control relates to transactions which simply result in a greater than 50% stake irrespective to the originally owned shareholding in the company¹⁹⁸, the pure control changes the originally held minority or no shareholding into a majority shareholding. Further aspects of this issue will be discussed in the section of the hypotheses’ testing. Based on our conviction we stated the first hypothesis as follows:

¹⁹⁸ According to our logic, it does not make any difference if the investor acquired e.g., 70% of the company’s shares when he or she had not owned any of the company’s shares before, or acquired only 30% in case he or she had been the owner of 40% stake of the company’s shares before.

Hypothesis 1: Acquirer of the pure control pays a higher control premium compared to an acquirer involved in the control transaction.

Our next hypothesis relates to the development of control premium paid in the M&A transactions in time. This aspect of the control premium has already been discussed by several researchers.¹⁹⁹ Seemingly, there might be at least two antagonistic effects causing either control premium to drop or rise. The first one, promoting the decrease of the control premium, might be represented by improving corporate governance and legal protection of minority shareholders. These attributes, as described in the theoretical part of the thesis, should jointly limit the benefits of the controlling owners from their prerogatives of control. Hence, economically rational actors acquiring control should be willing to pay lower control premiums for their controlling stakes.

The second effect, causing control premiums to rise in time, might be represented by the effect observable in the market for certain period of time even shortly prior to the market downturn in the second half of 2008. At that time, the M&A activity was growing rapidly.²⁰⁰ As the basic economic law of demand and supply determines, the increased demand must have been accomplished by increasing “prices” in the M&A market, *ceteris paribus*. As a consequence, also the control premium paid within the transactions had to increase.

We are not sure which out of these two effects outweighs the other, but stated our second hypothesis as follows:

Hypothesis 2: Control premium increases in time.

The legal and institutional factors can have a substantial impact on many areas which are usually considered within the M&A transactions. These areas can cover for example property rights, contractual duties of the contracting parties or the law enforceability as such.²⁰¹ As the legal and institutional situation substantially differs among the CEE countries, we stated our third hypothesis as follows:

¹⁹⁹ The topic of control premium development in time has been discussed e.g., by Hanouna, P., Sarin, A. and Shapiro, A.C. (2001) in their article called "Value of Corporate Control: Some International Evidence".

²⁰⁰ It is visible even from our sample of M&A transactions described further in this chapter.

²⁰¹ In Chari, A., Ouimet, P.P. and Tesar, L.L. (2007), "The Value of Control in Emerging Markets", NBER Working Paper, 2007.

Hypothesis 3: The control premium applicable within the CEE region differs among individual countries in the region.

Taking the aspect of transactions where foreign acquirers gained control into account, the literature argues that these represent transactions with considerably higher control premiums at stake compared to the rest of the M&A deals.²⁰² This might be explained for example by the fact that foreign acquirers usually face more competition. Their involvement in the respective deal implies that such a transaction was not limited to the pool of domestic investors only, but the potential pool of acquirers was enhanced by international players as well. As a consequence, the bargaining power of the seller increases. Hence, the transaction consideration including the control premium may rise. Our fourth hypothesis is constructed as follows:

Hypothesis 4: Cross-border transactions comprise larger control premiums compared to the domestic ones.

Until recently, many states classified in our research as CEE countries have also been included into the sample of emerging countries. The empirical results state that transfers of control from an emerging market target to a developed market acquirer are generally associated with higher positive returns for acquiring firms.²⁰³ This might be caused by higher benefits stemming from geographical diversification. In order to investigate whether such finding might be transferred to the CEE region, we compiled the following hypothesis:

Hypothesis 5: Transactions where the acquirer comes from a non-CEE country are connected with greater control premiums compared to transactions where both the target and the acquirer come from the CEE country.

With respect to the previous research which examines potential control premium determinants in the Italian market, the nature of the acquirer belongs to

²⁰² In Dyck A. and Zingales L. (2004a), "Control Premiums and the Effectiveness of Corporate Governance Systems", *Journal of Applied Corporate Finance*, Vol. 16, No. 2-3, Spring/Summer 2004, p. 51-72.

²⁰³ In Chari, A., Ouimet, P.P. and Tesar, L.L. (2007), "The Value of Control in Emerging Markets", NBER Working Paper, 2007.

statistically significant parameters of the model.²⁰⁴ In general, strategic investors have experience with the business segment of the target company and as such are usually more capable to exploit major synergies compared to the financial acquirer. In addition to that, strategic investors usually do not have such high requirements on the acquisition's internal rate of return in comparison with the financial acquirers. As a result of the above mentioned aspects, strategic buyers are usually prepared to offer a higher price if competing with a financial investor at the same transaction. The higher price offered by a strategic acquirer should be also reflected in the higher control premium. Our sixth hypothesis is stated as follows:

Hypothesis 6: Strategic investors are willing to pay higher control premium compared to financial investors.

We have not found it in any of the past studies but it seems to us to be interesting to examine whether the type of compensation of the transaction somehow influences the scope of the control premium. Taking the cash compensations as a reference point, other types of payments which are usually represented by compensations in shares,²⁰⁵ mean certain transfer of the risk from the buyer to the seller. Conversely, in case of pure cash compensation, the buyer is the only party taking the risk from the investment. As a result, an economically rational acquirer who intends to pay for the transaction fully in cash should pay lower control premium compared to an acquirer paying in stock or in stock together with some proportion of cash. We stated our seventh hypothesis as follows:

Hypothesis 7: Transactions compensated purely in cash encompass lower control premium compared to other types of remuneration.

5.2 Methodology

As mentioned in the initial section of this chapter, we decided to examine the value of the control premium applicable within the CEE region. In other words, our

²⁰⁴ In Massari, M., Monge, V. and Zanetti, L. (2005), "Control Premium in Legally-Constrained Markets for Corporate Control: The Italian Case (1993-2003)", SSRN Working Paper, March 2005.

analysis is distinguished from previous research in terms of the geographical coverage. While most studies, as already discussed, have focused prevalingly on the US market data, we focus on the control premium issue in the context of the CEE environment.

Generally, there are three methods used for quantification of the control premiums. These were already described in the subchapter called “Empirical Evidence of Control Premiums and Minority Discounts”. Due to lack of relevant CEE market data the only applicable approach to control premium estimation relates to controlling interests in the M&A market. We followed the formula for estimation of the control premium commonly used by this method. The formula might be simplified by the following Equation (8), as defined in the previous chapter of the thesis.

$$\text{Control Premium} = \left(\frac{\text{Share Purchase Price} - \text{Share Unaffected Price}}{\text{Share Unaffected Price}} \right)^{206} \quad (8)$$

The key issue in the process of control premium estimation is the definition of the control. There is no widely accepted consensus regarding which stake should be perceived as the controlling one. In case of considerably dispersed ownership, even an owner of a single digit stake can benefit from control rights represented by the so called prerogatives of control. For the purpose of our analysis we stated the threshold for control at the level of 50%²⁰⁷, which we assume to be a conservative estimate. Furthermore, we distinguished between control and pure control as described above.

For the econometric testing of the above mentioned research hypotheses we utilized “R”, a free software environment for statistical computing and graphics. In all of the cases we employed the Ordinary Least Squares (hereinafter also “OLS”) estimation technique. The basic OLS formula might be quoted as that one in Equation (9).

²⁰⁵ When the compensation in shares is put into place, it means that the transaction consideration is paid in stock, either 100% or in certain proportion with cash.

²⁰⁶ We arrived to this formula by making some marginal adjustments to the formula mentioned by Officer (2005). The “Control Premium” was explained in the chapter called “Control Premium”, the “Share Purchase Price” is the pro rata price per share which was paid for company’s stake in the underlying transaction, and “Share Unaffected Price” stands for the market price of the share prior to the transaction long enough not to be affected by the market rumours regarding the underlying transaction.

²⁰⁷ Our 50% threshold lies close to the assumption of Nicodano and Semebenelli (2004) who stated it at the level of 51%.

$$Y_i = \alpha + \beta X_i + \varepsilon_i, \text{ where} \quad (9)$$

Y_i stands for the endogenous vector of the dependent variable; X_i in our case denotes the exogenous matrix of the explanatory variable mostly represented by a column vector of one variable depending on the tested hypothesis; α accounts for the intercept, β the coefficient of the explanatory variable matrix and ε_i the vector of disturbances. In case of OLS several assumptions need to be verified – the most common one refers to heteroskedasticity of disturbances. For heteroskedasticity testing purposes we used Breusch-Pagan test and in case that heteroskedasticity is indicated, the Generalized Least Squares (hereinafter also “GLS”) estimators is obtained.

5.3 Data Sample

5.3.1 Initial Selection Criteria

The first step of our analysis consisted in selection of the most suitable M&A transactions fulfilling the following criteria:

- (i) **Deal type** – only acquisitions, divestitures or mergers were desired;
- (ii) **Announcement date of the transaction** – longest data series available until June 2009;
- (iii) **Characteristics of the target company** – public companies listed on any stock exchange all over the world,
- (iv) **Country of the target company has to be located in the CEE region** – we explicitly selected Albania, Belarus, Bosnia and Herzegovina, Bulgaria, Croatia, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Romania, Russia, Serbia, Slovakia, Slovenia, Turkey, and Ukraine;
- (v) **Industry of the target company** – no explicit limitations were set;
- (vi) **Deal Status** – both completed and pending transactions were of our interest; and
- (vii) **Acquirer’s characteristics** – no specific limitations were set.

5.3.2 Databases Used

The M&A as a mode of entry into emerging markets is being perceived as a relatively new phenomenon. Foreign participation in these markets was substantially restricted until the 1990s when many emerging countries deregulated their capital markets to foreign investors.²⁰⁸ To find the most suitable and extensive list of the M&A transactions with a target located in the CEE region we employed three databases in the first stage of our analysis; namely **Mergermarket**, **Zephyr**, and **Bloomberg**.

MERGERMARKET

Mergermarket is the electronic database administered by the Mergermarket Group, since 2006 part of the Financial Times Group. It is an independent M&A intelligence tool widely used by the M&A professionals all over the world. It provides different kinds of information ranging from proprietary intelligence on potential deal flow, company profiles, league tables, potential mandates and valuations to the comprehensive description of historically executed M&A transactions. For illustration, the Deal Search section being the most relevant part of the database for our research claims to cover every European transaction with an enterprise value of over EUR 5m since January 1998. The database identifies not only relevant exit multiples, dates, deals' description, but also the names and other characteristics of individual advisors acting either on the side of the target/seller or the buyer.

The whole database covers the M&A activity across Europe, Americas, Latin America and the Asia-Pacific region but not in the same scope. Whereas the western markets, being the most developed ones, have been investigated since the very beginning, the emerging markets on the other hand have been included into the database relatively recently, if included.

In our case, the selected M&A transactions, which materialized in the CEE region as characterized above, have been split over the period from January 1998 to July 2009. The data set we were able to gather while using the tools of the Mergermarket database encompasses the characteristics of 333 M&A transactions of publicly traded companies in the CEE region within the selected time framework.

²⁰⁸ In Chari, A., Ouimet, P.P. and Tesar, L.L. (2007), "The Value of Control in Emerging Markets", NBER Working Paper, 2007.

ZEPHYR

Besides the Mergermarket database we used Zephyr database, an information solution of Bureau van Dijk Electronic Publishing. In contrast to the Mergermarket it focuses purely on the historically executed transactions either in the form of M&A deals, IPOs, joint ventures, or venture capital deals with links to the detailed financial company information.²⁰⁹ Its geographical coverage is less limited than the one of Mergermarket and covers the key economies of the whole world.

The scope of information included in each respective transaction report is comparable to the one discussed in more detail in the case of Mergermarket with no minimum deal value constraint. Whereas the Zephyr database claims to include all the European companies' deals since 1997, in our case, according to our search criteria we identified 435 transactions split over the period from November 1999 to July 2009.

BLOOMBERG

The Bloomberg database, administered by Bloomberg L.P., is the most comprehensive interaction source of financial market data of almost any kind. Every one of its eligible users can utilize its integral platform of data, news, analytics, and multimedia reports. The scope of information covered by the database among others includes inputs from foreign exchange and commodity trading information to portfolio analytics and risk.

The part we exploited most during our research was the "MA" function, summarizing the M&A transactions in the world-wide perspective which allows for a unique utilization of both transaction and trading data present in the sample. Whereas the transactions somehow connected to the US, which is the Bloomberg's core market, were recorded since the database's origination in the 1980s, the M&A deals which materialized in the CEE region were added later on. As a result of the search limitations discussed above, we identified 1,401 transactions split over the CEE region from January 1997 to July 2009.

5.3.3 Final Sample

After a detailed comparison of the three above mentioned data sets we decided to proceed with our further analysis while using the sample obtained through the

²⁰⁹ As of 2008 year end the Zephyr database included over 600,000 world-wide transactions.

Bloomberg database. There were two major reasons for our final decision. First, the Bloomberg sample provided us with the most extensive and comprehensive set of transactions following the desired form (1,401 Bloomberg transactions versus 435 and 333 for Zephyr and Mergermarket, respectively). Second, Bloomberg's interconnection of transaction and trading schemes²¹⁰ provides us with the features essential for our research.

In order to receive the data set appropriate for testing our hypotheses we adjusted the sample of 1,401 transactions identified by Bloomberg. The most fundamental adjustments made to the sample were the following ones:

- (i) **Selection of completed transactions only** – as it is crucial for our control premium calculation to have the bid price for all of our transactions, we put aside all the pending transactions and kept only the terminated ones;
- (ii) **Deal price** – as it is indicated in the control premium formula, the implied equity value of the company is essential; all transactions where the deal equity price was not disclosed were erased accordingly;
- (iii) **Market capitalization issue** – the second part of the control premium formula asks for the pre-announcement date market capitalization. There does not exist a unified length of the pre-announcement period²¹¹ from which the unaffected market capitalization of the target company should be taken. Moreover, in the sample we gathered so far, the market capitalization for each of the companies stated some fixed number of days prior to the announcement date were not available. To overcome this obstacle and not to lose many observations we decided to state a “floating” unaffected pre-announcement period. This period ranges from 60 to 10 days before the announcement date. The respective unaffected pre-announcement market capitalization was calculated as a first market

²¹⁰ This interconnection is not provided either by the Mergermarket or Zephyr databases. These databases are mostly transaction ones and as such do not provide the comfort of personal adjustments to the trading data related to the underlying transactions. They offer irrevocably fixed trading data instead.

²¹¹ Sometimes the term “event window” can be seen as a synonym.

capitalization available while moving from the 60th to the 10th day of the unaffected preannouncement period;²¹²

- (iv) **Control transactions** – only control transactions were included; and
- (v) **Applicable premium range** – in order to have a sample with reasonable premium figures we decided to eliminate obvious outliers stated as all transactions where the calculated premium did not comply with the explicitly stated range of minus 50% to plus 150%.

As a result, our initial list of 1,401 transactions was reduced to 202 full-featured transaction sample. This new list of relevant transactions was used for the purposes of the control premiums analysis and was employed to test the hypotheses stated above.

5.4 Descriptive Statistics

Based on the sample of the 202 selected transactions, we arrived at the following descriptive statistics. In the following Table 17 we refer to the nature of the transactions in our sample. As can be seen, our sample consists of 161 acquisitions and 41 divestitures.

Table 17 - Nature of the Transactions in the Sample

Transaction Type	No. of Transactions
Acquisition	161
Divestiture	41
Total	202

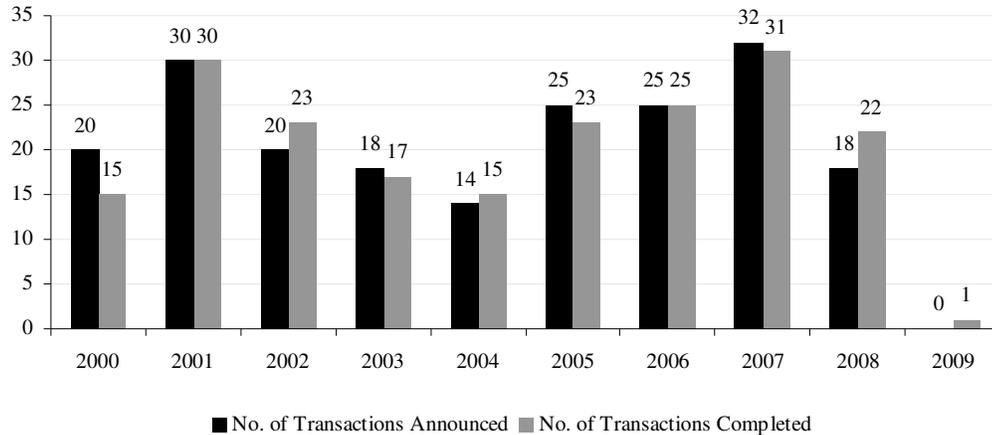
Source: Author

Whereas the initial list of 1,401 transactions identified by Bloomberg covered the period from 1997 to 2009, after the thorough examination of the sample we arrived at deals covering solely the period from 2000 to 2009. The following Figure 3 depicts the number of both announced and completed transactions in the respective year. As can be seen in the figure, the number of transactions in the sample has a peak in 2001, steadily decreases between 2002 and 2004 and recovers over the following period. This development might reflect the downturn of financial markets which followed the burst

²¹² For example, if there was not a market capitalization figure available for a certain transaction just 60 days before the announcement date we searched for the market capitalization figure 30, 15, and 10 days

of the so called dot.com bubble²¹³, assuming that its effects spilled over to the CEE region with some delay. The period beginning 2008, when the number of transactions started to diminish again might be caused by the current market crisis which hit the US, as well as the CEE region in the second half of 2008.

Figure 3 - Development of the Number of Transactions in Time



Source: Author

The following Figure 4 presents a matrix which displays the exact count of deals which realized between each respective CEE country as a domicile of the target company and the country of the acquirer, respectively. As the table shows, the CEE target with the most recorded transactions is Poland with 62, followed by Turkey, Russia and the Czech Republic with 33, 30 and 24 terminated deals, respectively. By far the most active acquirer country in the CEE region was Russia with 26 transactions in total, of which 23 were domestic transactions only. The non-CEE acquirers with the most deals closed in the region are Germany and Sweden with 15 and 14 transactions, respectively. The grey cells highlight the domestic transactions which amounted to the count of 56 with the rest of the 146 transactions representing cross-border transactions. Our further analysis discovered that 41 of all the deals in the sample constitute transactions among CEE countries on the side of the target, as well as on the side of the acquirer. The remaining 161 transactions denote operations where the target was from the CEE country and the acquirer's domicile could be found outside the CEE region.

before the announcement date, respectively. As the unaffected market capitalization figure we took the first available number that was the furthest from the announcement date.

²¹³ The dot.com bubble burst in the US and heavily negatively influenced local economy in 2000 and the following years.

Figure 4 - Transactions' Matrix According to Target's and Acquirer's Domiciles

		Acquirer Country																				Total							
		AUT	BEL	BER	GBP	CZE	DEN	FIN	FRA	GER	GRC	HUN	ISR	ITA	KUW	LIT	NET	POL	POR	RUS	SER		SVK	SPA	SWE	SUI	TUR	USA	N/A
Target Country	BUL		2							1										1							2		6
	CRO							1			1		2														2		6
	CZE	4				7		1	1				1				3	1					1	2	3				24
	EST						1																	4					5
	HUN	1								3		4	1	1												3		1	14
	LAT																							1					1
	LIT														1		2							1					4
	POL	1	5		1		7	1	7	5				2			4	13	1					1	5		6	3	62
	ROM	1								1							1									1	2		6
	RUS							2		2				1						23								2	30
	SER									1											1								2
	SVK											2																	2
	SLO								1						1											1			1
	TUR		4	1	4		3	2	1	1	2		1	2	1					1							7	3	33
	UKR									1			1								1								3
	Total	7	11	1	5	7	10	6	11	15	3	7	3	10	1	1	8	16	1	26	1	1	3	14	5	7	12	10	202

Note: AUT - Austria, BEL - Belgium, BER - Bermuda, BUL - Bulgaria, CRO - Croatia, CZE - Czech Republic, DEN - Denmark, EST - Estonia, FIN - Finland, FRA - France, GBP - Great Britain, GER - Germany, GRC - Greece, HUN - Hungary, ISR - Israel, ITA - Italy, KUW - Kuwait, LAT - Latvia, LIT - Lithuania, NET - Netherlands, POL - Poland, POR - Portugal, ROM - Romania, RUS - Russia, SER - Serbia, SVK - Slovakia, SLO - Slovenia, SPA - Spain, SWE - Sweden, SUI - Switzerland, TUR - Turkey, UKR - Ukraine, USA - United States of America, N/A - not available

Source: Author

The following Table 18 provides overview of the stakes which were newly acquired within the transactions included into the sample and the scale of final stakes the acquirers held in the respective companies after the transaction was closed. As our results indicate, in majority of our cases the acquirers bought at most 20% of the company. The most frequently reached shareholding in the firm after the transaction accounted for 90.1% to 100%.

Table 18 - Newly Acquired Stake and Final Stake of the Acquirer in the Company

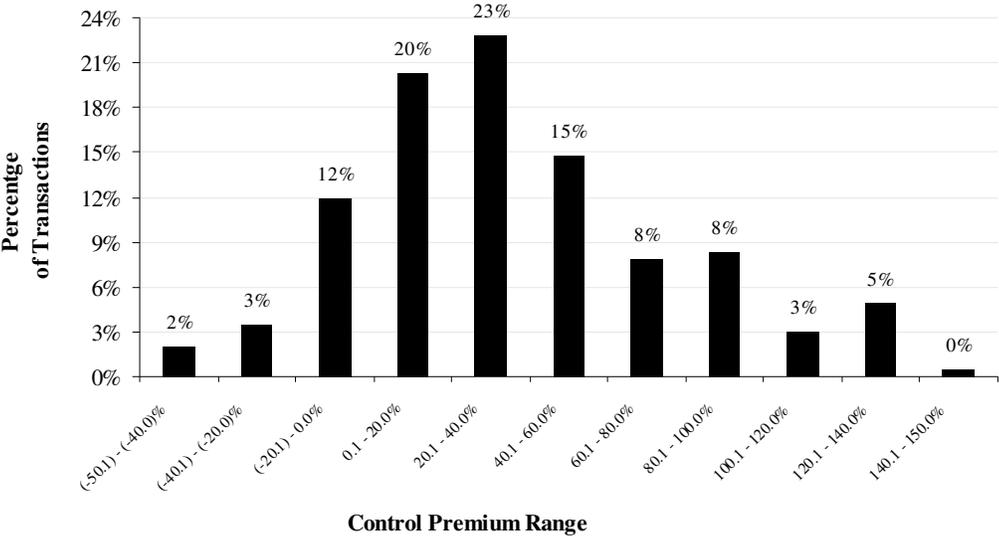
Stake Acquired in the Transaction	No. of Observations	Final Stake of the Acquirer after the Transaction	No. of Observations
0 - 20.0%	65	50.1 - 60.0%	42
20.1 - 40.0%	34	60.1 - 70.0%	26
40.1 - 60.0%	49	70.1 - 80.0%	29
60.1 - 80.0%	31	80.1 - 90.0%	23
80.1 - 100%	23	90.1 - 100%	82
Total	202	Total	202

Source: Author

The Figure 5 pointed below presents the distribution of the control premiums which were applied to the transactions in our sample. As is evident, the biggest part of the sample premiums, namely 43%, is situated in the range of 0.1% to 40%. The rest of

the values seem to be almost evenly distributed to the left and to the right hand sides from this range.

Figure 5 - Control Premiums Distribution



Source: Author

In order to better understand the control premiums applicable to our sample we designed two following tables (Table 19 and Table 20) which display the quantitative characteristics of the sample premiums with regards to the target company and the industry of the target, respectively. Table 19 shows the highest median of the control premium close to 68% applicable in Romania and the lowest value slightly exceeding 13% valid for Poland.

Table 19 - Control Premium Characteristics according to the Target's Country

Target Country Name	No. of Transactions	Control Premium Characteristics [%]			
		Median	Mean	Minimum	Maximum
Bulgaria	6	48.5%	64.0%	21.1%	135.1%
Croatia	6	46.4%	37.4%	-4.3%	59.2%
Czech Republic	24	20.4%	26.6%	-48.7%	129.4%
Estonia	5	18.8%	32.9%	1.6%	89.8%
Hungary	14	22.1%	28.3%	-17.2%	94.8%
Latvia	1	25.8%	25.8%	25.8%	25.8%
Lithuania	4	57.5%	68.6%	20.3%	139.1%
Poland	62	13.4%	21.4%	-49.0%	145.2%
Romania	6	67.7%	65.4%	4.3%	130.7%
Russia	30	51.1%	49.4%	-42.2%	128.5%
Serbia	2	60.5%	60.5%	54.0%	66.9%
Slovakia	2	29.4%	29.4%	-5.3%	64.1%
Slovenia	4	29.6%	28.3%	-2.0%	56.1%
Turkey	33	43.7%	52.9%	-30.0%	136.8%
Ukraine	3	31.5%	35.4%	-41.6%	116.3%
Total	202	28.7%	36.9%	-49.0%	145.2%

Source: Author

Table 20 below examines the scope of the premium based on the industry of the target. If we omit the only transaction where the industry of the target was not available, the highest median of the control premium close to 57% was detected in the segment of utilities. The lowest values, not even exceeding 17%, could be found among transactions where the target company was engaged in technologies.

Table 20 - Control Premium Characteristics according to the Target's Industry

Target Industry Name	No. of Transactions	Control Premium Characteristics [%]			
		Median	Mean	Minimum	Maximum
Basic Materials	14	43.4%	47.4%	-29.8%	135.1%
Communications	16	28.8%	37.3%	1.4%	129.4%
Consumer, Cyclical	16	42.7%	55.5%	-6.8%	136.8%
Consumer, Non-cyclical	36	27.8%	38.5%	-30.0%	145.2%
Energy	19	45.6%	45.8%	-48.7%	130.7%
Financial	51	20.4%	26.7%	-42.2%	139.1%
Industrial	31	17.6%	26.1%	-34.6%	107.5%
Technology	3	16.6%	16.8%	-1.4%	35.2%
Utilities	15	56.6%	51.5%	-49.0%	124.4%
N/A	1	57.1%	57.1%	57.1%	57.1%
Total	202	28.7%	36.9%	-49.0%	145.2%

Note: N/A - not available

Source: Author

5.5 Empirical Results

The purpose of this section is to provide our results on testing the hypotheses mentioned in at the beginning of this chapter.

To examine our **Hypothesis 1** regarding the acquisition of either control or pure control, we were forced to state a dummy variable called “Pure Control Dummy”. This variable equals 1 if the acquirer held a minority or no stake in the respective company prior to the transaction which brought the acquirer more than 50% stake in the company afterwards. Otherwise the dummy variable equals zero. Our regression produced the following Table 21.

Table 21 - Output of the OLS Regression Regarding the Pure Control Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.28811599	0.040760143	7.068571766	2.55505E-11
Pure Control Dummy	0.15875224	0.057081158	2.781167138	0.005934191

Source: Author

The estimated coefficient of the “Pure Control Dummy” is positive and the variable is statistically significant (p-value of 0.006). As a result, our hypothesis is supported.

In order to test our **Hypothesis 2** concerning potential increase in control premium in time, we introduced an independent variable called “Termination Year”. The variable takes the value of the year when the respective transaction was terminated. The “R” software produced the output which can be seen in Table 22.

Table 22 - Output of the OLS Regression Regarding the Termination Year Variable

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	-112.3545117	20.76176771	-5.41160624	1.7761E-07
Termination Year	0.0562428	0.01035897	5.42938701	1.6285E-07

Source: Author

The positive sign of the “Termination Year” variable together with its low p-value (below 0.001) promotes our hypothesis. Assuming that our considerations stated while formulating the Hypothesis 2 were correct, the effect of increasing demand in the M&A market outweighed the effect of improvements to the overall corporate governance and legal protection of the minority shareholders. As a result, the control premium applicable in the CEE region has been increasing over the past years.

For the purpose of testing our **Hypothesis 3** we decided to set in total 15 dummy variables which were tested one by one. All of them were called after one of the target CEE countries in our sample. The dummy variables equal one if the respective transaction realized in the country identical with the name of the dummy variable and zero otherwise.

All the outputs of our individual regressions related to this hypothesis can be found in Appendix 2. In this section we decided to mediate only outputs related to Poland and Turkey (Table 23 and Table 24, respectively), which were the only two statistically significant results from the sample of 15 countries.

Table 23 - Output of the OLS Regression Regarding the Poland Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.437623123	0.033818713	12.94026531	3.1778E-28
Poland Dummy	-0.223370307	0.061043148	-3.65921999	0.00032361

Source: Author

Table 24 - Output of the OLS Regression Regarding the Turkey Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.337817265	0.031320872	10.78569143	1.13055E-21
Turkey Dummy	0.191267405	0.077491232	2.468245768	0.014417051

Source: Author

Although, due to the statistically insignificant results in our regression we were not able either to support or reject our hypothesis, we utilized our partial regressions. The negative sign of the coefficient estimate of the “Poland Dummy” variable in the Table 23 implies statistically lower control premiums in Poland compared to other CEE countries in our sample. The positive sign of the coefficient estimate of the “Turkey Dummy” variable in the Table 24 indicates statistically higher control premiums applicable in Turkey in comparison to the rest of the CEE countries in our sample.

In order to examine our **Hypothesis 4** concerning the difference between control premium paid at domestic and cross-border transactions, we defined the “Cross-Boarder Dummy”. It equals one if the acquirer and the target come from different countries and zero otherwise. The regression output can be found in the Table 25.

Table 25 - Output of the OLS Regression Regarding the Cross-Boarder Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.3769683	0.055229135	6.825533353	1.02274E-10
Cross-Boarder Dummy	-0.0109362	0.064963213	-0.168344504	0.866482384

Source: Author

As the p-value in this case exceeds 0.86, we have to qualify this variable as statistically insignificant and our hypothesis cannot be supported.

For the purpose of testing our **Hypothesis 5** regarding the difference of control premiums recorded at transactions where the acquirer was a non-CEE resident compared to intra-CEE transactions, we determined the “Non-CEE Acquirer Dummy”. It equals one if the acquirer in the respective transaction comes from a non-CEE country and zero otherwise. The regression output related to this hypothesis is summarized in the Table 26.

Table 26 - Output of the OLS Regression Regarding the Non-CEE Acquirer Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.143819873	0.062036447	2.318312534	0.021443133
Non-CEE Acquirer Dummy	0.282604332	0.069487976	4.066952975	6.84845E-05

Source: Author

P-value below the 0.001 threshold denotes the statistical significance of this variable. The statistical significance and the positive sign of the coefficient estimate support our hypothesis.

To test our **Hypothesis 6** dealing with the difference of the control premiums in the transactions where the acquirer was a strategic buyer compared to the financial one, we designed the “Strategic Acquirer Dummy”. It equals one if the acquirer is a strategic buyer and zero in case the acquirer is a financial investor. Our software provided us with the econometric output summed up in the Table 27.

Table 27 - Output of the OLS Regression Regarding the Strategic Acquirer Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.335911728	0.086148502	3.899217276	0.000131697
Strategic Acquirer Dummy	0.037411971	0.091515973	0.408802634	0.683121904

Source: Author

The p-value in this case exceeds 0.40 and we have to evaluate this variable as statistically insignificant. As a result, our hypothesis cannot be supported.

In order to test our **Hypothesis 7** regarding the type of compensation for the transaction and its influence on the applicable control premium, we stated the “Cash Payment Dummy” variable. It equals one if the price consideration related to the respective transaction was paid fully in cash. The value of the dummy variable equals zero in all other cases of the payment. The regression output related to the seventh hypothesis can be found in the following Table 28.

Table 28 - Output of the OLS Regression Regarding the Cash Payment Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.494497346	0.077524527	6.378592271	1.21541E-09
Cash Payment Dummy	-0.145618115	0.083529558	-1.743312412	0.082815776

Source: Author

The p-value slightly exceeding 0.08 indicates statistical significance of our dummy variable on the 10% significance level. Taking the 10% significance level as a threshold, we can conclude that the negative sign before the estimated coefficient supports our final hypothesis.

5.6 Summary of the Results

Before we approach to the concluding phase of the thesis, we provide the brief summary of the main results of our empirical analysis. Table 29 gives the overview of our hypotheses including the empirical results based on our regression analyses. Except from Hypothesis 4 regarding the cross boarder transactions and Hypothesis 6 comparing the difference between the financial and strategic acquirer, all other hypotheses were supported by our models.

Table 29 - Summary of the Results

Hypothesis	Hypothesis' Formulation	Result
Hypothesis 1	Acquirer of the pure control pays a higher control premium compared to an acquirer involved in the control transaction	Supported
Hypothesis 2	Control premium increases in time	Supported
Hypothesis 3	The control premium applicable within the CEE region differs among individual countries in the region	Mixed result
Hypothesis 4	Cross-border transactions comprise larger control premiums compared to the domestic ones	Not supported
Hypothesis 5	Transactions where the acquirer comes from a non-CEE country are connected with greater control premiums compared to transactions where both the target and the acquirer come from the CEE country	Supported
Hypothesis 6	Strategic investors are willing to pay higher control premium compared to financial investors	Not supported
Hypothesis 7	Transactions compensated purely in cash encompass lower control premium compared to other types of remuneration	Supported

Source: Author

Conclusion

The aim of the thesis is to provide the reader with an overview of the discounts and premiums application in business valuation. We present a discussion of theoretical concepts related to control premium/minority interest discount and lack of marketability discount. We also examine the issue of control premium in the CEE region. For this purpose we use a sample of 202 control transactions which materialized in the CEE region from 2000 to 2009, as provided by the Bloomberg database. Our findings regarding the size of the control premium applicable in the CEE region suggest that 43% of all the transactions in our sample recorded a premium ranging from 0.1% to 40.0%. These figures are comparable with other empirical studies. Based on the selected data we statistically tested seven hypotheses regarding control premium determinants.

After distinguishing between control and pure control transactions, the regression analysis we carried out supported our initial hypothesis that pure control transactions are associated with significantly higher premiums compared to control transactions. As a pure control transaction newly brings controlling stake in the company to the acquirer, our finding is in line with both, our research and economic rationale.

In addition, we document that the control premium develops in time. We considered two antagonistic effects which can cause the control premium to go either up or down in time. The effect of improving legal environment in the CEE countries which would force the control premium to decrease was likely outweighed by the “psychological” effect of rising demand recently recorded in the M&A market. Such increasing demand also pushes up the size of the control premium in time.

Furthermore, we are not able to support our hypothesis that the control premium applicable within the CEE region differs among the countries in the sample. Nevertheless, we utilized the partial results of our analysis showing that the control premiums applicable to Poland is significantly lower compared to other countries in the CEE. On the other hand, the control premiums recorded in Turkey are significantly higher than in the rest of the CEE countries.

In contrast to the previous research, we are not able to support the hypothesis that cross boarder transactions comprise larger control premiums compared to the

domestic deals. On the other hand, we find evidence that transactions with an acquirer from a non-CEE country encompass greater control premiums compared to transactions where both, the target and the acquirer come from the CEE region. The very probable reason for this finding is that non-CEE acquirers pay higher control premiums because of a geographical diversification of their business activities.

Furthermore, we cannot support our hypothesis that strategic acquirers are on average paying higher control premiums in comparison to financial acquirers. Our assumption stems from the belief that strategic investor who already operates its business in the respective segment should be more able to exploit the synergistic effect, hence, should pay a higher control premium.

Our analysis also investigates the influence of the form of the transaction compensation on the applicable control premium. We support the hypothesis that the transactions compensated purely in cash involve lower control premiums. Our finding is in compliance with our belief that the pure cash remuneration makes the acquirer the only risk-taker in the whole transaction. On contrary, if at least part of the consideration is settled in shares, the risk transfers from the acquirer to the seller to certain extent. As a result, rational acquirer is willing to offer lower control premium in case the seller demands pure cash remuneration.

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DATABASES

Bloomberg

Mergermarket

Zephyr

Appendices

Appendix 1 – Black-Scholes Option Pricing Model Applied by Chaffe²¹⁴

$$V_0 = \frac{CF_1}{r-g} = \frac{CF_0(1+g)}{r-g}, \text{ where} \quad (10)$$

- (v) V_0 stands for the value of the underlying asset at time 0;
- (vi) CF_1 represents the cash flow in the next period of time;
- (vii) r means the company's discount rate; and
- (viii) g stands for the expected growth rate of the company's dividends.

$$P = Kr^{-t} N(Y + \sigma\sqrt{t}) - SN(Y), \text{ where} \quad (11)$$

- (i) $Y = \frac{\log(Kr^{-t} / S)}{\sigma\sqrt{t}} - \frac{1}{2}\sigma\sqrt{t}$;
- (ii) P stands for the put option price;
- (iii) S represents the stock price;
- (iv) K expresses the strike price;
- (v) r denotes the interest rate (+1.0% in yield);
- (vi) t stands for the time to expiration (% of year);
- (vii) σ represents the volatility;
- (viii) $N(z)$ denotes standard normal (distribution) density function, which can be expressed by the following formula:

$$N(z) = 1 - \left(\frac{1}{\sqrt{2\pi}} \right) e^{-z^2} (b_1k + b_2k^2 + b_3k^3 + b_4k^4 + b_5k^5), \text{ where} \quad (12)$$

$$k = \frac{1}{1+az}; \quad \mathbf{a} = 0.2316419; \quad \mathbf{b}_1 = 0.319381530; \quad \mathbf{b}_2 = 0.356563782;$$

$$\mathbf{b}_3 = 1.781477937; \quad \mathbf{b}_4 = -1.821255978; \quad \text{and} \quad \mathbf{b}_5 = 1.330274429.$$

²¹⁴ In Chaffe, D.B.H. (1993), "Option Pricing as a Proxy for Discount for Lack of Marketability in Private Company Valuations", Business Valuation Review, December 1993, p. 182-188.

Appendix 2 – Regression Outputs Related to the Hypothesis 3 (The Applicable Control Premium Differs among Countries in the CEE Region)

Table 30 - Output of the OLS Regression Regarding the Bulgaria Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.36077934	0.029326987	12.30195733	2.89362E-26
Bulgaria Dummy	0.27891419	0.170163928	1.63909117	0.102766335

Source: Author

Table 31 - Output of the OLS Regression Regarding the Croatia Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.368920625	0.029523247	12.49593653	7.36399E-27
Croatia Dummy	0.004824279	0.171302691	0.02816231	0.977560779

Source: Author

Table 32 - Output of the OLS Regression Regarding the Czech Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.383025792	0.030847432	12.41678068	1.28757E-26
Czech Dummy	-0.117512425	0.089493001	-1.31309066	0.190656964

Source: Author

Table 33 - Output of the OLS Regression Regarding the Estonia Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.37007767	0.02944475	12.56854368	4.40943E-27
Estonia Dummy	-0.04095544	0.18715378	-0.21883309	0.827003218

Source: Author

Table 34 - Output of the OLS Regression Regarding the Hungary Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.375467806	0.030095805	12.47575223	8.49192E-27
Hungary Dummy	-0.092398927	0.114318793	-0.80825667	0.419903186

Source: Author

Table 35 - Output of the OLS Regression Regarding the Latvia Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.36961583	0.029148511	12.68043584	1.9993E-27
Latvia Dummy	-0.11148532	0.414278184	-0.26910739	0.78812446

Source: Author

Table 36 - Output of the OLS Regression Regarding the Lithuania Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.36266864	0.029197502	12.42122141	1.24785E-26
Lithuania Dummy	0.32296163	0.207487238	1.55653731	0.121161354

Source: Author

Table 37 - Output of the OLS Regression Regarding the Poland Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.437623123	0.033818713	12.94026531	3.1778E-28
Poland Dummy	-0.223370307	0.061043148	-3.65922000	0.00032361

Source: Author

Table 38 - Output of the OLS Regression Regarding the Romania Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.360342694	0.029305668	12.29600674	3.01756E-26
Romania Dummy	0.293614601	0.170040232	1.72673606	0.085759126

Source: Author

Table 39 - Output of the OLS Regression Regarding the Russia Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.347219714	0.031259927	11.10750225	1.22699E-22
Russia Dummy	0.147084319	0.081115346	1.81327366	0.071288697

Source: Author

Table 40 - Output of the OLS Regression Regarding the Serbia Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.366709281	0.029178643	12.56772923	4.43488E-27
Serbia Dummy	0.237818522	0.293241731	0.81099822	0.418330734

Source: Author

Table 41 - Output of the OLS Regression Regarding the Slovakia Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.369813838	0.029221723	12.65544274	2.38579E-27
Slovakia Dummy	-0.075741705	0.293674678	-0.25791023	0.796741375

Source: Author

Table 42 - Output of the OLS Regression Regarding the Slovenia Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.370796111	0.029360921	12.62889928	2.87827E-27
Slovenia Dummy	-0.087475633	0.208648545	-0.41924871	0.675484322

Source: Author

Table 43 - Output of the OLS Regression Regarding the Turkey Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.337817265	0.031320872	10.78569143	1.13055E-21
Turkey Dummy	0.191267405	0.077491232	2.46824576	0.014417051

Source: Author

Table 44 - Output of the OLS Regression Regarding the Ukraine Dummy

	Coefficient Estimate	Standard Error	t-statistics	P-value
Intercept	0.369291426	0.029299626	12.60396385	3.43303E-27
Ukraine Dummy	-0.015318725	0.240423621	-0.06371555	0.949260354

Source: Author