Pavel Irinkov, “Covariant Loop Quantum Gravity”

This master thesis deals with a relatively new branch of Loop Quantum Gravity (LQG), namely Covariant LQG or “spin foams”. In contrast to the older canonical version, where spacetime is divided into space and time in order to define field variables - the connection - and conjugate momenta - triads - on three-dimensional manifolds, in the covariant theory one considers four-dimensional manifolds and a quantization of geometry mainly on their three-dimensional boundaries.

The present thesis concentrates on a model in one dimension lower (2+1 gravity), which shows many of the features of the 3+1 dimensional theory. In this approach a three-dimensional space-time manifold is first triangulated, i. e. divided into tetrahedra. Quantum theory is based on a dual triangulation, the so-called two-complex with two-dimensional faces, dual to the segments of the tetrahedra, and one-dimensional edges, dual to the faces of them. The faces of the two-complex intersect the boundary of the 3-d manifold in 1-dimensional curves, called links, and the edges intersect it in points, called nodes, where the links meet. So on the boundary there appears a graph of links and nodes, which representations of the group $SU(2)$ and the algebra $su(2)$ are attached to. These representations associate quantum numbers od length and area to the graph, in this way a quantization of the boundary geometry by 2-dimensional spin-networks arises, analogously to the mechanism in canonical LQG.

For the boundary states, in the spirit of Feynman’s path integrals, amplitudes can be calculated as sums over 2-complex configurations in the bulk, compatible with the surface geometry.

Chapter zero contains a detailed discussion about the why of quantum gravity and different approaches, in chapter one canonical LQG is introduced, from it’s beginning with the 3+1 split of space-time to Hamiltonian dynamics, which is the road to the spin-foams of the covariant theory.

In chapter two covariant LQG is introduced, by presenting first Feynman’s path integrals in quantum mechanics, and by discretizing space-time. Chapter three, the genuine part of the thesis, finally applies the dynamics to simple configurations of the Ponzano-Regge model: Amplitudes for quantum states of geometry are calculated on a tetrahedron, a double tetrahedron, an octahedron and a “bubble”, i. e. a tetrahedron with edges in the bulk. Unnormalized amplitudes of boundary states with a given maximal spin associated to a link are calculated with the result that with growing maximal spin the partition function, i. e. the inverse of the normalizing factor, grows according to a polynomial law. This would mean that in the limit of arbitrarily high spins included the partition sum would diverge and so the normalized amplitudes would all become zero, which is of course not in accordance with the literature. In the sequel possible reasons for this discrepancy are discussed.

Altogether, up to the normalization problem, the thesis gives a very widespread introduction into the problem of quantization of gravity, its different approaches and their aspects. Concretely the transition from canonical to covariant LQG is worked out very well and in very good English.

To the computational part I would like to ask a question: If all the spins j_l on the links on an octahedron are equal to one, why should the spin j_s on the one edge in the bulk be also equal only to one, and not to zero, two, three, four?

The work fulfills the standard requirements for a master thesis in theoretical physics, I suggest grade A.

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