Title: Integral and Supremal Operators on Weighted Function Spaces

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Abstract: The common topic of this thesis is boundedness of integral and supremal operators between function spaces with weights. The results of this work have the form of characterizations of validity of weighted operator inequalities for appropriate cones of functions. The outcome can be divided into three categories according to the particular type of studied operators and function spaces.

The first part involves a convolution operator acting on general weighted Lorentz spaces of types Λ, Γ and S defined in terms of the nonincreasing rearrangement, Hardy-Littlewood maximal function and the difference of these two, respectively. It is characterized when a convolution-type operator with a fixed kernel is bounded between the aforementioned function spaces. Furthermore, weighted Young-type convolution inequalities are obtained and a certain optimality property of involved rearrangement-invariant domain spaces is proved. The additional provided information includes a comparison of the results to the previously known ones and an overview of basic properties of some new function spaces appearing in the proven inequalities.

The second type of investigated objects are bilinear and multilinear operators defined as a product of linear Hardy-type operators or in a similar way. It is determined when a bilinear Hardy operator inequality holds either for all nonnegative or all nonnegative and nonincreasing functions on the real semiaxis. The proof technique is based on a reduction of the bilinear problems to linear ones to which known weighted inequalities are applicable. The use of this method to solve other questions concerning more general multilinear operators is described as well.

In the third part, the focus is laid on iterated supremal and integral Hardy operators, a basic Hardy operator with a kernel and applications of these to more complicated weighted problems and embeddings of generalized Lorentz spaces. Several open problems related to missing cases of parameters are solved, therefore completing the theory of the involved fundamental Hardy-type operators. The results have a standard explicit form of integral or supremal conditions which are compatible with those known previously. It allows for a straightforward application in various situations involving more complicated weighted inequalities.

Keywords: integral operators, supremal operators, weighted function spaces, Hardy inequality