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Report on the phd thesis

Integral and Supremal Operators on Weighted Function Spaces

by Martin Křepela

This phd thesis is devoted to the study of boundedness of certain integral and supremal operators between weighted function spaces (Lebesgue spaces, Lorentz spaces and its associate spaces). The main aim is to get associated two-weight inequalities under necessary and sufficient conditions.
The thesis consists of the Introduction, two chapters containing basic notation and a description of the content of nine papers which are attached. From these nine papers three have already appeared, four are accepted, one is submitted and one is just a preprint. Only one of these manuscripts is written with co-authors. For a phd student this list of publications is really impressive.
Papers I and IX deal with generalizations of the Young inequality in the form

$$\|f * g\|_{\Gamma^q(w)} \lesssim \|f\|_{\Lambda^p(v)} \|g\|_Y \,, \tag{1}$$

where $\Lambda^p(v)$ is a weighted Lorentz space and $\Gamma^q(w)$ belongs to the scale of associated spaces of weighted Lorentz spaces. Here the candidate has obtained far-reaching results of mainly optimal character. Optimal means that he could determine the largest r.i. space $Y = Y(p,q,v,w)$ such that (1) holds for the pair $(\Lambda^p(v), \Gamma^q(w))$. Furthermore, if g is radially decreasing, he proved

$$\|T_g|\Lambda^p(v) \to \Gamma^q(w)\| \asymp \|g\|_{Y(p,q,v,w)} \,,$$

where $T_g : \ f \to f * g$.
Papers II and III deal with a modification of (1) where $\Lambda^p(v)$ is replaced by either $\Gamma^p(v)$ or $S^p(v)$ (here $S^p(v)$ is determined in terms of the difference $f^{**} - f^*$, a quite interesting construction).
The bilinear and the multilinear Hardy operator are investigated in papers IV and V, again in connection with weighted Lorentz spaces $\Lambda^p(v)$ and $\Gamma^q(w)$, respectively. Essentially the

author is looking for operators of the type

$$T(f_1, \dots, f_n) := \prod_{i=1}^{n} T_i f_i ,$$

where the mapping properties of the T_i are known. He calls this type of operators product-based operators. Obviously they do not coincide with tensor product operators. Tensor product operators are well understood on Lebesgue spaces but probably much less is known on Lorentz spaces. Paper VI deals with a limiting situation, supremal Hardy-type operators.
Paper VII is devoted to the study of general embeddings between weighted Lorentz-type spaces. Finally, paper VIII deals with mapping properties of Hardy-type operators with respect to the pair $(L^p(v), L^q(w))$ for the critical range $0 < q < 1 < p < \infty$.
From this list it is clear that the candidate has a sufficiently wide knowledge in this field of research. The obtained results are new in the given generality and in almost all cases quite satisfactory. Definitely they prove the ability of the candidate for independent creative scientific work. With the submitted thesis Martin Křepela deserves to obtain the Phd degree of your faculty.

Sincerely yours

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