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Analysis of Profitability of Major World Lotteries

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Study programme: Mathematics
Study branch: General Mathematics

Prague 2017
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In Prague, May 17, 2017

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Abstract: Lottery tickets cost the same for every given jackpot, which might present an opportunity to make a profitable bet for very high jackpots. This work analyses whether buying a lottery ticket might be profitable in the mean value, for a given number of tickets sold, for four major American and European lotteries: Mega Millions, Powerball, EuroJackpot, Euro Millions. A regression of the sales on the jackpot is carried out for the American lotteries to find out whether some combination of the jackpot and the tickets sold, which was determined to be profitable, can be expected to happen.

Keywords: lottery, random ticket, syndicate, regression
I would like to thank Jan Večer for inspiring me to conduct a research on the topic of lotteries, advising me during my work on the thesis and providing me the support needed.
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Introduction

Lotteries are one of the most unfair games. Since the companies running a lottery must make a profit, it is clear that in general, betting in a lottery can never be profitable in the mean value. What presents an opportunity is that the price of a lottery ticket is constant, while the jackpot varies, which implies that when the jackpot is high enough, buying a lottery ticket is profitable for a given amount of sold tickets in expectation. On the other hand, when the jackpot rises, it attracts more people to bet and makes the risk of sharing the jackpot prize with someone higher, which decreases the expected value of the ticket.

This thesis theoretically analyses 4 of the largest lotteries in the world: Mega Millions, Powerball, EuroJackpot, Euro Millions. I compute the minimal jackpot for a given amount of random tickets sold while considering two scenarios: buying a single ticket, the syndicate option (buying all the combinations).

For the American lotteries (Powerball, Mega Millions), where data are easily available, I build a regression model consisting of sales and the jackpot to find out how many people bet given the value of the jackpot.

Finally, combining the result of the computation and the regression, we can find out, whether there could be an opportunity to make a profit by betting in the lottery.

I assume that all tickets bought are random. This assumption should be almost satisfied in the real life, because tickets are bought by a large number of individuals, which makes the set of bought combinations similar to the obtained by random sampling. The main assumption is that there is no one, who buys a substantial amount of non-random tickets (no identical combinations).
1. American Lotteries

I focus on two major American lotteries: Mega Millions and Powerball. Unlike in many European countries, American government imposes tax on lottery winnings, which greatly influences the profitability of buying a ticket and must be counted with. The tax is different for residents and non-residents. For non-residents, the federal tax rate is 30% from the taxable income, which is \( \text{prize} - \text{cost} \), where the cost is the cost of buying one or more lottery tickets. Although the tax rate is lower than the rate in the highest bracket for residents, it is hard to buy a ticket as a foreigner. The ticket must be bought by someone personally and you must have the physical copy of the ticket. The service of buying a ticket for you is provided for example by thelotter.com, but the price of tickets is about 2 times higher.

For US residents, there are 2 types of taxes, federal and state-specific. There are states with no lottery tax and it can go up to almost 10%. For example there is no lottery tax in Pennsylvania or California. The federal tax is progressive and paid according to the following table:

<table>
<thead>
<tr>
<th>Taxable Income</th>
<th>Tax Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0—$9,275</td>
<td>10%</td>
</tr>
<tr>
<td>$9,276—$37,650</td>
<td>$927.50 plus 15% of the amount over $9,275</td>
</tr>
<tr>
<td>$37,651—$91,150</td>
<td>$5,183.75 plus 25% of the amount over $37,650</td>
</tr>
<tr>
<td>$91,151—$190,150</td>
<td>$18,558.75 plus 28% of the amount over $91,150</td>
</tr>
<tr>
<td>$190,151—$413,350</td>
<td>$46,278.75 plus 33% of the amount over $190,150</td>
</tr>
<tr>
<td>$413,351—$415,050</td>
<td>$119,934.75 plus 35% of the amount over $413,350</td>
</tr>
<tr>
<td>$415,051 or more</td>
<td>$120,529.75 plus 39.6% of the amount over $415,050</td>
</tr>
</tbody>
</table>

I assume no state tax for the calculations. I also assume that the outcome \( \text{prize} - \text{cost} \) equal to the taxable income, no tax deductions will be considered. Other income is not considered, which means it falls into tax deductions or it is taxed after.

1.1 Mega Millions

Mega Millions is a lottery game, where 6 numbers are drawn in total. The first 5 are from a field of 75 numbers, ranging from 1 to 75, the last one is from 15 numbers, from 1 to 15. The ticket cost is 1$. Prizes are stated in the following table:
Table 1.2: Mega Millions Prizes

<table>
<thead>
<tr>
<th>#of matching numbers (from first field, second field)</th>
<th>Category Prize</th>
<th>Prize</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,1</td>
<td>Jackpot</td>
<td>Jackpot</td>
</tr>
<tr>
<td>5,0</td>
<td>Second</td>
<td>$1,000,000</td>
</tr>
<tr>
<td>4,1</td>
<td>Third</td>
<td>$5,000</td>
</tr>
<tr>
<td>4,0</td>
<td>Fourth</td>
<td>$5,000</td>
</tr>
<tr>
<td>3,1</td>
<td>Fifth</td>
<td>$50</td>
</tr>
<tr>
<td>3,0</td>
<td>Sixth</td>
<td>$5</td>
</tr>
<tr>
<td>2,1</td>
<td>Seventh</td>
<td>$5</td>
</tr>
<tr>
<td>1,1</td>
<td>Eighth</td>
<td>$2</td>
</tr>
<tr>
<td>0,1</td>
<td>Ninth</td>
<td>$1</td>
</tr>
</tbody>
</table>

Jackpot prize is divided evenly among all jackpot winners, other prizes are paid to all winners in full amount in all states but California. I will suppose that the full amount is paid to all winners.

There is also an option to buy Megaplier, which multiplies all other prizes but the jackpot (set prizes) by the megaplier number, which is drawn from the set of numbers: 2, 2, 3, 3, 3, 3, 4, 4, 4, 5, 5, 5, 5, 5 and 5. The cost of Megaplier is another $1.

Table 1.3: Megaplier odds

<table>
<thead>
<tr>
<th>Megaplier</th>
<th>Odds</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1:7.5</td>
</tr>
<tr>
<td>3</td>
<td>1:3.75</td>
</tr>
<tr>
<td>4</td>
<td>1:5</td>
</tr>
<tr>
<td>5</td>
<td>1:2.5</td>
</tr>
</tbody>
</table>

The jackpot prize has 2 options: cash and annuity. The lump sum option (single cash payment) is now around 60% of the advertised jackpot. The annuity is an option of 30 payments, one right after winning and 29 more payments each year, increasing by 5% each year with the total sum of the jackpot. Which one to choose?

I carry out a calculation for the jackpot from Friday, Jan 27, 2017. The annuity option was $191 million and the cash option was $114.1 million.

First to determine the annuity: let $a$ denote the annuity

$$
\sum_{i=0}^{29} (1.05)^i a = 191,000,000
$$

Solving the equation for $a$ gives us $a = 2,874,820$. Now I will determine the internal rate of return, compared to the cash option.

$$
\sum_{i=0}^{29} \frac{(1.05)^i a}{(1 + r)^i} = 114,100,000
$$
Solving the equation for $r$ gives us $r \approx 0.0308$. In addition, choosing the annuity option lowers the tax responsibility, because we pay the lower tax rate up to $415050$, according to the tax table 1.1 the difference is:

$$415050 \cdot 0.396 - 120529.75 = 43830.05$$

So every year after the first one, 43830.05 dollars saved on taxes $\Rightarrow 1,271,071.45$ dollars in 29 years. So comparing after-tax amounts with the internal rate of return: (I will not subtract the $1$ ticket cost for comparison, it is not taxed in both options in the first time period right after winning and both fall into the highest tax rate)

**cash option:**

$$114100000 - 120529.75 - (114100000 - 415050) \cdot 0.396 = 68,960,230.05$$

**annuity:**

$$\sum_{i=0}^{29} \frac{(1.05)^i a - 120529.75 - ((1.05)^i a - 415050) \cdot 0.396}{(1 + r)^i} = 68960230.05$$

Solving for $r$ gives us $r \approx 0.0316$. The IRR is high enough to make the annuity to be a reasonable choice. For my computation, I consider taking the cash option, which means I count how much the jackpot cash option has to be so the mean value of prizes equals the price of the ticket. This assumption is neutral, because everyone could have different demands on the IRR and the cash option makes the cost and the prize comparable.

### 1.1.1 Single Ticket

Let $P$ denote the amount won, $S$ the amount of set prizes and $R$ the jackpot prize. $E$ denotes mean value. $P = S + R \Rightarrow E\ P = E\ S + E\ R$.

Let us compute the $E\ S$:

First, the prizes have to be taxed. If we look at the tables of prizes and the tax rate, the only prizes to be taxed at a higher rate than 10% are the jackpot and the second prize. So after-tax second prize $= 1000000 - 120529.75 - (999999 - 415050) \cdot 0.396 = 647830$ (only $999999$ is taxed, because the cost of the ticket is $1$). Other prizes are taxed at the rate of 10%. (50 is taxed in the way that: $50 - (50 - 1) \cdot 0.1$ etc.)

$$E\ S = \frac{(14)}{1} \cdot 647830 + \frac{(70)}{1} \frac{(5)}{4} \cdot 4500.1 + \frac{(70)}{1} \frac{(5)}{4} \frac{(14)}{1} \cdot 450.1 + \frac{(70)}{2} \frac{(5)}{3} \cdot 45.1$$

$$+ \frac{(70)}{2} \frac{(5)}{3} \frac{(14)}{1} \cdot 4.6 + \frac{(70)}{1} \frac{(5)}{2} \frac{(14)}{1} \cdot 4.6 + \frac{(70)}{1} \frac{(5)}{2} \frac{(15)}{1} \cdot 1.9 + \frac{(70)}{1} \frac{(5)}{2} \frac{(15)}{1} \cdot 1 = 0.149971$$
We can now decide whether the Megaplier option is profitable in the mean value. Even if the Megaplier number was 5 all the time, which means 4 times the value of a normal ticket added by buying Megaplier, it would not be profitable in the mean value \((4 \cdot 0.149971 < 1)\). The real expected prize is even lower, because of the progressive tax.

Let \(N\) be the number of tickets. To compute \(ER\), I will compute the total expected jackpot for all tickets and divide it by \(N\) to get \(ER\). Let \(p\) denote the probability of winning the jackpot.

\[
p = \frac{1}{\binom{75}{5} \binom{1}{1}} = \frac{1}{258890850}
\]

Now let us count the bonus of not taxing the whole prize at the rate of 39.6% when \(prize > 415050\) and denote it \(B\), taxing a prize \(A\) (1 is the cost of the ticket):

\[
A - 120,529.75 - (A - 1 - 415050) \cdot 39.6 = A(1 - 0.396) + 43830.446 \\
\Rightarrow B = 43830.446
\]

The probability that nobody wins the jackpot is \((1 - p)^N\) so the probability of somebody winning the jackpot is \(1 - (1 - p)^N\). The amount contributed to the jackpot is 32.577% of sales (according to Mega Millions rules [3]), but minimum is $5 million. Let \(J\) denote the last jackpot amount, the new jackpot is:

\[
max(J + 5 \cdot 10^6 \cdot 0.61, J + N \cdot 0.32577 \cdot 0.61)
\]

The 0.61 is a constant to calculate the cash option from the annuity option. So cash option = 0.61 · annuity This is according to jackpot reports [1] and empirical data on the cash option and the annuity option. The real ratio may differ a little, but I will suppose 0.61 to be the ratio for calculations. The real ratio is determined by the securities price.

Moreover the amount won by all players depend on the number of winners, for every winner, a tax bonus \(B\) is added, because they pay the tax separately (I suppose every ticket owner to have only one ticket, same situation that I assume - buying one ticket - so I can compute the expected value this way). Let \(W\) represent the number of winners, \(W \sim Bi(N, p)\), because I suppose \(N\) sold tickets, each with probability of winning \(p\), then summing for all the tickets gives us \(W\). Then the jackpot after tax is:

\[
max(J + 5 \cdot 10^6 \cdot 0.61, J + N \cdot 0.32577 \cdot 0.61) \cdot (1 - 0.396) + W \cdot B
\]

\[
ER = \frac{(1 - (1 - p)^N) \cdot (max(J + 5 \cdot 10^6 \cdot 0.61, J + N \cdot 0.19872) \cdot 0.604 + EW \cdot B)}{N}
\]
Theorem 1. Under the assumptions stated above, the minimum jackpot for Mega Millions single ticket to be profitable is a function of $N$ and for $N \leq 15, 348, 252$:

$$J_{\text{min}} = \frac{0.844813 \cdot N - (N \cdot p \cdot B)(1 - (1 - p)^N)}{(1 - (1 - p)^N) \cdot 0.604} - 3050000$$

For $N > 15, 348, 252$:

$$J_{\text{min}} = \frac{0.850029 \cdot N - (N \cdot p \cdot B)(1 - (1 - p)^N)}{(1 - (1 - p)^N) \cdot 0.604} - 0.19872 \cdot N$$

Proof. Let us find the value $J$ so the $E P = 1$ (cost of the ticket). Then for every greater jackpot, the ticket will be profitable.

$$0.149971 + ER = 1$$

$$(1 - (1 - p)^N) \cdot (\max(J + 5 \cdot 10^6 \cdot 0.61, J + N \cdot 0.19872) \cdot 0.604 + N \cdot p \cdot B) = 0.850029 \cdot N$$

The formulation of the amount won by all the winners is not exact, because the tax will be even lower then the fraction of the jackpot for each winner will be less than $415050$, so the tax would be lower. But the minimum jackpot we would need for the ticket to be profitable in the mean value will be at least 300 million and the number of winners would have to be more than 600. But the probability that there will be more than 100 winners if we consider a billion tickets is:

$$1 - \sum_{k=0}^{100} \binom{10^9}{k} p^k (1 - p)^k = 0$$

with precision to 20 effective numbers. So no exactness is lost.

For $N \leq \frac{5 \cdot 10^6}{0.32577} \doteq 15, 348, 252$:

$$J_{\text{min}} = \frac{0.850029 \cdot N - (N \cdot p \cdot B)(1 - (1 - p)^N)}{(1 - (1 - p)^N) \cdot 0.604} - 5 \cdot 10^6 \cdot 0.61 = \frac{0.844813 \cdot N - (N \cdot p \cdot B)(1 - (1 - p)^N)}{(1 - (1 - p)^N) \cdot 0.604} - 3050000$$

For $N > 15, 348, 252$:

$$J_{\text{min}} = \frac{0.850029 \cdot N - (N \cdot p \cdot B)(1 - (1 - p)^N)}{(1 - (1 - p)^N) \cdot 0.604} - 0.19872 \cdot N$$

$\square$
If we consider buying multiple random tickets, the expected value will differ from just the number of tickets times the expected value of a single ticket. There are two main factors: first is the fact, that the tax is progressive, which would make the expected value lower. Second is the fact, that buying more tickets would lower my tax responsibility by the cost of the tickets). In conclusion, the effect of lowering the tax responsibility should be greater, but the difference would be really small. Also, there is no reason to buy multiple random tickets, when I can buy multiple non-random tickets, which will be considered in the next section.

### 1.1.2 Syndicate

Now let us consider buying all possible combinations. This should give me advantage over the random tickets, because I will not buy any combination twice, which gives me no additional value for the jackpot price, due to the prize sharing.

First to compute the costs: there are 258,890,850 combinations, so the cost is $258,890,850.

The set prizes equal to the expected value, because all the combinations are bought. Let $S$ again represent the set values, but now the prizes are before
taxing, because we do not know the total amount won/lost:

\[
E S = S = \binom{14}{1} \cdot 1000000 + \binom{70}{1} \binom{5}{4} \cdot 5000 + \binom{70}{1} \binom{5}{4} \binom{14}{1} \cdot 500 + \binom{70}{2} \binom{5}{3} \cdot 50 + \binom{70}{2} \binom{5}{3} \binom{14}{1} \cdot 5 + \binom{70}{3} \binom{5}{2} \cdot 5 + \binom{70}{3} \binom{5}{2} \binom{14}{1} \cdot 2 + \binom{70}{4} \binom{5}{1} \cdot 2 + \binom{70}{5} \cdot 1
\]

\[= 45,106,964\]

Now we can just assume the cost to be \(258890850 - 45106964 = 213,783,886\) and no set prizes, because the set prizes are won every time. Let \(C\) represent the cost. Again, the constant 0.19872 represents the amount added to the jackpot with every additional ticket, so with buying all the combinations, \(258890850 \cdot 0.19872 = 51,446,789\) is added to the jackpot. So now we know, that the option of just adding the 5 million to the annuity jackpot will never happen, because it happens only when the jackpot increase from ticket sales is less than 5 million.

To compute the minimal jackpot so that the mean value is zero, I will use following functions: First, defining the tax rate function \(f\)

\[
f(x) : = \begin{cases} 
0, & x < 0 \\
0.1, & 9275 \geq x \geq 0 \\
0.15, & 37650 \geq x > 9275 \\
0.25, & 91150 \geq x > 37650 \\
0.28, & 190150 \geq x > 91150 \\
0.33, & 413350 \geq x > 190150 \\
0.35, & 415050 \geq x > 413350 \\
0.396, & x > 415050 
\end{cases}
\]

Next, defining the function \(g\), which is the fixed amount paid before taxing everything more than certain amount by the appropriate tax rate:

\[
g(x) : = \begin{cases} 
0, & x < 0 \\
9275 \geq x \geq 0 \\
37650 \geq x > 9275 \\
91150 \geq x > 37650 \\
190150 \geq x > 91150 \\
413350 \geq x > 190150 \\
415050 \geq x > 413350 \\
x > 415050 
\end{cases}
\]

Finally, defining the function \(h\), which represents the tax, which I do not have to
pay if I would tax the whole amount by the appropriate tax rate.

\[ h(x) : \begin{align*}
  & = 0, \quad x < 0 \\
  & = 0, \quad 9275 \geq x \geq 0 \\
  & = 9275 \cdot f(x) - g(x) = 463.75, \quad 37650 \geq x > 9275 \\
  & = 37650 \cdot f(x) - g(x) = 4228.75, \quad 91150 \geq x > 37650 \\
  & = 91150 \cdot f(x) - g(x) = 6963.25, \quad 190150 \geq x > 91150 \\
  & = 190150 \cdot f(x) - g(x) = 16470.8, \quad 413350 \geq x > 190150 \\
  & = 413350 \cdot f(x) - g(x) = 24737.8, \quad 415050 \geq x > 413350 \\
  & = 415050 \cdot f(x) - g(x) = 43830.1, \quad x > 415050
\]

Now, the function

\[ x \cdot (1 - f(x)) + h(x) \]

represents the function of taxing, when \( x \) is your income (prize - cost). The function is continuous and increasing.

Let \( W \) again denote the number of winners, now let it denote the number of other winners (aside me, which is certain), \( W \sim Bi(N, p) \), where \( N \) denotes the number of other tickets. Because we consider \( N \) large (for real data) and \( p \) is very small, I approximate \( W \) by Poisson distribution \( Bi(N, p) \approx Po(N \cdot p) \). So now I consider \( W \sim Po(N \cdot p) \). For the computation of the mean value (like in the single ticket case), the probability of more than 100 random tickets to win is negligible, so I approximate the infinite sum with a partial sum to 100.

We know that 51446789 is added to the previous jackpot for all the combinations bought and \( 0.19872 \cdot N \) is added for other tickets bought, for simpler notation, let \( J^+ \) represent \( 0.19872 \cdot N + J + 51446789 \).

**Theorem 2.** Under the assumptions stated above and \( N > 0 \), the minimal prof-
**itable jackpot for Mega Millions syndicate must satisfy following equation:**

\[
\sum_{i=0}^{100} (P(W = i) \cdot \left( \frac{J^+}{i+1} - C \right) \left( 1 - f \left( \frac{J^+}{i+1} - C \right) \right) + h \left( \frac{J^+}{i+1} - C \right) = 0
\]

where \( J^+ = 0.19872 \cdot N + J_{\text{min}} + 51446789. \)

**Proof.** We want the mean value to be equal to zero and using the computation and consideration above, we get the equation.

There is no easy way to express \( J \), so the equation is solved numerically for \( N = Z \cdot 10^6, Z \in \{1, 2, ..., 500\} \).

For \( N = 0 \), the equation does not make sense, there are no other tickets, so minimal \( J \) must satisfy:

\[
(J^+ - C)(1 - f(J^+ - C)) + h(J^+ - C) = 0
\]

And we know that we have 0 after-tax income if and only if we have 0 income before tax, so:

\[
J^+ = 0 \Rightarrow J + 51446789 - 213783886 = 0 \Rightarrow J_{\text{min}} = 162,337,097
\]
1.1.3 Regression

Finally, I use polynomial regression to find out the relation between number of tickets sold and the jackpot. For Mega Millions, I use polynomial regression of the third degree, because higher degrees do not give an increasing function and lower degrees do not approximate the observations with high sales that well. The data are taken from lottoreport.com [4], from October 2013 to April 2017, because rules were changed in October 2013. The data contain: date, sales (only Mega Millions sales, Megaplier is not included) and jackpot. Sales are stated in dollars and since each ticket costs $1, the sales equal to number of tickets. Jackpot is the annuity option and it is the jackpot on that day, so is needs to be adjusted. Firstly, $0.32577 \cdot \text{sales}$ or 5 million, whichever is greater, is subtracted to obtain the previous jackpot. Secondly, the jackpot is multiplied by 0.61 to obtain the cash option.

To explain more the role of the constant 0.61 and its use for the regression. I want to achieve an estimation for the cash value, but I am not using the data on the cash value, because the constant varies throughout time, so the $0.61 \cdot \text{annuity}$ does not equal to the cash option. Its value depends on the capital yield, which is not important for us. An important thing is the fact, whether people react to the annuity option value or to the cash option value. I use the annuity option, because it is the main number published and advertised by the lottery and I believe it provides a better estimation. What is more, data is not that easily available for the cash option. Furthermore, the value of the constant does not vary much, so the result I get today should be usable, or at least approximately, for a certain time period.

To sum up, I suppose that the sales react to the annuity option, so I try to predict the annuity option using the regression and then multiply it by 0.61, which is the constant now to obtain the current cash option.
The regression table is presented in the appendix, because the coefficients are not important. The important is the R-squared, which is 0.8931, which means around 90% of the variation in the number of tickets was explained by the value of the jackpot (annuity option).

Finally, combining the results of the previous chapters and the regression, we can find out, whether it was ever profitable to buy Mega Millions tickets and when we should expect it to be profitable in the future.
Comparison of empirical data to theoretical results

The results clearly show, that buying a single ticket is never profitable in the mean value. The regression line intercepts the syndicate profitability line when the jackpot is around 194 million. This shows us that when the jackpot is more than 194 million, it is likely to be profitable to buy all combinations, nevertheless, we can see that there is one observation with the jackpot more than 194 million which is below the profitability line and vice versa, there is an observation of the jackpot below 194 million still above the profitability line. Theoretically, there should be another intersection, which would make betting with higher jackpot not profitable again, but we have no observations of higher jackpots/sales and there has never been much higher jackpot in the history of Mega Millions (only 10 million higher than our biggest jackpot, on 3/30/2012). We cannot make any conclusions about the upper bound on the jackpot, we just know that when the jackpot is over 194 million, we could expect the syndicate to be profitable.

1.1.4 Profit and Loss Analysis

Now I analyse, what would have happened, if I had bought all combinations in some of the cases that are above the profitability line. The results are biased, because the last observation of high jackpots string (the jackpot string means growing jackpot until there is a winner, which resets the jackpot) will usually not be profitable, because there are other winner/s, important is the probability of being in a profit (in black numbers). Another bias is that when I would bet in some of the cases, there would surely be a winner, so other observations of high jackpots in this string would not happen. Last comment is more from the social point of view, if I would buy so many tickets, it would probably be published in news etc., which would lower the incentive for others to buy tickets, making my
chances better, but this is purely speculative.

I use the real cash option values (which means the ratio of the cash/annuity option might be more or less favourable) which are adjusted for extra tickets bought. Probability of profit represents the probability, that the prize would be greater than the costs, theoretical profit represent the outcome for buying all the combinations for current draw. Values are in millions:

Table 1.4: Profit and Loss Analysis

<table>
<thead>
<tr>
<th>date</th>
<th>cash option</th>
<th>mean value</th>
<th>probability of profit</th>
<th>number of winners</th>
<th>theoretical profit/loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>13/12/2013</td>
<td>228.2</td>
<td>-25</td>
<td>0.52</td>
<td>0</td>
<td>36</td>
</tr>
<tr>
<td>17/12/2013</td>
<td>347.6</td>
<td>-13</td>
<td>0.27</td>
<td>2</td>
<td>-83</td>
</tr>
<tr>
<td>18/3/2014</td>
<td>230.9</td>
<td>-5</td>
<td>0.65</td>
<td>2</td>
<td>-120</td>
</tr>
<tr>
<td>14/6/2016</td>
<td>198.2</td>
<td>10</td>
<td>0.86</td>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td>17/6/2016</td>
<td>211.1</td>
<td>15</td>
<td>0.85</td>
<td>0</td>
<td>33</td>
</tr>
<tr>
<td>21/6/2016</td>
<td>226.2</td>
<td>21</td>
<td>0.83</td>
<td>0</td>
<td>42</td>
</tr>
<tr>
<td>. . . . . .</td>
<td>. . . .</td>
<td>. . . .</td>
<td>. . . .</td>
<td>. . . .</td>
<td>. . . .</td>
</tr>
<tr>
<td>5/7/2016</td>
<td>319</td>
<td>53</td>
<td>0.66</td>
<td>0</td>
<td>99</td>
</tr>
<tr>
<td>8/7/2016</td>
<td>378</td>
<td>66</td>
<td>0.88</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

The ratio was much less favourable in 2014, around 0.55, so the results are worse, on the other hand, in 2016 it was around 0.67 and in July even 0.7, which would allow to make a profit even if there was another winner. The skipped observations represent other drawings between the dates, there were no winners.

1.2 Powerball

Powerball is a lottery game, where 6 numbers are drawn in total. The first 5 are a field of 69 numbers, ranging from 1 to 69, the last one is from 26 numbers, from 1 to 26. The ticket cost is $2.

Table 1.5: Powerball Prizes

<table>
<thead>
<tr>
<th>#of matching numbers (from first field, second field)</th>
<th>Category Prize</th>
<th>Prize</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,1</td>
<td>Jackpot</td>
<td>Jackpot</td>
</tr>
<tr>
<td>5,0</td>
<td>Second</td>
<td>$1,000,000</td>
</tr>
<tr>
<td>4,1</td>
<td>Third</td>
<td>$50000</td>
</tr>
<tr>
<td>4,0</td>
<td>Fourth</td>
<td>$100</td>
</tr>
<tr>
<td>3,1</td>
<td>Fifth</td>
<td>$100</td>
</tr>
<tr>
<td>3,0</td>
<td>Sixth</td>
<td>$7</td>
</tr>
<tr>
<td>2,1</td>
<td>Seventh</td>
<td>$4</td>
</tr>
<tr>
<td>1,1</td>
<td>Eighth</td>
<td>$4</td>
</tr>
<tr>
<td>0,1</td>
<td>Ninth</td>
<td>$4</td>
</tr>
</tbody>
</table>
Powerball very much resembles Mega Millions, the jackpot prize is divided evenly among all jackpot winners, other prizes are paid to all winners in full amount in all states but California. I suppose that full amount is paid to all winners. In Powerball, 50% of sales goes to the prize pool, and out of this, 68.0131% goes to the jackpot prize.

Power Play option is available, a random number out of (2, 3, 4, 5, 10) is drawn. 10 is not always available, but we can ignore it. Then all prizes but the jackpot and the second prize are multiplied by this number, the second prize is always $2 million and the jackpot is not influenced. Power Play option can be bought in addition to Powerball for $1. Again, we should not anticipate Power Play to be profitable, because we know that the key is in the jackpot, which is not affected and the lottery must make a profit of this game, which is proven by a calculation later.

There are again two options, how to receive the jackpot prize, the annuity and the lump sum. The calculation of IRR was shown in the Mega Millions section and the relation between the annuity and the cash option is the same. I again suppose 0.61 to be the ratio between the cash option and the annuity option and I suppose the cash option to be chosen.

### 1.2.1 Single Ticket

Again, Let $P$ denote the amount won, $S$ the amount of set prizes and $R$ the jackpot prizes. If we look at the table of prizes, the first two are taxed by the highest tax, third at the rate of 25% and all other at the rate of 10%. (again, from the prize, 2 is subtracted as the cost)

\[
\frac{\binom{25}{1}}{\binom{69}{5}} \cdot \frac{\binom{64}{5}}{\binom{26}{1}} \cdot 647831 + \frac{\binom{5}{1}}{\binom{69}{5}} \cdot \frac{\binom{6}{1}}{\binom{26}{1}} \cdot 41729.3 + 90.2 \left( \frac{\binom{5}{1}}{\binom{69}{5}} \cdot \frac{\binom{64}{1}}{\binom{26}{1}} + \frac{\binom{5}{3}}{\binom{69}{5}} \cdot \frac{\binom{64}{3}}{\binom{26}{1}} \right) \\
+ 6.5 \left( \frac{\binom{5}{1}}{\binom{69}{5}} \cdot \frac{\binom{25}{1}}{\binom{26}{1}} + \frac{\binom{5}{2}}{\binom{69}{5}} \cdot \frac{\binom{64}{2}}{\binom{26}{1}} \right) + 3.8 \left( \frac{\binom{5}{1}}{\binom{69}{5}} \cdot \frac{\binom{64}{4}}{\binom{26}{1}} + \frac{\binom{6}{1}}{\binom{69}{5}} \cdot \frac{\binom{64}{6}}{\binom{26}{1}} \right) = 0.270767
\]

For Power Play, the mean value of (2, 3, 4, 5, 10) is 4.8, which means 3.8 times the set prizes value is added. $3 \cdot 0.270767 < 1 = cost$, so even if the second prize was affected by all multipliers, it would not be profitable to buy Power Play.

Let $N$ be the number of tickets, I use the same technique to compute $E R$ as I used in Mega Millions section. Let $p$ denote the probability of winning the jackpot.

\[
p = \frac{1}{\binom{69}{5} \cdot \binom{26}{1}} = \frac{1}{292201338}
\]

The tax bonus $B$ is almost the same:

\[
A - 120529.75 - (A - 2 - 415050) \cdot 39.6 = A(1 - 0.396) + 43830.842
\Rightarrow B = 43830.842 \div 43831
\]
Let $W$ represent the number of winners, $W \sim Bi(N, p)$. We know that the price of each ticket is $2$, 50% of sales goes to the prize pool and out of this, 68.0131% goes to the jackpot prize. This means $0.680131 \cdot N$ is added to the jackpot, but the minimal rise of the jackpot is 10 million, so the new jackpot is:

$$\max(J + 10^7 \cdot 0.61, J + N \cdot 0.680131 \cdot 0.61) = \max(J + 6100000, J + N \cdot 0.41488)$$

$$E R = \frac{(1 - (1 - p)^N) \cdot (\max(J + 6100000, J + N \cdot 0.41488) \cdot 0.604 + EW \cdot B)}{N}$$

**Theorem 3.** Under the assumptions stated above, the minimal jackpot for Powerball single ticket to be profitable is a function of $N$ and for $N \leq 14703046$:

$$J_{min} = \frac{1.72923 \cdot N - (N \cdot p \cdot B)(1 - (1 - p)^N)}{(1 - (1 - p)^N) \cdot 0.604} - 6100000$$

For $N > 14703046$:

$$J_{min} = \frac{1.72923 \cdot N - (N \cdot p \cdot B)(1 - (1 - p)^N)}{(1 - (1 - p)^N) \cdot 0.604} - 0.41488 \cdot N$$

**Proof.** Let us find the $J$ value so the $EP = 2$ (cost of the ticket). Then for every greater jackpot, the ticket will be profitable.

$$0.270767 + ER = 2$$

$$(1 - (1 - p)^N) \cdot (\max(J + 6100000, J + N \cdot 0.41488) \cdot 0.604 + N \cdot p \cdot B) = 1.72923 \cdot N$$

For $N \leq \frac{6100000}{0.41488} \approx 14703046$:

$$J_{min} = \frac{1.72923 \cdot N - (N \cdot p \cdot B)(1 - (1 - p)^N)}{(1 - (1 - p)^N) \cdot 0.604} - 6100000$$

For $N > 14703046$:

$$J_{min} = \frac{1.72923 \cdot N - (N \cdot p \cdot B)(1 - (1 - p)^N)}{(1 - (1 - p)^N) \cdot 0.604} - 0.41488 \cdot N$$

$\blacksquare$
1.2.2 Syndicate

First to compute the costs: there are 292201338 combinations, so the cost is

\[ 292201338 \cdot 2 = 584402676 \]

The set prizes equal to the expected value, because all the combinations will be bought. Let \( S \) again represent the set values:

\[
\mathbb{E} S = S = \binom{25}{1} \cdot 1000000 + \binom{64}{1} \binom{5}{4} \cdot 50000 \\
+ 100 \left( \binom{5}{4} \binom{25}{1} \binom{64}{1} + \binom{5}{3} \binom{64}{2} \right) + 7 \left( \binom{5}{3} \binom{25}{1} \binom{64}{2} + \binom{5}{2} \binom{64}{3} \right) \\
+ 5 \left( \binom{5}{1} \binom{64}{4} + \binom{64}{5} \right) \approx 93,466,048
\]

Now we can just assume the cost to be 584402676 - 93466048 = 490,936,628 and no set prizes, because the set prizes are won every time. Let \( C \) represent the cost. Again, the constant 0.41488 represents the amount added to the jackpot with every additional ticket, so with buying all the combinations, 292201338 \cdot 0.41488 \approx 121,228,491 is added to the jackpot. So now we know, that the option of just adding the 10 million to the annuity jackpot will never happen, because it happens only when the jackpot increase from ticket sales would be less than 10 million.

Let \( t \) denote the tax function \( t(x) := x \cdot (1 - f(x)) + h(x) \).

**Theorem 4.** Under the assumptions stated above and \( N > 0 \), the minimal prof-
itable jackpot for the Powerball syndicate must satisfy the following equation:

\[
\sum_{i=0}^{100} (P(W = i)) \cdot t \left( \frac{J + 121228491 + 0.41488 \cdot N}{1 + i} - 490936628 \right) = 0
\]

Proof. Same as for Mega Millions.

I will solve the equation numerically for \( N = Z \cdot 10^6, Z \in \{1, 2, ..., 700\} \).

For \( N = 0 \):

\[ J + 121228491 - 490936628 = 0 \Rightarrow J_{\text{min}} = 369708137 \]

1.2.3 Regression

I use a polynomial regression (this time the second degree, for the same reasons as in the Mega Millions section) to find out the relation between the number of tickets and the jackpot. The data are taken from lottoreport.com [6], from July 2015 to April 2017, because rules were changed in July 2015. There are less observations than we had in Mega Millions, but in this time period, the highest jackpot ever was observed, so we are not limited by the shorter time period. The data contain: date, sales, jackpot. Sales are stated in dollars and since each ticket costs $2, the sales/2 equal to the number of tickets (Power Play is not included in sales). Jackpot is the annuity option and it is the jackpot on that day, so is needs
to be adjusted. Firstly, $0.41488 \cdot tickets$ or 10 million, whichever is greater, is subtracted to obtain the previous jackpot. Secondly, the jackpot is multiplied by 0.61 to obtain the cash option.

The regression table is given in the appendix, because the coefficients are not important. The important is the R-squared, which is 0.9002, which means around 90% of the variation in the number of tickets was explained by the value of the jackpot (annuity option).

Finally, combining the results of previous chapters and the regression, we can find out, whether it was ever profitable to buy Powerball tickets and when we should expect it to be profitable in the future.
We can see that there are not enough observations with high number of tickets, so the regression does not really help us. What is more, the observations are very far from the profitability line, so there would have to be a large change in the cash option ratio for Powerball to be profitable. Despite the great resemblance of Mega Millions and Powerball, the main difference is in the price of the ticket, which is $1 for Mega Millions and $2 for Powerball, while the probabilities for winning jackpot are almost the same. This clearly shows that people do not take the price of the ticket into consideration as much as they should. Buying lottery tickets is not rational, but entertaining, and even though the jackpot is proportionally lower compared to the ticket cost than it is in Mega Millions, people bet Powerball more, which shows that higher jackpots attract more.
2. European Lotteries

The European lotteries analysis will be simpler. I assume no tax from the winnings, because for example in the Czech Republic, we do not pay any taxes from the lottery prizes. Moreover, lotteries do not state the annuity option, but the cash option is the only option, which makes the analysis simpler again. On the other hand, all prizes are pari-mutuel, which means the prize is shared among all prize winners in current tier (the jackpot, the second prize, etc.), which brings more randomness into the process.

2.1 Euro Millions

Euro Millions is a lottery game, where 7 numbers are drawn in total. The first 5 are a field of 50 numbers, ranging from 1 to 50, then 2 numbers from a field of 12 numbers, from 1 to 12. The ticket cost is €2.5. The jackpot cap is set to €190 million.

2.1.1 Random Ticket

Let $N$ denote the number of the tickets sold. We know that the prize pool consists of 50% of the sales, from which 43.2% is added to the jackpot, 4.8% is added to the reserve fund and the rest is added to the other prize tiers. We actually do not need to know the actual tiers, because I count the expected prize pool and divide it by the number of tickets to get the expected prize (there is an expected prize pool for $N$ tickets, so each ticket gets $\frac{1}{N}$ of it). If a prize tier other than the jackpot does not have a winner, the amount is added to the next tier, which makes all the money in the pool for set prizes given to the players. When the jackpot reaches 190 million, the extra amount is added to the second tier.

\textbf{Theorem 5.} Under the assumptions stated above, the minimal jackpot for Euro Millions to be profitable is a function of $N$ and

$$J_{min} = 1.31 \cdot N + 190 \cdot 10^6(1 - p)^N$$

\textit{Proof.} We now know that $2.5 \cdot N \cdot 0.5 \cdot 0.52 = 0.65 \cdot N$ is always redistributed among the players and the jackpot will be:

$$\min(J + 0.54 \cdot N, 190 \cdot 10^6)$$

and the total amount of set prizes will be

$$0.65 \cdot N + \max(J + 0.54 \cdot N - 190 \cdot 10^6, 0)$$

Let $p$ denote the probability of winning the jackpot.

$$p = \frac{1}{\binom{50}{5} \binom{12}{2}} = \frac{1}{139838160}$$

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So the expected prize pool is:

\[(1 - (1-p)^N) \cdot \min(J + 0.54 \cdot N, 190 \cdot 10^6) + 0.65 \cdot N + \max(J + 0.54 \cdot N - 190 \cdot 10^6, 0)\]

Firstly, let us consider \(J + 0.54 \cdot N \leq 190 \cdot 10^6\) (the jackpot cap is not achieved). Then the expected prize pool is:

\[(1 - (1-p)^N) \cdot (J + 0.54 \cdot N) + 0.65 \cdot N\]

Setting equal to the cost of \(N\) tickets, \(2.5 \cdot N\), we will get the minimal jackpot when the random ticket is profitable in the mean value.

\[(1 - (1-p)^N) \cdot (J + 0.54 \cdot N) + 0.65 \cdot N = 2.5 \cdot N\]

so the minimal jackpot is:

\[J_{\text{min}} = 1.85 \cdot \frac{N}{(1 - (1-p)^N)} - 0.54 \cdot N\]

For \(N = 1\) the minimal jackpot is 258,700,595, which tells us that the inequality

\[J + 0.54 \cdot N \leq 190 \cdot 10^6\]

never holds, so \(J + 0.54 \cdot N > 190 \cdot 10^6\)

Then the expected prize pool is:

\[(1 - (1-p)^N) \cdot 190 \cdot 10^6 + 0.65 \cdot N + J + 0.54 \cdot N - 190 \cdot 10^6\]

Setting equal to the cost of \(N\) tickets, \(2.5 \cdot N\):

\[(1 - (1-p)^N) \cdot 190 \cdot 10^6 + 0.65 \cdot N + J + 0.54 \cdot N - 190 \cdot 10^6 = 2.5 \cdot N\]

\[\rightarrow J_{\text{min}} = 1.31 \cdot N + 190 \cdot 10^6(1-p)^N\]

The fact that much more money is returned to the payers right away is very strong, because we do get some possible outcome \((J < 190\, \text{million})\).
Notice that the y axis is not scaled from zero, because nothing would be visible. The rolled over jackpot would have to be in fact 190 million and no more than 10 million tickets would had to be sold to get a positive mean value (for $N=5$ million, $J=190$ million, the expected value of a random ticket is 2.52471, so after subtracting the cost, 0.52471).

### 2.1.2 Syndicate

Given the results of previous part, let us first consider the option of nobody else placing a bet and the jackpot 190 million. This would mean the costs are $2.5 \cdot 139838160 = 349595400$ and the revenues are $190000000 + 1.19 \cdot 139838160 = 356407000$. So the syndicate option can be profitable and the maximum profit is around $7$ million.

We can see that the case when the jackpot cap is not reached will not give any results (minimal profitable jackpot would be over 190 million), so I assume the jackpot cap will be achieved.

Let $Q$ denote $\frac{1}{p}$, which is the number of tickets bet by the syndicate. Firstly, each ticket of the syndicate wins approximately 0.65 from the prizes other than the jackpot and extra money exceeding the jackpot cap. The bonus from betting non-random numbers is neglected on other prizes than the jackpot and extra money from the jackpot, according to Večerě, 2012, when $N = Q$ the share of syndicate is 0.507 for the second prize.
Secondly, I compute the fraction of the jackpot that the syndicate is expected to earn (I use approximation by the Poisson distribution and $W$ denotes the number of other winners).

$$E\ [\text{jackpot fraction}] = \sum_{i=0}^{\infty} \frac{1}{i+1} \cdot P(W = i) = \sum_{i=0}^{\infty} \frac{1}{i+1} \cdot e^{-Np}(Np)^i \cdot \frac{i}{i!} = \frac{1 - e^{-Np}}{Np}$$

Which means each ticket will earn $\frac{1-e^{-Np}}{N} \cdot \frac{1}{N} \cdot \frac{1}{p}$ share of the 190 million jackpot. We know that $J + 0.54 \cdot (N + Q)$ will exceed 190 million, which moves the extra money to the second prize. So $J + 0.54 \cdot (N + Q) - 190 \cdot 10^6$ is split among all winner of the second prize. The second prize winners must guess correctly 5+1, which makes our syndicate to have 20 combinations winning the second prize (one of the 2 correctly, 10 other numbers left - there are 12 numbers in the second drawing).

Let $q$ be the probability of a random ticket winning the second prize:

$$q = \frac{20}{139838160}$$

Then the extra money from second prize:

$$\frac{J + 0.54 \cdot (N + Q) - 190 \cdot 10^6}{Q} \cdot \sum_{i=0}^{\infty} \left( \frac{1}{i + 20} \cdot e^{-Nq}(Nq)^i \cdot \frac{i}{i!} \right)$$

Summing and setting equal to 2.5 gives:

$$J_{\text{min}} \approx 190 \cdot 10^6 - 0.54 \cdot (N + Q) + \frac{(1.85 - 190 \cdot 10^6 \cdot \frac{1-e^{-Np}}{Np})Q}{\sum_{i=0}^{\infty} \left( \frac{1}{i + 20} \cdot e^{-Nq}(Nq)^i \cdot \frac{i}{i!} \right)}$$

For $N = 0$:

$$J_{\text{min}} = 190 \cdot 10^6 - 0.54 \cdot Q + (1.85 - 190 \cdot 10^6 \cdot p)Q$$
Again, notice that the y axis is not scaled from 0. As we can see, the results are slightly better, but still very limited. According to Večer, 2012 [11], the amount of tickets sold, when the jackpot was around 190 million, was 100 million, which is much more than the maximum of around 10 million, which is theoretically computed. In conclusion, the Euro Millions lottery does not give us any opportunity to make a profit.

Now to observe the change given by the last change of rules, in terms of profitability. Using the results from Večer, 2012 [11] for the profitability lines of a single ticket and the syndicate:
2.2 EuroJackpot

EuroJackpot is the same type as Euro Millions, but has different "numbers". You choose 5 numbers out of 50 and 2 out of 10. The jackpot cap is €90 million, the price is typically €2 per ticket (varies throughout countries - I assume Slovakia) and the prize pool consist of 50% of the sales, where 36% of the prize pool is given to the jackpot prize and 12% is given to the reserves, while the rest is for the lower prizes. Everything else works the same as it was in the Euro Millions.

Firstly, let us consider the best case, which is when the jackpot is €90 million and there is no other bet. This means I get all the contributions to the lower bets, which is 0.88 and now I just need to compute the mean value of the jackpot prize, which is:

$$
\frac{90 \cdot 10^6}{\binom{50}{5} \binom{10}{2}} = 0.943948
$$

Which gives us the total expected value of a ticket:

$$
0.943948 + 0.88 = 1.82395 < 2
$$

This means that EuroJackpot can never be profitable, because even under the best circumstances, the mean value of a ticket is less than its price.
Conclusion

I have analysed the four major lotteries in America and Europe. We have seen that American lotteries have better chances to make a profit, because there is no jackpot cap. Very high jackpots may appear and are more likely to be profitable, because most of the expected value of the ticket comes from the jackpot prize.

Mega Millions brings the opportunity to make a profit in the mean value by buying all the combinations under some high jackpot and as is shown in the P&L analysis, the odds might be very favourable.

Powerball due the the higher ticket price and almost the same probability of winning the jackpot does not present any real chance to be profitable, but there is no jackpot cap, which gives us theoretical results, when it would be profitable to bet.

The European lotteries now present no real opportunity, Euro Millions do have some theoretical combinations of the jackpot and the number of tickets sold, when it would be reasonable to buy tickets, but according to the real sales such situations are unlikely to occur. For EuroJackpot, there is no way to make a profitable bet.

To conclude, in the analysed lotteries, it is never profitable to buy a ticket and only Mega Millions present a real possibility of making a profit in the mean value by buying all the combinations.

Syndicate option presents various problems. First is the incredible amount of capital needed to buy all the combinations and given the high risk, banks would not be willing to borrow money. Another problem is with the tickets. In America, the tickets must be physically printed, it would be challenging to get more than 250 million tickets. In history, there are examples of lottery syndicates, for example according to NY Times, there was a syndicate in 1992 in Australia, which bought 5 out of 7 million combinations (they did not buy all because of time issues with the printing), each ticket for $1. They were lucky and won the jackpot, which was $27 million.
3. Appendix

3.1 Mega Millions regression

I used polynomial regression of third degree, $a_1, a_2, a_3$ represent coefficients corresponding to power degree of the independent variable (jackpot).

Table 3.1: Mega Millions regression

| Coefficients | Estimate | Std. Error | t value | Pr(>|t|) |
|--------------|----------|------------|---------|----------|
| Intercept    | 1.128e+01| 9.672e-01 | 11.660  | <2e−16  *** |
| $a_1$        | 3.129e-01| 4.042e-02 | 7.742   | 1.02e-13 *** |
| $a_2$        | -2.971e-03| 4.032e-04| -7.370  | 1.21e-12 *** |
| $a_3$        | 1.480e-05| 1.034e-06 | 14.316  | <2e−16  *** |

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Multiple R-squared: 0.8931

3.2 Powerball regression

I used polynomial regression of second degree, $a_1, a_2$ represent coefficients corresponding to power degree of the independent variable (jackpot).

Table 3.2: Mega Millions regression

| Coefficients | Estimate | Std. Error | t value | Pr(>|t|) |
|--------------|----------|------------|---------|----------|
| Intercept    | 8.975e+00| 3.026e+00 | 2.966   | 0.0035   ** |
| $a_1$        | 9.641e-04| 3.538e-02 | 0.027   | 0.9783   |
| $a_2$        | 1.053e-03| 5.711e-05 | 18.446  | <2e−16  *** |

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Multiple R-squared: 0.9002
Bibliography


