MASTER'S THESIS

Identifying the Conditions of Instability in Macromodels of Financial Cycles

Author: Bc. Aleš Zenáhlík
Supervisor: PhDr. Jaromír Baxa, Ph.D.
Academic Year: 2016/2017
Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature, and the thesis has not been used to obtain a different or the same degree.

The author grants to Charles University permission to reproduce and to distribute copies of this thesis document in whole or in part.

Prague, May 19, 2017

Signature
Acknowledgments

I would like to express my sincerest gratitude to my supervisor PhDr. Jaromír Baxa for his patience and refreshing ideas that gave me direction when working on this thesis. My special thanks belongs to my family for their unwavering dedication and continuous support during my university studies and time spent writing this thesis, without whom I would never have been able to finish these tasks.
Abstract

The purpose of this thesis is to construct an endogenous macroeconomic model explaining the cause of financial cycles and systemic instability based on the financial instability hypothesis (FIH) published by Hyman Minsky (1982). FIH maintains that capitalist financial systems have an inherent disposition to financial instability because periods of economic prosperity encourage borrowers and lenders to be increasingly reckless which in turn lead to the formation of financial bubbles. The problem is approached by employing an adaptive expectations model based on stylized facts from Kaldor’s and Kalecki’s models with addition of behavioral equations implemented in an attempt to simulate market expectations.

JEL Classification  E02, E11, E32
Keywords  Instability, Macromodel, Cycles

Author’s e-mail  ales.z@hotmail.com
Supervisor’s e-mail  jaromir.baxa@fsv.cuni.cz

Abstrakt

Účelem této práce je sestavit endogenní, makroekonomický model vysvětlující příčinu vzniku finančních cyklů a systematické nestability dle hypotézy finanční nestability (HFN) zveřejněné Hymanem Minskym (1982). HFN stanovuje, že kapitalistické finanční systémy mají vrozený předpoklad k finanční nestabilitě, jelikož období ekonomické prosperity nabádají dlužníky a věřitele ke stále větší lehkovážnosti, což následně vede k utváření finančních bublin. Rešení problému je založeno na použití modelu adaptivních očekávání na základě Kaldorova, a Kaleckiho modelů ve spojení s behaviorálními rovnicemi za účelem simulování tržních očekávání.

Klasifikace JEL  E02, E11, E32
Klíčová slova  Nestabilita, Makromodel, Cykly

E-mail autora  ales.z@hotmail.com
E-mail vedoucího práce  jaromir.baxa@fsv.cuni.cz
## Contents

List of Figures vii

Acronyms x

Thesis Proposal xi

1 Introduction 1

2 Review of Literature 3

3 Financial Instability Hypothesis 6
   3.1 Keynes’ Theory of Investment Cycle ..................... 6
   3.2 Minsky’s Theory of Investment .......................... 8
   3.3 The Role of Profits ........................................ 9
   3.4 Minsky’s Model ............................................ 11

4 Modelling the FIH 15
   4.1 Kaldor’s Model ............................................. 15
   4.2 Kaldor-Kalecki Model ....................................... 18
   4.3 Four Functional Kaldor-Kalecki Model .................... 20
   4.4 Kaldor-Kalecki-Minsky Model ............................. 23
      4.4.1 Time Dependent Expectations ......................... 26
      4.4.2 Adaptive Expectations ................................ 28
      4.4.3 Thwarting Systems ................................. 31
   4.5 Empirical Validity ....................................... 39

5 Discussion 50

6 Conclusion 52

Bibliography 55
A Appendix

A.1 Chapter 3 ...................................................... I
A.2 Chapter 4 ...................................................... II
A.3 Impulse Response Functions ............................... X
List of Figures

3.1 Level of Investment ........................................... 9
4.1 The evolution of production $y$ and capital $k$ .................. 17
4.2 The phase portrait of production $y$ and capital $k$ ............... 17
4.3 The phase portrait of production $y$ and capital $k$ with dimension of time (note that the units on the time axis are divided by 100) 18
4.4 A Kaldor model with zero lag .................................. 19
4.5 A Kaldor-Kalecki model with a lag of $\theta = 8$ ................. 19
4.6 An attractor Kaldor model with zero lag ....................... 20
4.7 A Kaldor-Kalecki model with identical parameters and a lag of $\theta = 2$ .................................................. 20
4.8 Four functional Kaldor model ................................... 22
4.9 Four functional Kaldor-Kalecki model with an investment delay of $\theta = 30$ ............................................... 23
4.10 Exponential debt spiral given temporal decrease in prudence .. 27
4.11 A stable economy with a positive lending leniency .............. 29
4.12 A semi-stable economy with a positive lending leniency ...... 29
4.13 A low risk aversion market with an average inflation of 3.5% .. 29
4.14 A low risk aversion market with an average inflation of 5.5% 29
4.15 A highly prudent deflatory economy ............................ 30
4.16 Deflatory economy with low risk aversion .................... 30
4.17 Model crisis with high leniency ............................... 33
4.18 Fig. 4.17 model with an intervention of a lender of last resort (LOLR) providing zero real interest loans ..................... 35
4.19 Fig. 4.17 model with inflation targeting and a constant rate of overinvestment ........................................ 37
4.20 Inflation targeting in a system with an increasing rate of overinvestment ................................................. 37
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.21</td>
<td>impulse response functions (IRFs) of an inflation impulse in a restricted model</td>
</tr>
<tr>
<td>4.22</td>
<td>IRFs of an inflation impulse in the general model</td>
</tr>
<tr>
<td>4.23</td>
<td>Interest rate to debt IRF</td>
</tr>
<tr>
<td>4.24</td>
<td>Risk premium to debt IRF</td>
</tr>
<tr>
<td>4.25</td>
<td>80s depression in the US</td>
</tr>
<tr>
<td>4.26</td>
<td>4.25 first differenced</td>
</tr>
<tr>
<td>4.27</td>
<td>Private debt to GNP</td>
</tr>
<tr>
<td>4.28</td>
<td>Debt/GNP comparison</td>
</tr>
<tr>
<td>4.29</td>
<td>Japanese asset price bubble</td>
</tr>
<tr>
<td>4.30</td>
<td>gross national product (GNP) to debt ratio IRF</td>
</tr>
<tr>
<td>4.31</td>
<td>Trade IRFs</td>
</tr>
<tr>
<td>A.1</td>
<td>Propensity to invest as defined in Kaldor’s model, parameters: a=2, b=8, i₀=1/8</td>
</tr>
<tr>
<td>A.2</td>
<td>A phase portrait comparison of four functional Kaldor-Kalecki models with lags of θ = 0 (left) and θ = 30 (right)</td>
</tr>
<tr>
<td>A.4</td>
<td>Stabilization and improved performance due to inflation</td>
</tr>
<tr>
<td>A.3</td>
<td>Exponential debt spiral given temporal decrease in prudence</td>
</tr>
<tr>
<td>A.5</td>
<td>A highly prudent deflatory economy</td>
</tr>
<tr>
<td>A.6</td>
<td>Deflatory economy with low risk aversion</td>
</tr>
<tr>
<td>A.7</td>
<td>Stabilization achieved through a change in market culture</td>
</tr>
<tr>
<td>A.9</td>
<td>Fig. 4.17 model with inflation targeting and a constant rate of overinvestment</td>
</tr>
<tr>
<td>A.8</td>
<td>Model crisis with an intervention of a LOLR</td>
</tr>
<tr>
<td>A.10</td>
<td>Model identical to A.9 but with a higher real interest rate</td>
</tr>
<tr>
<td>A.11</td>
<td>Inflation targeting in a system with an increasing rate of overinvestment</td>
</tr>
<tr>
<td>A.12</td>
<td>80s depression in the US</td>
</tr>
<tr>
<td>A.13</td>
<td>A.12 first differenced</td>
</tr>
<tr>
<td>A.14</td>
<td>Private debt to GNP</td>
</tr>
<tr>
<td>A.15</td>
<td>Debt/GNP comparison</td>
</tr>
<tr>
<td>A.16</td>
<td>Trade IRFs</td>
</tr>
<tr>
<td>A.17</td>
<td>Savings IRFs</td>
</tr>
<tr>
<td>A.18</td>
<td>Prime rate IRFs</td>
</tr>
<tr>
<td>A.19</td>
<td>GNP IRFs</td>
</tr>
<tr>
<td>A.20</td>
<td>Inflation IRFs</td>
</tr>
</tbody>
</table>
A.21 Private debt IRFs  
A.22 Unemployment rate IRFs  
A.23 Risk premium IRFs
Acronyms

**FED** Federal Reserve System
**FIH** financial instability hypothesis
**FRED** Federal Reserve Economic Data
**GDP** gross domestic product
**GNP** gross national product
**HAM** heterogeneous agent model
**IRF** impulse response function
**LOLR** lender of last resort
**VAR** vector autoregression
## Master's Thesis Proposal

Institute of Economic Studies  
Faculty of Social Sciences  
Charles University in Prague

<table>
<thead>
<tr>
<th>Author</th>
<th>Bc. Aleš Zenáhlík</th>
<th>Supervisor</th>
<th>PhDr. Jaromír Baxa, Ph.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-mail</td>
<td><a href="mailto:ales.z@hotmail.com">ales.z@hotmail.com</a></td>
<td>E-mail</td>
<td><a href="mailto:jaromir.baxa@fsv.cuni.cz">jaromir.baxa@fsv.cuni.cz</a></td>
</tr>
<tr>
<td>Phone</td>
<td>721 418 068</td>
<td>Phone</td>
<td>777 152 097</td>
</tr>
<tr>
<td>Specialization</td>
<td>FFM&amp;B</td>
<td>Defense Planned</td>
<td>February 2017</td>
</tr>
</tbody>
</table>

### Proposed Topic:
Identifying the Conditions of Instability in Macromodels of Financial Cycles

### Motivation:

The financial instability hypothesis (FIH), published by Hyman Minsky (1982), maintains that capitalist financial systems have an inherent disposition to financial instability because periods of economic prosperity encourage borrowers and lenders to be increasingly reckless which leads to a "Minsky moment" – a transformation of a stable economy into an unstable state and the formation of financial bubbles. In such processes the economic system’s reactions amplify the movement – inflation feeds upon inflation and debt deflation feeds upon debt deflation (Minsky, 1994). Minsky thus went even further than his precursors, Schumpeter and Fisher, who accepted that capitalism was always in disequilibrium, by arguing that the equilibrium itself is unstable and that the financial sector is the exclusive source of instability in a capitalist economy.

Minsky’s work on capitalist instability consists of two key building blocks: his theory of investment and his theory of profit determination. His theory of investment is based on theory of profit associated with Michal Kalecki, introducing a time delay between the decision to invest and actual spending of investment, which can lead to "involuntary" demand for investment funds (Wolfson, 1986). This demand is implied to be an important source of persistent instability in endogenous business cycles, as illustrated by the Kaldor-Kalecki model with delayed investments (Kodera, 2012). However, since the FIH is based on the assumption of changing preferences under long periods of prosperity, a model capable of simulating the impact of these preferences is needed to fully discern the causes of instability.

### Hypotheses:

1. Hypothesis #1: The ratio of private debt to GDP is the main source of instability
2. Hypothesis #2: The ratio of investment and speculation agents changes significantly during the cycle
3. Hypothesis #3: It is possible to predict crises and prevent them using monetary policy

### Methodology:

In the first step of modelling the FIH I will create a model framework under the assumption of homogeneous agents. Minsky himself was unsuccessful in specification of a mathematical model of his hypothesis. According to Keen, it was because the foundation he used, the multiplier-accelerator model, was itself flawed (Keen, 2000). Keen defined a monetary Minsky model using nonlinear functions for key monetary relationships: the rate at which existing money is circulated, the rate of loan repayment, and the rate of investment (and hence the rate of growth of the money stock) (Keen, 2011). This model, adjusted for investment gap and government spending as an extension to Keen’s original model, will, along with the implications from the Kaldor-Kalecki model, serve as a basis for the homogeneous agent model framework.

Building on top of this framework, I will devise a variable preferences model to illustrate the continuous increase in propensity to risky investments and its impact on the length of the Minsky cycles. I will then use this model to define the conditions under which a stable economy becomes unstable. I will also discern the impact of financial regulations and government interventions on the growth and stability of the economy, as Minsky stressed the
importance of the Federal Reserve as a lender of last resort, while also arguing, that government interventions could potentially lead to a slower overall growth of the economy.

In order to verify the implications of the model, the final part of the thesis will be dedicated to verification of the findings from the model by means of simulating impulse responses of a VAR model based on historical US economy data.

Expected Contribution:
The aim of this thesis is to define the dynamics of the Minsky financial instability hypothesis by abandoning various simplifications and possibly set the cornerstone of a new research agenda. Although the 2007-08 crisis has brought the FIH into spotlight and garnered much attention from many influential economists, the role of heterogeneous financial conditions is still under-researched. Albeit the FIH can be formulated by homogeneous models, at a deeper level Minsky develops his ideas in a heterogeneous agents’ setting, characterized by hedge, speculative and Ponzi units. I will model this relationship and define the model framework for future research. Additionally, I will also evaluate the efficiency of different approaches to financial regulation.

Outline:
1. Introduction
   (a) motivation, hypotheses, methodology
   (b) literature review
2. Model Definition
   (a) a model of economic cycles
   (b) variable preferences model
   (c) extended regulatory model
3. Model Inference
   (a) conditions of instability in the models
   (b) discussion on government policies
4. Analysis of impact of financial regulation
   (a) modelling impulse responses to monetary policies
5. Conclusions
   (a) summary, policy recommendations

Core Bibliography:


__________________________  ____________________________
Author                      Supervisor
Chapter 1

Introduction

Starting from the Great Depression, crises have been the main focus of modern economic research, resulting in not only strictly traditional but also less mainstream theories, particularly the post-Keynesian school of thought that was deeply influenced by economists such as Michal Kalecki or Nicholas Kaldor.

These theories garnered rising interest which peaked during the so–called credit crunch of 1966 which was subsequently recognized as the first significant postwar financial crisis. These times of turmoil provided the data and opportunity for economists to try and define a framework that would include crises as a part of modern day economics and not just exogenous, unpredictable occurrences.

Hyman Minsky and his key work, the financial instability hypothesis (FIH)\(^1\), have played a pivotal role in this revolutionary approach to economic modelling. Minsky directly opposed neoclassical school of thought, stating that its models, unable to generate instability, a state displayed by the US economy during the Great Depression, cannot correctly describe real economic environment. Minsky instead combined insights from Schumpeter, Fisher and Keynes to develop a theory of financially driven business cycles which can lead to an eventual debt-deflationary crisis.\(^2\) This theory was mostly ignored by the contemporary scientific society up to the crash of 1987 that displayed aspects in line with the reasoning that serves as the basis of FIH. This fact spurred up new interest about Minsky’s work that further peaked when the financial crisis of 2007–2008 once again provided facts supporting the FIH that was capable of predicting the crisis when classical models could not.

While the theory seems to be empirically sound, aside from the strictly

\(^1\)Minsky (1992)  
\(^2\)Keen (1995)
monetary models proposed by Steve Keen\textsuperscript{3}, the model framework is not well defined yet. The primary aim of this master thesis is to describe FIH by building a model framework based on the work of Kaldor, Kalecki and the insights provided by Keen in combination with a dummy variable approach of modelling Ponzi debt, which is essential in correctly modelling and understanding the underlying processes in the FIH as well as predicting future crises due to the assumption that crises come as the result of the ratio of speculative agents exceeding a tolerable limit which forces the economy into a deflationary spiral.

Of special interest is the focus on uncovering the effects of the so-called investment lag defined by Michal Kalecki as a lag between the investment decision and installation of investment goods.\textsuperscript{4} It can be assumed that longer investment lags or even the possibility of investments providing returns over time affect not only the length of the business cycles but also the decision making ability of the agents and thus the structure of the cycles, potentially leading to a chaotic motion even in fully endogenous models.\textsuperscript{5}

The secondary aim is to determine the impact of various approaches to financial regulation and government interventions on the growth and stability of the economy, as Minsky stressed the importance of the Federal Reserve as a lender of last resort while also arguing that government interventions could potentially lead to a slower overall growth of the economy. This assumption will be explored in the final part of the thesis with the goals of validating the findings and claims obtained from the model and attempting to discover the main driver of instability in the current global market.

\textsuperscript{3}Keen (2013)
\textsuperscript{4}Kalecki (1937)
\textsuperscript{5}Lorenz (2012)
Chapter 2

Review of Literature

Although the popularity of Minsky’s theories has started to rise in the recent years due to the latest crises, the past few decades have left us with rather scarce literature concerning the topic of FIH, especially any kind of research pertaining to macroeconomic models, as Minsky did not manage to produce a general model himself leading to a lack of interest from mainstream economists.

Minsky’s FIH builds on the foundation established by ”The General Theory of Employment, Interest and Money” (Keynes, 1937) and ”Alternative Theories of the Rate of Interest” (Keynes, 1937) which significantly deviated from the conventional theories at the time, introducing the duality of supply and demand price, a volatile basis for the formation of expectations, which determines the desire to invest, as well as finance based demand for money.

Minsky was heavily influenced by Joseph Schumpeter, Minsky’s professor at Harvard University, and his credit view of money and finance described in ”The Instability of Capitalism” (1928), ”Review of Keynes’s General Theory” (1936), ”Business Cycles: A Theoretical, Historical, and Statistical Analysis of the Capitalist Process” (1939) and many other publications. Despite the fact that Minsky viewed Schumpeter’s business cycle theory as ”banal”, since it was based on Walrasian principles of economy which Minsky considered unrealistic, the FIH nonetheless borrows many ideas introduced in Schumpeter’s essays.

The cornerstone of FIH was laid down by Irving Fisher in his publication named ”The Debt-Deflation Theory of Great Depressions” (1933) which attributed the crises to the bursting of a credit bubble, providing Minsky with the core idea for his own theory of systemic instability.
Over the following decades, Minsky published a large amount of academic articles and papers on the topic of financial instability which were later consolidated and published under the name "Can "It" Happen Again?: Essays on Instability and Finance" (1982). Despite the fact that this publication did not prove to be very popular with contemporary economists due to its unorthodox approach, it established Minsky as one of the foremost figures in the post-Keynesian school. A decade later and four years before his death, Minsky published his working paper no. 74 titled "The Financial Instability Hypothesis" (1992), where he fully fleshed out his theory by identifying the ratio of hedge, speculative, and Ponzi units as the source of instability in the economy – the greater the weight of speculative and Ponzi finance, the greater the likelihood that the economy is a "deviation-amplifying" system.\(^1\) Unfortunately, not even his magnum opus provided a satisfactory mathematical model of the hypothesis, once again preventing its widespread adoption.

The most significant attempt at modelling the FIH was undertaken by Steve Keen (1995), resulting in a set of endogenous monetary models that he later expanded in a working paper named "A monetary Minsky model of the Great Moderation and the Great Recession" (2000). Keen argued, that Minsky’s own attempts to formulate a mathematical model of his hypothesis were unsuccessful because the multiplier-accelerator model he used as the basis of his effort was flawed. Keen instead opted for the use of Goodwin’s growth cycle model, modified by introducing a non–linear investment function and extending the model by including an explicit monetary sector.

This model framework generated qualitative characteristics that matched the real macroeconomic and income-distributional outcomes of the testing period, serving as a proof that a macroeconomic model based on Minsky’s FIH is indeed possible and may yield practical results as it allowed Keen to be one of the few economists to anticipate the impeding financial crisis of 2007–08.\(^2\) However, these models are currently still in a research stage and do not enjoy widespread use. Moreover, they are based on many simplifications, most importantly the lack of heterogeneous agents framework that is central to the FIH.

\(^1\)The Financial Instability Hypothesis (1992)
Another approach, bearing similarity to the approach taken in this thesis, was demonstrated in working paper named "Inside Money in a Kaldor–Kalecki–Steindl Fiscal Policy Model: The Unit of Account, Inflation, Leverage, and Financial Fragility" (2015) by Greg Hannsgen and Tai Young-Taft. This paper uses the Kaldor–Kalecki–Steindl model, based on Kaldor's stylized growth facts ("Strategic Factors in Economic Development", 1967), Kalecki’s investment lag ("A Macrodynamiopic Theory of Business Cycles", 1935) and Steindl’s profit margin adjustment theory ("Maturity and Stagnation in American Capitalism", 1952), in addition to a rather unorthodox solution of introducing a proxy variable named 'financial prudence' to simulate the behavior of economic agents.

A paper "Financial Instability and Agents' Heterogeneity: A Post Minsky Research Agenda" (2010) and its subsequent extension "Financial Instability after Minsky: Heterogeneity, Agent Based Models and Credit Networks" (2012) by Demenico Delli Gatti then explore the possibility of modelling the FIH with HAMs, using the Greenwald-Stiglitz variant of the Financial Accelerator to model individual behavior of the economic agents, going far beyond the scope of this thesis by introducing random elements of failure and accounting for the robustness and "noise" of the financial environment.
Chapter 3

Financial Instability Hypothesis

In his essay "Can "It" Happen Again?: Essays on Instability and Finance", Minsky criticizes the mainstream economic theory as a reliable guide to national policy and calls for the radical restructuring of economic institutions. Drawing heavily on the work of John Maynard Keynes, Minsky attempts to uncover the mechanisms behind the 1966 crisis in the domestic economy and in the international financial system in order to prevent or at least minimize the negative impact of further financial crises. His work on capitalist instability consists of two key building blocks: his theory of investment, based on Keynes, and theory of profit determination, based on Kalecki’s equations.

3.1 Keynes’ Theory of Investment Cycle

Keynes’ General Theory establishes investment as driving force behind the economy. The amount of invested capital determines the aggregate level of effective demand, which, in turn, is the primary factor generating the equilibrium level of employment and output.\(^1\)

The level of investment is dependent on two variables: the efficiency of capital and the market interest rate, or, in other words, the monetary supply and demand. When the marginal efficiency is above the market interest rate, investment is undertaken, raising income, output, and employment through the spending multiplier.\(^1\) Conversely, when the opposite is true, no incentive to invest exists and the economy remains in equilibrium.

\(^1\)Wray & Tymoigne (2008)
Keynes further expanded this theory by introducing his liquidity preference theory of asset prices, stating that interest rates are proprietary and every commodity comes with its own rate of interest. This is expressed by the equation

\[ r = q - c + l + a \]  

(3.1)

where \( r \) is the expected return on holding the asset, where \( q \) is the asset’s expected yield, \( c \) is carrying costs, \( l \) is liquidity, and \( a \) is expected price appreciation (or depreciation). The total return is then used to calculate a marginal efficiency for each asset.\(^2\)

In this model, expectations play a significant role in determining the marginal efficiency of the asset, producing different results according to the distribution of the returns. Optimistic expectations about future returns will raise the \( q \) on capital assets while lowering the values of liquid positions represented by \( l \), leading to an increase in capital efficiency in comparison to assets that get a large fraction of their return from \( l \). If the expectations cause the return of this asset to significantly rise above all other returns on the market, the production of the asset will be stimulated, leading to a multiplier effect on aggregate demand and therefore growth of investment, consumption and output until the system reaches a new equilibrium (i.e. until there is no such asset that holds a higher marginal efficiency than liquid financial assets).

While Keynes’ theory describes the mechanism behind the financial cycle, it does not explain how the investments are financed when the marginal efficiency of a capital asset exceeds the marginal efficiency of money.\(^2\) Keynes argued that investment is motivated by the desire to produce assets with higher demand price than supply price. In addition, Keynes acknowledges the existence of financial demand for money, which must be satisfied before investments can be undertaken. Despite that, it was difficult for him to explain how the two price levels could diverge or how expectations play role in the question of supply of finance. This lead Minsky to fill these gaps in Keynes’ theory and formulate his own financial theory of investment.

\(^{2}\)Wray & Tymoigne (2008)
3.2 Minsky’s Theory of Investment

Borrowing from Keynes, Minsky distinguished between current output prices, determined as cost plus mark-up, set at a sustainable level that will generate profits and asset prices. To explain the price disparity, Minsky abandoned the notion of asset prices being bound to the cost of their production and instead assumed they are defined by the net present value of their anticipated cash flows. Depending on the state of economy, these expectations can vary wildly in the long term and thus produce a gap between the two price levels. Similarly, short term fluctuations are the result of "the subjective value placed upon the insurance against uncertainty embodied in quick money or cash".3 The alignment of the capital asset prices, influenced by long term expectations, and current output prices, influenced by short term expectations, is the main driving force behind investment – when the current output prices are higher than the capital asset prices, the propensity to invest increases and vice versa.

When trying to determine the level of investment, Minsky was met with another obstacle: current output and capital asset prices prices effectively represent supply and demand prices of capital only in a system where purchases of capital are financed purely out of internal funds. In the case of external financing, the supply price also includes additional costs in form of various financial fees – mainly the interest rate – represented by the upward slope in figure 3.1.

The demand price for new capital assets, directly obtained from the price of old capital assets, undergoes similar treatment due to "borrower’s risk" – firms have to provide sufficient cash flow in the future to finance the debt, while the amount of cash flow from internal funds depends on the state of the economy. Firms therefore require a specific margin of safety dependent on the expectations about future market development. This leads a decrease in demand price represented by the downward slope in figure 3.1. Investment is then carried out when the demand price exceeds the supply price of capital assets (including the effect of expectations and uncertainty).

---

3Minsky (1978)
Figure 3.1: Level of Investment

*Source: Minsky (1986). Stabilizing an Unstable Economy*

Figure 3.1 represents how expectations influence the level of investment. The supply and demand prices of capital are initially equal to the capital asset ($P_k$) and current output ($P_t$) prices. When the amount of available internal funds or "quasi rents" ($Q_n$) falls below the supply price of current assets ($I_t$), internal funds are no longer sufficient to finance new investments. The amount of investment financed by external funds (distance from $I_t$ to $I^*$) experiences the effects of increasingly more severe lender’s and borrower’s risks, which are additionally subject to expectations. Should the economy undergo a positive development, the need for safety margin is lessened and so are the risk effects, leading to a higher level of investment illustrated by $I_2$.

### 3.3 The Role of Profits

Although both Keynes and Kalecki shared the view that private investment determines private savings (and not vice versa), Keynes assumed that the relationship between savings and gross domestic product (GDP) is the result of a pre-determined multiplier of the investment. Kalecki instead proposed that the relationship between savings and GDP is determined by profits, the driving force of economy, and put together his famous equation: $^4$

$$W + \prod + T = C + G + I + NX$$  \hspace{1cm} (3.2)

where $W$ represents worker’s wages, $\prod$ the total profit of the economy – GDP, $T$ taxes, $C$ is consumption, $I$ the volume of investment and $NX$ denotes net export ($export - import$). This equation can be modified by subtracting

$^4$Kalecki (1971)
wages and taxes from each side and dividing consumption into consumption of capitalists \((C_C)\) and consumption of workers \((C_W)\) to obtain

\[
\Pi = C_C - S_W + I + DEF + NX \tag{3.3}
\]

where \(S_W\) denotes workers’ savings and \(DEF\) the government’s deficit. While this equation serves to formalize the relationship between various parts of a national balance sheet, it is not practical for use in economic models due to the complexity and exogenous nature of the individual variables. This lead Kalecki to define a simplified model under the following assumptions:

- There is no public sector
- There is no international trade
- Workers do not save

thus the equation is \(W + \Pi = C_W + C_C + I\) and since \(W = C_W\) then

\[
\Pi = C_C + I \tag{3.4}
\]

meaning that profits are equal to the sum of capitalists’ consumption and investment. As profits minus consumption are equal to savings, the equation can be rewritten to

\[
S = I \tag{3.5}
\]

This, combined with Kalecki’s belief that investment determines savings, begs an interesting question of causality – Kalecki notes ”in a sense, investment finances itself” (Kalecki, 1976). Minsky adopted Kalecki’s view of profits and incorporated it into his investment theory. He argued that investment today is the source of today’s profit and thus validates the investment decisions made yesterday. In the same manner, investment decisions made tomorrow decide the outcome of investment decisions made today. This gives a great deal of power to expectations – a pessimistic view of future leads to a lower volume of investment today, therefore producing profit below the level necessary to finance the past investments. This subsequently leads to increased levels of safety margins, decreasing the demand price of capital and therefore even less investment today, potentially forming a cycle leading the economy into a recession, although this can be avoided by a correct use of fiscal policy.
The equation shows that a deficit in profits can be balanced by government subsidies and increase of net export. Kalecki noted that: "It is the export surplus and the budget deficit which enable the capitalists to make profits over and above their own purchases of goods and services." (Kalecki, 1971)

3.4 Minsky's Model

Unlike most of his peers, who assumed that the market is always trying to reach an equilibrium, considering crises to be an unnatural state of the economy, Minsky considered instability to be a regular part of modern capitalist cycle. He argued that economic stability influences behavior of market participants, leading to an increase in risk appetite which promotes excessive speculation and, in turn, increases the instability of the system.

The Root of Systemic Fragility

The first period of Minsky’s cycle is defined by its stability due to the high risk aversion as a result of the previous financial crisis. Although the risk premiums are high and financial institutions are willing to fund only low-risk investments, the economy is experiencing growth with most investment projects succeeding. This does not go unnoticed by the banks and investors, who very soon acknowledge that “it pays to lever” (Minsky, 1982) and start decreasing their safety margins.

This decrease in risk aversion leads to an increase in investment, asset prices and speculation – the attempt to bet on the future direction and psychology of the market. The rising profits accelerate the growth of the economy, setting off a spiral of decreasing risk aversion, where, due to recent experiences, both borrowers and lenders share blind optimism that most investment projects will succeed. This investment boom demands an ever-growing amount of external finances, resulting in high debt-to-equity ratio of firms, increasing their susceptibility to high interest rates and therefore the fragility of the system.

---

5 Some influential economists, such as Robert Barro and Herschel Grossman, viewed the problem from the opposite angle, researching disequilibrium models – an approach completely different from Minsky’s.

6 Explained by figure 3.1

7 Keynes (1936)
The Change of Balance Leading to The Crisis

The low risk aversion and race for higher profits lead to the development of new, high yield, low liquidity financial products, increasing the demand for investment and simultaneously devaluing low yield, high liquidity financial assets, forcing an increase in interest rates. The reduced interest rate cover then turns investments that were originally conservatively funded into speculative ones and those that were speculative into "Ponzi." Minsky states that "speculative and Ponzi-finance units are vulnerable to changes in interest rates . . . increases in interest rates will raise cash-flow commitments without increasing prospective receipts" (Minsky 1986). These terms are used by Minsky to differentiates between levels of speculative positions depending on the firm’s ability to repay debt:

(a) **Hedge** financing represent firms that are capable of satisfying all their liabilities at any time. This liquidity protects them from most asset price and interest rate fluctuations.

(b) **Speculative** positions are adopted by firms that have some difficulty repaying a part of their liabilities, usually those due in the short term. They choose to refinance it by taking a loan while having a large enough cash flow to cover the interest payments, expecting to repay all debts in the future.

(c) **Ponzi** firms find themselves in a similar position as speculative ones, albeit with one important difference – they must borrow to meet current interest payments and are therefore continuously increasing their outstanding debt.

Ponzi financed firms, stuck in a borrowing loop with a growing debt, are forced to sell their assets to finance their interest payments, increasing the supply side of the market. Flooding the market with new assets eventually counterbalances the asset price growth caused by the investment boom, leaving the Ponzi firms with no way to pay the debt installments, serving as the foundation for the upcoming crisis.

---

8Keen (1995). Finance and economic breakdown: modeling Minsky’s "financial instability hypothesis"
9Wolfson (2002)
The Beginning of The Crisis

With such a fragile system, sustained purely by overly optimistic expectations, it is only a matter of time before a spark, such as a default of a large bank, causes a widespread change in these expectations, plunging the economy into a debt–deflation crisis as defined by Fisher:\textsuperscript{10} the falling expectations cause a rise of safety margins, decreasing capital asset prices and investment, forming a cycle of lowering expectations. As profits are bound to the amount of investment while the debt accumulated to obtain these assets is not, the continuous fall of investment causes a growing amount of firms to fall into insolvency. These defaults lead to a decline in aggregate demand, which reduces prices, increases the real value of debt commitments, and further accelerates the downward spiral.

The Aftermath of The Crisis

Minsky notes that the severity of the crisis is significantly influenced by the rate of inflation. With low rate of inflation, the economy does not manifest any self–correcting mechanism and finds itself in the position described in the previous section – indebted firms have no other choice but to either move continuously closer towards the Ponzi financing or go bankrupt, with both choices fueling the negative cycle of the recession.

With high rate of inflation the real value of debt falls, allowing the indebted firms to repay the debt accumulated during the investment phase. This possibility still results in an economic recession with low growth and high inflation but the amount of defaults and liquidity in the system is kept at a manageable level, avoiding the fall into a prolonged crisis. It is important to note that this does not mean that a period of prosperity is to follow. Minsky states that "the propensity to expand into a boom will be atrophied as bankers become wary of Ponzi schemes":\textsuperscript{11} soon giving a way to another repeat of the cycle.

While the process described in this section outlines the fundamental mechanisms behind the formation of crises, it is not completely realistic as it omits two important insitutions: large banks and strong government.\textsuperscript{12} For this reason, the full debt–deflation process, as Fisher had described it, has not been present in any United States crisis since the Great Depression. By serving as a lender of last resort, the Federal Reserve System (FED) has managed to sustain the

\textsuperscript{10}Fisher (1933)  
\textsuperscript{11}Minsky (1982)  
\textsuperscript{12}Wolfson (2002)
financial sector’s profits, preventing firms from slumping towards Ponzi financing. Minsky argued, however, that while government interventions through FED have the power to prevent the debt–deflation, the likelihood of crises can only be reduced by regulating the risk appetite and inevitable financial innovations that flood the market during the favorable times.\textsuperscript{13} Strong government interventions can have the opposite effect of validating wrong investment decisions – “once the doctrine of salvation through investment becomes deeply ingrained into our political and economic system, the constraints on foolish investments are relaxed”.\textsuperscript{14}

\textsuperscript{13}Wolfson (2002)
\textsuperscript{14}Minsky (1982)
Chapter 4

Modelling the FIH

4.1 Kaldor’s Model

To start modelling the FIH, we first need to define the model put forward by Nicolas Kaldor. Kaldor studied the evolution of production and capital formation, eventually building a model of national income and capital with nonlinear investment and saving functions. His model was later expanded into a four dimensional model involving not only the former variables but also price levels and inflation, providing us with an endogenous model capable of generating cyclic behaviour. The basic two–functional model has the following form:

\[
\dot{Y}(t) = \alpha [I(Y(t), K(t) - S(Y(t))] \tag{4.1}
\]

\[
\dot{K}(t) = I(Y(t), K(t) - \delta K(t) \tag{4.2}
\]

where \(Y(t)\) denotes real production, \(K(t)\) denotes capital, \(I(Y(t), K(t))\) labels investment which is an increasing function of production and is decreasing in capital. The savings function is an increasing function of production and is denoted as \(S(Y(t))\). The second equation describes capital formation which is given by the difference between investment and capital consumption \(K(t)\) where \(\delta\) denotes the rate of depreciation.

Let us assume that the investment function \(I\) is a product of two functions \(J(Y, K)\) and \(Y\). Then \(J(Y, K)\) is in fact an investment–production ratio frequently called the propensity to invest. It is common to assume that it is an increasing function of productivity of capital, therefore we have \(J(Y, K) = \)
4. Modelling the FIH

In our model, for convenience, we will use the logarithmic form and rewrite the original function \( J(Y, K) \) as \( J(Y/K) = J(e^{y-k}) = i(y - k) \), where \( y = \log(Y) \), \( K = \log(K) \). Due to this, the propensity to invest\(^1\) can be defined as:

\[
i(y(t) - k(t)) = \frac{ai_0}{bi_0 + (a - bi_0)e^{-a(y(t) - k(t))}} \quad (4.3)
\]

Under the assumption that \( iY = I \), the investment function is then

\[
I(Y(t), K(t)) = \frac{ai_0}{bi_0 + (a - bi_0)e^{-a(y(t) - k(t))}} Y(t) \quad (4.4)
\]

Similarly, the real savings function is defined as a product of the propensity to save and the production. The form of propensity to save is therefore

\[
s(\log(Y)) = s_0 + s_1 \log(Y(t)) = s_0 + s_1 y(t) \quad (4.5)
\]

And the real savings function is

\[
s(y(t)) = (s_0 + s_1 y(t)) Y(t) \quad (4.6)
\]

Under the previous assumptions the equations 4.1 and 4.2 can be modified to

\[
\dot{Y}(t) = \alpha [i(y(t) - k(t))Y(t) - s(y(t))Y(t)] \quad (4.7)
\]

\[
\dot{K}(t) = i(y(t) - k(t))Y(t) - \delta K(t) \quad (4.8)
\]

Dividing equation 4.7 by \( Y \) and 4.8 by \( K \) we get

\[
\dot{y}(t) = \alpha [i(y(t) - k(t)) - s(y(t))] \quad (4.9)
\]

\[
\dot{k}(t) = i(y(t) - k(t))e^{y(t) - k(t)} - \delta \quad (4.10)
\]

Finally substituting 4.3 and 4.5 into 4.9 and 4.10 and assuming \( \dot{y}(t) = \frac{\dot{Y}(t)}{Y(t)} \), \( \dot{k}(t) = \frac{\dot{K}(t)}{K(t)} \) we have

\[
\dot{y}(t) = \alpha [\frac{ai_0}{bi_0 + (a - bi_0)e^{-a(y(t) - k(t))}} - (s_0 + s_1 y(t))] \quad (4.11)
\]

---

\(^1\)Appendix A.1

\(^2\)Kodera & co. (2012)
4. Modelling the FIH

\[ \dot{k}(t) = \frac{a_i t_0}{b_i t_0 + (a - b_i t_0)e^{-a(y(t) - k(t))}}e^{b(t) - k(t)} - \delta \] (4.12)

The solution for this system of differential equations for \( a = 2, b = 8, s_0 = 0.2, s_1 = 0.05, \delta = 0.15 \) is illustrated on the following graphs:

**Figure 4.1:** The evolution of production \( y \) and capital \( k \)

Source: Author’s computations

**Figure 4.2:** The phase portrait of production \( y \) and capital \( k \)

Source: Author’s computations
4. Modelling the FIH

Figure 4.3: The phase portrait of production y and capital k with dimension of time (note that the units on the time axis are divided by 100)

Source: Author’s computations

This simple two-equation model demonstrates the innate cyclicity of economy as described by Kaldor’s growth facts, allowing us to observe non-linear and periodical behaviour of production and capital. Under the specified parameters, the system is stable and the peaks of both production and capital approximately coincide with each other. The fluctuations of production are marginally higher compared to the ones of capital and the dynamics of capital experience slower decay and comparatively faster growth whereas the cycles of production are relatively symmetrical.

4.2 Kaldor-Kalecki Model

In an attempt to analyse the investment process in a capitalist economy, Kalecki\(^3\) introduces a concept of a time-lag between investment decisions and investment output, as he believes that the present investment output is not the result of the present but the former investment decisions. He calls this time lag a ”gestation period”, which is ”the average time required for the completion of investment decisions”.

---

\(^3\)Kalecki (1937)
4. Modelling the FIH

We will introduce this time lag, denoted as $\theta$, into the model by modifying the equation for capital formation. Previously, the capital formation equation (equation 4.2) was assumed to respond to decisions made at time $t$, the time when both the investment decision was made and its effect was observed, the new capital equation will react to investment decisions made in time $t-\theta$. This leads to the following system of equations:

$$\dot{Y}(t) = \alpha[I(Y(t), K(t) - S(Y(t))] \quad (4.13)$$

$$\dot{K}(t) = I(Y(t - \theta), K(t - \theta) - \delta K(t) \quad (4.14)$$

Adopting the method of solution from the basic model we arrive at

$$\dot{y}(t) = \alpha\left[\frac{ai_0}{bi_0 + (a - bi_0)e^{-a(y(t)-k(t))} - (s_0 + s_1y(t))}\right] \quad (4.15)$$

$$\dot{k}(t) = \frac{ai_0}{bi_0 + (a - bi_0)e^{-a(y(t-\theta)-k(t-\theta))}e^{y(t-\theta)-k(t-\theta)}} - \delta \quad (4.16)$$

Following graphs illustrate the comparison of non-lagged and lagged Kaldor models.

![Figure 4.4: A Kaldor model with zero lag](image)

![Figure 4.5: A Kaldor-Kalecki model with a lag of $\theta = 8$](image)
4. Modelling the FIH

Figure 4.6: An attractor Kaldor model with zero lag

Figure 4.7: A Kaldor-Kalecki model with identical parameters and a lag of $\theta = 2$

In the two dimensional model the lag changes the dynamics of the production and capital growth while also significantly slowing down the business cycle. It does not induce any instability to a stable system by itself. On the contrary in the case of an attractor it stabilizes the system, hinting at the possibility that models assuming instant reaction to investment are overstating the volatility and instability of the system.

Despite its usefulness to determine the viability of self sufficient endogenous models based on Kaldor’s facts, this model is overly simplified to serve any real purpose beyond that as it omits the monetary sector. This issue will be addressed by extending the two-functional model to a four functional model.

4.3 Four Functional Kaldor-Kalecki Model

To add capital dynamics the two functional model of production and capital outlined above, it is possible to expand the model by price level and expected inflation equations\(^4\). First, let us assume that the velocity of money in the demand for money is an increasing function of expected inflation:

\[
M^d = \frac{1}{V(\pi)}PY
\]  

\(^4\)Kodera & co. (2013)
where $M^d$ stands for demand for money, $P$ is the price level, $V$ the velocity of money and $\pi$ denotes inflation. Taking the logarithm of the equation we get

$$m^d = p + y - v(\pi) \quad (4.18)$$

Logarithm of the velocity of money is assumed to be given by the following equation

$$v(\pi) = v_0 + \kappa \phi(\pi) \quad (4.19)$$

where constant $v_0$ is determined by a technological level of the banking sector. Parameter $\kappa$ is a constant and $\phi$ is a logistic function solving the logistic equation

$$\frac{d\phi(\pi)}{d\pi} = \phi(\pi) * (g - h\phi(\pi)) \quad (4.20)$$

where $g$ and $h$ are parameters. With an initial condition $\phi(0) = \phi_0$ we get a particular solution of the above differential equation

$$d\phi(\pi) = \frac{g\phi_0}{h\phi_0 + (g - h\phi_0)e^{-g\pi}} \quad (4.21)$$

The rate of inflation or an instant increment of the logarithm of price level $p$ is affected by the disequilibrium in money market equal the difference between the constant supply of money $m$, and the demand for money $m^d$, so we get

$$\dot{p} = \sigma(m - m^d) \quad (4.22)$$

where $\sigma > 0$ is an adjustment parameter and $p$ is the derivative of the logarithm of the price level with respect to time. Replacing $m^d$ from 4.18 and using 4.19, we get

$$\dot{p} = \sigma(m - p - y + v_0 + \kappa \phi(\pi)) \quad (4.23)$$

The instant increment of expected inflation $\dot{\pi}$ is then

$$\dot{\pi} = \omega(\dot{p} - \pi) \quad (4.24)$$

Finally we assume that propensity to invest is a subject to an arbitrary constant $\mu$:

$$i = \frac{ai_0 \mu}{bi_0 + (a - bi_0)e^{-a(y(t) - k(t))}} \quad (4.25)$$

Combining the simple two functional model with the newly derived capital dynamics sector yields a system of four differential equations formulating the
extended Kaldor-Kalecki model:

\[
\dot{y} = \alpha \left[ \frac{ai_0\mu}{bi_0 + (a - bi_0)e^{-\alpha(y(t) - k(t))}} - (s_0 + s_1y(t)) \right] \tag{4.26}
\]

\[
\dot{k}(t) = \frac{ai_0\mu}{bi_0 + (a - bi_0)e^{-\alpha(y(t) - k(t))}} e^{\sigma(y(t) - k(t))} - \delta \tag{4.27}
\]

\[
\dot{p}(t) = \sigma \left( m - p - y + v_0 + \kappa \frac{g\phi_0}{h\phi_0 + (g - h\phi_0)e^{-g\pi}} \right) \tag{4.28}
\]

\[
\dot{\pi}(t) = \omega \left( \sigma \left( m - p - y + v_0 + \kappa \frac{g\phi_0}{h\phi_0 + (g - h\phi_0)e^{-g\pi}} \right) - \pi \right) \tag{4.29}
\]

Solving the system with the following parameters \( \alpha = 35, a = 1, b = 1, i_0 = 18, s_0 = 0.2, s_1 = 0.05, \delta = 0.1, \sigma = 0.6, \omega = 0.8, m^d = 2, v_0 = 1, g = 1, h = 1, \phi_0 = 0.5, \kappa = 30, \theta = 0 \) yields the following graphical solution

![Graphical Solution](image)

Figure 4.8: Four functional Kaldor model

These parameters produce a very stable series of data depicting the cyclical nature of productivity, capital, price level and inflation assuming zero investment lag. Introducing a positive investment lag leads to a significant bifurcation in the model, creating periods of low market prices leading to an increase in propensity to invest and therefore decrease in capital and increase in productivity which persists until the capital reaches a breaking point. The impact of an investment delay is demonstrated by the following figure\(^5\):

\(^5\)Phase portraits better illustrating the bifurcation: Appendix, fig. A.2
4. Modelling the FIH

Figure 4.9: Four functional Kaldor-Kalecki model with an investment delay of $\theta = 30$

Despite the model’s ability to explain endogenous economic cycles, for the purpose of modelling Minsky’s instability hypothesis it has many shortcomings. It provides no explanation of risk or risk aversion, does not differentiate between external and internal financing and, most notably, does not take market expectations into consideration. For this purpose, the model requires to be further expanded to accommodate for these requirements, either by expanding the financial sector with a measure of debt level and a market expectations dummy variable or by switching to an adaptive expectations heterogeneous agent model (HAM).

4.4 Kaldor-Kalecki-Minsky Model

This section will attempt to further expand the base model with debt level and market expectations dynamics while simultaneously introducing a simplification in order to implement a more realistic scenario and prevent the model from being overly-complicated: the previously described equation for inflation will be supplanted by a simple cosine function, yielding very similar results while allowing for a simpler implementation of inflation as a depreciation multiplier:

$$\pi = \omega_1 \ast \cos(\eta \ast t) + \omega_2$$

The approach of expanding a general model with a level of debt was already utilized by Steve Keen in his paper titled A monetary Minsky model of the
Great Moderation and the Great Recession⁶ but, despite bringing new findings and contributing to economic theory, Keen’s model, based on Goodwin’s model, does not depend on expectations as the main driving force of the economic collapse, effectively omitting a central piece of Minsky’s theory. The following model, despite being less factually descriptive and taking many liberties, attempts to verify the possibility of instability of an otherwise stable market purely through the power of market expectations or more precisely, gradually decreasing risk prudence, resulting in an unsustainable spiral of increasing debt. The change in the level of debt is defined as the difference between actual investment level and investment level desired by the market, modified with depreciation and Ponzi effects. This is described by the following equation

\[
debt_t = i(y - k) \cdot \rho - \text{debt} \cdot \pi + ir \cdot \text{debt} \cdot (1 - \tau)
\]

where \( \rho \) stands for the desired amount of borrowings expressed as a fraction of internal money investment, \( \pi \) stands for inflation, \( ir \) is interest rate, defined as a sum of inflation and a constant real interest rate, and the parameter \( \tau \) describes the fraction of borrowers not capable of repaying their debt and resorting to Ponzi financing. These borrowers can only repay their interest by further borrowing, which leads to an increase of debt by the amount of unpaid interest, serving as one of the main forces behind the debt induced breakdown.

One of the reasons why modelling Minsky’s FIH is so problematic is the fact that modelling factors which are based on market psychology, such as \( \rho \) and \( \tau \), requires arbitrary equations for market reaction to the various states of the economy. In this work we will take inspiration from a paper named Investor Psychology and Security Market Under- and Overreactions written by Daniel Kent and co.⁷ to define an equation for financial prudence. Kent attributes price fluctuations mainly to two psychological regularities: overconfidence and attribution bias, citing DeBondt and Thaler⁸: "Perhaps the most robust finding in the psychology of judgment is that people are overconfident.". Additionally, evidence suggests that experts tend to be more overconfident than relatively inexperienced individuals. Psychological evidence also indicates that overconfidence is more severe for diffuse tasks, e.g. making diagnoses of illnesses, and more severe for tasks with delayed feedback as opposed to tasks that provide immediate and conclusive outcome.

---

⁶Keen (2013)  
⁷Kent (1998)  
⁸DeBondt & Thaler (1995)
In the simplest version of his approach, Kent assumes a static confidence level, leading to a model where the expected security prices, despite being initially over- or understated depending on private signals and noise, eventually asymptotically converge to their rational expected value, more precisely the prices satisfy the following conditions:

\[ P_1 = E_C[\theta|\theta + \epsilon] \]  
\[ P_2 = E_C[\theta|\theta + \epsilon, \theta + \eta] \]

where \( E_C \) denotes trader’s expectations based on his confidence, \( \theta \) is the riskiness of the security, \( \epsilon \) is the deviation of the price signal and \( \eta \) is noise. These conditions define equilibrium prices satisfying

\[ P_1 = \frac{\sigma^2_{\theta}}{\sigma^2_{\theta} + \sigma^2_C}(\theta + \epsilon) \]  
\[ P_2 = \frac{\sigma^2_{\theta}(\sigma^2_C + \sigma^2_p)}{D} \theta + \frac{\sigma_C \theta^2 \sigma^2_p}{D} \epsilon + \frac{\sigma_C \theta^2 \sigma^2_C}{D} \eta \]

\[ D = \sigma^2_{\theta}(\sigma^2_C + \sigma^2_p) + \sigma^2_C \sigma^2_p \]

A similar concept, albeit without any noise, will be introduced to model financial prudence in our model. The prudence or desireability of debt is modelled in the following manner

\[ \rho_t = 1 - prudence_t \]  
\[ prudence_t = \frac{debt_t \ast \xi}{|debt_t| + \text{elast.}} + \gamma \]

where \( \zeta \) is the multiplier of debt induced reaction, \( \text{elast.} \) is the reaction elasticity modifier and \( \gamma \) describes an overreaction to market risk. We assume that prudence is never negative. Furthermore, an equation for \( \tau \), the ratio of people capable of meeting their commitments, will be defined as

\[ \tau_t = \frac{\text{elast.}(\gamma-1)}{\text{debt} + \epsilon}, \quad \tau \in <0,1>, \quad \xi + \gamma \neq 1 \]

or, as the ratio of desired debt level over the actual outstanding debt level plus an arbitrary constant. The value of this variable should be close to 1 as long as the market behaves rationally and does not over-borrow due to leniency.
4. Modelling the FIH

4.4.1 Time Dependent Expectations

To examine the effects of various macroeconomic variables, first a model of gradually and exogenously decreasing expectations will be built. Revisiting the 4.26 to 4.29 Kaldor-Kalecki model equations, with the modification to inflation, in addition to the 4.30 debt equation and 4.35 to 4.37 market behavior equations produces the following behavioral Kaldor-Kalecki-Minsky model

\[ i_r = i_r + \pi \quad (4.38) \]

\[ \text{prudence}_t = \frac{\text{debt}_t \ast \xi}{|\text{debt}_t| + \text{elast.}} + \gamma \quad (4.39) \]

\[ \tau_t = \frac{\text{elast.} \ast (\gamma - 1)}{|\text{debt}| + \epsilon}, \quad \tau \in <0, 1>, \quad \xi + \gamma \neq 1 \quad (4.40) \]

\[ \rho = 1 - \text{prudence} \quad (4.41) \]

\[ \dot{\text{debt}}_t = \frac{a_0 \mu}{b_0 + (a - b_0) e^{-a(y(t) - k(t))}} - \text{debt} \ast \pi + i_r \ast \text{debt} \ast (1 - \tau) \quad (4.42) \]

\[ \dot{y} = \alpha \left[ \frac{a_0 \mu (1 + \rho)}{b_0 + (a - b_0) e^{-a(y(t) - k(t))}} - (s_0 + s_1 y(t)) \right] - i_r \ast \text{debt} \ast \tau \quad (4.43) \]

\[ \dot{k}(t) = \frac{a_0 \mu (1 + \rho)}{b_0 + (a - b_0) e^{-a(y(t - \theta) - k(t - \theta))}} e^{y(t - \theta) - k(t)} - \delta - i_r \ast \text{debt} \ast \tau \quad (4.44) \]

\[ \dot{\pi} = \sigma \left( m - p - y + v_0 + \kappa \frac{g_0}{h_0} + (g - h_0) e^{-y} \right) \quad (4.45) \]

\[ \pi = \omega_1 \ast \cos(\eta \ast t) + \omega_2 \quad (4.46) \]

In this model, the market prudence or wariness plays a major role in forcing the debt to asymptotically converge to a value equal to \( \frac{\text{elast.} \ast (\gamma - 1)}{|\text{debt}| + \epsilon} \), as any debt level above this bound will result in zero or even negative "over-investment", leading to a decrease in the outstanding debt. The model is stable under most conditions\(^9\) but becomes naturally unstable after introducing a gradual decrease either in market prudence or market "optimism" symbolized by increasing leniency, leading to borrowing even when the debt has already reached or exceeded its desirable level, and decreasing \( \tau \), symbolizing a level of debt that is unsustainable without Ponzi financing.

The simpler version of the model based on a gradual decrease in risk aversion over time can be simulated by introducing such a decrease into the equation of

\(^9\)Instability in an absence of exogenously induced time dependent decrease in prudence is the subject of the following section (4.4.2)
prudence, for example in the following way:

\[ prudence_t = \frac{debt_t \times \Xi}{|debt_t| + elast.} + \gamma - m_t \times t \quad (4.47) \]

where \( m_t \) stands for a time multiplier representing the severeness of decrease in prudence. Simulating the model with such a modification yields predictable results: the gradual decrease in prudence leads to unsustainable borrowing, promoting an increasing amount of Ponzi financing which in turn feeds the debt spiral creating an exponential debt growth over time illustrated by fig. 4.10\(^{10}\):

**Figure 4.10:** Exponential debt spiral given temporal decrease in prudence

![Exponential Debt Spiral](image)

*Source: Author’s simulation*

While this example is not very realistic due to significant simplifications such as the decrease in prudence being stable over multiple economic cycles, constant real interest rate or infinite external money supply, it nonetheless serves to demonstrate the theory behind Minsky’s FIH and some underlying mechanics. The increase in Ponzi financing is very disproportional to the decrease in prudence, leading to a state where the economy keeps providing similar or even improving results with higher producion over new cycles while actually being on the brink of collapse hidden behind a large mob of Ponzi borrowers. This model does not provide random shocks due to its endogenous nature but it is simple to imagine the large collapse resulting from a minute shock or dwindling external money supply no longer being able or willing to provide funds

\(^{10}\)A full page size figure is available in appendix (A.3)
for further Ponzi debt financing, leading to defaults of a significant share of debtors.

4.4.2 Adaptive Expectations

To answer the concern of lack of realism when simulating a temporal decrease in prudence, this section will introduce an alternate version of the model capable of simulating the behavior mentioned in FIH. The model’s original equation of prudence (4.39) will be left untouched. Instead, we will assume that there is always a specific amount of market participants that seek out risky borrowing opportunities or outright attempt to exploit the system. Considering such an assumption, the equation of the ratio of new debt to investment (4.39), represented by $\rho$ will be modified in the following manner

$$\rho = 1 - \text{prudence} + l$$  \hspace{1cm} (4.48)

where $l$ stands for leniency, symbolizing the constant amount of irrational over-borrowing which will, under specific circumstances, lead to the destabilization of an otherwise completely stable system. Nonetheless, simulating the model with a significant lending leniency is not guaranteed to lead to a debt spiral. On the contrary, especially in cases with high $\gamma$ and $\xi$, i.e. high natural prudence and reactivity to debt levels, or in cases with high inflation or low real interest rate, the system will remain stable indefinitely or at least over long periods of time. This corresponds to the economic theory developed by Irving Fished in his work titled The Debt-Deflation Theory of Great Depressions\textsuperscript{11}. In his theory, Fisher stated that "Thus over-investment and over-speculation are often important; but they would have far less serious results were they not conducted with borrowed money. That over-indebtedness may lend importance to over-investment or to over-speculation. The same is true as to over-confidence. I fancy that over-confidence seldom does any great harm except when, as, and if, it beguiles its victims into debt.", meaning that with a high enough market prudence, or risk aversion, keeping stability with otherwise rather high levels of debt remains possible.

\textsuperscript{11}Fisher (1933)
One of the major findings of simulating the model in this way is that inflation is a defining force in economy with a high level of debt, as can be seen on fig. 4.11. More precisely, if inflation is present then it is one of the main drivers of economic cycles due to its effect on the level of debt and, in some cases, can lead to a gradual stabilization and increase in performance of the whole system, as demonstrated by fig. A.4 in the appendix. In the absence of inflation, the opposite is true and debt level is purely defined by the natural cycles of the Kaldor-Kalecki model, slightly augmenting the performance of economy by increasing production compared to an economy with no debt.

Inflation plays another important role in the model by lessening the burden on debtors by depreciating the outstanding debt and allowing for more borrowing while keeping the model from devolving into a debt spiral. The importance of inflation or deflation in the creation of debt bubbles is another cornerstone in the theory of debt-deflation with Fisher noting that "When over-indebtedness stands alone, that is, does not lead to a fall of prices, in other words, when its tendency to do so is counteracted by inflationary forces (whether by accident or design), the resulting "cycle" will be far milder and far more regular.”\textsuperscript{12}

\textsuperscript{12}Fisher (1933)
Figures 4.13 and 4.14\textsuperscript{13} illustrate the impact of inflation on a market with a low risk aversion, simulated by a positive leniency towards debt level and a low value of $\xi$, faithfully reflecting the theory put forward by Irving Fisher. Taking the opposite approach and modelling a very rational and risk averse economy under the effect of deflation further confirms the idea of deflation serving as a primer for the creation of debt bubbles as shown on figure 4.15:

![Figure 4.15: A highly prudent deflationary economy](image1)

![Figure 4.16: Deflationary economy with low risk aversion](image2)

Figure 4.15 represents a system with zero lenience or overconfidence intending on keeping a low level of debt with very strict arguments of the behavioral equations (4.39 and 4.40). Despite that, the deflation leads to a decrease in the value of underlying assets or rather an increase in the level of debt. This level of debt is not sustainable purely by the internal funds of the economy, leading to a decrease in $\tau$, meaning an increase in Ponzi financing where the debt feeds itself through unsustainable interest. Despite that, the model remains stable as the high prudence of the market enforces under-investment in favor of partial debt repayment, eventually leading to a high enough investment levels capable of lowering the debt. On the contrary, figure 4.16 represents a market with lower risk aversion and higher overconfidence, leading to an exponential rise in debt and decrease in $\tau$. This markets soon becomes financed purely through recursive Ponzi loans, the burden eventually becoming unbearable to the lenders and resulting in a large financial crash. Despite the possibility of such an explosive instability, according to Minsky, such a situation rarely, if ever, comes to pass, due to what Minsky calls the "thwarting systems"\textsuperscript{14}.

\textsuperscript{13}Larger versions in appendix under A.5 and A.6
\textsuperscript{14}Minsky & Ferri (1991)
4.4.3 Thwarting Systems

Thwarting systems are represented by institutionary interventions that "thwart the instability breeding dynamics that are natural to market economies by interrupting the endogenous process and starting the economy again with non market determined values as initial conditions"\textsuperscript{15}. In this way, the thwarting systems prevent the economy from exploding. These mechanisms are not static in time and evolve according to the needs of the economy. Furthermore, Minsky states that "Policy agents and law makers need to interpret what is happening and need to understand how their actions can affect the behavior of endogenous agents and thus the economy. If the economy is endogenously unstable, then policy based upon the assumption that the economy is endogenously stable is likely to be inept.". Such policies can lead to further destabilization through more reckless behavior due to the belief that "they wont let it happen". To incorporate such behavior into his theory, Minsky introduces two theorems:

1. The Anti-Laissez Theorem The first theorem postulates the possibility of regulating the market through conventions, constraints and interventions, leading to results which could not be attained naturally by the market, such as forcefully adjusting and maintaining a specific level of inflation or imposing limitations and conditions on new loans. A specific example would be the Glass-Steagall legislation of 1933 preventing securities firms and investment banks from taking deposits and imposing many other limitations on Federal Reserve member banks. While such an approach deals with the immediate risk of economy going explosively out of control, introducing crude interventions to the system every time the economy exhibits an unfavorable performance is very likely to send the whole system into viscious oscillations. Furthermore, a situation

2. The Limitation Upon Performance Theorem The second theorem states that while the constant attempts to stabilize the economy are capable of keeping the values within a desirable range, they might not be able to influence the tendency of the economy to diverge from these values, leaving the stability of the system purely depending on these regulations. If we then consider the presence of an observation and implementation lags and less than perfect adjustments, such a system will never perform

\textsuperscript{15}Minsky & Ferri (1991)
at an optimal level, implying that "the practical best for an economy falls short of the abstract best". A solution would be to gradually modify the market rules and culture to soften the sources of market destabilizing influences.

Intuition suggests that a careful combination of both hard, interventive and soft, gradual solutions would be ideal to steer the market towards stability and lessen the damage caused by future crises. This choice does not only have an impact on the borrowing side but also influences the bankers who yearn for a mechanism that limits the exposure of clients to downside profit risks, potentially leading to lower interest rates but also, unless enforced otherwise, lower safety margins, influencing the system on many fronts.

This in turn raises the question of whether the financing efficiency gained through regulations offsets the allocational inefficiency and decreased growth resulting from such an environment or rather whether there is an arrangement which is optimal for a specific market for an extended period of time. A lot of recent research into this topic suggests that there is indeed no optimal policy system and that markets are condemned to repeatedly go through monetary policy revolutions, forming a policy cycle above the financial one. A 2015 IMF working paper titled Recognizing the Bias: Financial Cycles and Fiscal Policy\textsuperscript{16} describes a so-called "debt bias" or the tendency of debt to increase over the cycle, pointing out that the debt bias is larger for countries that have lower policy buffers, leading to a gradual increase in debt and decrease in performance. These cycles can also prove to be recursively spiralling downwards themselves, such as:

1. The government experiences rising deficits and missed targets leading to
2. Spending cuts and tax increases which
3. Slow down growth, increase unemployment and decrease profits resulting in
4. Falling tax revenue and rising transfer costs

leading, mainly as a byproduct of the third step, to an increase in the amount of Ponzi financing. The instability of a specific thwarting system framework can also be attributed to the eventual adaptation of the market participants, who gradually learn the effects of constraints, institutions, and interventions

\textsuperscript{16}Budina et al. (2015)
and modify their behavior accordingly as there are always incentives to evade and avoid them. This change in their behavior in turn modifies the systemic effect of the interventions, decreasing their effectiveness over time and eventually requiring an implementation of a new, different set of rules and interventions, starting the cycle anew. Unfortunately, modelling such a behavior seriously would require making use of machine learning and is therefore outside the scope of this thesis. Modelling specific thwarting systems and their short term impact, though, is still readily possible within the outlined model. Taking a simple unstable situation caused by high lending leniency will produce the basic testing environment upon which the results of various approaches can be shown. Such a model crisis with spiralling debt is shown on figure 4.17:

![Figure 4.17: Model crisis with high leniency](image)

This model experiences rising debt due to a sharp increase in Ponzi financing right at the start while lacking any control mechanisms to stop or at least control the borrowing cycle. Although the stability of this system is theoretically achievable purely through a change in agent behavior or market culture and yields far better results if achieved this way\[^{17}\], Minsky notes several times\[^{18}\] that such a change is very improbable as market agents typically care only about their own short term profit at all times and it is therefore essential to implement controlling mechanisms capable of neutralizing the causes of the debt spiral.

\[^{17}\]In the depicted model, stability is achievable through modifying the behavioral equation, e.g. by increasing prudence elasticity. Such a modification is depicted on figure A.7 in the appendix.

\[^{18}\]Minsky & Ferri (1991)
Lender of Last Resort

One such mechanism is implementing a stopgap measure in the form of an emergency LOLR with zero real or even nominal interest rate. The responsibility of being a LOLR is one of central bank’s most controversial roles. Citing a BIS Paper No 79 Re-thinking the lender of last resort\(^{19}\): *On the one hand, emergency liquidity assistance to financial institutions is a core responsibility of central banks. This is because of central banks’ unique ability to create liquid assets in the form of central bank reserves, their central position within the payment system and their macroeconomic stabilisation objective. On the other hand, central bank LOLR is seen as very risky, as it potentially creates moral hazard on a massive scale, exposes the central bank to large financial risks, and blurs the boundary with fiscal policy. Moreover, liquidity assistance to individual institutions is typically deeply unpopular, creating reputation risks.*

This unpopularity stems from the stigma attached to borrowing from discount windows and has a surprisingly large impact on the utilization of such an option, as shown during the 2007-2008 financial crisis. During the crisis, FED provided discount window loans to depository institutions against a wide range of collateral but the discount window was severely underused and was proving ineffective for stabilizing the economy due to this reluctance to borrow. This was eventually partially solved through FED’s change of strategy by introducing a crisis specific lending institution, the Term Auction Facility, and allowing lending even to non-bank financial institutions with the intention to decrease interest rates by decreasing liquidity risk premium.

While this approach substantially decreases the interest rates and allows for easier borrowing with the increased money supply which in turn helps the economy stabilize, it also resurfaces the problem of moral hazard, adverse selection and actual credibility and competence of the providing state institutions. The question stands: “Will central banks keep their promise to lend into a liquidity crisis? But also, will central banks lend when they shouldn’t, when the underlying problem is one of solvency? In other words, will they oversupply liquidity re-insurance by setting soft terms?”\(^{20}\)

---

\(^{19}\)BIS (2014)  
\(^{20}\)See footnote 1.
There are four main views\textsuperscript{21} on which path should the LOLR take:

1. **The free banking view**: abolish the central bank as a state LOLR.

2. **The Richmond FED view**: lend only through open market operations to the market as a whole and abolish the discount window.

3. **The New York FED view**: lend to anybody, solvent or insolvent, and sometimes on soft terms, where necessary to keep the credit system going.

4. **The Bagehot\textsuperscript{22} view**: lend freely to illiquid firms at high rate of interest but require a sufficient collateral to ensure solvency.

Out of these options, this thesis will consider only the approach outlined in the third point, the school of thought of the New York FED. To model the effect of LOLR the provides the option of a low interest rate credit to all market participants we will assume that all agents but most importantly agents succumbed to Ponzi financing will without exception prefer to acquire loans from this source as agents in this model do not care about their reputation but only about the final output. This can be interpreted as adding a condition to the model maintaining that when the debt reaches unbearable levels and the economy finds out it is already knee-deep in a crisis, it adopts measures outlined above, aiming to reduce the interest rates in hopes of preventing a debt spiral. The question of whether a simple reduction in interest rates is capable of controlling or at least lessening the impact of the crisis is answered by the following figure\textsuperscript{23}:

---

\textsuperscript{21}BIS (2014)  
\textsuperscript{22}Bagehot (1873)  
\textsuperscript{23}Larger version under A.8 in appendix
Intuitively, while the conditions under which this example was made are not very realistic – it is inconceivable that a majority of short term investors would switch to and receive the support of a LOLR – this fact does not diminish the explanatory value of such an approach. Furthermore, for the sake of highlighting the implications, the above depicted example is rather extreme with significant inflation and lending leniency. The result is a sharp increase in debt which soon becomes sustainable only though Ponzi financing. This spiral continues up to a point where the responsible LOLR is forced to intervene in order to keep the market from further spiralling away from equilibrium by introducing a zero rate real interest loans\(^{24}\). Assuming this service is widely adopted, it eliminates the increases in debt level caused by Ponzi financing by removing the need for further borrowing to pay the interest. Furthermore, the high prudence of the economy causes a part of the production to, instead of investment, go towards debt settlement which, coupled with a significant inflation, results in fast stabilization of the debt level.

However, such a solution has three major difficulties accompanying it. First, as can be seen on figure 4.18, the low level of debt causes the economy to significantly underperform compared to a less regulated environment, increasing the recovery time period after the crisis. Second, this situation falls under the cases mentioned by Minsky’s second thwarting theorem, the limitation upon performance theorem. While enforcing a lower interest rate indeed brought stability to the system, the system retained its tendency to escape from its equilibrium by overborrowing which results in periodical debt bubbles and oscillations in debtor solvency. Third, introducing a low interest rate policy may lead to a liquidity trap, eventually making any further monetary policies ineffective. While this third point is not a problem in our model, where we assume a positive inflation and do not implement a zero nominal rate but a zero real rate of interest, it is an important cause for concern when applying such a strategy in reality.

Nevertheless, despite all the mentioned issues, lowering the average interest rate proves to be a rather efficient method of stabilizing the market in case of an otherwise uncontrollable situation. On the other hand, it is important to consider that this solution can not, or rather should not, be in effect for an extended period of time and lifting it without using other means to stabilize the market first would only lead to the exact same situation which this method was adopted to prevent. It should therefore be used only temporarily in synergy with other stabilizing mechanisms to prevent or stop the propagation of a crisis.

\(^{24}\)In this case the interest rate is equal to inflation.
Inflation Targeting

Another method to consider when attempting to influence the course of a crisis is inflation targeting. This approach is exceedingly controversial among contemporary economists due its nature of a double edged sword and the fact that the large amount of unpredictable changes it brings affect the economy not only immediately but also in a long time horizon after the change with potentially disastrous results such as an earlier onset of a new crisis. The positives, on the other hand, are undeniable – higher inflation transfers wealth from creditors to debtors, which, as unfair as it may seem, remains preferable to overborrowed debtors outright defaulting on their loans. Moreover, higher inflation spurs production by lowering the incentive to save and thus increasing investment. Lastly, it also allows the implementation of lower or even negative nominal interest rates providing more control over the level of debt and amount of borrowing.

To simulate an ideal inflation targeting policy in our model, equation 4.46 will be modified in the following way

\[ \pi = \omega_1 \ast \cos(\eta \ast t) + \omega_1 + \omega_2 \ast (1 - \tau) \]  

(4.49)

While \( \tau \) represents the amount of solvent debtors, it is derived from comparing the actual level of outstanding debt to its perceived adequate level and thus measures over-indebtedness. To counter the over-indebtedness, policy makers resort to increasing the rate of inflation through means such as an increase in money supply or prices, which in turn increases the rate at which the debt depreciates. The following figures\(^{25}\) depict two possible situations that can occur in this scenario:

---

\(^{25}\)Larger versions under A.9 and A.11 in appendix.
The scenario depicted in figure 4.19 outlines the result of optimal inflation targeting in a system with a constant rate of overinvestment. In this case the outcome is unambiguously positive, resulting in a stable levels of debt and production significantly above the levels achieved by implementing a freely available LOLR in the previous subsection. Unfortunately, such a scenario is rather unrealistic for many reasons: the decision and implementation lags of policymakers are rather significant and relevant information is not available as readily or cleanly as in the model to achieve optimal targeting precision, the required level of adjustment scales with real interest rates\textsuperscript{26} which are even less predictable than level of debt and finally, as mentioned when discussing thwarting systems, market participants adjust their behavior in unforeseen ways when faced with policy interventions – should they become convinced that policymakers will tolerate the overborrowing behavior and adjust inflation accordingly to save the situation, the tendency to overborrow might increase.

This more realistic outcome, shown on figure 4.20, leads to a constant rise in required inflation to cope with the rising propensity to overinvest, resulting in a gradual rise in prices. Increasing the $\omega_2$ parameter in our equation leads to a decrease in the slope of debt but also a faster rise in prices with potentially severely negative consequences as shown during the 1960s and 1970s, when high inflation resulted in an unexpectedly fast rise in prices and wages, eventually doubling interest rates from 4 percent in 1962 to 8 percent in 1978. The natural reaction of public was then to increase their savings rate, further stifling the productivity of the economy.

Another potential problem with this approach is that in a real world economy with a significant amount of short term debt the power of inflation to reduce the overall debt is limited as short term debt will eventually be refinanced with higher interest rates, nullifying the effect of inflation, or requiring an even larger increase in inflation. The investors, having learned of the inflationary spiral, would take projected forecasts into consideration and rise the interest rates even further resulting in a vicious circle, unless paired with guaranteed low interest rates achievable through a LOLR or other policy means.

Paul Krugman, a laureate of a Nobel Prize in Economic Sciences, is one of the important proponents of this measure despite the large amount of economists who seem to disagree with him on this matter, e.g. Manuel Hinds who, in his

\textsuperscript{26}An increase in real interest rate once again throws the economy into a debt spiral unless accounted for. Such an adjustment is not linear and would be difficult to implement in reality. The situation is shown in figure A.10 in appendix.
book Economic Effects of Financial Crises, denies the possibility of inflation being an effective solution to crises putting forward the example of Argentina in the early 1980s implementing a similar inflationary strategy and consequently suffering a financial crisis after the real value of then existing stock of debts was wiped out by a sudden acceleration of inflation. Since the early 1980s, 179 financial institutions have failed while 19 have come under the management of the central bank. ... The country was demonetized, real interest rates rose over 50% in real terms and the average maturity of deposits dropped to less than seven days,”, resulting in a situation no better than before.

Under these circumstances, it is difficult to assume any such approach to be a panacea for financial crises, especially considering the incompetence of policymakers to deal with economic problems, whether unintentional or fuelled by political ambitions, as well as policymaking lags that limit the effectiveness of situational measures in such a highly dynamic system potentially full of various feedback loops and could even become sources of instability themselves as they become obsolete. To eliminate the problem of credibility and lagging performance, government would be required to stabilize the economy not discretely but continuously, through structural policies that directly manage investment projects, employment, asset pricing and other driving forces behind the economy. Specifically, Minsky envisioned the government assuming the role of an economic planner while not utilizing the socialist approach – instead of central planning, the government would restrict certain substantial spheres of activity from the hands of private enterprise. This approach sounds exceedingly similar to the implementation of the U.S. Banking Act of 1933, also known as the Glass-Steagall legislation, which led to the separation of commercial and investment banking while imposing many additional limitations such as preventing investment banks from taking deposits and commercial Federal Reserve member banks from dealing with securities.

### 4.5 Empirical Validity

Despite the fact that the model outlined in this thesis is purely theoretical, explaining mechanisms that may or may not be hidden deep beneath the surface of intricate machinery of a real world market, the findings and ramifications uncovered in this chapter, if valid, ought to have counterparts in a real economy.

---

27 Hinds (1988)
28 Robinson (1943)
4. Modelling the FIH

In order to examine their validity, this section will introduce a few hypotheses that will be checked against IRFs from a vector autoregression (VAR) model by utilizing historical US data.

These hypotheses are as follows:

1. **Inflation reduces debt and negatively impacts production**

   Thoroughly described in the Inflation Targeting section, high inflation transfers the debt burden from debtors to creditors, increasing the stability of our model. On the other hand, this limits the opportunities of entrepreneurs, leading to a decrease in performance which is potentially further amplified by an unintuitive increase in the rate of savings, as noted by Angus Deaton who proposed a disequilibrium model of aggregate demand in the presence of unanticipated inflation\(^{29}\), the basic premise being that consumers notice inflation through the increase in current prices and react by purchasing less with the intention to delay the purchase or seek out substitutes, unknowingly increasing the rate of savings.

2. **An increase in interest rates leads to an increase in debt level**

   As interest rates influence nearly every part of the economy, it is not simple to predict what effect will a rise in interest rates have. Higher interest rates will result in higher interest payments for loans with floating rates, leading to an increase in payments and overall more financial burden for debtors, potentially leading to an increase in Ponzi financing. Additionally, high interest rates will reduce the return on investment and opportunities for expansion, making market participants more apprehensive and cautious, leading to less borrowing but also a decrease in the price of assets as market participants show decreased appetite for investment, creating problems for private equity investors. Very low interest rates, on the other hand, as seen recently in the 2009-2016 period when the US prime rate was equal to 3.25%, can lead to overabundance of capital, creating extreme competition and decreasing the amount of successful ventures.

3. **An increase in risk premium leads to a decrease in growth rate of debt level**

   Not only is an increase in risk premium equal to an increase in general interest rates, it can also be considered an indicator of risk aversion and

\(^{29}\)Deaton (1977)
overall cautiousness of the market, with higher values intuitively leading to lower growth rates of debt and decrease in debt feedback cycle given the harsher conditions for Ponzi financing. While our model does not include independent risk premium dynamics, risk premium can be considered a reflection of prudency of the market.

4. **Debt positively influences production**

Similarly to the previous hypothesis, debt influences production both positively and negatively. The positive side of debt is that the borrowed funds are used to finance larger investments, stimulating the economy and producing more output. Conversely, this development also comes with a negative side of larger interest payments, meaning lower returns on investment and potentially more financial difficulties, possibly resulting in more defaults. The findings from the model lead us to believe that there is a level of debt that could be considered optimal for a specific market but as determining such a level for a real market is outside of the scope of this thesis, an assumption of positive influence of debt will be made instead.

5. **An increase in production leads to a decrease in relative level of debt**

In the model we consider the market participants to behave rationally and responsibly, keeping the debt at sustainable levels by "buying out the debt" should it overreach the optimal level. The higher the production is, the more resources there are available for such a buyout, resulting in lower relative debt levels in periods of high production. This assumption can be expected to not hold under unconventional circumstances, especially should the behavior of investors become highly irrational.

6. **Trade balance influences debt**

While the model in this thesis uses only a simplified Kalecki’s equation\(^{30}\) \(\Pi = I - S\), Kalecki also accounted for government sector by assuming that \(\Pi = I - S + DEF + NX\) where \(DEF\) is the government deficit and \(NX\) are net exports. Assuming that both this equation and hypothesis 5 hold, a positive trade balance should have a positive impact on production and a negative impact on the level of debt.

\(^{30}\)Kalecki (1971)
Verifying the hypotheses

To check the validity of these hypotheses, a simple VAR model with one lag due to annual frequency of the data will be used to generate specific IRFs. This model has the following form:

\[ x_t = \Gamma u_t + \phi x_{t-1} + w_t \]

where \( x_t \) is the matrix of variables, \( u_t = (1, t)' \) fits a constant and a trend and \( \Gamma \) and \( \phi \) are matrices of constants.

The data used consists of the following first differenced series: inflation, private debt, GNP, savings, prime rate, risk premium, trade balance and unemployment. All the data has been obtained from the Federal Reserve Economic Data (FRED) public online database and was further normalized to produce legible IRFs. As such, the results of IRFs cannot be taken literally on a percentual basis.

Hypothesis 1: Inflation reduces debt and negatively impacts production

Two outputs will be provided in this case to ascertain the results and limit external interference: a limited model with data series consisting of only private debt, GNP and inflation and a general model outlook with all the series. The IRFs from the limited model are shown\(^{31}\) on figure 4.21 and the general model on figure 4.22:

---

\(^{31}\)All IRFs are also included in the appendix.
The results do not significantly differ across the restricted and unrestricted models, both suggesting that inflation does indeed negatively influence the level of debt but, contrary to expectations, the effect of inflation on GNP, albeit negative in the results, is not statistically significant. In this case, GNP has only one influence that could be considered significant – the rate of savings:

Table 4.1: Results of the VAR model for GNP

|                | Estimate | Std. Error | t value | Pr(>|t|) |
|----------------|----------|------------|---------|----------|
| Trade.l1       | 0.09960  | 0.33448    | 0.298   | 0.7676   |
| Savings.l1     | -0.45783 | 0.26613    | -1.720  | 0.0940   |
| Prime.l1       | 0.38358  | 0.25438    | 1.508   | 0.1403   |
| GNP.l1         | 0.53075  | 0.40036    | 1.326   | 0.1933   |
| Inflation.l1   | -0.18117 | 0.18612    | -0.973  | 0.3368   |
| Private.l1     | -0.09956 | 0.06407    | -1.554  | 0.1290   |
| Unemployment.l1| 0.34783  | 0.33685    | 1.033   | 0.3087   |
| RiskPr.l1      | -0.73968 | 0.64736    | -1.143  | 0.2607   |
| const           | 0.32101  | 0.56251    | 0.571   | 0.5718   |
| trend           | 0.07792  | 0.03952    | 1.969   | 0.0567   |

---

Multiple R-Squared: 0.6023, Adjusted R-squared: 0.5028
F-statistic: 6.057 on 9 and 36 DF, p-value: 3.811e-05

Furthermore, figure 4.22 suggests that, as proposed by Deaton’s unanticipated inflation theory, inflation does indeed increase the rate of savings which in turn decreases GNP. Thus it can concluded that while the effects of inflation on GNP are rather complex and arguably negative, it leads to a decrease in the growth rate of private debt, slowing down the potential debt spiral.
**Hypothesis 2:** An increase in interest rates leads to an increase in debt  
**Hypothesis 3:** An increase in risk premium leads to a decrease in growth rate of debt

The results of the VAR model for debt shown in table 4.2 are in line with expectations for both of these hypotheses, although the results are not very statistically significant.

**Figure 4.23:** Interest rate to debt IRF  
**Figure 4.24:** Risk premium to debt IRF

| Estimate  | Std. Error  | t value | Pr(>|t|) |
|-----------|-------------|---------|----------|
| Trade.l1  | 0.004318    | 0.562205| 0.008    | 0.9939   |
| Savings.l1| -0.647431   | 0.447322| -1.447   | 0.1565   |
| Prime.l1  | 0.532532    | 0.427571| 1.245    | 0.2210   |
| GNP.l1    | 1.265363    | 0.672946| 1.880    | 0.0682   |
| Inflation.l1| -0.491332  | 0.312829| -1.571   | 0.1250   |
| Private.l1| 0.522976    | 0.107692| 4.856    | 2.33e-05 |
| Unemployment.l1| 0.294907 | 0.566180| 0.521    | 0.6056   |
| RiskPr.l1 | -0.799233   | 1.088096| -0.735   | 0.4674   |
| const     | -0.571296   | 0.945490| -0.604   | 0.5495   |
| trend     | -0.048647   | 0.066424| -0.732   | 0.4687   |

---

Multiple R-Squared: 0.8181, Adjusted R-squared: 0.7727  
F-statistic: 17.99 on 9 and 36 DF, p-value: 7.821e-11

This lack of statistical significance can largely be attributed to the early 80s depression which led to a massive inflation in the US while the level of private debt was affected to a significantly lesser extent. This is further amplified by the fact that while changes in inflation coincide with changes in debt rather accurately, risk premium seems to experience a time lag compared to inflation, shown on figure 4.25\(^{32}\) in the period until 1986, or even move in the opposite direction compared to inflation in the following period up to 2009.

\(^{32}\)Larger versions in appendix under A.12 and A.13
The correlation coefficient between risk premium and inflation is equal to 0.622 in the 1966-1985 period but subsequently drops to -0.302 for the period of 1985-2013. A plausible explanation was provided by a paper published by FED titled Inflation Risk Premium: Evidence from the TIPS Market\textsuperscript{33} that estimates the 10 year inflation risk premium to be mostly negative in the period between 2000 and 2009, varying between -16 and 10 basis points over the full sample, attributing it to a possible combination of illiquidity and deflation scare in addition to efficient FED policy which led to a decrease in risk premium volatility.

These findings hint at the possibility that while risk premium dynamics are capable of influencing the level of private debt as expected, they are not being utilized correctly as a record increase in private debt post 2000 was met with an overall decrease in risk premium, possibly becoming one of the main drivers of the following financial crisis.

**Hypothesis 4: Debt positively influences production**

Debt can be considered a double edged sword, the dosage making it either a remedy or a poison. Both the potential advantages of debt and catastrophic effects of over-indebtness have been sufficiently discussed in the previous sections, leaving us with the question whether and when the positives outweigh the negatives.

Contrary to the hypothesis, utilizing both the general and limited models leads to similar results that, empirically in the case of the US market, higher levels of private debt resulted in a lower GNP.

\textsuperscript{33}Grishchenko & Huang (2012)
These results are in line with findings published in a 2011 BIS working paper titled The Real Effects of Debt\textsuperscript{34} which concluded that "For government debt, the threshold is around 85\% of GDP. Our examination of other types of debt yields similar conclusions. When corporate debt goes beyond 90\% of GDP, it becomes a drag on growth. And for household debt, we report a threshold around 85\% of GDP, although the impact is very imprecisely estimated.". Attaining these values might have been realistic until 1990 but the following economic boom in the US led the economy in a completely opposite direction. While the federal debt remained at acceptable levels, at least until the financial crisis of 2007-2008 when it experienced similar growth, the distance between GNP and private debt grew continuously larger until it peaked at the time of the crisis.

Comparing figure 4.28 and figure 4.25 provides interesting insight into the origins of a large macroeconomic asset bubble or a "super-bubble" that could be considered the root of the 2007-2008 crisis – in 1989-1990 inflation took a significant plunge in the opposite direction compared to both risk premium and unemployment as a result of various global developments such as the gradual opening of the Chinese economy, the fall of the Soviet Union leading to positive labour supply shocks, the widespread decline in the bargaining strength of labour unions or innovations in the IT industry which led to an increase in productivity and reduced the overall pressure on price growth in many sectors\textsuperscript{35}. This impulse started a trend where public debt instead of following the development of GNP started rising significantly, likely further amplified by smaller asset bubbles such as the Japanese real estate and stock markets bubble between 1986-1991, the "dot-com" bubble during 1995-2000 and the 1997-2006 US housing bubble, all of which noticeably influenced the curve of private debt.

\textsuperscript{34}Cecchetti \textit{et al.} (2011)
\textsuperscript{35}Ramskogler (2014)
on figure 4.28. This private debt overhang, which itself could be considered a super-bubble, became an increasingly destabilizing factor until the "too big to fail" mindset was no longer sustainable, resulting in a large crisis. This was not the first historical case in modern economy either, as the Japanese asset price bubble experienced identical development before the following onset of economic recession, depicted on figure 4.29:

Figure 4.29: Japanese asset price bubble

Source: Government Debt Isn’t the Problem – Private Debt Is, The Atlantic (2014)

The difference being that after the Japanese private debt peaked, the decision made by the Bank of Japan to sharply raise interbank lending rates prevented private debt from further increasing. While the soundness of such a decision is highly debatable for it led to two decades devoid of any GDP growth, the decreasing ratio of private debt to GDP is a definite sign of the slowly increasing health of Japanese economy. The repercussions of not undertaking such a resolute healing process are yet to be seen in the case of the US economy.

The answer to the hypothesis question whether debt stimulates production or not is thus, unlike in the previous cases, rather straightforward: yes, borrowing stimulates economy but overborrowing, with the two decades following 1989 of US market developments serving as an example, does not. Overborrowing hurts the production not only directly, as concluded by Cecchetti et al. (2011), but also destabilizes the market as a whole and increases the severity of financial crises.
4. Modelling the FIH

Hypothesis 5: An increase in production leads to a decrease in relative level of debt

The approach taken with the model in this thesis becomes highly polarizing given the examples in the previous section. The assumption behind it being that market participants behave rationally, at least partially in the manner of being willing to sacrifice a part of their production to quell the overindebtedness in order to prevent a meteoric rise in the level of private debt. This is the exact approach unintentionally taken by the Japanese economy, which is in a stark contrast to recent developments in the US economy.

The stagnation experienced by Japan can be described crippling in the face of the current extremely competitive global market and could hardly be considered positive or desireable. Despite that, the result is a continuously decreasing debt to GDP ratio that could, should Japan keep it that way while dealing with the stagnation, lead to an unprecedently stable economy, unachievable by the opposite approach of ignoring rising levels private debt. While such an outcome is very unlikely, the problem of rising private debt will have to be globally addressed sooner or later.

Unlike the Japanese debt, the US private debt, after decreasing slightly immediately following the financial crisis, returned to its previous rise in 2013. Furthermore, the IRFs depicted on figure A.19 show that an increase in GNP results in a statistically significant increase in private debt. In this way, the intuitive assumption that borrowing has a positive impact on production and increase in production has a negative impact on the relative level of debt is completely reversed.

Figure 4.30: GNP to debt ratio IRF
Hypothesis 6: Trade balance influences debt

Keeping Kalecki’s equation\textsuperscript{36} in mind, the ever-growing demand for profit can be satisfied in five ways: by increasing productivity, by borrowing to directly increase investments, by modifying market expectations to decrease the rate of savings, by increasing government spending and finally by an increase in trade balance. The question is: can trade balance substitute debt? Does an increase in trade balance result in less borrowing and vice versa a decrease in trade balance result in more borrowing? The following IRFs suggest that a low trade balance encourages further borrowing, deepening the private debt overhang in accordance with the hypothesis:

Figure 4.31: Trade IRFs

This along with different approach to government spending, could be one of the possible explanations for the fast rate at which US private debt grows compared to Japan. Considering that the US government debt has nearly overtaken its GNP on figure 4.28, the United States may soon be facing another depression, whether small or large, should nothing be done with the dependence on import, high unemployment\textsuperscript{37}, high private debt and rising federal debt. Those are the prime ingredients in a recipe for disaster.

\textsuperscript{36}Π = I − S + DEF + NX

\textsuperscript{37}Not discussed in the text but implied by figure A.22 in the appendix.
Discussion

The direction this thesis has taken in modelling the FIH, while in many ways seemingly arbitrary, has not only produced results in accordance with the general economic theory but also with empirical data and can therefore be considered at least partially successful in the way of goals set by the thesis. Despite that, from the outset these goals could only be considered as a pebble on the road to more detailed modelling of Minsky’s FIH, as such an undertaking would require the use of both full-fledged HAMs and machine learning which are indispensable when attempting to delve deeper into the inner mechanisms of FIH. Nonetheless the findings inferred from the model provide an interesting insight into the mechanisms of private debt accumulation, pinpointing it as a definite source of instability and an imminent threat to long term performance.

The findings in the last section are especially relevant at the current historical crossroads in global economy. Private debt, if not the sole cause, is one of the leading issues when it comes to economical stability. Inflation, interest rates, trade balance, unemployment, risk aversion stemming from both socio-cultural background and government policies, low confidence in market, limitations imposed by legislation and more – those are all factors that contribute to market stability or lack thereof. The question is: do these factors influence market stability directly or is their influence cumulative over time in the form of snowballing levels of debt? There is no question about the fact that individual factors, coupled with impulses such as defaults of large corporations, can cause the creation and popping of temporary asset bubbles but do the following depressions reset the stability counter?

The results of this thesis, both theoretical, represented by the modelled output, and practical, inferred from the US data IRFs, point in the latter direction of market stability based solely on accumulation of debt. It may be unscrupulous to call this debt overhang a "super-bubble" but the existence and influence of this phenomenon cannot be denied. One could even go as far as to argue that the global trend of shorter business cycles can be attributed to
rising levels of private debt instead of advancements in information technology or globalization itself. Citing a statement released by Deutsche Bank: "The Western authorities "maxed" out on the benefits of this inflationary decline by pumping monetary and fiscal stimulus into their economies whenever they had an economic problem. Given the lack of inflationary pressures, they had a rare ability to do this without the normal subsequent price rises. So every business cycle threatening incident was dealt with using aggressive intervention. This led to more and more confidence in the ability of the authorities which, coupled with lower and lower interest rates, increased public and private leverage to previously unthinkable levels." additionally noting that further recession would follow in 2020 (The Economist 2011).

This leaves policymakers with two options: introduce drastic measures in order to deal with the skyrocketing private debt and risk causing a drawn out stagnation or attempt to maintain status quo while dealing with the ever rising instability of the market, potentially resulting in a far more damaging depression in the future. Nonetheless, the debt question will need to be resolved in the near future. To cite Vague (2014a): "What we need to do is remove some of the debt burden weighing down middle-income and low-income people. You can call it debt "restructuring" or you can call it (partial) debt forgiveness. Either way, it’s needed."

Even with the recent rise in interest in FIH, its research is nowhere near sufficient despite its relevance to the current global economical situation. A word-by-word implementation of Minsky’s theory through HAMs has not yet been attempted and, despite the repeated occurrence of ”debt overflow” in Japan and the US, not enough attention is being paid to the issue of unsustainable private debt growth and its impact on the instability of global markets. It is of paramount importance to focus further research in these directions in order to be able to better understand and deal with these issues to achieve a growing, stable economy.
Conclusion

This thesis has successfully achieved the primary goals it set out for itself, namely the construction of a theoretic, endogenous model of Minsky’s FIH, ascertaining the sources of instability in said model and comparing its results to a real world economy. The secondary goals of investment lag analysis and possible "Minsky moment" or breaking point of over-indebteness have unfortunately not been achieved due to computational difficulties as adding such features to the model after its finalization proved to be not feasible, requiring a rework of a significant part of the code, if possible at all.

The implemented model, despite providing only limited explanatory value and results that most experienced economists would consider to be obvious, does not aim to bring a revolution to the world of economic theory but rather an evolution, serving as stepping block, an inspiration for future aspirations in the field of research of market instability.

It also serves as a warning that, should the problem of rising private debt not be taken care of, the only direction in which the health of financial markets can move is downwards, into more instability, faster cycles and more noticeable depressions. While market reactions to various impulses are not identical, as proven by the cases of the US and Japanese markets, the economic response to over-indebteness is, if we are to believe historical developments. There is no guarantee that the market culture will not undergo a worldwide shift in the direction of completely ignoring debt or even adopting debt restructurialization, gradually transferring the debt burden to high-income population, partially or completely invalidating the findings in this thesis. But until that happens, the above findings lead to a simple conclusion: the source of instability is private debt and it needs to be taken care of as soon as possible.
Bibliography


BIS (2014): “Re-thinking the lender of last resort.” BIS Monetary and Economic Department.


Appendix A

Appendix

A.1 Chapter 3

Figure A.1: Propensity to invest as defined in Kaldor’s model, parameters: \( a=2, \ b=8, \ i_0=1/8 \)

Source: Author’s computations
A.2 Chapter 4

Figure A.2: A phase portrait comparison of four functional Kaldor-Kalecki models with lags of $\theta = 0$ (left) and $\theta = 30$ (right)

Figure A.4: Stabilization and improved performance due to inflation
Figure A.3: Exponential debt spiral given temporal decrease in prudence
Figure A.5: A highly prudent deflationary economy

Source: Author’s computations

Figure A.6: Deflationary economy with low risk aversion

Source: Author’s computations
Figure A.7: Stabilization achieved through a change in market culture

Figure A.9: Fig. 4.17 model with inflation targeting and a constant rate of overinvestment
Figure A.8: Model crisis with an intervention of a LOLR
Figure A.10: Model identical to A.9 but with a higher real interest rate

Figure A.11: Inflation targeting in a system with an increasing rate of overinvestment
Figure A.12: 80s depression in the US

Figure A.13: A.12 first differenced
Figure A.14: Private debt to GNP

Orthogonal Impulse Response from Private

95% Bootstrap CI, 100 runs

Figure A.15: Debt/GNP comparison
A. Appendix

A.3 Impulse Response Functions

Figure A.16: Trade IRFs

Orthogonal Impulse Response from Trade

Figure A.17: Savings IRFs

Orthogonal Impulse Response from Savings
Figure A.18: Prime rate IRFs

Orthogonal Impulse Response from Prime

Figure A.19: GNP IRFs

Orthogonal Impulse Response from GNP
Figure A.20: Inflation IRFs

Orthogonal Impulse Response from Inflation

Figure A.21: Private debt IRFs

Orthogonal Impulse Response from Private
Figure A.22: Unemployment rate IRFs

Orthogonal Impulse Response from Unemployment

Figure A.23: Risk premium IRFs

Orthogonal Impulse Response from RiskPr