

Charles University in Prague

Faculty of Social Sciences

Institute of Economic Studies



MASTER THESIS

**Natural Interest Rate: Is 2 % CPI
Inflation Still the Right Target?**

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Academic Year: **2016/2017**

Declaration of Authorship

1. Hereby I declare that I have compiled this master thesis independently, using only the listed literature and sources.
2. I declare that the thesis has not been used for obtaining another title.
3. I agree on making this thesis accessible for study and research purposes.

Prague, May 17, 2017

Signature

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Abstract

This paper uses the semi-structural Laubach and Williams model to estimate the time-varying natural rate of interest by Kalman filter and Maximum Likelihood method, applying it for the first time to Czech data. The results show a significant decrease of the natural interest rate during the past decade, which constitutes further evidence for the wide-spread notion that structural factors in many countries have shifted after the global financial crisis. The paper's contribution is mainly represented by preparing ground for further research. It concludes that the basic version of the Laubach and Williams model is not optimal for the Czech environment and suggests appropriate adjustments to it. It discusses and analyzes sources of potential problems with the estimation, notably the issues of singularity and model specification. Eventually the paper concludes that due to the low significance of results and the uncertainty of gains and losses related to a policy switch, the best reaction of the central bank would be to keep the current regime and inflation target.

JEL Classification C32, E43, E52, O40

Keywords natural real interest rate, inflation target, inflation measurement, monetary policy, Kalman filter, trend growth

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Abstrakt

Práce odhaduje přirozenou úrokovou míru podle semi-strukturálního modelu Laubacha a Williamse za pomoci Kalmanova filtru a metody Maximální Věrohodnosti, a jako první aplikuje tento model pro česká data. Výsledky naznačují významný pokles přirozené úrokové míry v posledním desetiletí, což představuje další důkaz ke známé hypotéze, že došlo ke změně strukturálních faktorů ekonomik v mnoha zemích po globální finanční krizi. Práce je přínosná v tom, že reprezentuje odrazový můstek pro budoucí výzkum. Dochází k závěru, že základní verze modelu Laubacha a Williamse není optimální pro českou ekonomiku, a navrhuje možné úpravy, které by lépe reflektovaly česká specifika. V práci se diskutují a analyzují zdroje problémů s odhadem, především riziko singularity a specifikace modelu. Následně, z důvodu nízké signifikance odhadnutých hodnot v kombinaci s nejistou mírou přínosů a nákladů v případě změny monetární politiky, je navrženo ponechat současný režim i dvouprocentní inflační cíl.

Klasifikace JEL

C32, E43, E52, O40

Klíčová slova

přirozená reálná úroková míra, inflační cíl, měření inflace, monetární politika, Kalmanův filtr, růst potenciálního produktu

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Master's Thesis Proposal

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Proposed Topic:

The Natural Interest Rate: Is 2% CPI Inflation Still the Right Target?

Motivation:

The main objective of monetary policy in Czech Republic is sustaining stable prices. There has been an ongoing global discussion about whether the inflation targeting should not be reassessed, especially after the recent financial crisis that might have structurally changed the economic conditions which the 2% CPI target is based on.

The way to investigate whether the discussion is relevant for Czech Republic, I intend to empirically test if the natural interest rate has significantly changed in the long term, what is its current estimated level and relationship to other specific macroeconomic variables.

In the Czech Republic, short-term interest rates are one of the primary tools of monetary policy. Natural level of interest rate is a level at which the output converges to its potential and the inflation remains stable. Short term real interest rates different from the natural interest rate, ceteris paribus, have either stimulative or contractionary effect. Therefore, the natural interest rate has important implications for conducting the monetary policy.

If the natural interest rate in Czech Republic has changed notably in recent past, it would suggest that the inflation target has a potential to be updated and the discussion about it is relevant.

Hypotheses:

The null hypotheses are set as follows:

1. Hypothesis #1: The natural interest rate tends to be constant in a long term
2. Hypothesis #2: The natural interest rate has not significantly changed after the financial crisis
3. Hypothesis #3: The relationship between the growth rate of GDP and the natural interest rate has not changed after the financial crisis

Methodology:

Macroeconomic data will be gathered from the Czech Statistical Office

Following Laubach & Williams (2003), Kalman filter in a combination with the Maximum Likelihood method will be used for the estimation of natural rate of interest, output and trend growth. Kalman filter allows to partially adjust the estimate of natural interest rate (burdened by noise and errors) based on how far off the model's estimate of GDP is from the actual GDP.

Expected Contribution:

The knowledge of the level of natural interest rates serves as an important input into the discussion about the right level of the target and the relevance of the discussion itself. Moreover, the knowledge of natural interest rate is a vital information for monetary policy makers, because it implies what level of short term interest rate stimulates and what level contracts the economy.

I am going to contribute to the current research by investigating the data related to Czech Republic, since the Czech economy has been covered relatively little.

Outline:

Introduction

Theoretical part

1. Literature review
2. Economic specifics of Czech Republic and its implications relevant to the topic (devaluation, zero-lower bound)
3. Issues regarding the inflation measurement (CPI, indexes)
4. Natural interest rate in a theory
5. Inflation targeting in theory (how it should be set relative to economic variables, incl. natural int. rate)

Empirical part

1. Data description
2. Specification and estimation of the empirical model
3. Analysis of the results
4. Implications for conducting of monetary policy

Conclusion

Core Bibliography:

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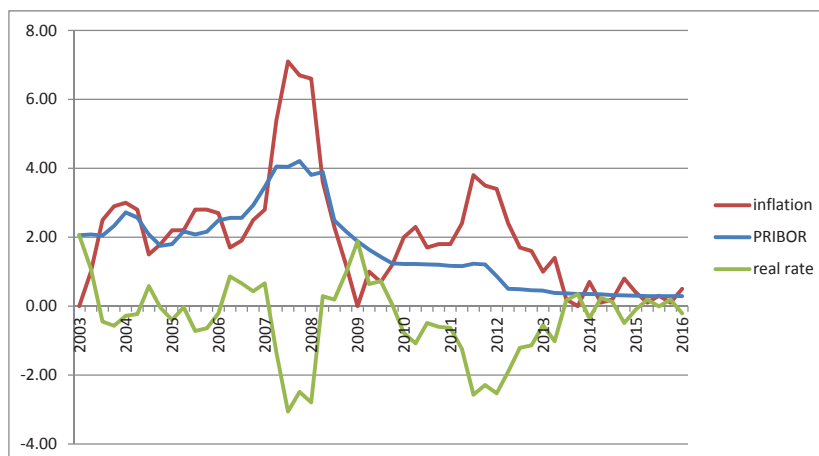
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1. Introduction

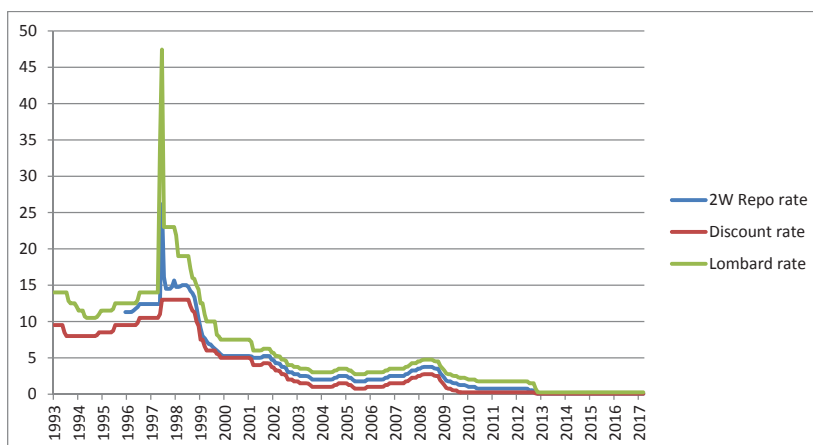
It has been observed in recent years that a large number of developed economies including the Czech Republic, are experiencing very low, often even negative real interest rates (Figure 1.1). The nominal policy rates are approaching or have even hit zero (Figure 1.2), so the topic of secular stagnation and zero lower bound is being heavily discussed. Moreover, the real rates across many countries have been gradually decreasing for a longer period - as Carvalho et al. (2016) mention, at least since 1990's. This empirical fact suggests the change might be permanent – a so called “new normal”, due to the shifts in various structural factors. Is this really the case? If so, should we be reassessing the current monetary policy and inflation targeting altogether? The concept of natural interest rate can be used to examine some or all of these questions.

Figure 1.1.: Czech Republic inflation rate, nominal PRIBOR and real PRIBOR



source: data from Czech Statistical Office, CNB's database ARAD, and author's calculations

Figure 1.2.: Czech Republic deposit interest rate



source: data from CNB's website

The main task of monetary policy in the Czech Republic is keeping price stability in the economy and the short-term interest rate represents the primary instrument of the central bank for doing so. In order to achieve price stability it is important to know what the neutral (equilibrium) stance of the monetary policy is, otherwise the policy could not be conducted effectively. The neutral policy stance can be defined as the one having neither inflationary nor contractionary effects, and therefore it can serve as a benchmark for specific actions of a central bank. It is the stance when the short-term interest rate is set at the level of the natural interest rate, since the natural rate of interest represents the rate of interest at which the inflation is constant and the output is at its potential level. If a central bank chooses short-term interest rate lower than what the natural interest rate in the economy is, the consequence would be an increase in inflation and output, and vice versa. Some researchers, for instance Taylor (1999) or Woodford (2003) highlight the importance of the natural interest rate and call for central banks to be enacting policies to track its value. The importance of the natural interest rate concept is only amplified by a shift from money to inflation targeting in many industrial economies, the increasing use of short-term interest rates as a main tool of monetary policy, and the wider implementation of a policy rule introduced in Taylor (1993), the Taylor rule, requiring the estimate of an equilibrium interest rate.

This said however, the natural interest rate concept faces severe obstacles. Not only

the natural rate of interest itself, but also some factors influencing it - notably the potential output growth - are not actually observable and must be estimated. The risk of misspecification and eventual costly mis-measurement, summarized thoroughly for instance by Orphanides and Williams (2002) are the main causes for central banks not to be using the time-varying natural interest rate on a large scale and in their day-to-day policymaking efforts.

In this study, the main focus is directed at the natural interest rate estimation for the Czech Republic, discussing its shortcomings and implications for the monetary policy. Using the semi-structural model originated by Laubach and Williams (2003), I show that there is evidence for a considerable decrease in natural interest rate and potential growth rate during the past decade. The results, however, should be considered as tentative due to the risk of misspecification in the original model, the small size of the data sample, and possibly due to the specifics of Czech data. Moreover, a potential misspecification is identified and adjustments to this model proposed.

The paper is constructed as described herein. In Section 2 the literature review summarizing the research done on the topic is included. Section 3 provides an explanation of the natural interest rate concept in economic theory. In Section 4 the empirical model and methodology are introduced, data described and estimation performed. Section 5 includes the discussion of the results together with their shortcomings. In Section 6 the link between natural interest rate and inflation targeting is reviewed and potential implications for conducting monetary policy are identified and assessed. Lastly, Section 7 includes the open economy considerations and Section 8 provides the conclusion of the paper.

2. Literature Review

The current booming research related to the natural interest rate began with the publishing of Laubach and Williams (2003). Until then most of the authors had primarily been focusing on the actual estimation of potential GDP. The Laubach and Williams' surprising results encouraged other researchers to investigate the issue and since then natural interest rate estimation has become the subject of a number of studies especially in United States. Nevertheless, Central and Eastern European Countries have not gained much attention in research and debate, despite the fact that many of them target inflation as their primary monetary goal (this is true for the Czech Republic, but also for Poland, Hungary, Romania, and for Slovakia before adopting the Euro currency). One of the studies is for instance Brzoza-Brzezina (2006) investigating the case in Poland. Some papers focus on Western European data, notably Manrique and Marques (2002) who apply the Laubach & Williams methodology on the German economy, Winttr et al. (2005) for Luxembourg, or Danielsson et al. (2016) for Iceland.

The common feature shared by most research papers is the highlighting of evidence for historically low natural interest rate levels, specifically Hamilton et al. (2015) or Lubik and Matthes (2015). Also, research suggests the rate is not at a constant level – for instance Manrique and Marques (2002) argue that the historical averages change by a large extent over longer periods of time, and Laubach and Williams (2003) find that the equilibrium real interest rate fell almost by a factor of two - from 4.5 percent in the 1960's to nearly 2.5 percent in the 1970's.

The time horizon over which estimations of the natural interest rate are calculated appears to be critical since its development is affected by different determinants, as shown in the Section 3.2. Part of the research focuses on the estimation of short-term natural interest rate levels (Giammarioli and Valla (2003), Curdia et al. (2015), Goldby et al. (2015)) which are considered to be dependent on business cycles. Such models are designed in a way to avoid the low frequency movements of the natural interest rate which is achieved by detrending the series. They concentrate solely on the response of the natural interest rate to temporary shocks and the results show it

to have a strongly cyclical behavior. It has been pointed out by Levin et al. (2003), however, that the models are not robust to changing model assumptions. The other part, notably Laubach and Williams (2003) and Holston et al. (2016), but also many others, concentrate on changes in the long-term natural interest rate. Models focusing on the long term development suffer from potential inaccuracy and low robustness especially due to the presence of a large number of unobserved variables in them.

The selection of a time frame relates to a major challenge in the research - deciding which model formulation and which method of estimation to use for the investigation of natural interest rate. Giammarioli and Valla (2003) provide a useful overview of the methods.

Some authors tried to use models that do not relate the natural rate of interest to the real factors. Enders and Siklos (2001) chose ARIMA model assuming that the real interest rate is based on a GARCH process. Kozicki and Tinsley (2001) or Bomfim (2001) have decided for the estimation of equilibrium short-term real interest rates using forward rates, particularly data gathered from the yield curve of inflation-indexed government securities. As Manrique and Marques (2002) claim, such models do not seem appropriate since they do not reflect changes in any structural characteristics of the economy and therefore do not seem useful for decisions concerning monetary policy.

Current research also contains papers that adjust the Laubach-Williams model, while retaining the basic concept. For example, Mesonnier and Renne (2007) allow for stationarity with high persistency in the potential output growth and natural interest rate, while Laubach and Williams (2003) assume non-stationary processes. Larsen and McKeown (2003) do not link the natural interest rate to potential growth at all. The high number of parameters needed to be estimated was, however, causing problems. Rudebusch and Svensson (1999) have used a similar equation for the output gap, the difference being they only assumed the natural rate of interest to be constant. Orphanides and Williams (2002) assume the natural interest rate to follow a random walk instead of linking it to the trend growth rate. In Kozicki (2004) the IS curve alone is used for the estimation while the development of the natural rate of interest is defined as a random walk, and the output gap is constructed using the Congressional Budget Office's estimates.

The reduced-form structural model of Laubach and Williams used in this paper links the natural rate of interest to the trend growth rate (potential growth of the economy). The authors are followed by a number of authors, namely Crespo Cuaresma et al.

(2003), Basdevant et al. (2004), Larsen and McKeown (2003) or Garnier and Wilhelmssen (2009). Laubach and Williams get inspired by models that decompose the evolution of output into the trend and cycle components, specifically Kuttner (1994), Clark (1987) or Watson (1986). None of them, however, included the interest rate variable into the model. Watson (1986) introduces a specification that assumes a constant trend growth and output gap following an AR process. Clark (1987) modifies Watson's model by allowing for time variation of the trend growth rate. The equation specifying the dynamics of inflation is added by Kuttner (1994).

An important issue to consider is the robustness of the models introduced. Clark and Kozicki (2004) thoroughly investigate the difficulties related to estimating the natural rate of interest in real time. They find that the models are exposed to a high level of specification uncertainty and that the relationship between the natural rate of interest and the trend growth rate seems to be weaker than other papers assume. For instance, they point out that even though Laubach and Williams highlight the robustness of their model, certain changes in specification, as in Orphanides and Williams (2002), lead to significantly different estimates. Moreover, different difficulties related to one-sided filtering have been addressed in a number of studies, for example in Orphanides and Williams (2002) or Laubach and Williams (2003). Additionally, there is a part of the research being devoted solely to investigating the shortcomings of the natural interest rate estimation. For example Kiley (2015) who states that "*the co-movement of output, inflation, unemployment, and real interest rates is too weak to yield precise estimates of r^** " (page 2) and he goes on to suggest introducing various control variables as a potential solution.

The monetary policy implications of the results are the most important part and the actual purpose of the research related to the natural interest rate. In the current research, the opinion that we live in the environment defined by persistent, unprecedentedly low natural interest rate, prevails. Williams (2016) names potential solutions to such situation. One of the solutions presented is a rise in the inflation target, which has been analyzed and supported for instance in Ball (2014). The commonly mentioned alternative targeting regimes competing with inflation targeting are price-level targeting and nominal-GDP targeting. Price-level targeting has been investigated for instance in Svensson (1996), and nominal GDP targeting in Sivak (2013). Moreover, the low natural interest rate and potential growth rate are likely related to the global financial crisis which started in 2008. Blanchard et al. (2010) examine whether the macroeconomic policy should be reassessed after the crisis and according to their results, the general monetary policy framework should stay the same.

3. Theoretical Background

Natural interest rate, sometimes called equilibrium interest rate, represents a concept introduced by the Swedish economist Knut Wicksell in 1898. In particular, Wicksell (1898) defined the natural interest rate in the following way: *“There is a certain rate of interest on loans which is neutral with respect to commodity prices, and tends neither to raise nor to lower them.”* (1936 translation from 1898 text, p.102.) Nowadays, there are various definitions of natural interest rate. However, all of them commonly define it as an equilibrium real interest rate that has neither inflationary nor deflationary effects, when the economy is operating at its full potential (output being at its potential level). This is the interest rate which persists when the economy is in a state of equilibrium, when savings equal investment. Thus the natural interest rate is the rate yielding price and economic stability.

3.1. Transmission Mechanism

The relationship between the interest rates, inflation and output is summarized by Wicksell as follows: When the interest rate at which banks lend money to firms is lower than the natural rate, firms increase profits. As a consequence they expand production, demanding more production inputs like capital or labor. Together with the employment increase, households can borrow more cheaply. As a consequence, the demand of both firms and households rises, implying the increase of general price level. According to the author, the measurement or estimation of the natural interest rate is not necessary, since it should be enough to simply observe the development of the inflation.

Nowadays, however, the transmission process became much more difficult and, especially in small open economies, more channels exist that could connect interest rates with inflation. For instance, the low short-term interest rate implies a higher supply of credit since the adverse selection problem gets partially eliminated so the banks loosen their credit conditions. The higher credit supply increases the aggregate

demand, and eventually the inflation. Moreover, as a result of the low interest rate the exchange rate depreciates as the domestic currency becomes less attractive and demand for it decreases. Depreciation in turn implies an inflation increase. Asset prices also play a role – the low interest rate makes asset purchases more attractive relative to money saving, so their prices go up. The higher asset prices imply a rise in wealth, thus it also contributes to the increase of the aggregate demand and inflation. These channels are not exclusive, and could actually be synergistic, since one influence and complement the other.

As seen, the transmission mechanism is not straightforward and has many interdependent channels. This is the reason why nowadays, the natural interest rate cannot simply be inferred from the inflation development, as Wicksel proposed.

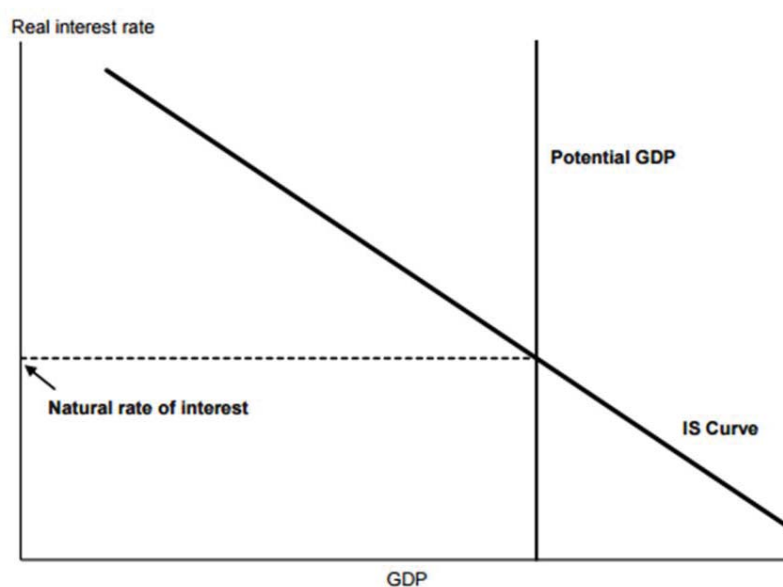
3.2. Natural Interest Rate Determinants

The natural rate of interest can be looked at from two perspectives. The medium-term rate is being influenced by structural factors on both the supply and demand side. The supply side includes the potential growth of the economy (determined predominantly by technology and factor endowments). The demand side is related to demographical shifts, population growth and the effects of aging on the elasticity of substitution and time preference. The short-term natural rate is understood as the one yielding period-to-period price stability. It can be affected by temporary real shocks regarding government spending, asset prices or credit conditions. In the medium-run, though, the short-term shocks fade away and the natural rate is affected primarily by changes in productivity or demography. In this paper we will use the term natural real rate of interest to refer to the medium-term one.

For illustration, in the Figure 3.1 it is visible, how the medium-term natural interest rate can be determined. The changes in the real interest rate affect consumption and investment, as represented by the IS curve. The natural interest rate is then defined by the intersection of the IS curve with the potential output. The natural interest rate can vary in time due to the low frequency shifts in the IS curve as a consequence of the significant structural changes mentioned above. It is worth mentioning that the graph relates to the medium-term natural interest rate, thus such interpretation is not applicable to the cyclical shifts in the IS curve – for instance the precautionary savings creation during recession periods. The Laubach and Williams model, which will be used in this paper, follows the rationale behind Figure 3.1 and derives the

medium-term natural interest rate based on the neoclassical growth theory. It states that the equilibrium real rate is an increasing function of the trend growth rate of the output.

Figure 3.1.: Stylized determination of natural interest rate



source: Laubach and Williams (2003)

However, there is a notable difference between open and closed economies. The positive relationship between the trend growth rate and natural interest rate is applicable especially for the closed regions. For small open economies the real version of the uncovered interest rate parity should be considered. As a result, the higher trend growth rate is related to the real appreciation of the domestic currency, which eventually lowers the natural interest rate. Therefore, contrary to the neoclassical growth model, the relationship can be inverted.

Specifically in the Czech Republic, the natural interest rate is expected to experience a decrease in its value due to the transition accompanying the membership of Czech Republic in the European Union. As Lipschitz et al. (2006) mention, before the transition process took place the ratios of capital to labor were relatively low in the future EU members. Therefore their marginal product of capital and the natural interest rate exceeded significantly Western countries' values. Due to the accumulation of domestic capital together with the significant inflow of the foreign capital (especially the foreign direct investment) after that, the fall in natural interest rate is expected to be seen in the sample. The other factors related to the EU transition, for instance the decrease of exchange rate risk, potentially also the country risk premium, or the fall

in foreign natural interest rate may play a role in the development of natural interest rate in Czech Republic, as explained in Horvath (2006).

3.3. Shortcomings of Natural Interest Rate Estimation

As mentioned, since the natural interest rate is unobservable, it must be estimated. However, the estimation of the time-varying natural interest rate is not straightforward and there are various issues the estimation process must address. As a result, policy decisions, did not take the time-varying estimates of the natural interest rate into much consideration. If we briefly summarize the potential issues, we find out that all known methodologies face some obstacles.

The general problem of the source data arises due to a common revision of available economic data that is gathered and consequently used in an estimation. Thus the estimates resulting from the estimation process performed in real time can differ considerably with the ones based on the revised data.

Estimating the natural real rate of interest using Taylor rule leads to tautology. This problem is investigated in the Section 6 in more detail.

Moreover, the estimation based on the co-movement of inflation, output and real interest rate suffers from some notable issues too. Most importantly, the empirical relationship seems to be weak and the estimation might be subject to an omitted variable bias. Evidence of the weak link between the variables, especially the flatness of the Phillips curve, has been found also in this paper. The issue is discussed more in detail in the Section 5. The potentially omitted factors affecting the output can be for instance fiscal policy, asset prices or credit conditions. In the neoclassical approach, they are considered as shorter-term phenomena and therefore regarded as noise. However, Kiley (2015) finds that those factors, especially the credit spread, are important for the estimated link between output and interest rates. The bias doesn't have to exist only due to the omitted variable, but even due to the omitted equations in the model. The omitted variable bias issue is further examined in Section 5 of this paper, where the analysis concludes that the estimation issues are likely not caused by omitted variables alone, and that the omitted equations may be a problem too. As a consequence, the estimated developments of the unobserved variables like natural interest rate or trend growth rate can represent developments of other macroeconomic

variables affecting the economy, which were not included in the model, thus the relationships become spurious.

The biases are closely related to the robustness issue. As Clark and Kozicki (2004) point out, the estimates are highly sensitive to the concrete form of the natural interest rate equation. For instance, Laubach and Williams (2003) highlight that their estimates proved to be robust to certain changes in model specification. However, Orphanides and Williams (2002) who investigated the Laubach and Williams model adjusted in a way that the natural interest rate follows a random walk instead of being related to the trend growth rate, show that their natural interest rate estimates differ considerably from the ones of Laubach and Williams. Moreover, the variability of the natural interest rate components (trend growth rate and other factors) allowed in the model, or the value of initial parameters can also significantly alter the resulting estimates. The other potential reason for the impreciseness is a one-sided filtering technique for real-time estimates, in which the estimate at certain point in time is based solely on the data available at that time. The problem is pointed out for instance in Orphanides and Williams (2002), who claim that the difference between the one-sided and two-sided (smoothed) filtered estimates can be considerable, since the one-sided estimates typically include much more noise.

4. Methodology

4.1. Overview of the most common methods

The estimation of the natural rate of interest can be a challenging task. The simplest possible ways to estimate natural interest rate are univariate methods including averages and statistical filters. According to Laubach and Williams (2003), if natural interest rates were more or less constant over time and the inflation rate were not trending, we could estimate it by the sample mean (long-term average) of actual real interest rates. Nevertheless, the natural interest rate is most probably not constant over a longer period of time. For instance Rapach and Wohar (2005) discovered evidence of a number of structural breaks in the real interest rates mean in the past decades for many countries. Consequently the authors suggest using estimation methods allowing for significant persistent shifts in the natural interest rate. The other way of estimation allowing to account for persistent shifts in natural rate is a short-term moving average calculation - as shown in Figure A.1. The striking fact is that the interest rate behaves nearly cyclically and there are large swings lasting for decades.

The more advanced and precise estimation methods are using time-series filters that are attempting to separate long and short-term deviations. Such techniques include Hodrick-Prescott or Band-pass filters.

However, all the univariate estimation methods just track a longer term development of the short-term rate and they do not account for the actual definition of the natural interest rate as the rate that does not produce any inflationary or deflationary pressures. Inflation rate and economic activity rises when the actual short-term interest rate is below the natural rate of interest, and vice versa – when the short-term rate is above the natural rate, inflation and economic activity decreases. The above mentioned swings can occur due to large shifts in the inflation rate or economic activity - which in fact means that the short-term interest rate departs from the natural interest rate and is not a reliable representation of it. Therefore, in times when inflation was

rising, the short term rates were below the natural interest rate and thus the univariate methods would have underestimated the natural interest rate. Analogically, in periods of falling inflation rates, these methods would overstate the natural rate.

The next possible approach is to consider financial market indicators, for instance forward rates implied from the yield on longer-term government securities, rather than the actual interest rates. Such forward rates bear information about the market participants' expectations of the future interest rate which is more reasonable than just recording past values of the actual rates. The problem regarding such approach is that the forward rates contain a term and liquidity premiums adding noise into the measurement. The issue was investigated for instance by Kim and Wright (2005) who find that the premiums contribute significantly to the variation of forward rates.

The other possibility is to use structural models because they, together with the interest rate information, incorporate also relevant information regarding the state of the economy. Their power is in the possibility to provide a "full picture" of the economy in which natural interest rate plays a part - they are not purely statistical as the models mentioned previously. On the other hand, such models tend to be sensitive to model specification and choice of model assumptions.

Ideally, the model needed for the estimation has several specific properties. Firstly, it should allow the estimates to vary with time. The natural interest rate is required to be directly linked to the real factors in the economy. Moreover, the estimates need to be robust to changes in the model specification. Nevertheless, as already pointed out, all models introduced until now face certain obstacles and do not fully meet all of the mentioned requirements.

I have decided to use the well-known Laubach and Williams reduced-form structural model (Laubach and Williams (2003)). The structural models are evaluated in Garnier and Wilhelmsen (2005) as follows: *This method has become popular since it strikes a compromise between the theoretically coherent DSGE approach and ad-hoc statistical approaches.*" (page 6) Concretely, the Laubach and Williams model is a simple model that essentially links the interest rates to economic activity measures, specifically the output-inflation dynamics. It accounts for a development of the potential output level and its trend growth rate together with the natural interest rate, and estimates the three variables simultaneously using Kalman filter and Maximum Likelihood methodology. The Laubach and Williams model has gained significant attention in the literature worldwide. It is valuable especially for closed economies

since it does not include any factors defining the international effects (exchange rate, terms of trade, foreign macroeconomic variables and so on). However, even for countries like the Czech Republic it carries an added value - it represents a stepping stone enabling further research leading to its more complex and country specific versions, as discussed in Section 7.

4.2. The Model

In this paper an adjusted version of the Laubach and Williams model from Holston et al. (2016) will be used. The authors build on the unobserved components model specification introduced by Watson (1986) which decomposes a series of the real output into a trend and a cycle component. The trend is consequently interpreted as a level of potential output and the cycle as the output gap.

$$y_t = y_{t,trend} + y_{t,cycle} = y_t^* + y_{gap}$$

The model is common in the research literature, and it provides a ground for various extensions. The difference between the Laubach and Williams model and Watson's, is that Laubach and Williams add the inflation and real interest rate changes into Watson's specification. Thanks to that, as noted by Manrique and Marques (2002), the system of equations avoids the bias present in the single-equation methods when there are prolonged inflationary or deflationary periods, for instance in the case of the Hodrick Prescott filter.

The inflation and output gap behavior in the Laubach and Williams model is inspired by a neo-Keynesian approach, where the dynamics of the natural interest rate follow the optimal growth theory. The model includes an equation defining an IS curve or aggregate demand equation (1), Phillips curve (2) and the equation relating the natural rate of interest to the trend growth rate (3).

IS curve is a Keynesian concept that provides the combinations of output and real interest rate satisfying the equilibrium condition in the market for goods and services – that the total demand affected by the income and interest rate must equal total supply (income):

$$Y^d(Y, r) = Y$$

$$Y^d(Y, r) = C + I + G + NX = \bar{A} + c(1 - t)Y - bi$$

where $Y^d(Y, r)$ stands for aggregate demand, Y for national income, C for consumption, I for investment, G for government expenditure and NX for net exports. \bar{A} is a notation for exogenous demand, c for marginal propensity to consume, t for tax rate, b for the sensitivity of investment to the real interest rate and i for the interest rate. Compared to the generally known IS curve (as shown in Figure 3.1) in which output represents the X axis and the real interest rate occupies the Y axis, the axes are reversed in the current model setting. This difference is important to mention in order to avoid confusion related to the discussion about the IS curve slope in next sections.

The Phillips curve is a curve representing an empirical inverse relationship between inflation and unemployment. It is a model developed by William Phillips in 1958. Unemployment is directly related to the output gap since the output gap is defined as the difference between the potential and actual output and potential output - as the output in a state of full employment. Thus, the Phillips curve can be also interpreted as a positive relationship between inflation and output gap. In the long term, according to the generally accepted theory, the Phillips curve should become vertical at the natural rate of unemployment, so that the expansionary policies have no effect on unemployment. Natural unemployment is defined as the rate of unemployment when the economy is at an equilibrium and the output gap is closed.

The third equation is defined according to the neoclassical growth theory which is a product of inter-temporal utility maximization in a steady state for a representative household. The equation used is implied from the relationship:

$$r_t^* = \frac{1}{\theta} q_t + \sigma$$

where θ represents the inter-temporal elasticity of substitution in consumption, q_t the technological change, and σ is a notation for time preference. The technological change determines the potential growth, also called trend growth rate, g_t . The inter-temporal elasticity of substitution and rate of time preference determine the savings rate. In the model used, the time preference and elasticity of substitution are then replaced by the more general term σ , which includes also other factors potentially affecting the permanent shifts in natural interest rate, for instance the growth of the population.

4.3. Model Specification

The model used in this paper consists of six equations:

$$ygap_t = a_{y,1}ygap_{t-1} + a_{y,2}ygap_{t-2} + \frac{a_r}{2} \sum_{j=1}^2 (r_{t-j} - r_{t-j}^*) + \varepsilon_{1t} \quad (4.1)$$

$$\pi_t = b_\pi \pi_{t-1} + (1 - b_\pi) \pi_{t-2,4} + b_y ygap_{t-1} + \varepsilon_{2t} \quad (4.2)$$

$$r_t^* = c g_t + z_t \quad (4.3)$$

$$z_t = z_{t-1} + \varepsilon_{3t} \quad (4.4)$$

$$y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_{4t} \quad (4.5)$$

$$g_t = g_{t-1} + \varepsilon_{5t} \quad (4.6)$$

The variables that are unobservable are potential GDP y_t^* together with the output gap $ygap_t$, natural interest rate r_t^* and trend growth rate g_t . They will be jointly estimated in the course of the estimation process. Output gap is the difference between the logarithms of real GDP and potential output measured as a percentage deviation, which is expressed by the equation:

$$ygap_t = 100(y_t - y_t^*)$$

The output gap variable is explained by its own lags and the equally weighted average of the first two lags of the “interest rate gap”, where the interest rate gap represents the difference between the short-term real interest rate r_t and the unobserved natural rate of interest r_t^* . Moreover, the equation includes the serially uncorrelated error term ε_{1t} that refers to short-term shocks and movements. The long-term output gap dynamics is, on the other hand, included in the variations of the natural interest rate. The number of output gap lags included in the IS equation was chosen as two. Laubach and Williams (2003) let the estimation itself determine the number of lags to be included and they, in line with other relevant research, conclude that two lags are appropriate.

The inflation term π_t is explained again by its own lags, which is mostly due to the existence of certain price stickiness. Also, the lagged output gap, and the serially uncorrelated disturbance ε_{2t} are included in the equation. For the purpose of parsimony, the relative energy prices described by x_t in the original Laubach and Williams

model were not included. This adjustment was made also due to the fact that the core inflation used in this paper excludes the energy prices, which implies it is not necessary to take the energy prices explicitly into consideration in the model specification. The lag variable $\pi_{t-2,4}$ represents the moving average of the second to fourth quarter:

$$\pi_{t-2,4} = \frac{\sum_{i=2}^4 \pi_{t-i}}{4}$$

The natural rate of interest is assumed to be determined only by real factors – the trend growth rate, as implied by the standard optimal growth model, and the other factors captured in the term z_t following a random walk, in which the serially uncorrelated disturbance ε_{3t} stands for an innovation. The other factors, as mentioned, include the change in rate of time preference of households, changes in intertemporal elasticity of substitution or population growth – factors very much influenced by demographics.

The potential output y_t^* follows a random walk process with a stochastic drift defined by lagged trend growth rate, which itself follows a random walk. The shocks are allowed to affect both the potential output and its trend growth rate. In both equations the error terms allowing for temporary shocks are serially uncorrelated and mutually contemporaneously uncorrelated with ε_{3t} . Moreover, the disturbances are assumed to be normally distributed.

4.4. Data

The quarterly data covering the period from the year 1996 until 2016 has been gathered. Therefore the sample consists of 80 observations. The sample period is relatively short compared to the samples used in studies investigating other countries. Thus, the size of the sample together with its potential implication on the estimation process must be taken into consideration when interpreting the results. The variable y_t represents the natural logarithm of the real GDP. The real GDP data with quarterly frequency, quoted in millions of CZK, seasonally adjusted, is gathered from the website of the Czech Statistical Office.

The annualized quarterly inflation rate is calculated from the monthly core CPI index downloaded from OECD website using the formula:

$$\pi_t = 400 \ln\left(\frac{CPI_t}{CPI_{t-1}}\right)$$

Since the inflation rate used is the one measuring quarter to quarter CPI index value changes, they should be seasonally adjusted. The seasonal adjustment is done in the beginning of the estimation process. CPI index is the main measure of inflation in the Czech Republic. It is based on the Classification of Individual Consumption by Purpose (COICOP) sorting the prices into 12 main groups. The classification allows separating the effect of the more volatile price developments (energy, food) from the less volatile ones. From each category a certain number of representative products is chosen according to how well they reflect the price movements of the group and how dominant they are in the households' consumption. Their price is then tracked. The core CPI index represents the price development excluding prices of food and energy, since the volatile movements of food and energy prices are less informative, contribute substantially to the increase of variance, often don't move in line with the development of the general price level, thus may distort its trend. Moreover, due to its short-term nature, the movements are usually reversed, so they don't require any monetary response.

Looking into the nominal short-term interest rate variable, the studies investigating the US market have used federal funds rate as a proxy, in Europe some authors picked the main policy rates of central banks, others proxied the short term interest rate by the interbank offered rates. For Czech Republic, monthly three-month PRIBOR rate has been gathered from Czech National Bank's system ARAD, in line with Manrique and Marques (2002). Consequently, the time series have been transformed into quarterly data. PRIBOR has a close relationship to the policy rates set by Czech National Bank, but since it has greater variance, the eventual estimation becomes more "up-to-date". Moreover, the PRIBOR rate is set in the interbank market as opposed to policy rate determined directly by national banks. Therefore, PRIBOR is considered to be a more relevant representative of the short-term interest rate in for the purpose of this paper.

Following Holston, Laubach, Williams (2016), the 4-quarter moving average of past inflation is used as a proxy for backward-looking inflation expectations. The inflation expectations are then used for the construction of the ex-ante real interest rate.

4.5. Estimation Methodology

I am going to simultaneously estimate the natural rate of interest, output gap and the trend growth rate of potential output. Firstly, it is needed to get a preliminary output

gap estimate. Linear trend is applied to the output variable, which will proxy the potential output.

Output gap implied from the proxy is then used to estimate by OLS the parameters of the system of equations in a simplified form, where the output gap is not affected by the interest rate gap:

$$\text{IS equation} \quad ygap_t = a_{y,1}ygap_{t-1} + a_{y,2}ygap_{t-2} + \varepsilon_{1t}$$

$$\text{Phillips curve equation} \quad \pi_t = b_\pi \pi_{t-1} + (1 - b_\pi) \pi_{t-2,4} + b_y ygap_{t-1} + \varepsilon_{2t}$$

The estimation should give us appropriate starting values of parameters and error variances for the maximum likelihood method applied in next steps.

Consequently, Following Kuttner (1994), the similar simplified system of equation is estimated, now by using MLE method and Kalman filter. The difference from the system estimated by OLS is that the potential output is considered as an unobserved variable and the trend growth rate as a constant:

$$ygap_t = a_{y,1}ygap_{t-1} + a_{y,2}ygap_{t-2} + \varepsilon_{1t}$$

$$\pi_t = b_\pi \pi_{t-1} + (1 - b_\pi) \pi_{t-2,4} + b_y ygap_{t-1} + \varepsilon_{2t}$$

$$y_t^* = y_{t-1}^* + \bar{g} + \varepsilon_{4t}$$

The parameter vector being estimated is:

$$\theta = (a_{y,1}; a_{y,2}; b_\pi; b_y; \bar{g}; \sigma_{ygap}; \sigma_\pi; \sigma_{y^*})$$

Kuttner (1994) explains the rationale behind the above stated model specification simply. The potential output is assumed to be an unobserved stochastic trend, and deviations of the output from its potential influence inflation through a standard aggregate supply relationship. However, instead of directly analyzing the supply side (labor supply, productivity), the joint behavior of inflation and output, represented by the Phillips curve equation, is used. The process yields an estimate of the potential output \hat{y}_t^* which will be used consequently.

Next, it is needed to address the problem investigated by Stock and Watson (1998). The natural interest rate and trend growth rate are likely to be subject to the pile-up

problem for non-stationary processes explained by Stock (1994). In most of the samples, variations of natural interest rate and trend growth rate (σ_3 and σ_5) representing the permanent components of the time series are likely to be very small relative to the large transitory components included in the series. Because of this, the MLE estimator of the standard deviations of their changes (innovations) will be biased towards zero. This happens due to the large portion of the probability being piled up at zero in the probability density function. The problem is tackled by median unbiased estimation of coefficient variance/ standard deviation in a time-varying parameter model (Stock and Watson (1998)). To start with, for each time t , the potential output growth Δy_t^* where y_t^* is the estimate of the potential output from the previous step, will be regressed on a constant and a dummy variable with a break at time t . T-ratios related to the dummy coefficients are obtained, and consequently the Exponential Wald statistic for a structural break at unknown time is computed, following Andrews and Ploberger (1994). The statistic, which sums the obtained t-ratios, is computed for each t . Its formula is expressed below:

$$EW = \ln \left[\frac{1}{T} \sum_{i=1}^T \exp \left(\frac{s_i^2}{2} \right) \right]$$

where s_i stands for the t-ratio related to a break in time i .

The values of the Exponential Wald statistic are compared with the table provided in Stock and Watson (1998), Table 3., and converted into the median unbiased signal-to-noise ratios λ_g using the table. The ratio is then simply plugged into the formulas provided by Stock and Watson:

$$\lambda_g = \frac{\sigma_5}{\sigma_4} = \frac{sd(g_t)}{sd(y_t^*)} \rightarrow \sigma_5 = \lambda_g \sigma_4$$

as a relationship between the standard deviations of trend growth rate g_t and the output gap $ygap_t$.

As a next step, in order to estimate λ_z , we use the system of equations:

$$ygap_t = a_{y,1} ygap_{t-1} + a_{y,2} ygap_{t-2} + \frac{a_r}{2} \sum_{j=1}^2 (r_{t-j} - r_{t-j}^*) + \varepsilon_{1t}$$

$$\pi_t = b_\pi \pi_{t-1} + (1 - b_\pi) \pi_{t-2,4} + b_y ygap_{t-1} + \varepsilon_{2t}$$

$$r_t^* = c g_t + \bar{z}$$

$$y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_{4t}$$

$$g_t = g_{t-1} + \varepsilon_{5t}$$

Now, the real interest rate gap is already included, and z_t is assumed to be constant. Further the restriction implied from the estimated signal-to-noise ratio $\sigma_5 = \hat{\lambda}_g \sigma_4$ is imposed. The system is estimated and a new \hat{y}_t^* obtained. Exponential Wald statistic is computed again and, in similarity to the λ_g , we arrive to an estimate of λ_z . $\hat{\lambda}_z$ gives us the second restriction that needs to be imposed to the general model:

$$\lambda_z = \frac{\sigma_3}{\sigma_1} \frac{a_r}{\sqrt{2}} = \frac{sd(z_t)}{sd(ygap_t)} \rightarrow \sigma_3 = \lambda_z \sigma_1 \frac{\sqrt{2}}{a_r}$$

as a relationship between the standard deviations of z_t and the output gap $ygap_t$. The equation differs from the one related to λ_g due to following reasons. The term $\sqrt{2}$ is used since in the equation (1) of the general model it is assumed that the output gap is influenced by two lags of the interest rate gap, and the current interest rate gap is determined by the current z_t , as mentioned in the description of the variables in the general model. Such specification implies that the potential output is affected by z_{t-1} and z_{t-2} via the coefficient a_r . At this point in the estimation process, the parameter vector being estimated by MLE is represented by:

$$\theta = (a_{y,1}; a_{y,2}; a_r; a_0; a_g; b_\pi; b_y; g; \sigma_{ygap}; \sigma_\pi; \sigma_{y^*})$$

In the final stage of the estimation the above mentioned restrictions on variances are imposed to the whole system of all equations in the general model, and the Maximum Likelihood method is used to estimate its parameters. The final parameter vector estimated by MLE is as follows:

$$\theta = (a_{y,1}; a_{y,2}; a_r; b_\pi; b_y; g; \sigma_{ygap}; \sigma_\pi; \sigma_{y^*})$$

As a last step, the constraints are imposed on the model. Specifically, the fundamental requirement is that the IS curve should be restricted to have a negative slope and the Phillips curve to have the positive slope. Over the estimation process, more concrete restrictions on the slopes are examined.

4.6. Kalman filter

Kalman filter has been chosen to compute the likelihood function. Generally, Kalman filter is a recursive technique widely used not only in econometrical time series analysis, but many other areas, especially engineering. Unlike Hodrick-Prescott and other univariate filters, Kalman filter incorporates both the statistical technique and economic theory. Therefore the results provide also an economic interpretation for the estimated variable. In order to estimate a model by multivariate Kalman filter, it needs to be phrased in the form of state-space representation – set of inputs, outputs and state variables which are related to each other by first-order differential equations.

The state space is a full set of values the process can take. It is the Euclidean space in which the variables located on the axes are the state variables. State variables are the ones that describe the state of the dynamic system – knowing them is enough to determine the future state of an object in the system. The basic state space includes a signal (measurement) equation relating the unobserved state variable (vector) α_t to the observed one (y_t), and state equations determining the assumed behavior of the unobserved variable. Following notation of Tusell (2011) and Harvey (1989) the state space model is specified as:

$$\text{State equation} \quad \alpha_t = T_t \alpha_{t-1} + R_t \eta_t$$

$$\text{Signal equation} \quad y_t = d_t + Z_t \alpha_t + \varepsilon_t$$

where $\eta_t \sim N(0, Q_t)$ and $\varepsilon_t \sim N(0, H_t)$

Matrices T_t, R_t, Z_t, Q_t and H_t can be dependent on a parameters vector θ , and may change or be constant over time. It is assumed that the errors η_t and ε_t are mutually and serially uncorrelated. The process consists of two steps. In the prediction part, Kalman filter yields estimates of the current state (unobserved) variables together with their noise – this part is essentially an estimation of the state equation, where we are estimating the “next state” of the system. This yields also an estimation of the observed variable. The updating part takes place when the new observation y_t , can be applied. The new observation is necessarily also burdened by noise, and therefore the initial estimates are updated based on a weighted average where more weight is given to the more certain estimates. The procedure yields an estimate that is an average of the state predicted by the system (state equation) and the new measurement. The Kalman filter algorithm in a formal form is defined as follows. The variable a_t represents the estimate of the state vector and P_t its variance-covariance matrix:

$$a_{t-1} = E [\alpha_{t-1} \mid y_0, y_1, \dots, y_{t-1}]$$

$$P_{t-1} = E [(\alpha_{t-1} - a_{t-1})(\alpha_{t-1} - a_{t-1})^T]$$

Time update equations determine future state in the form of $a_{t|t-1}$ and $P_{t|t-1}$ as a result of basic statistical identities:

$$a_{t|t-1} = T_t a_{t-1} + c_t$$

$$P_{t|t-1} = T_t P_{t-1} T_t^T + R_t Q_t R_t^T$$

Measurement update equations define the future state in the form of $a_{t|t-1}$ and $P_{t|t-1}$ influenced by a new measurement of the observed vector y_t available. If we define the variance-covariance matrix of y_t given estimated $a_{t|t-1}$ as $F_t = Z_t P_{t|t-1} Z_t^T + H_t$, we can write the measurement equations in the form:

$$a_t = a_{t|t-1} + P_{t|t-1} Z_t^T F_t^{-1} (y_t - Z_t a_{t|t-1} - d_t)$$

$$P_t = P_{t|t-1} - P_{t|t-1} Z_t F_t^{-1} Z_t^T P_{t|t-1}$$

The part $y_t - Z_t a_{t|t-1} - d_t$ represents the difference between the value of the observed variable measured and predicted by the state equation. $K_t = P_{t|t-1} Z_t F_t^{-1}$ is the weight given to this difference. The weight K is determined by the estimate error and the estimate error covariance matrix. When the estimate error covariance matrix $P_{t|t-1}$ ceteris paribus, approaches zero and therefore the reliability of the estimate using the state equation is high, there is little or no weight given to the difference between the measured and estimated y_t . As a result, the update to the estimated observed variable is small. On the other hand, if the measurement error covariance matrix H_t approaches zero, the weight given to the difference grows.

By combining the time-update and measurement equations, we obtain the set of two

equations that directly relate a_t to a_{t-1} and P_t to P_{t-1} . The only information necessary is the initial values a_{-1} and P_{-1} .

In order to relate the general knowledge of Kalman filter to the Laubach and Williams model, let's clarify the state space model in this particular setting. The signal (observation) equations are represented by IS and Phillips curve:

$$ygap_t = a_{y,1}ygap_{t-1} + a_{y,2}ygap_{t-2} + \frac{a_r}{2} \sum_{j=1}^2 (r_{t-j} - r_{t-j}^*) + \varepsilon_{1t}$$

$$\pi_t = b_\pi \pi_{t-1} + (1 - b_\pi) \pi_{t-2,4} + b_y ygap_{t-1} + \varepsilon_{2t}$$

The state equations are:

$$r_t^* = c g_t + z_t$$

$$z_t = z_{t-1} + \varepsilon_{3t}$$

$$y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_{4t}$$

$$g_t = g_{t-1} + \varepsilon_{5t}$$

Based on the equation stated above, the estimate of the natural rate of interest, potential output and its trend growth rate is partially adjusted depending on the distance of the model's estimates of real GDP and inflation from its actual measured values. Therefore, assuming a positive output gap, in case the observed value of the output gap is lower than the estimate, it means that the estimate of the interest rate gap is underestimated. Therefore, the natural interest rate is understated and should be revised up.

There are two versions of the Kalman filter method. Traditional Kalman filter is a technique known as a one-sided filter. One-sided filtering takes into account only the information which is available at time t in order to generate estimates of states at time t . Although such method is convenient to model a behavior of someone who learns the values of state variables in real time, we have a full history of data available at hand, both before and after time t . The availability of the "future" data for any time t in the sample is a reason for using two-sided version called Kalman smoother. It uses not only the data before time t , but also after it to calculate the expected values of the state variables. Using Kalman smoother is advised for instance by Mise et al. (2005), or by Clark and Kozicki (2004). Due to these reasons, two-sided Kalman smoother is applied in this work to produce the estimates.

4.7. Maximum Likelihood Estimation

Combining the MLE technique with Kalman filter is an efficient way to estimate the model. Kalman filter algorithm allows to compute estimated values of the unobserved variable and predicted observed variable. Nevertheless, there are still the unknown parameters that need to be estimated before the Kalman filter technique is used, specifically T_t, R_t, Z_t, Q_t and H_t . This is done using the Maximum Likelihood Estimation (MLE) method.

According to Wooldridge (2012), Maximum Likelihood method is consistent in most cases, and it should be “*the most asymptotically efficient estimator when the population model $f(y, \theta)$ is correctly specified*” (page 769), and sometimes “*the minimum variance unbiased estimator; that is, it has the smallest variance among all unbiased estimators of θ* ” (page 769). If $f_{Y_1, \dots, Y_n}(\theta)$ represents a joint density function of series $\{Y_1, \dots, Y_n\}$ depending on a parameter vector, the maximum likelihood estimate of θ , denoted $\hat{\theta}^{MLE}$, is a value which maximizes the density function $f_{Y_1, \dots, Y_n}(\theta)$. In other words it is a value of θ that maximizes the probability of getting the observed sample data. If Y_t is independent in time, the joint density function can be respecified as:

$$f_{Y_1, \dots, Y_n}(\theta) = f_{Y_1}(\theta_1) * f_{Y_2}(\theta_2) \dots * f_{Y_n}(\theta_n) = \prod_{i=1}^n f_{Y_i}(\theta_i)$$

Taking the logarithm of the likelihood function, we get the log-likelihood function, which will be maximized:

$$\Lambda(\theta) = \max_{\theta} \sum_{i=1}^n \log f_{Y_i}(\theta_i)$$

In case Y_i is not independent in time, the log-likelihood function being maximized is as follows:

$$\Lambda(\theta) = \max_{\theta} \sum_{i=1}^n \log f_{Y_i|Y_1, \dots, Y_{i-1}}(\theta_i)$$

The concrete form of the log-likelihood function is derived as described below. Let's assume the matrices T_t, R_t, Z_t, Q_t and H_t (all or only some of them) included in the state space model setup in the previous chapter do not change over time and are dependent on the parameters vector θ , which we want to estimate. We are further assuming that the unobserved vector's distribution has properties $\alpha_0 \sim N(a_0, P_0)$ where

a_0, P_0 are known. Following Tusell (2011), the log-likelihood function is then defined as:

$$L(\theta) = \log p(y_0, \dots, y_n | \theta)$$

$$L(\theta) = -\frac{(N+1)p}{2} \log(2\pi) - \frac{1}{2} \sum_{t=0}^n (\log |F_t| + e_t^T F_t^{-1} e_t)$$

where $F_t = Z_t P_{t|t-1} Z_t^T + H_t$, as mentioned in the Kalman filter section, and $e_t = y_t - Z_t a_t$. Once we have defined the log-likelihood function, the MLE method is implemented by running an optimization algorithm, prominently one of the quasi-Newton methods.

4.8. BFGS Estimation

Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm is a numerical optimization technique, one of the most efficient quasi-Newton optimization methods. The goal of the process is to find an extremum of the function which is being optimized by way of iteratively seeking an optimum – starting with an arbitrary solution and then incrementally changing it with the goal of finding a better solution. BFGS is used for the nonlinear multidimensional unconstrained functions, in which the analytical solution is not possible.

Quasi Newton processes are specific in a fact that they take into account both the gradient (first derivative) and Hessian matrix (second derivative) of the function, but as opposed to Newton's method, the Hessian matrix is approximated rather than calculated directly. Therefore, only the gradient of the function is needed at each iteration.

The iterative process begins with starting position and determines the quadratic model of the objective function – calculates gradients and approximates the inverse Hessian matrix. Consequently it determines the search direction using the quadratic model, and calculates an optimum step size by running a backtracking line search. As a next step, it updates the position and the approximation of the Hessian matrix. This process repeats until no better optimum than the current one can be found. The iterative scheme is defined as follows:

$$x_{k+1} = x_k - \alpha_k H_k g_k$$

where H_k is an approximation of the inverse Hessian matrix, g_k is a gradient of the objective function at point x_k and α_k is the step size.

5. Results

The structural models for time-varying natural interest rate provide a useful economic intuition behind the estimates. Even though they are considered more robust and generally better than the univariate filtering methods (for instance the end-point bias is handled better), their dependence on the model assumptions may still significantly undermine their performance. Therefore, they are often combined with statistical filters into so called semi-structural models, as in Laubach and Williams (2003). Together with the partially structural nature of the model, the small sample size and the small open economy considerations should be taken into account and emphasized when interpreting the results.

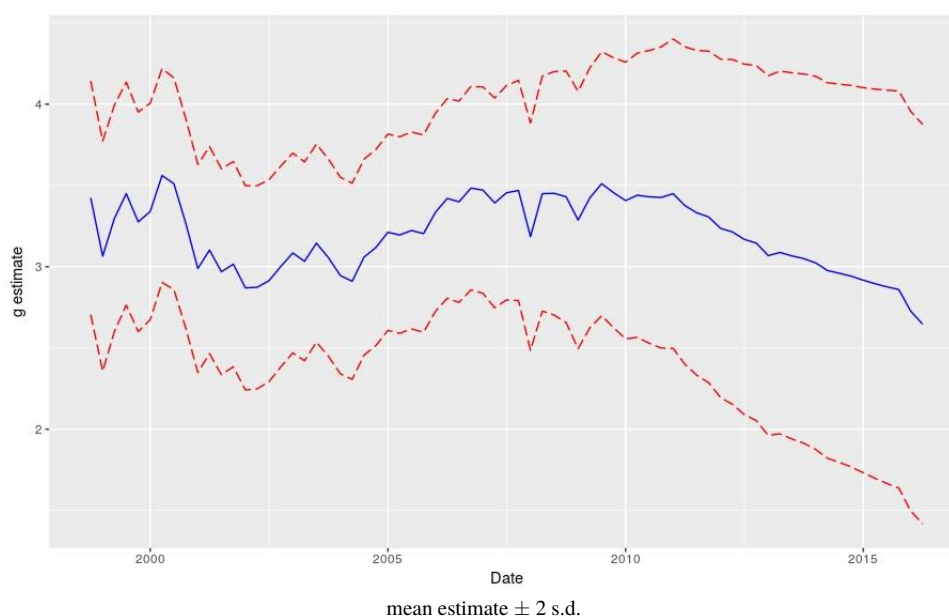
In this section the estimated natural interest rate, trend growth rate and output gap is presented. Most importantly, it is visible from the graphs that both natural interest rate and trend growth rate has been exhibiting a considerable downward trend until the recent years, although r^* experiences a slight revision at the end of the sample.

5.1. Trend growth rate development

The trend growth rate experienced a decline shortly after 2000 and remained low in the period between years 2000 and 2005 (Figure 5.1). A more persistent decrease occurred after the global financial crisis, which even accelerated in recent years, reaching 2.6 percent in 2016. The results are partially consistent with the other research done for the European Area and OECD countries, notably Cetto et al. (2016) and OECD (2015). According to the authors, the European potential growth has been experiencing a downturn not only after the global financial crisis, but also in the period before that. This happened mostly due to the productivity slowdown, which is a fundamental force of the potential growth. The mid-1990s produced a surge in the innovation related to the information and communications technology (ICT) in the United States, but most of Europe has been lagging behind substantially and no efficiency gains or other positive effects from the ICT revolution were observed there.

The reasons for the European inability to keep up with the innovation were substantial under-investment in knowledge-based capital, and the weak diffusion of the technologies and knowledge from U.S and other front-runners. The other possibility, although less probable, is what Gordon (2012) points out. Maybe the slowdown represents a persistent or even permanent phenomenon due to the fact that the innovations which occurred until now, for instance the invention and application of electricity, were by far the most important and there is a little chance to come across a technological revolution which would exceed their contribution and promote a long-term potential growth.

Figure 5.1.: Trend growth rate estimate

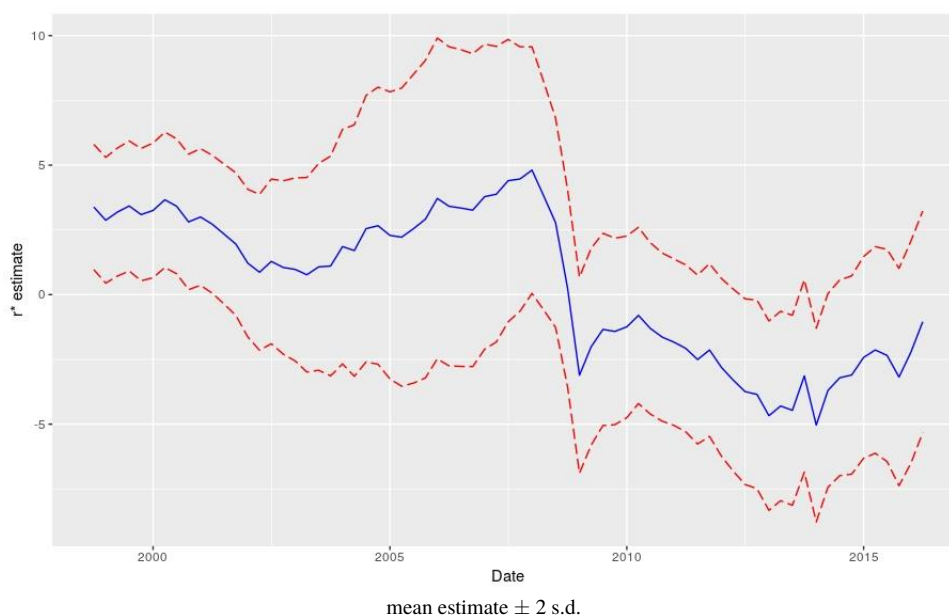


5.2. Natural interest rate development

The natural interest rate seems to enter into negative territory during the global financial crisis and remains negative since then (Figure 5.2). The average estimated natural interest rate over the whole sample period is 0.393%. However, it is visible that a structural break occurs in 2008. The average estimated rate for the period 1996-2008 is 2.635% and for the period 2009-2016 is -2.671%. The less severe dip around the year 2004 can be the result of the Czech Republic joining the European Union, in line with the expectations regarding the higher accumulation of domestic capital and inflow of foreign capital expressed in the Section 3. After the adjustment to the European level, the natural interest rate is rising until the time when the global

financial crisis begins, having a similar trend as other European members. Observing the decomposition of the natural interest rate movements in Figure A.2 we can see that most of its variability is explained not by the trend growth rate, but by the other structural factors captured in the term z . This finding is in line with the result of the Exponential Wald statistic showing much higher variation of term z compared to g . The striking weak importance of the trend growth rate in the natural interest rate variation is also consistent with other related research. For instance, in case of the U.S. economy, Hamilton et al. (2015) notes that “*other factors play a large, indeed dominant, role in the determination of average real rates*” (page 16) . The importance of other factors captured in z , for the Czech economy, may be amplified by its small open nature.

Figure 5.2.: Natural interest rate estimate



5.3. Output gap and real interest rate gap

Figure A.3 shows the development of the estimated output gap over the sample period. The graph shows that in the period reaching from the start of the sample to the first years of the new millennium, circa 2003, the real output was relatively close to the potential and output gap was converging to zero. The period from 2003 until 2008 was characterized by overheating the economy and rising output gap. Eventually, the results suggest that the Czech economy experienced a negative output gap following the global financial crisis, which is a consistent conclusion with the economic theory.

The graph for the real interest rate gap $r_t - r_t^*$ presented in the Figure A.4 should serve as an indicator for the monetary policy stance. As mentioned in the introduction, when the real short-term interest rate set by the central bank is above the natural interest rate, the monetary policy is contractionary. When the real rate is below its equilibrium counterpart, the monetary policy becomes expansionary. The estimated values show that the monetary policy until the year 2008 has been mostly expansionary. The second part of the sample starting from 2008 is characterized by contractionary monetary policy.

Such result indicates a development which is in contrary to what has been expected. The real rate gap is defined as the difference between the real interest rate gathered as input data, and the estimated natural interest rate. As Figure 1.1 shows, the real interest rate has decreased considerably twice - during the years 2008 and 2009, and also around 2011. The natural interest rate follows a similar trend, however its volatility is higher and it decreases by a greater amount than the real interest rate. Such dynamics cause the unexpected real rate gap result. There are two possible explanations for such dynamics. If the natural interest rate estimate were precise and significant, the results would suggest that the monetary policy is not responsive enough and it changes the interest rates in the right direction, but by less than what is needed. This problem is formally analysed in Section 6.3. Such explanation would be in line with the fact that the Czech National Bank does not explicitly work with the time-varying natural interest rate estimate, but rather with the constant estimate. However, since the estimated values of the natural interest rate in this paper show a considerable variation and little significance, the opposite sign of the real rate gap is expected to be more caused by the impreciseness and high variation in natural interest rate estimate - it moves in the correct direction, but by a greater amount than it should.

Moreover, starting from the end of the year 2012 the Czech National Bank has been limited by the zero lower bound, so it could not sufficiently loosen the interest rate component of the monetary conditions. Therefore, to further ease the monetary conditions, the Czech National Bank decided in November 2013 to declare a one-sided exchange rate commitment - an unconventional monetary instrument based on potentially unconstrained foreign exchange interventions. The central bank's decision is related to the period in which the estimate of the natural interest rate together with its whole confidence interval appears in the negative territory. It suggests that the central bank was aware of the potential decline of the natural interest rate and indeed, Franta et al. (2014) shows that it was one of the additional arguments for choosing the exchange rate commitment tool. This fact is another evidence of the need to include

the exchange rate variable into the model for Czech Republic, which is suggested in the Section 7.

5.4. Median Unbiased Estimators

Using the Andrews and Ploberger (1994), Table 2, critical values allows for the interpretation of the resulting Exponential Wald statistics, which test for an intercept shift in the potential output growth $\Delta \hat{y}_t^*$ representing the trend growth rate, and an intercept shift in the term z_t . The results are presented in Table 5.1. In the first stage of the estimation process the Exponential Wald statistic of 0.6906 shows a weak significance of the trend growth rate variation. The value is not significant even at 10% significance level. On the other hand, the variation in z_t – part of natural interest rate not related to g_t , which has been estimated in the second stage of the process - is much higher, and significant at 5% significance level. Such evidence, following Stock and Watson (1998), implies median unbiased estimators λ_g and λ_z having values of 0.0454 and 0.1486 respectively.

Table 5.1.: Median unbiased estimates

Variable	EW statistic	Median Unbiased Estimate
g	0.6906	0.0454
z	3.6640	0.1486

5.5. Parameter estimates

In the Table 5.2 the parameter estimates are presented, t-statistics mentioned in parentheses. Indeed, in line with the general concerns and the discussion about the shortcomings of the time-varying natural interest rate estimation in Section 3.3, the estimates are weakly significant and are subject to high standard errors. The estimated value of the coefficients related to the output gap lags $\sum a_y$ is estimated as 0.943, implying considerable persistence of the output gap. Nevertheless, the value is less than unity and the hypothesis of the sum equaling one is rejected at conventional significance level. If it wasn't possible to reject that the sum of output gap lags yields unity, an identification problem would arise. The lags alone would be enough to explain the development of the output gap, leaving no room for interest rate gap to affect it.

The slope parameters a_r and b_y stay at the level of constraints imposed in the model suggesting that the real slope parameters converge more to zero. However, when they are allowed to be closer to zero, the singularity problem arises. The singular covariance matrix issue is discussed more in detail in Section 5.7.

Table 5.2.: Values of estimated parameters

Parameter	Value
$\sum a_y$	0.943
a_r	-0.1
	(0.784)
b_y	0.1
	(1.423)
λ_g	0.045
λ_z	0.149
$\sigma_{\bar{y}}$	0.571
	(4.189)
σ_π	2.206
	(17.019)
σ_{y^*}	0.227
	(0.838)

Table 5.3.: Sample averages of estimated standard errors

Standard errors – sample averages	
r^*	1.961
g	0.407
y^*	2.596

5.6. Slopes of IS and Phillips Curve

The most problematic part of the estimation process is related to the parameters of the IS and Phillips curve slopes $\frac{1}{a_r}$ and b_y , respectively. If the original constraints on the slope parameters $a_r \leq -0.0025$ and $b_y \geq 0.025$ are imposed on the Czech data in line with Holston et al. (2016), the estimation produces a covariance matrix of state variables converging to singularity and provides uninterpretable, very weakly identified estimates with unusually high average standard errors. The results are presented in the Table A.1 and A.2. The estimated values of the slope parameters remain

−0.0025 and 0.025, and are not significant at any conventional or even unconventional significance level. The lack of significance in IS and Phillips curve slopes is one of the reasons why the estimates of the natural interest rate and the potential output are highly imprecise and barely identified.

It is also important to look at the estimated magnitude of the slope parameters. The values of −0.0025 and 0.025 suggest that if the constraints were not imposed, the parameters would converge to zero even more, thus the Phillips curve would be even flatter and the IS curve steeper. It is also visible on the large part of the Monte Carlo random draws excluded from the standard error computation due the violation of one of the slope parameter constraints. Such findings are partially in line with Holston et al. (2016), who noted that, compared to other regions, one of the largest portions of the draws excluded due to the above mentioned reasons was for the Euro Area. In addition to the lack of significance of the estimated parameters, the slope parameters considerably close to zero for the IS and Phillips curves in Czech Republic are the second reason for the high time variation, standard errors and impreciseness of the natural interest rate and potential output estimates.

5.7. Singularity

The singularity issue deserves some further investigation. In order to see whether the problem is partially caused by a small sample size, the Laubach and Williams estimation process is applied firstly to the short sample of the US data and secondly, to the Czech data, but now with monthly frequency where the monthly industrial production index serves as a proxy for GDP. This estimation of US data doesn't face the problem of singular covariance matrix anymore, however the Czech data with four times longer sample due to the switch of the frequency still results in singularity. Therefore, the small sample size does not seem like a real reason behind the singular covariance matrix issue.

The variance-covariance matrix of the state vector in Kalman filter $P_{t|t-1}$ has a zero determinant in case the variance of the estimated potential output $\sigma_{y^*}^2$ included in the matrix:

$$Q = \begin{pmatrix} \sigma_{y^*}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (\lambda_g \sigma_{y^*})^2 \end{pmatrix}$$

converges to zero. In that case λ_g is estimated as 0 due to negligible potential output variation, as seen in Table A.1. Indeed, when the variance of the potential output is constrained by a sufficiently low upper bound, the process always yields singularity. Moreover, when the constraints on the slope parameter of the Phillips curve change, the estimate of the potential output variance does as well. It has been observed that the closer to zero the constraint is set, the more probably it gets for the process to estimate very low potential output variation, which in turn yields singularity of the covariance matrix. The estimation of $\sigma_{y^*}^2$ comes from the non-linear BFGS optimization process. A suggested direction for further research could be a further analysis of how the constraint on the slope parameter b_y affects the estimated potential output variance. The singularity problem encountered in the current work was also mentioned in the paper of Danielsson et al. (2016) who used the LW model for Icelandic data. Unfortunately, the authors do not provide any potential explanation for why the issue may arise. There is a possibility that the issue related to singular covariance matrix is typical for certain countries.

5.8. Omitted Variable Bias

The estimated values of the slope parameters close to zero for the IS and Phillips curves might be caused by the open nature of the Czech economy which is not taken into account in the model. In this section, the basics of the open economy biases are introduced, and carried forward into Section 7, where the more complex open economy adjustments are suggested.

First of all, it is important to investigate the potential biases related to omitting the open economy variables, notably the exchange rate (and terms of trade). The guess on the direction of the bias follows Wooldridge (2012):

$$E(\tilde{\beta}_1) = \beta_1 + \beta_2 \tilde{\delta}_1$$

where β_1 is a true value of the parameter, β_2 the effect of the omitted explanatory variable on the explained variable, and $\tilde{\delta}_1$ is the parameter from the regression of the omitted explanatory variable on the included explanatory variable. The bias is thus represented by $\beta_2 \tilde{\delta}_1$.

Let's apply the general equation representing bias to the Phillips curve. We are assuming here that the omitted exchange rate is positively related to inflation, and also,

as mentioned, to the output gap. Thus, the bias of the estimated Phillips curve slope parameter b_y is positive. This in turn implies that the true b_y is even closer to zero than the estimated one, and the Phillips curve is likely even flatter than the estimated one. Such result actually corresponds to current research for foreign economies. Slope of the Phillips curve has gained a wide-spread attention among academics. Its significant flattening among advanced economies in recent years represents a phenomenon shown by many research papers, for instance Kohn (2006), Pain et al. (2006), or Gaiotti (2010). The evidence can be a result of a structural shift in the inflation-output gap trade-off, or a notable success of the monetary policy in its efforts for stabilizing the inflation.

Some researchers suggest that in small open economies like the Czech Republic, the international competition reduces the pricing power of domestic firms so the inflation becomes less responsive to domestic demand, and more to the foreign demand conditions. Moreover, the labor market faces an inflow of cheap labor which contributes to the low responsiveness of the inflation to demand at the home country. These factors have some effect on the flattening of the Phillips curve. The theory is empirically supported by Gali and Monacelli (2005). Nevertheless, there are also authors claiming that the openness should actually have the opposite effect – since the costs of keeping prices constant in such environment rise significantly, the firms will actually revise their prices more often.

Regarding the IS curve, since theory suggests that the exchange rate is positively related to the output gap and negatively to the interest rate, we can conclude the bias of the estimated slope parameter a_r is negative. The negative bias implies that the true parameter a_r is even closer to zero than the estimated value. Now, after the exchange rate is included in the equation in question, the effect of the interest rate alone (keeping the effect of exchange rate fixed) is naturally lower in absolute value, which would imply a steeper IS curve. Nevertheless, the exchange rate is just one of the channels in which interest rate influences the output gap, so the IS curve slope should include the combined effect of the interest rate alone and the exchange rate. Therefore, despite the true parameter a_r being closer to zero, the overall effect of the interest rate on the output gap captured by a_r together with the parameter related to the exchange rate variable, is expected to be further from zero. Thus, the expected result would present as a flatter IS curve.

There is much less research done focusing on the slope of the IS curve. The critical variable influencing the slope, the interest rate elasticity of demand dependent on

the terms of trade and exchange rate in the open economy, has been mentioned. As Knutter and Wagner (2008) mention: *“The effect of expansive monetary policy is amplified through the depreciation of the exchange rate which in turn boosts exports and domestic output in addition to the (positive) output effect of monetary policy”* (page 11). The other possible explanation of the flatter IS curve for open economies, mentioned in the literature, is that the real interest rates becomes less sensitive to the monetary policy itself and rather responds to the global forces. This can happen due to the increasing financial integration and the recent phenomenon of persistent low real interest rates caused by the so called “global savings glut” – situation described by the former chairman of United States Federal Reserve Ben Bernanke, in which the desired saving becomes higher than the desired investment in certain economies from the Asian and Middle Eastern regions, implying excess capital inflows from these countries.

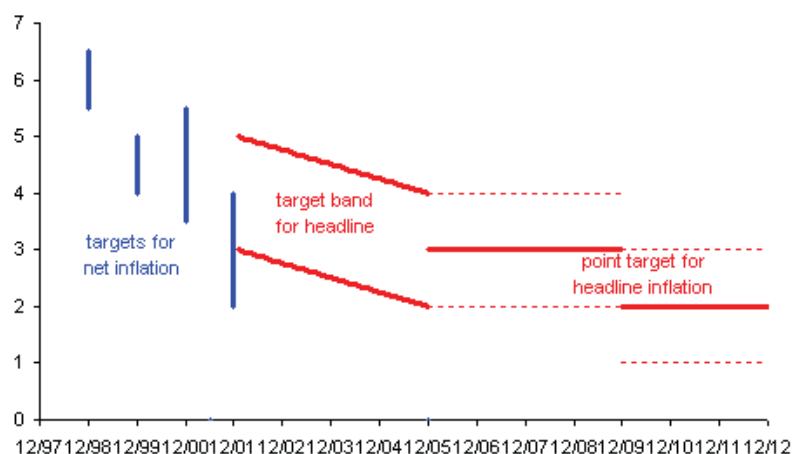
We can conclude that the slope parameters of the IS and Phillips curve would likely not be greater in absolute value after including the exchange rate, or terms of trade (which would work analogically), thus they would not solve the singularity problem on their own. It is possible that the more sophisticated small open economy adjustments to the current model would yield better estimates. These adjustments are proposed in Section 7.

6. Natural Interest Rate and Inflation Targeting

The secular stagnation is a topic which has been discussed heavily in recent years. The concept was first popularized by Summers (2014) which describes the situation when low demand works against potential output growth. Therefore, in contrast to a temporary recession from business cycle theory, it is a condition when very sluggish or completely absent output growth becomes a standard in a market-based economy. The demand for investments is very weak and on the other hand there is a strong appetite to save. As a result, the interest rate, at which the savings would equalize the investment, is negative. The central bank would have to decrease its policy rates below the negative value defined by the natural rate in order to stimulate the economy. The paper by Kiley and Roberts (2017) shows that the Federal Reserve can currently face zero-lower bound as often as 40% of times. The risk of secular stagnation and zero-lower bound brings forth the discussion about the appropriateness and actuality of the current monetary policy regime.

An inflation targeting policy was first adopted by the Czech Republic in 1998. The policy is based on the explicit public announcement of a numerical inflation target, or a sequence of targets, as a tool to achieve price stability – the main policy goal in the medium term - most importantly by anchoring long-term inflation expectations. The inflation targets for the Czech Republic set by Czech National Bank are presented in Figure 6.1. Together with the target value, the horizon in which it should be achieved is also announced. Due to the monetary transmission mechanism, which in the Czech Republic lasts around 4 quarters (Borys and Horvath (2009), forecasts play a major role. Moreover, the target represents a nominal anchor of the economy, and it increases the central bank's transparency and accountability.

Figure 6.1.: Inflation targets in Czech Republic



source: CNB's website

6.1. Why Inflation Targeting and at What Level

The logic behind setting the inflation targets in general is well explained for instance in Horvath and Mateju (2011). They find that “*not only are macroeconomic fundamentals important for the level of inflation targets, but some institutional characteristics as well*”(page 2). Among the institutional factors, credibility in achieving low inflation is crucial. Specifically in the Czech Republic, the macroeconomic variables that matter for the central bank in setting the inflation target are “*past inflation, inflation expectations, price convergence, wage rigidities, zero interest rate bound and measurement error*”(Table 1).

In many countries, the inflation target of, or around, 2% has been chosen in a belief that it is not too high to distort business decisions or destabilize the economy, but also not exceedingly low. There are several reasons why to keep the positive inflation target, some of them coinciding with the factors influencing the target decision of the Czech National Bank. Bank of Canada (2016) provides a useful discussion about the level of the inflation target and its consequences. Firstly, as already mentioned, it is prevention against secular stagnation and zero lower bound. Since the nominal interest rates set by banks cannot be negative, it reduces the central bank's ability to stimulate the economy by decreasing them. Nevertheless, when the inflation is positive and exceeds the nominal interest rate value, the real interest rates still

can be negative. Negative real interest rates in turn create incentives for households to save less and borrow more. This has a positive effect on the economic activity. When the inflation target is set at zero, even the option of negative real interest rates is lost. The other reason is difficulties in the inflation measurement. For instance Consumer price index faces certain measurement biases that result in an overstating of the true inflation rate. An example of this may be the substitution bias which refers to the situation when the consumer substitutes the product in the consumer basket by a cheaper alternative if its price increases sharply. The CPI cannot capture the dynamics and therefore doesn't accurately measure the effect of a price increase on the consumer's budget. Therefore, if zero inflation was targeted, the consequence could be an actual decrease of prices. Nominal wage rigidity also contributes to the belief that a zero inflation target is not a good idea. It is very difficult and usually practically impossible to decrease nominal wages. Therefore, in poor performing industries the inflation is a tool to cut real wages of workers as a substitute for them losing their jobs.

6.2. Taylor rule

Together with the setting of the inflation target, the price stability is ensured by choosing the short-term nominal interest rates. This is the step where the natural interest rate comes into place. In most countries, rates are decided on by using the well-known Taylor rule introduced by Taylor (1993). The central bank's reaction function in form of the Taylor rule in its general form is defined as follows:

$$i_t = \pi_t + r_t^* + \alpha_1(\pi_t - \pi_t^T) + \alpha_2(y_t - y_t^*)$$

The policy maker reacts to the economic conditions by setting the short-term nominal interest rate i_t . Specifically, the central bank responds to changes in the output gap, difference between the actual inflation and its fixed target level, and the natural interest rate in its nominal form. The parameters should be positive so that the monetary policy is countercyclical.

Czech Republic is not an exception, in Czech National Bank's g3 model the modified Taylor rule is implemented:

$$i_t = \sigma i_{t-1} + (1 - \sigma) [i_t^* + \psi(\pi_{t+4} - \pi_{t+4}^T)] + \varepsilon_t$$

where σ represents a smoothing parameter facilitating rather gradual approaching to the inflation target, and $i_t^* = \pi_t^e + \hat{r}_t^*$ is the natural nominal interest rate implied from the sum of the estimated natural real interest rate and the inflation expectations. The rule suggests that the nominal interest rate set by a policy maker in a situation of inflation at its target level would be the one equal to the estimated natural rate in nominal terms at the end of the adjustment horizon:

$$i_t = i_t^*$$

6.3. Natural Interest Rate and Inflation Targeting Regime

Laubach and Williams (2003) provide a simple example for understanding the effect of natural interest rate mis-perception. Let's assume a permanent decrease of the natural interest rate from r^* to r_L^* . If the central bank recognizes this decline, it sets the nominal interest rate so that the real interest rate $i_t - \pi_t$ converges to the new level of the natural interest rate r_L^* :

$$i_t = \sigma i_{t-1} + (1 - \sigma) [\pi_t^e + \hat{r}_L^* + \psi(\pi_{t+4} - \pi_{t+4}^T)] + \varepsilon_t$$

so that at the end of the smoothing horizon:

$$(i_t - \pi_t) - \hat{r}_L^* = \psi(\pi_t - \pi_t^T)$$

$$0 = \psi(\pi_t - \pi_t^T)$$

$$\pi_t = \pi_t^T$$

The equation tells us that only if the natural interest rate considered in the reaction function represents its true, in our case lower actual, value, the inflation target will be achieved. On the other hand, if the central bank still assumed the initial higher value of the natural interest rate and set the nominal interest rate accordingly, the inflation would eventually get below the inflation target by the amount specified by:

$$\pi_t - \pi_t^T = \frac{r^* - r_L^*}{\psi}$$

The results in the Section 5.3 showing the real interest rate gap estimate suggest that

the Czech National Bank may face this problem. As mentioned, the reason is that the natural interest rate estimate in this paper is considerably volatile, but the Czech National Bank doesn't explicitly consider a time-varying estimate of the natural interest rate in its monetary decisions.

6.4. Reassessing the Inflation Targeting?

Low natural interest rates can lead to serious problems like secular stagnation as described above, and this is why the central bank should address the issue. There are several suggested ways how to react to the low natural rate environment.

The first potential solution is to simply increase the inflation target (Ball (2014) or Williams (2009)). The broad public discussion on this was initiated by an economist Olivier Blanchard suggesting an inflation target of 4%. The economist and Nobel laureate Paul Krugman also belongs to the group of higher target defenders. The higher inflation target would increase the inflation expectations and the average level of interest rates which would give policy-makers more room for reacting to current economic conditions without hitting zero lower bound. Such a measure would be accompanied by costs that need to be considered, as distortions of relative prices, destabilization of inflation expectations or loss of the central bank's credibility.

The other possibility is switching from inflation targeting to some of its alternatives like nominal GDP or flexible price-level targeting. Nominal GDP represents real GDP multiplied by the price level. Under nominal GDP targeting, the central bank's aim is to hit either a certain level of nominal GDP, or its rate of growth. Its advantage lies in its ability to decrease the output fluctuations, thus stabilizing the economy and its business cycles. Proponents of the nominal GDP targeting claim that sometimes it is a good idea to let inflation fluctuate. In case of a positive supply shock, for example an increase in energy prices, the inflation would rise and the output would respectively decrease. The economy would enter a stagflation and the economy under inflation targeting would accommodate a contractionary monetary policy in order to address the high inflation. As a result, inflation would decrease, but the output, which was already low, would too. On the contrary, under the nominal output targeting the central bank would let the inflation rise temporarily, in order to hit the nominal GDP. The issue might arise with the predictability of inflation and inflation expectations.

The monetary authority under price level targeting changes its short-term interest rates in order to hit the predetermined level of a certain price index. Compared to

inflation targeting, it is believed to imply more inflation and employment volatility in the short-run. The reason for this is that under the price-level targeting the unexpected price shocks result in corrective actions that bring the price level back to its predetermined path. If there was a target path of the price index rising 2% every period, and the price shock would cause the price index to increase by more than 2%, the correction would result in temporarily bringing the inflation down until the prices get back to its previous path. However, under the inflation targeting corrections do not happen, the aim of the monetary authority would be bringing the inflation back to its 2% level and leaving the prices shifted to their higher level. In the Figure A.5 the difference is clearly visible. Such corrective actions can lead to macroeconomic instability. On the other hand, inflation targeting implies an uncertainty in a way that the future price level is not defined. Based on the available research related to the topic, for instance Svensson's so called "Free lunch" (Svensson (1996)), the price-level targeting eliminates inflation and nominal interest rate variability, and on the other hand it produces only slightly higher variability of the output. However, as for instance Deutsche Bundesbank (2010) shows, price-level targeting might be an optimal strategy only under specific circumstances, for instance the degree of how much the economic agents are forward-looking.

Generally, there are high potential costs related to the change of the monetary policy regime, and in many cases only limited past experience with alternative regimes. For instance, price-level targeting has been implemented only in Sweden in the 1930's. The communication to the public is of key importance and might cause severe disruptions in the credibility of policy makers and inflation expectations. Due to these reasons, switching from inflation targeting to another regime seems like a last resort type of option.

Also, the countries committed to the inflation targeting regime do not follow strict targeting requiring to set the short-term interest rates to equate the inflation target with the forecasted future inflation at the end of the transmission horizon. On the contrary, the central banks, including the Czech National Bank, pursue a flexible inflation targeting regime in which the goal is to set the interest rate which will help to gradually return inflation to target, taking into consideration the output variability. In practice, escape clauses, choice of targeted index or interest rate smoothing can be used to allow for the flexibility of the inflation targeting procedure. In particular, escape clauses allow policy makers to defer or modify attaining the inflation target in order to respond to adverse economic shocks. The choice of targeted index also allows a certain flexibility. Although headline CPI index is the choice most of the

time, the policy maker can track also the core index excluding volatile components which is less sensitive to the economic shocks. Therefore, since the Czech National Bank does not follow the strict inflation targeting subject to the drawbacks described above, the switch to another targeting regime seems like an unnecessary and costly solution.

One of the ways how to fight the low natural interest rate combined with the zero-lower bound constraint without changing the policy regime is a use of unconventional policy tools. The Czech National Bank, as mentioned in the Section 5.3, opted for the exchange rate commitment at the end of the year 2013. The choice of the exchange rate tool was a result of the central bank reaching a zero-lower bound on nominal interest rates approximately year before. Franta et al. (2014) mention that due to zero-lower bound existence the negative real interest rates may be achieved solely through substantially positive inflation expectations. In the times when the aggregate demand is low and central bank faces the problem of policy rates being close to zero, the inflation expectations anchored at the inflation target level are not enough to yield sufficiently low real interest rate that would be able to stabilize the economy with the output close to its potential. In this case the output gap becomes even more negative and the inflation more below its target value, implying the risk of deflation. Through a commitment to weaken the czech currency by intervening on the FX market in order to keep the exchange rate close to CZK 27 per 1 EUR, the Czech National Bank pushed to increase the inflation expectations above the target value. Therefore, the existence and experience with the unconventional monetary policy rules may serve as another argument against the monetary policy switch.

Moreover, as John Williams from the Federal Reserve of San Francisco (Williams (2016)) mentions: *“While price level or nominal GDP targeting by monetary authorities are options, fiscal and other policies must also take on some of the burden to help sustain economic growth and stability”* (page 1). Long-term investments in areas of research and development, or education directly address a long-run output growth, and the strong predictable countercyclical fiscal policy can prevent the economy from falling into the recession accompanied by low interest rates, inflation and output growth.

To conclude, keeping the flexible inflation targeting combined with a properly managed countercyclical fiscal policy focused on long-term investments into human capital and research and development seems like the best solution to the current potential low level of natural interest rates.

7. Small Open Economy Adjustments

Small open economies are characterized by their high volume of international trade and cross-border capital flows, and by their size which has an effect on the degree to which the economy can influence its macroeconomic variables, such as the domestic interest rate. Unlike large open economies, which are to a high degree “price setters”, the small open economies represent rather “price takers”.

In small open economies the exchange rate and monetary policy responding to its value is of particular interest. Changes in exchange rates have a direct effect on the prices of goods. As the exchange rate depreciates, the prices of imported goods increase, which in turn increases the price index and inflation. Moreover, the depreciation has the same effect also on the production inputs of firms, thereby increasing their production costs. Such an increase naturally leads to the rise of consumer prices. From the real economy point of view, the depreciation of real exchange rate increases the demand for domestically produced goods, since consumers replace the costly goods from abroad for relatively cheaper domestic goods. Raising demand has a consequence of an increase in prices. Thus, the exchange rate has a significant effect on the economy in small open environments. Open-economy models based on microeconomic foundations, as in Galí and Monacelli (2005) or Kirker (2008) provide a useful example of how the structural model for small open economies can be defined.

In its simplest form, focusing on backward-looking type of models, it is needed to include especially foreign variables affecting the domestic economy and a version of the uncovered interest rate parity condition to model the development of the exchange rate. The stylized model, assuming 2 countries – domestic and foreign, can look like

the one below:

$$ygap_t = A_y(L)ygap_{t-1} + A_r(L)(r_{t-1} - r_{t-1}^*) + A_x(L)(x_{t-1} - x_{t-1}^*) + A_f(L)(y_{f,t-1} - y_{f,t-1}^*) + \varepsilon_{1t} \quad (7.1)$$

$$\pi_t = b_\pi \pi_{t-1} + (1 - b_\pi) \pi_{t-2,4} + b_1 [b_2 ygap_{t-1} + (1 - b_2)(x_{t-1} - x_{t-1}^*)] + \varepsilon_{2t} \quad (7.2)$$

$$x_t = x_t^* + \gamma_1 x_{t-1} + \gamma_2 x_{t-1}^* + \gamma_3 [(r_{t-1} - r_{t-1}^*) - (r_{f,t-1} - r_{f,t-1}^*)] + \varepsilon_{3t} \quad (7.3)$$

$$r_t^* = c g_t + z_t \quad (7.4)$$

$$x_t^* = x_{t-1}^* + \varepsilon_{4t} \quad (7.5)$$

$$z_t = D_z(L)z_{t-1} + \varepsilon_{5t} \quad (7.6)$$

$$y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_{6t} \quad (7.7)$$

$$g_t = g_{t-1} + \varepsilon_{7t} \quad (7.8)$$

The equations include lag polynomials $A_y(L)$, $A_r(L)$, $A_x(L)$, $A_f(L)$ and $D_z(L)$ in general form so that the user can define the number of lags that best fit the economy of interest. The IS curve is adjusted by the real exchange rate gap where x_t represents the real exchange rate and x_t^* the equilibrium real exchange rate, both in logarithmic form. Moreover, the term $y_{f,t-1} - y_{f,t-1}^*$ defined as foreign output gap, which serves as a proxy for foreign demand, is added into the IS curve specification. For the sake of parsimony, x_t^* is following a random walk.

The uncovered interest rate parity states that the real interest rate in the home country should equal the real interest rate abroad less the expected change in the real exchange rate, so that any arbitrage opportunity is eliminated. The uncovered interest rate parity in its original form has been thoroughly investigated and a number of papers show poor evidence for the relationship. In case of the Czech Republic, for example Cuestas et al. (2015) come to the same conclusion. Therefore, following Kirker (2008) and Berg et al. (2006), the modified parity condition can be used, as shown in equation (3). The equation is implied, as a backward looking version, from the general UIP relationship in the form $x_t - E(x_{t+1}) = r_t - r_t^f + RP^* + \varepsilon$, in which the term RP^* describes risk premium.

Some authors also include an equation based on the Taylor rule into their model. Such specification, however, has a significant drawback. The Taylor rule equation in its general form, as mentioned in the previous section, takes into account the natural

rate of interest in the following way:

$$i_t = \pi_t + r_t^* + \alpha_1(\pi_t - \pi_t^T) + \alpha_2(y_t - y_t^*)$$

The central bank reacts to the economic conditions by setting the short-term nominal interest rate i_t . The intercept here is the central bank's estimate of the nominal equilibrium interest rate $i_t^* = \pi_t + r_t^*$. After rearranging we end up with an equation for r_t^* :

$$r_t^* = i_t - \pi_t - \alpha_1(\pi_t - \pi_t^T) - \alpha_2(y_t - y_t^*)$$

The problem for using r_t^* from the Taylor rule is the very fact that the natural interest rate in the Taylor rule is also not known by the central bank - it is only estimated. As a consequence, we would be estimating short-term equilibrium rate dependent on the central bank's decisions, rather than the underlying medium-term rate based on structural developments of the economy. The purpose of estimating the natural rate of interest is to give an exogenous benchmark for monetary decisions. If we model the natural rate development by the Taylor rule, we are essentially using circular reasoning – estimated natural rate is implied from the equation implied from the estimated natural rate. Moreover, using the equation is intrinsically valid if and only if the nominal interest rate is really set according to the Taylor rule, which does not have to be the case all the time.

8. Conclusion

The estimation of the natural interest rate represents an important concept in macroeconomic theory. Knowing the real value of the natural rate of interest would be a useful tool for policymakers in conducting an effective monetary policy. Nevertheless, its estimation based on currently used methods is still considered difficult and uncertain.

In this paper the natural interest rate for the Czech Republic has been estimated using the semi-structural Laubach and Williams model leveraging Kalman filter and Maximum Likelihood methods. The model jointly estimates the natural interest rate, the output gap and the trend growth rate. The estimated natural interest rate shows a notable decrease in the past decade, which is in line with most of the relevant research from recent years, and with the heavily discussed risk of secular stagnation and zero-lower bound. Moreover, the global financial crisis in 2008 seems to represent a structural break causing the natural interest rate to change trend and drop into the negative territory reaching -2.6% on average. The estimated trend growth rate has been experiencing a short decline in the beginning of the new millenia, and a persistent one after the global financial crisis, partially copying the European trend. The striking fact is that the estimated real interest rate gap is mostly negative in the first part of the sample (1996-2008) implying the expansionary monetary policy, and significantly positive after the financial crisis, suggesting the policy being contractionary. If the natural interest rate really has a volatile behaviour as estimated in this paper, the results would imply systematically insufficient responses of monetary policy to current economic conditions.

However, the results show low significance and implied uncertainty of the estimated values, which is most likely caused by the model specification and Czech economy specifics. Most importantly, the omitted open economy variables likely play a role in the estimation process. For instance, including the exchange rate in the model would account for the exchange rate commitment of Czech National Bank from 2013, which could correct for the strikingly positive real interest rate gap in that period.

Moreover, the potential solutions to the "low r-star conditions" are investigated. The possibility of the inflation target increase or the switch to another targeting regime like the price-level or nominal-GDP targeting is discussed. Eventually, keeping the flexible inflation targeting regime with the current inflation target is considered to be the best reaction even to the significantly low natural interest rate and trend growth rate values. The potential benefits and costs of the switching to a different regime are uncertain, and most importantly, this paper only provides additional evidence that the natural interest rate estimation is not straightforward and is subject to various obstacles. Additionally, the research related to the natural interest rate, as mentioned in the Section 2, is relatively new and needs more investigation. To conclude, the estimated natural interest rate at the current stage of the research should not represent a trigger for a change in monetary policy.

There has not been a research specializing in natural interest rate estimation using the popular Laubach and Williams model or any of its versions for Czech Republic. Therefore, this paper can be considered as a stepping stone for further research. It attempts to replicate the basic version of the model and concludes that its original form is not optimal for Czech economy. The issues related to the estimation are addressed, discussed and explained, which may ease the further research related to the Laubach and Williams model in case of Czech Republic and other small open economies. The issues discussed are the risk of the covariance matrix singularity, the omitted variable bias and the slope parameters of IS and Phillips curves close to zero. The paper consequently suggests the adjustments which are better fitting the Czech environment. In particular, the open economy variables such as the exchange rate, and the forward-looking version of the model are encouraged to be used.

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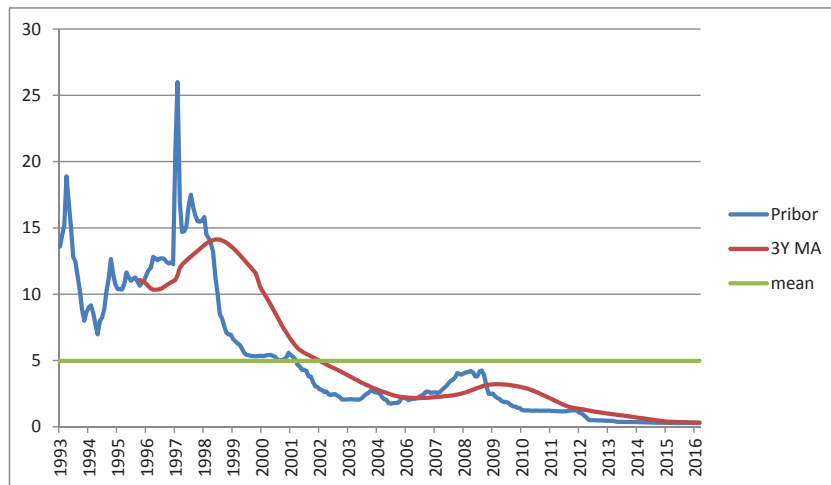
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A. Figures and Tables

Figure A.1.: Averages of PRIBOR



source: data from CNB's database ARAD, and author's calculations

Figure A.2.: Decomposition of natural interest rate

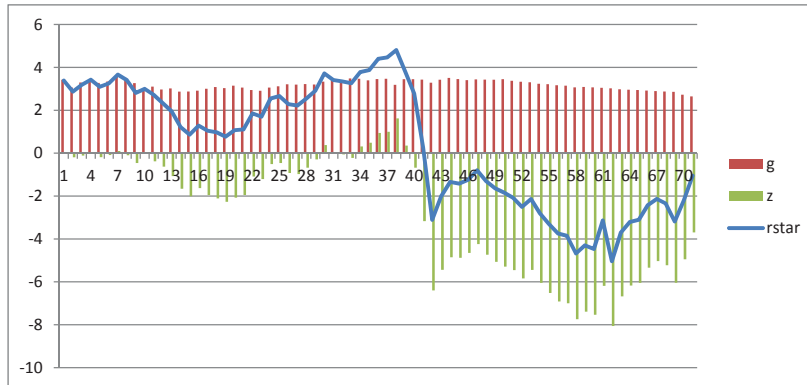


Figure A.3.: Output gap estimate

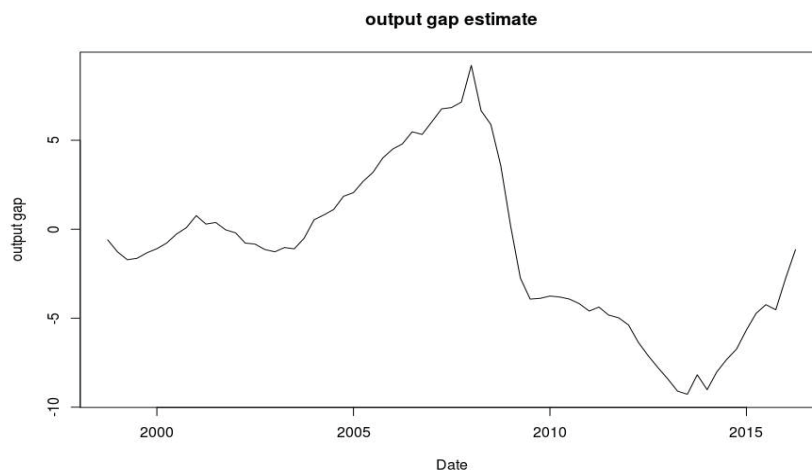


Figure A.4.: Real interest rate gap estimate

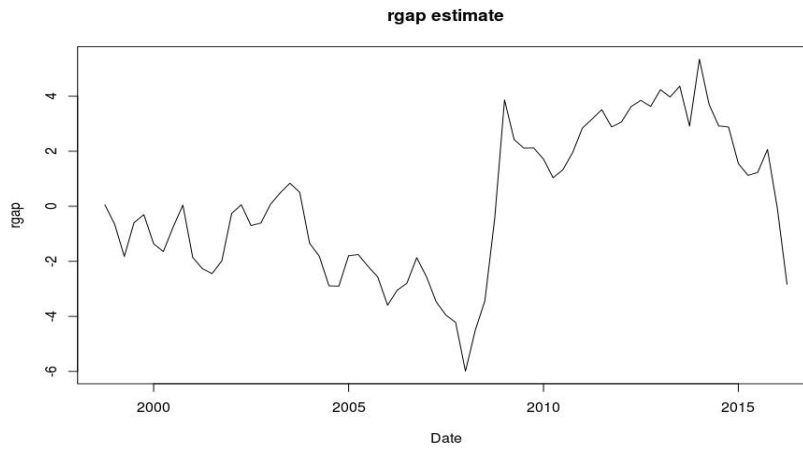
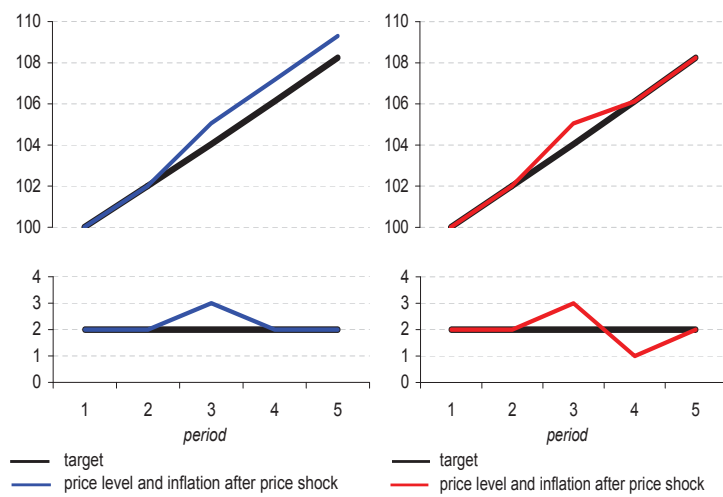


Figure A.5.: Difference between price-level and nominal-GDP targeting



source: Bohm, et al. (2012)

Table A.1.: Estimated parameters when original constraints imposed

Parameter	Value
$\sum a_y$	0.979
a_r	-0.0025
	(0.027)
b_y	0.025
	(0.352)
λ_g	0
λ_z	0.042
$\sigma_{\bar{y}}$	0.625
	(4.993)
σ_{π}	2.139
	(18.306)
σ_{y^*}	0.144
	(0.838)

Table A.2.: Sample averages of estimated standard errors when original constraints imposed

<i>Standard errors – sample averages</i>	
r^*	15.258
g	0.507
y^*	4.382