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2017 **Radim Kašpárek**

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Scale of Market Movements for US stock market

Bachelor thesis

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Abstract

Currently, there is no singular, codified, and widely accepted approach towards measuring the depth of financial crises. One of the approaches applied towards this problematic has been to build on the observed similarity between financial markets and dynamic systems in physics and to create analogous systems. The Scale of Market Shocks originally proposed for foreign exchange markets has been adapted for the US stock market in order to provide US policy makers with a tool to assess the severity of such crises. Using methodology adapted from relevant research and literature we used volatilities calculated with different sampling resolution as the basis for our scale as we believe that these capture the behavior of different market agents. The resultant scale correctly identifies sharp movements and assign them a numerical value that denotes the importance of a crash. This scale is applicable for US policy makers to assess outcomes of proposed policies, however, the use of Principal Component Analysis to ease the computational complexity proved to not yield required results.

Abstrakt

V současné době neexistuje široce akceptovaná škála, která by klasifikovala finanční krize. Jeden z přístupů použitých v literatuře staví na analogii mezi šoky na finančních trzích a šoky v dynamických systémech známých z fyziky, protože tyto spolu sdílí jisté charakteristiky. V této bakalářské práci jsme adaptovali Scale of Market Shocks, která byla původně navržena pro forexové trhy, pro americký akciový trh. Tato škála by měla být jedním z nástrojů, pomocí kterých američtí ekonomové mohou posoudit vážnost finanční krize. Jako základní kámen naší škály jsme použili volatilitu měřenou na datech lišící se granularity. Výstupem naší práce je škála, která správně identifikuje všechny výrazné nenadálé pohyby na americkém akciovém trhu a přiřazuje jim číselnou hodnotu, která odráží jejich význam. Během psaní této práce jsme také zjistili, že Principal Component Analysis, pomocí které jsme chtěli snížit výpočetní náročnost problému, není pro tento účel vhodná, protože nedokáže odlišit signál v datech od šumu.

Keywords

stock market, crash, market scale, econophysics, volatilities, Principal Component Analysis, US stock market

Klíčová slova

akciové trhy, šok, škála pohybu pro akciový trh, škála pohybu pro americký akciový trh, ekonofyzika, volatilita, Analýza hlavních komponentů

Declaration of Authorship

I hereby proclaim that I wrote my bachelor thesis on my own under the leadership of my supervisor and that the references include all resources and literature I have used.

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Prague, 13 May 2017

Signature

Acknowledgment

I would like to express gratitude to my parents, who were of endless support during my studies, to my siblings, who always believed in me, to my girlfriend, who motivated me during my studies, to my friends Samuel Kozuch, Peter Kovalkovic, Daniel Kolar, Marek Vareka, Jan Hynek, Stepan Svoboda, Zbynek Stursa and Ivan Balogh, who were always there for me when I needed an advice or a kind word of support. Lastly I would like to thank to my supervisor doc. PhDr. Ladislav Krištoufek, Ph.D., who showed tremendous patience and introduced me to the beauties of econometrics and data science.

Bachelor Thesis Proposal

Author	Radim Kašpárek
Supervisor	doc. PhDr. Ladislav Krištoufek, Ph.D.
Proposed topic	Scale of Market Shocks for US stock market

Preliminary scope of work

We aim to create a comparison of a few used measurement methods of financial crises and describe how they differ, how would these methods rank major financial earthquakes of the last century and whether their measures of financial shocks magnitude would follow power laws used in seismology.

In 2003 Gabaix et al. proposed a theory of power-law distributions of stock markets fluctuations. Since then, there has been a number of papers concerning this topic and we believe that further research in this area would bring better insight into financial markets crises and theoretical background for policies aiming to minimize the losses caused by these. However, there are several measurement methods of financial crisis magnitude which leads to some difficulties when linking research done by different researchers. The desired output of my thesis would be a comparison of different approaches to expressing financial shocks magnitude.

In this thesis I?d like to take a look at the measurement methods of financial crises and their impacts on stock markets. I?d like to compare their performance and asses whether their provide a useful tool to work with when using power laws from seismology to describe developments on stock markets.

Methodology

Regression analysis of time series will be used to asses whether the values of shocks on these scales used in seismology. One of the scales I want to compare was proposed by Zumbach et al. In 1999 - The Scale of Market Shocks (SMS) is based on mechanics. In mechanics, the kinetic energy is given as:

$$E = \overrightarrow{v}^2$$
$$\overrightarrow{v} = d\overrightarrow{x}/d\overrightarrow{t}$$

In finance, the possible analogy must be related to price changes. Therefore they defined $Dx(t) = (x(t) - x(t-\tau)) \sim \overrightarrow{v}$ as a financial markets analogy to velocity, where x is the logaritmic middle price and τ is some given time range. After including volatility and stochastic nature of τ, they proposed Scale of Market Shocks as a logarithm of the volatility at the time ranges τ, integrated over different time ranges:

$$S(\mu, f, x) = \int \mu(\log \tau) * f(v[\tau, t]) d \log \tau \tag{1}$$

This SMS was proposed for development on FX markets, that?s why the information about the market is limited. They propose, that with more information about the markets, for example exchanged volumes would be incorporated into the definiton of energy and later into their SMS. Another scale of financial shocks gravity is IMS (Index of market shocks) proposed by Betrand Maillet and Thierry Michel in 2002. This scale is a modified version of SMS – the authors left the assumption of zero correlations between volatilities. The data I want to use are stock market indices given by DJIA, S&P 500 and NASDAQ.

Outline

1. Introduction - Why we need to measure financial shocks magnitude
2. Literature review
3. Short introduction of measurement methods that will be compared
4. Data
5. Methodology
6. Empirical findings
7. Discussion
8. Conclusion

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Contents

1 Introduction

In the history of stock markets, there has been a number of crashes. These crashes often had great impact on lives of individuals as well as on political situation. However, the definition of market crash remains vague - shock or unexpected movement is described as crash, when the incident lasts "a long time" or is "severe" enough (Maillet et al., 2003). It is not an easy task to establish a definition that would be based on some objective measure, as the financial crashes can be of a very different nature. Some may start slowly and gradually take on severity, whereas other may start with one sharp move.

With such diversity in nature of crashes, it is rather difficult to analyze them offhand, therefore most of the analyses come "post mortem". The time period between the crash itself and the analysis allows for deeper analyses of the causes and the dynamics, nevertheless, the economists rarely agree on the severity of the crash. This is given partly by different approaches to measuring the "absolute" impact of the crash (see Gandrud and Hallerberg (2015)) and partly by the importance, the single economists attach to different indexes. Chancellor (1999) dissects a few of well-known crises and on their examples proves that even though the decline in stock prices could be sharp, the perceived "severity" would differ dramatically. For example, the crisis in years 1929-1933 was perceived as extremely severe as many market agents were indebted and therefore directly influenced by the crash whereas the collapse of the South Sea Bubble in 1720 was not seen as particularly dramatic, because most of the merchants had sold their stocks before the crash occurred and therefore were not greatly influenced (Chancellor, 1999).

The possibility to objectively estimate the depth of financial crisis is crucial for the regulatory bodies. The central banks as well as national governments or supranational institutions need to have an instrument to measure how much were the financial markets affected. This knowledge is essential for understanding and describing the dynamics of the crisis and consequently for proper assessment of the policies taken. Without understanding the very nature of market crashes, it is not possible to find reliable links between policies and their outcomes as the circumstances can differ dramatically and what could be a good step in one situation, could be deadly in another.

Whereas the question, how to assess the importance of the experience of different groups of market agents, is subjective and as such not possible to answer, the problematics of measuring the "absolute" impact is rather a question of developing sufficient tools. There has been a number of attempts to frame such measurements, ranging from simple methods such as assessing the percentage drop in market instruments prices to advanced methods from the field of quantitative finance assessing the market dynamics. One of the most interesting methods is trying to tie the shocks in the dynamic systems, such as earthquakes or hurricanes, to shocks on financial and stock markets. This method lies within the field of econophysics, which links physics and quantitative finance and applies the knowledge from physics to financial markets.

The research in this field is rather new as the accessibility of high frequency data as well as the sufficient computational power emerged in only several antecedent decades. The first instrument for measuring market shocks that used the similarity of dynamic systems from physics and financial markets was introduced in 1999 (Zumbach et al., 1999). Zumbach et al. developed this instrument called *Scale of Market Shocks* for FX markets and tested it on the USD/DEM and USD/JPY exchange rates in the period between January 1997 and September 1998. His scale was good for the purposes of describing the development on the market and it correctly identified all time points, where the crises based on the traditional views occurred.

In this thesis, we would like to construct analogous scale for US stock market, namely for S&P 500 and Dow Jones Industrial Average indexes and check, whether this scale would be able to capture the sudden movements in stock prices. The time frame for which we will construct the scale is

1/2015-12/2015 for S&P 500 and 3/2016-3/2017 for Dow Jones Industrial Average.

This thesis is organized as follows. In the first chapter we would like to introduce the topic of econophysics, summarize research that has already been done in this field, as well as state some concepts, which we will use later on and some assumptions needed for the analysis. In the second chapter we will describe the data and their basic properties. In the third part we plan to describe in detail the methodology of constructing the scale, possible issues and necessary assumptions. In the fourth part we will assess, whether the constructed scale captures the stock prices movements and examine the periods, where our scale attains its peaks. Lastly, a comparison between the scale's performance on the stock market over the FX market will be drawn and the usefulness to policy makers of thusly obtained new information analysed.

2 Literature Review

The term econophysics was first used by H. Eugene Stanley in the Kolkata conference on statistical physics in 1995 to applying knowledge from physics to economics (Ghosh, 2013). It describes “an interdisciplinary research field, where the tools of physics are applied to understand the problems of economics” (Ghosh, 2013). However, it is not new that physicists contribute to research in economics. For example Irving Fisher, who greatly contributed to the neoclassical economic theory, was a student of physicist Josiah Willard Gibbs, or Jan Tinbergen, the first Nobel laureate in economics, did his Ph.D. in statistical physics.

Nevertheless, these people in the end always left physics and started to publish in economical journals. On the contrary, the new approach, which econophysics brings, is that the researchers have their articles published by both economical as well as physics journals. This interdisciplinary approach is rather unique in social sciences and once could become an example of successful combination of natural sciences and social sciences.

The econophysics as such was started in 1990s by physicists, who were originally doing research in statistical mechanics. With the utilization of large amounts of accessible historical financial data, they were attempting to create more practical framework for investigating financial markets. The methods used by standard economists were based on theoretical models and assumptions of homogeneous market agents and mostly concerning equilibrium situations, however, in the real world application to financial markets, these methods often failed as the agents were heterogeneous and the situations were often far from equilibrium.

In 1935, seismologists Charles Francis Richter and Beno Gutenberg of the California Institute of Technology, proposed a scale for measuring earthquakes (Richter, 1935). This scale defines magnitude as the logarithm of the ratio of the amplitude of the seismic waves.

As the earthquakes magnitudes are distributed with power law probability (see Figure 1), the scale also measures the inverse probability of an event.

$$mb = \log_{10}\left(\frac{A}{T}\right) \qquad , \tag{2}$$

where A is the amplitude of ground motion (in microns); T is the corresponding period (in seconds), mb is the magnitude on Richter's scale;

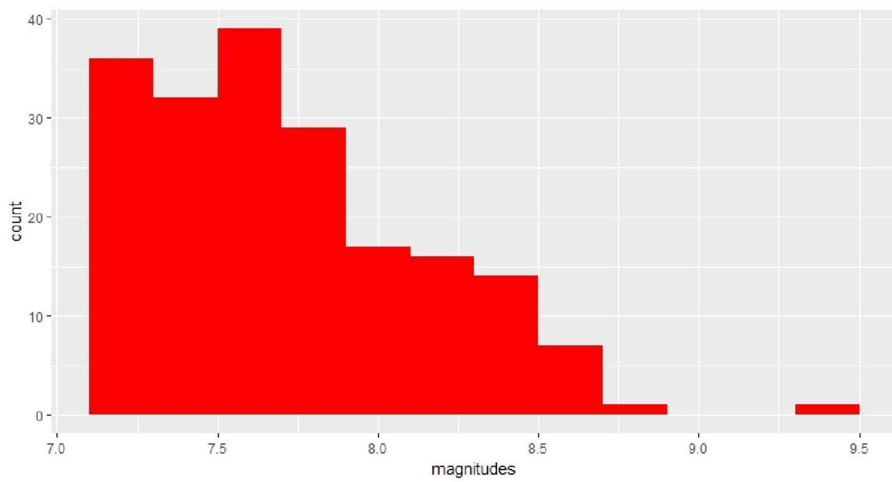

Figure 1: Earthquakes in South America between 1900 and 2000 (based on dataset acquired from web[1])

Zumbach et al. (1999) drew analogy between shocks in dynamic systems, such as earthquakes or hurricanes, and shocks on the financial markets. On the example of Richter scale of earthquake magnitudes they demonstrated the similarity, however, the analogy is not perfect.

For the analogy with the Richter scale, they define an equivalent for financial markets of energy and total energy released in a shock. As the energy for unit mass is in mechanics given as $\frac{\partial x}{\partial t}$, the possible analogy for the speed is linked to changes in price. They define price change as $Dx(t) = (x(t) - x(t-\tau)) \sim \vec{v}$. However the speed itself is not sufficient, therefore one more parameter is defined, namely the time range τ. They also state that

[1] http://www.johnstonarchive.net/other/quake1.html

to the definition of energy would be further incorporated other information, for example the volume of traded goods.

For the foreign exchange markets, it was very difficult to estimate the traded volume as these are over-the-counter markets and there were no centralized place, where the transactions were recorded. Therefore Zumbach et al. did not incorporated the volume into their definiton of energy. However, nowadays, when the automated dealing systems are used, it is possible to make good estimates of traded volume – for example the US Foreign Exchange Committee releases estimates[2] of foreign exchange volumes every month from which could be possible to estimate daily volumes.

They also define an equivalent for total energy released during a shock. Whereas shocks in mechanical systems are often clearly denoted by some initial event and some terminal event, in case of shocks on financial markets it is rarely so. Therefore Zumbach proposed continuous indicator based on the analogy with mechanical work $\frac{\partial E}{\partial t}$.

This research was further developed by Bertrand Maillet and Thierry Michel in the paper called *An index of market shocks based on multiscale analysis* (Maillet et al., 2003). The authors attempted to create a scale analogous to the *Scale of Market Shocks* that would be used for describing movements on the French stock market using historical values of CAC 40 representative index.

The main idea behind their scale was that the crisis emerges, when the perceived instability of the market is high by all market agents - by the day traders as well as by long-term investors. They decided to use volatility to estimate this perceived instability. By measuring volatility of stock prices in different time ranges, they were able to capture the market moves of all market agents.

The authors used high-frequency data from French stock market (1995-2002) and compared the performance of IMS with the performance of mul-

[2] https://www.newyorkfed.org/fxc/volumesurvey/index.html

tifractal spectrum width method. They concluded that the IMS correctly identified all crises that had emerged between 1995 and 2002 and that the crisis after the 9/11 had been severe, even though the recovery was rather quick. The authors suggest that the next step in this research should concern possible use of IMS in prediction of market crashes. They state, as well as Zumbach did, that their scale would benefit from adding the volume of trade into the formulas as this information would improve the analogy between mechanical energy in dynamic systems and the energy of financial markets.

Negrea (2014) published a paper called *A statistical measure of financial crisis magnitude* (Negrea, 2014). In this paper he proposed a new way how to measure financial crises - he drew up on the analogy between Gutenberg-Richter Law from seismology and the financial markets. The scale is based on the concept of financial market energy, which is defined from return on the stock market index and trading volume.

In this thesis I would like to construct similar scale for US stock market and test it on high frequency (1 min granularity) data containing the values of Dow Jones Industrial Average index between 3/2016 and 3/2017 and the trading volume. The other dataset, we will work with, contains the values of S&P 500 index between 1/2015 and 12/2015 in 1 min granularity. My aim is to answer the question, whether this scale could be useful for US policy makers and whether the inclusion of trading volume enhances the performance of the scale.

3 Dataset description

In this thesis S&P 500 and DJIA data are used. For the aim of the thesis, which is to test a tool that would be of use for US policy makers, these two indexes are the best data sources available as they capture the development of stock prices of the leading companies traded on NYSE or NASDAQ. The development of these indexes is closely related to the development on the US Stock Market as such and therefore these indexes are commonly used as proxies for the US Stock Market.

For constructing a scale of market shocks, high frequency data are needed. Unfortunately, the accessibility of such data from these indexes is limited. It would be optimal to use tick data (Bisig et al., 2012), i.e. data that contain multiple data points for one minute of trading, and to work with dataset comprising longer time frame. However, given the computational power we have access to, we decided to use minute-granularity data from one year long time frames. Both of the datasets contain roughly around 100 000 observations, which should be sufficient for constructing the desired scale.

3.1 Standard & Poor's 500 Index

S&P 500 is an American stock market index based on market capitalization of 500 firms listed on the NYSE or NASDAQ. These firms are selected by a committee in order to represent all the industries in United States economy, therefore the index should adequately mirror the state of the whole economy (see Figure 2). The index was developed and is maintained by S&P Dow Jones Indices, a joint venture majority-owned by S&P Global.

The value of the index is given as a weighted average of market capitalizations of the companies listed[3]. Since 2005 the value is calculated only from stocks available for public trading. Technically the index is calculated

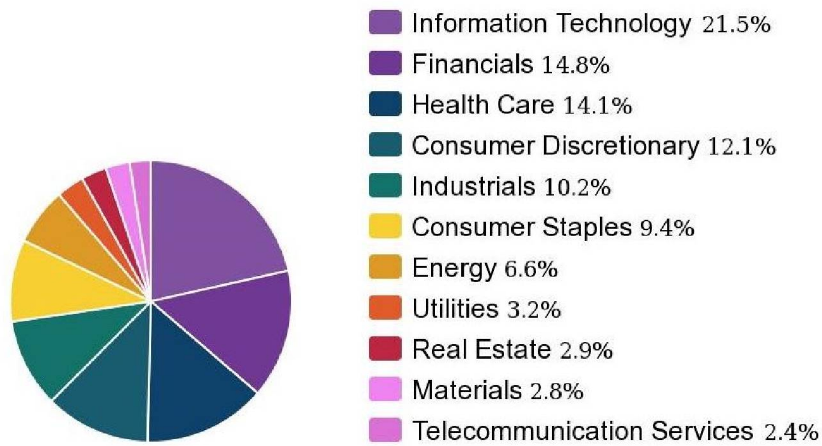

Figure 2: S&P 500 strucure by GICS sectors

source: http://us.spindices.com/indices/equity/sp-500

as $\frac{\sum(P_i * Q_i)}{Divisor}$, where P is the price of the stock, Q is the number of shares publicly available for the stock market and *Divisor* is a parameter which is adjusted in case of stock issuance, spin-offs or other structural changes in order to maintain the numerical value of the index.

In 2015, there were 252 trading days. For 247 of these days we have complete set of 411 observations that range from 08.31 AM to 3.21 PM. For three days, specifically 11th of March, 24th of June and 26th of June we miss two or four observations and finally, on 24th of December and 27th of November the NYSE is opened just till 1 PM, therefore we have just 210 observations for these days.

The values of Standard & Poor's 500 ranged from 1868 to 2135, the maximum was attained on 20th of May and the minimum on 21st of August. The average realized daily volatility was computed by Close-to-Close method and is nearly 1%. We had to correct for the days with missing observations,

[3]for the components in 2015 see wikipedia.org/wiki/List_of_S\%26P_500_companies

we chose the last observations in respective days.

The biggest drop in the values on the S&P 500 index occured between 19th of August and 24th of August (see Figure 3). The index plummeted by nearly 230 points, which was about 11% of the value before the plunge. This dive was probably a result of uncertainty brought by concerns about slow down of China's economy, oil prices drop under 40$/barrel and mixed signals from U.S. Federal Reserve about raising interest rate. The S&P 500 did not recover until 2nd of November when the index attained a value over 2100 for the first time after the aforementioned drop.

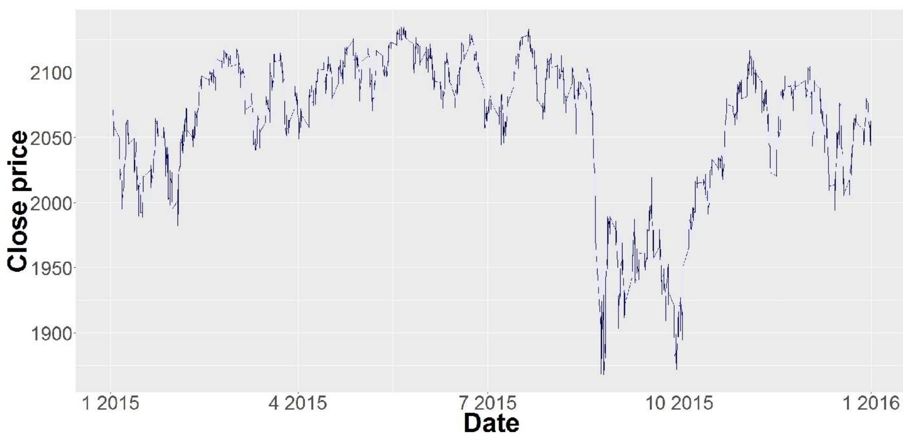

Figure 3: S&P index development in 2015 (please note that the values of y don't start with 0 in this figure)

We expect that in the beginning of this period the market energy will be highest in the whole time span we are working with. Unfortunately, our dataset does not contain information about volume of stocks traded, therefore we won't be able to incorporate this variable into the definiton of market energy and we will have to rely on the properties of price development. In the rest of the inspected period the movements are significantly smaller and we expect our scale not to attain high values.

We didn't check for seasonality in the dataset as we rely on *Efficient Market Hypothesis* (Fama, 1970). This hypothesis says that every pattern would be soon discovered by market agents, utilized and through this utilization would diminish. Even though since the proposition of this hypotheses there were concerns about its accuracy (Malkiel, 2003) and attempts to revise it, verifying these is beyond the scope of this thesis. Also the seasonality would not be an important issue as we are trying to describe market energy and propose a scale to measure it.

The same holds for other features of our time series, such as stationarity, trend stationarity or weak dependency. We would need to address them in case we were trying to describe the drivers behind market energy and measure their impact on it, but since we are not, we are not interested in these characteristics.

3.2 Dow Jones Industrial Average

The Dow Jones Industrial Average is an American stock market index consisting of stocks of 30 major publicly owned companies based in US. The word "Industrial" is rather a historical relict, as most of the companies are not directly related to traditional heavy industry. As well as in the case of the S&P 500 index, the components of this index are traded on NYSE or NASDAQ, therefore the development of the index could be used as a proxy for the development on the US stock market and indirectly as a proxy for the development of US economy.

The value of the index is a price-weighted average of the values of the components. The components of the index changed 51 times since its creation, for the list of current components see Table 9 in the Appendix. Technically, the Dow Jones is calculated as $\frac{\sum(p_i)}{d}$, where p are the prices of component stocks and d is the *Dow Divisor*.

In case of stock splits or change in the composition of the index, the divisor guarantees consistency of the index. Technically, the adjustment is rather simple $DJIA = \frac{\sum(p_{old})}{d_{old}} = \frac{\sum(p_{new})}{d_{new}}$

Although some argue that the index does not represent the market (Edelman, 2005), it is still commonly used and recognized as an accurate representation of the market. The critics most often point out the price-weighing methodology, which favors highly prized stocks and gives them greater weigh than to the lower prized stocks. For example if stock goes from 10$ to 11$, i.e. rise of 10%, the effect on Dow Jones is the same as rise from 100$ to 101$, which is a rise of only 1% of the value of the stock.

Our dataset contains minute-granular data from the period between 07/03/2016 and 03/03/2017. This specific scope was chosen due to specific process of data extraction from the webpage finam.ru. The dataset does not have perfect quality, nevertheless, we decided to use it as this dataset is the best one accessible. The analysis would probably yield better results if data from longer time period were used, however as discussed earlier the computational power limits us. This dataset contains the same variables as the one we have for S&P 500 plus information about trading volume. We assume that the quality of the construction of these indexes does not differ dramatically and that we will be able to correctly assess the contribution of the information about market volume. We assume that the performance will be enhanced by adding this variable, however since we are working with estimates of the volume, this might not be the case. We also were not able to find the methodology of estimating the volume, therefore we cannot confirm its correctness.

In the period, we decided to look into, there were 249 trading days. The time scope is from 09.31 AM till 16.21 PM, however for the last twenty minutes of trading we have some missing values. We have to correct for these as we need not to have complete days of observations.The missing values are always a few observations from the very end of day. We decided to approximate these values as a weighted average of the last price observation and the very first one from next day. The weight assigned to the last known observation has to be greater than the weight assigned to the morning value, as opening prices of stocks tend to be relatively high (Berkman et al., 2012).

We would like to utilize the information about average daily volume of trade for DJIA and incorporate this into our Scale of Market Movements.

The average daily volume of transactions ranged from 51.22 attained on 15th of June to 59.84 on 19th of October. There is a significant increase in volume on 14th of October. We concluded that this growth is not linked to the development of price as no sudden movement is present around this date and we are not sure what caused this growth. After thorough search on the web, we were still not able to determine to which was this growth linked to and we decided conduct a separate analysis on dataset containing the volume information as well as on dataset from which we exclude the information, because we are not sure whether the information about volume traded is accurate.

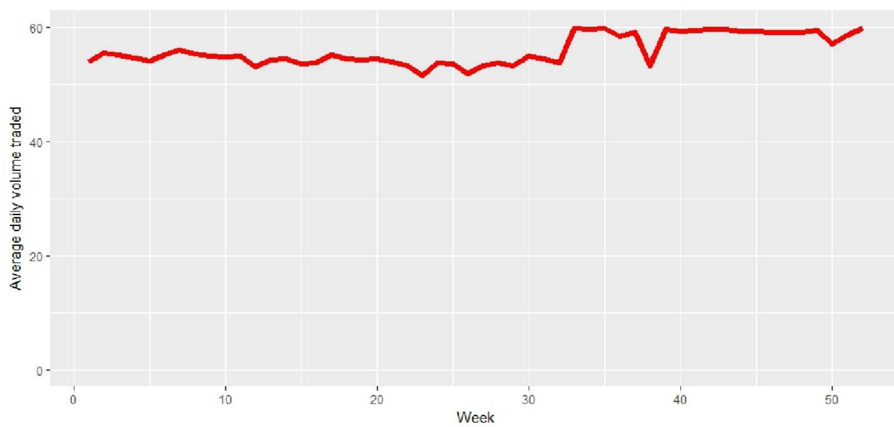

Figure 4: Avg daily volume traded by weeks, DJIA

The prices in this period ranged from 16822 to 21169, the minimum was attained on 10th of March 2016 and the maximum on 1st of March 2017. It is not a coincidence that these are at the beginning and at the end of the period as the data shows more or less stable growth over the examined time range. The average daily volatility was computed by Close-to-Close method and is 0.61%. We had to correct for days with missing observations, we decided to use last observation available as the closing price for the day.

There is a significant decline in DJIA values on 24th of June, which is clearly linked to Britain voting "leave" in the referendum regarding the membership in European Union. The decline was rather steep from 18011 points at the end of 23th to 17070 on 27th of June, which is circa 5.5 percentage points drop. On the other hand, in the week of the US presidential elections there was a rapid growth in the prices of shares and the DJIA grew from 17888 points on 4th of November to 18871 points in just six days, which is a growth of 5.5 percentage points. Apart from these two events, the prices did not experience any extremely sharp moves, therefore we expect the market energy to attain the highest values around these aforementioned dates.

We will not address other features of the time series such as stationarity, trend stationarity or weak dependency. For the justification refer to the description of the second dataset used.

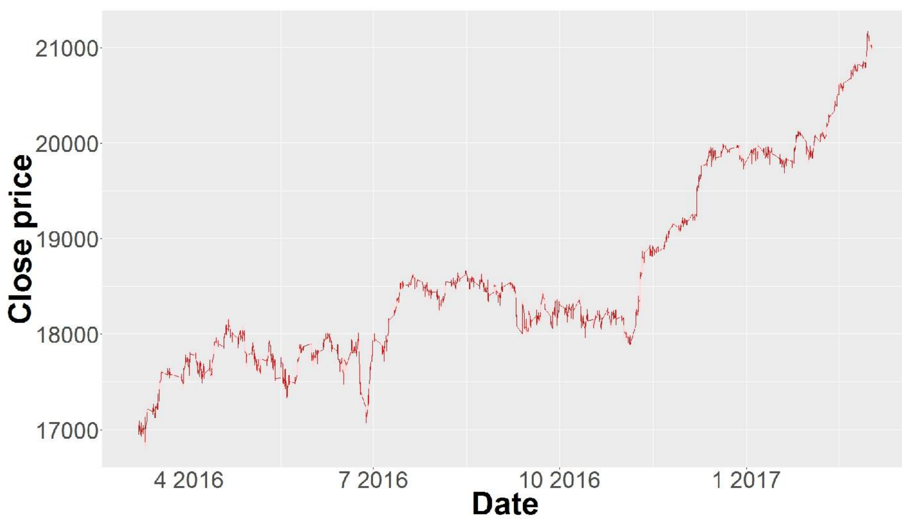

Figure 5: DJIA index development between March 2016 and March 2017

4 Methodology

In this section we will present the technical aspects of conducting the analysis and the construction of Scale of Market Movements for US stock market. Firstly, we will examine closer how the original Scale of Market Shocks was constructed (Zumbach et al., 1999). Secondly, our approach will be presented. Thirdly, we will examine closer the volatility distributions and lastly, we will outline the theoretical concepts behind the mathematical methods used.

We built on the pioneering proposal of Zumbach et al. and extend it on the US stock market. Nevertheless, the output differs in form as our scale lost the nice feature of continuity. This is given by different approach to incorporating volatility into our scale as we chose to use non-overlapping time horizons (more on this decision in the following sections). Moreover, we utilized the information about trading volume we had for our DJIA dataset.

4.1 The Scale of Market Shocks

The proposal of Scale of Market Shocks contained basically two scales, one *universal* which allowed for direct comparison of different assets and one *adaptive* which was calibrated for typical behavior of each asset. These scales are closely connected and the mechanisms behind calculation are very similar, therefore we will investigate only the *universal* one. We chose the *universal* one, as we aimed to create a scale that would also be able to capture movements on different market indexes (DJIA, S&P 500).

The proposed scale was building on the information gathered from volatilities calculated over different time ranges. The authors used these volatilities as proxies for behavior of different market agents, from day traders to long-term investors. They constructed the scale as an integral of the logarithm of the volatility calculated over multiple time ranges:

$$S[\mu, f; x] = \int d\log(\tau) * \mu(\log(\tau)) * f(v[\tau; x]) \qquad , \tag{3}$$

where the measure $\mu(\log(\tau))$ fixes the weights of contribution at different time horizons τ and the f is a mapping function that should distinct between exceptional market events and background fluctuations.

In addition to that, they further specified the components of the formula, namely the derivative and volatility operators, the form of mapping function f and the measure μ. In this thesis we will further inspect mainly the volatility operators used, as our approach builds on volatility.

4.1.1 Volatility operators

For the Scale of Market Shocks the authors used logarithmic middle prices. The time series was then in following form:

$$x = \frac{1}{2}(\log(p_{bid}) + \log(p_{ask})) \tag{4}$$

The annualized return in a given time range τ was defined as:

$$r[\tau; x] = \frac{x(t) - x(t-\tau)}{\sqrt{\tau/1y}}. \tag{5}$$

The denominator was used in order to remove the Gaussian random walk scaling. This definition, $E[r[\tau]^2] = \sigma^2$, leads to a very good approximation (Zumbach et al., 1999). The *1y* stands for the one year, therefore the $\frac{\tau}{1y}$ denoted the fraction of the year and annualizes the returns.

The volatility v of a time series x can be then measured by:

$$v[\tau; x](t) = \sqrt{\frac{1}{16}\sum_{i=1}^{16}(r[\tau/16; x](t_i))^2} \tag{6}$$

where $t_i = t - (i-1)\tau/16$. This denoted the annualized volatility, as the return was already scaled by $1/\sqrt{\tau}$. However, these two last formulas suffer from number of drawbacks when working with high frequency data (Zumbach and Müller, 2001) (in the original paper the authors cited probably some working paper that had been internally published in 1998, however, we were not able to find a mention earlier than in the paper cited here). Therefore the authors in the paper decided to modify the approach and use formula based on exponential moving average (EMA) technology:

$$v[\tau; x] = MNorm[\tau/2, p = 2; D[\tau/16; x]] \tag{7}$$

For details on the use of EMA technology please refer to Zumbach et al. (1999) and Zumbach and Müller (2001) as a detailed description of these is beyond the scope of this thesis.

Further concern about this approach is in the fact that the authors were working with intra-day time range and the volatilities show strong seasonality (Baillie and Bollerslev, 1991). Without accounting for these issues, a peak would had been observed every European afternoon which could had been simply a consequence of overlap of trading times in Europe and US. The authors had decided to use $\vartheta - scale$ as introduced by (Dacorogna et al., 1993).

The authors computed the volatility for every time point using the aforementioned formula. They used different sampling resolutions over different time horizons ranging from one hour to 42 days. Therefore they obtained a vector of volatilities for every time point in the dataset and due to the fact that no restrictive transformation was later used, their scale was continuous. Nevertheless, we decided to add several constraints for our Scale of Market Movements. The most significant one is that we decided not to allow for data overlap between two successive shock measurements.

4.2 The Scale of Market Movements

Our scale builds extensively on the volatilities similarly to other scales from literature (see Maillet et al. (2003)). For every week in our dataset we obtained a value that captures the movements on the market and the sequence of these values presents the quantities of market energy on our scale for weeks in the examined period. Whereas the original Scale of Market Shocks worked with volatilities measured over varying time horizon, we measured volatility only over the preceding week to obtain a measurement at a specific time point. We varied the sampling resolution to be able to capture the market moves of different agents on the stock market. The main consequence is that since the volatilities were measured over the same time period, their correlation could have been no longer overlooked.

Moreover, the seasonalities within the time horizon used for computation does not affect the measure since the possible bias were constant over the time horizon and the usage of whole weeks corrected for them. We also incorporated an explanation for the weighting scheme of volatilities which the original proposal lacks.

For dealing with the non-zero correlation between volatilities we used their property of multinormality. We used the logarithms of volatilities, as these are close to normality (more on the properties of volatilities distribution in following section). Then we computed the variance-covariance matrix of the log-volatilities and for each log-volatility we calculated the mean. These we used for a Monte Carlo simulation and we had a million of observations generated for every log-volatility generated. By using these, we were able calculate the probability that our observation of specific volatility in specific week would not have been exceeded. Then we assessed the whole vector of probabilities (one for each week) and based on its values we defined the first form of Scale of Market Movements as follows:

$$SMM_t = \alpha \log \left(\frac{1}{1 - (F[\log \hat{\sigma}_t(\underline{\Delta t_r}), \ldots, \log \hat{\sigma}_t(\overline{\Delta t_r})])} \right), \tag{8}$$

where $F()$ is the multivariate normal cumulative density function, $(\underline{\Delta t_r}, \overline{\Delta t_r})$ is the lowest and the highest sampling resolution used. Unfortunately, this approach has the hindrance of assigning the same weight to all the volatilities, which is rather inconvenient as we believe that the amount of information is not the same. In addition to that, the accuracy of results is also decreased by the use of Monte Carlo simulations in the process.

In the original proposal of SMS (Zumbach et al., 1999), the authors chose as the weighting scheme for volatilities a function:

$$\mu(ln\tau) = ce^{-x}(1 + x + x^2/2) \tag{9}$$

The theory behind choosing this form is not very clear from the paper therefore our aim was not to modify this existing function, but rather use the same approach as had been used for developing a scale for French stock market (Maillet et al., 2003). We found this approach more transparent and the theory behind sounder.

The authors in this paper used variance reduction technique and they assigned the weights according to the contribution to the global variance. We will use Principal Component Analysis, which allowed us to reduce the number of dimensions. We chose the significant components and assigned the weights according to the relative contribution to the overall variance.

Moreover, we utilized the information we have about the trading volume in the Dow Jones Industrial Average dataset. We incorporate the information into the alpha coefficient as we believe that when a market comes into a turbulent state the trading volume grows as market agents try to react in the fast-changing environment (Negrea, 2014). Technically, we computed the average volume for a minute of trading in a given week. This average volume traded was then scaled (we assigned the mean volume traded value 1) and this scaled average volume traded we used as the alpha value.

In the following sections the structure of volatilities distribution will be further examined, description of the mathematical theory behind PCA and Monte Carlo simulations will be provided too.

4.3 Volatility

For the purpose of this thesis, we needed high-quality dataset. Whereas the S&P 500 dataset this requirement definitely met, the Dow Jones Industrial

Average had had some missing observations for which we had to correct. We corrected for missing values by approximation described in section 'Dataset description'.

After approximation, we calculated the realized volatility for every Friday in the dataset. We calculated the volatility over the preceding week, over different sampling resolutions. Our sampling resolutions range from 1 minute to 1026 trading minutes, which corresponds to two and a half trading days. We chose these resolutions based on evidence found in the literature (Maillet et al., 2003) and by the number of observations we had for each week. The resolutions were chosen as divisors of 2052, as we wanted to capture the whole week by every sampling resolution. We had to neglect the first two minutes of trading for every week in order to be able to choose appropriate number of sampling resolutions. The varying sampling resolution along with the stable time horizon means that the number of observations, which we used for calculating the volatility, differs - for 1 minute we have 2053 observations whereas for the calculation of 1026 minute (2.5 days) volatility we used only 3 observations.

We decided to use the annualized realized volatility:

$$Vol = \sqrt{\frac{252}{5} \sum_{t=1}^{n} R_t^2} \quad , \tag{10}$$

where we used 252 as a constant representing the approximate number of trading days in a year, 5 as the measurement time frame (which is five days for every calculation), t as the counter representing each observation and finally R_t as the continuously compounded returns calculated by formula $R_t = ln\left(\frac{P_t}{P_{t-1}}\right)$.

For every week in our datasets we obtained 23 annualized volatility measurements, one for each sampling resolution. For further analysis we used logarithms of the volatilities, as these come from distributions that are close to normal, which is in accordance with literature (Andersen et al., 2001).

4.3.1 Normality of log-volatilities

The normality was proposed by use of *Cullen and Frey Graph* and confirmed by Shapiro-Wilk Normality test, Cramer-von Mises test and Anderson-Darling test. We also plotted the Q-Q and P-P plots and density and cumulative density functions for visual confirmation. We decided to use several tests in order to maximize the probability of detection of non-normal distribution in our log-volatilities, because the normal distribution was a key assumption for our further work.

The Cullen and Frey Graph was proposed in literature (Cullen and Frey, 1999) as an aid for determining the underlying distributions in data sets. It is an skewness-kurtosis plot given for the dataset and on this plot, skewness-kurtosis ratios for common distributions are marked. Whereas for some distributions there is only one possible value for skewness and kurtosis (normal, uniform, logistic, exponential) some distributions don't have specific unambiguous values (gamma, lognormal) of skewness and kurtosis. These are represented by lines or areas. Because the estimated values of skewness and kurtosis for the dataset are not exact, the data set is often bootstrapped and the average value is reported.

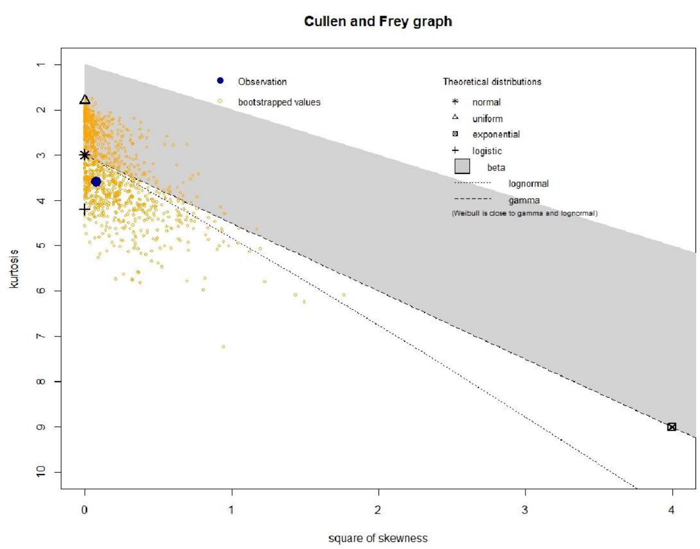

Figure 6: C&F graph for DJIA one-minute volatility distribution

For our datasets we set the number of iterations to 1000. We did this for all the measured volatilities vectors and the Cullen and Frey graphs looked very similar, therefore we will present here just one (see Figure 6). It is obvious that our datasets have very similar skewness and kurtosis as normal distribution and therefore we tested a fit to univariate normal distribution.

The Shapiro-Wilk Normality Test was introduced in 1965. It has become the preferred test because of its good power properties (Mendes and Pala, 2003). The original Shapiro-Wilk test statistic is defined as:

$$W = \frac{(\sum_{i=1}^{n} a_i y_i)^2}{\sum_{i=1}^{n} (y_i - \overline{y})^2}, \tag{11}$$

where y_i is the i$^{\text{th}}$ observation and $\overline{y}$ is the sample mean, $a_i = \frac{m^T V^{-1}}{\sqrt{m^T V^{-1} V^{-1}}}$ and $m = (m_1, \ldots, m_n)^T$ are the expected values of the order statistics of independent and identically distributed random variables sampled from the standard normal distribution and V is the covariance matrix of those order statistics (Razali et al., 2011). The value of W is between 0 and 1, small values leads to rejection of normality whereas one indicates normality of the underlying distribution of data. The test was originally restricted to samples with up to 50 observations, nevertheless in 1995 Royston provided improved algorithm AS R94 that can be used on samples of size between 3 and 5000 observations. We used this algorithm for our calculations (R function *shapiro.test* from R package {stats}).

The Cramer-von Mises test was developed by Cramer (1928), von Mises (1931) and Smirnov (1936) according to (Conover, 1999). It is used for judging the goodness of fit of a cumulative distribution function F^* compared to a given empirical distribution function F. The test statistics is:

$$T = n\omega^2 = \frac{1}{12n} + \sum_{i=1}^{n} \left[\frac{2i-1}{2n} - F(x_i) \right]^2, \tag{12}$$

where $x_1, x_2, \ldots, x_n$ are the observed values. If the T value is larger than the tabulated value, we can reject the hypothesis that the data come from normal distribution (we used the function *cvm.test* from R package {nortest}).

The Anderson-Darling test is a modification of the Cramer-von Mises test. It differs from the Cramer-von Mises test in such a way that it gives more weight to the tails of the distribution. The test statistic of this test is:

$$W_n^2 = n \int_{-\infty}^{\infty} [F_n(x) - F^*(x)]^2 \psi(F^*(X)) dF^*(x) \tag{13}$$

Later on, following formula was proposed to make the computation easier (Arshad et al., 2003)

$$W_n^2 = -n - \frac{1}{n} \sum (2i-1) \log F^*(X_i) + \log(1 - F^* X_{n+1-i}) \quad , \tag{14}$$

where $F^*(X_i)$ is the cumulative distribution function of the distribution against we test, x_i's are the ordered data and n is the sample size. We used the function *ad.test* from R package {nortest}.

As these has the null hypothesis of normal distribution, we decided to state only the p-values (see Table 1 and Table 2). We set the α to 0.99 and since all the p-values are higher than 0.01 we believe that these tests along with the aforementioned evidence in literature provide sufficient justification for the assumption of normal distribution of *log*-volatilities.

Table 1: DJIA *log*-volatilities normality test outcomes

Sampling resolutions	Shapiro test p-value	A-D test p-value	C-VM test p-value
1	0.35	0.62	0.73
2	0.49	0.68	0.75
3	0.25	0.39	0.40
4	0.31	0.70	0.84
6	0.27	0.44	0.47
9	0.17	0.23	0.23
12	0.23	0.40	0.44
18	0.27	0.09	0.06
19	0.54	0.36	0.33
27	0.47	0.34	0.27
36	0.67	0.42	0.30
38	0.40	0.26	0.26
54	0.44	0.21	0.17
57	0.62	0.37	0.31
76	0.11	0.09	0.10
108	0.46	0.49	0.48
114	0.30	0.30	0.27
171	0.53	0.47	0.38
228	0.05	0.11	0.15
342	0.04	0.25	0.34
513	0.70	0.53	0.51
684	0.43	0.46	0.53
1026	0.34	0.46	0.43

Table 2: S&P 500 *log*-volatilities normality test outcomes

Sampling resolutions	Shapiro test p-value	A-D test p-value	C-VM test p-value
1	0.03	0.07	0.05
2	0.03	0.09	0.07
3	0.06	0.19	0.18
4	0.06	0.18	0.17
6	0.10	0.17	0.13
9	0.13	0.22	0.18
12	0.13	0.26	0.20
18	0.10	0.17	0.14
19	0.10	0.27	0.24
27	0.11	0.45	0.50
36	0.23	0.20	0.16
38	0.31	0.29	0.22
54	0.18	0.39	0.37
57	0.05	0.09	0.08
76	0.96	0.96	0.93
108	0.96	0.92	0.91
114	0.32	0.37	0.31
171	0.13	0.12	0.14
228	0.91	0.71	0.61
342	0.84	0.87	0.85
513	0.53	0.39	0.29
684	0.04	0.03	0.02
1026	0.16	0.12	0.15

4.3.2 Other properties of volatility vectors

By S&P 500 dataset we obtained 23 volatility vectors, each with 52 observations. These correspond to particular weeks in 2015. We plotted the means and variances of the obtained log-volatility vectors in order to compare them with each other (see Figure 7 and Figure 8). The means of these annualized volatility vectors range from 0.10 to 0.12, which translates into values around -2.3 after the log transformation. The means of log-volatilities do not differ dramatically, however, the variances do. The variance of log-volatilities range from 0.13 to 0.47. As we can observe, the log-volatilities calculated with longer time steps have significantly higher variance than those with shorter time steps.

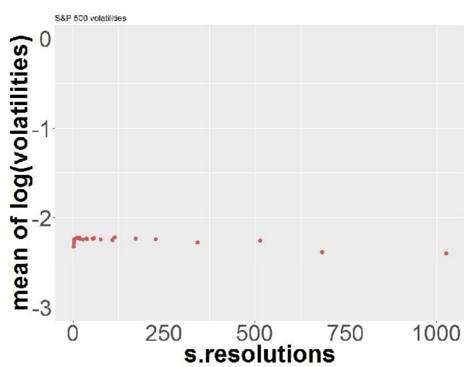

Figure 7: S&P 500 log-volatilities in 2015 means by sampling resolution

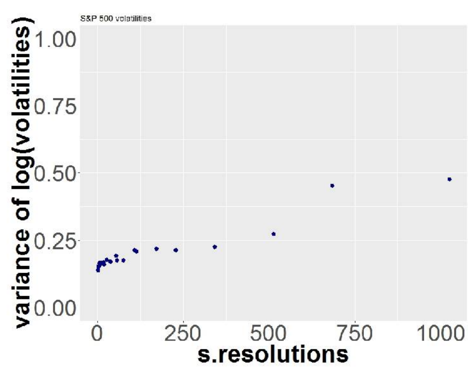

Figure 8: S&P 500 log-volatilities in 2015 variances by sampling resolution

We expected the correlation of log-volatilities to decrease with increasing difference in the sampling resolution used for calculation. For the visual check, we used a heat map (see Figure 9). In this heatmap the colour of given square denotes the correlation of the log-volatilities with sampling resolution marked on the x axis and on the y axis. This heatmap is by construction diagonally symmetrical. It is obvious that the correlations of volatilities measured over shorter time steps are much higher than correlations of those measured with longer steps. This is in line with our intuition.

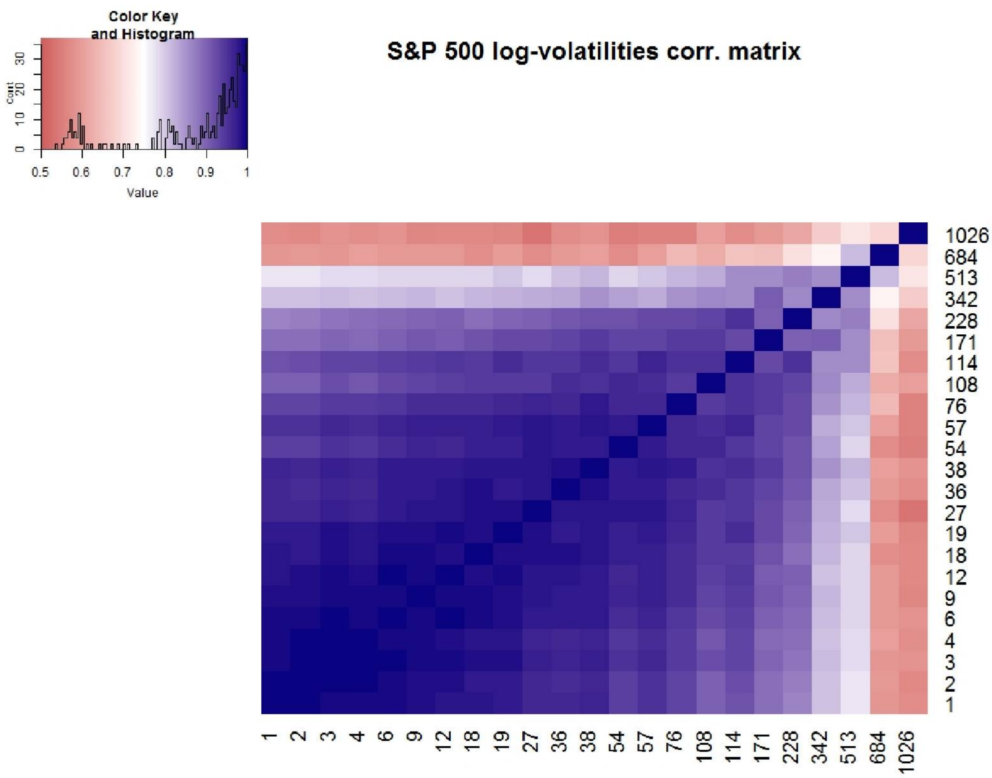

Figure 9: S&P 500 log-volatilities correlation matrix. The color denotes the correlation of respective log-volatilities

We also plotted how the volatilities evolved throughout the examined period (see Figure 10). This gives us the first impression of how our scale will capture the market development. The colour denotes the volatility with specific sampling resolution (rows) in specific week (columns). We can detect enlarged volatility in long term investors behavior that came in the week 33 and that the rest of market agents reacted with one week delay. We can also notice that the long term investors preceded the market once again and that the volatility amongst them decreased first and the others followed again with small delay.

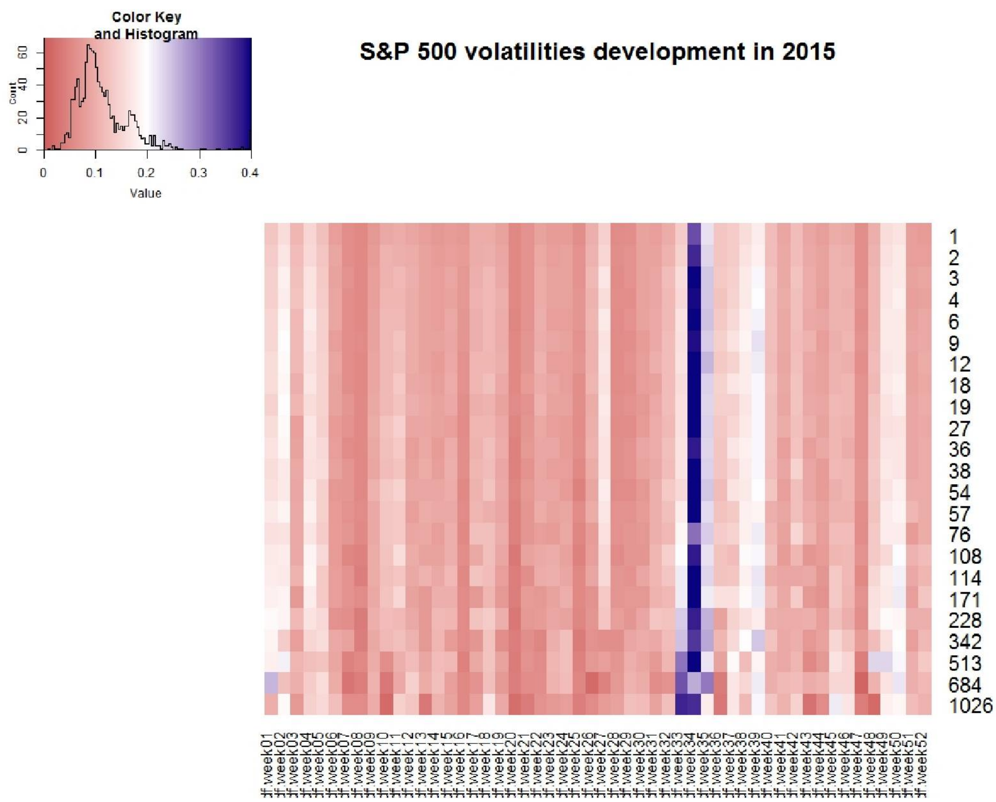

Figure 10: S&P 500 volatilities development in 2015. The color denotes the value of volatility calculated with respective granularity in respective week

By DJIA dataset we obtained 23 volatility vectors, each with 52 observations. These correspond to particular weeks between March 2016 and March 2017. We plotted the means and variances of the obtained log-volatility vectors in order to compare them with each other (see Figure 11 and Figure 12). The means of these annualized volatility vectors range from 0.072 to 0.085, which translates into values around -2.5 after the log transformation. The means of log-volatilities do not differ dramatically, however, it can be noticed that the means slightly decrease with increasing time step used for calculation of log-volatilities.

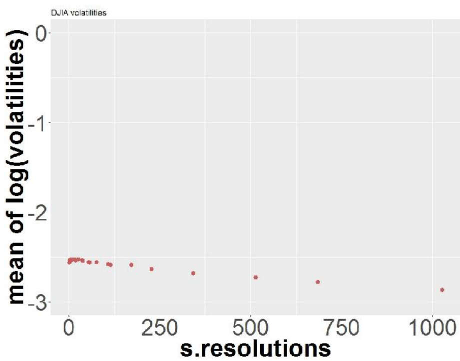

Figure 11: DJIA log(volatilities) in 3/2016-3/2017 means by sampling resolution

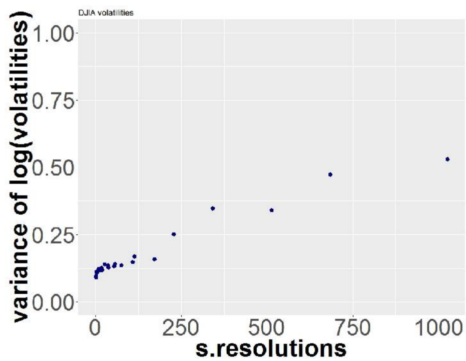

Figure 12: DJIA log(volatilities) in 3/2016-3/2017 variances by sampling resolution

The variances of log-volatilities range from 0.13 to 0.47. As we can observe, the log-volatilities calculated with longer time steps have significantly higher variance than those with shorter time steps - we observed this also in the S&P 500 dataset.

We expected the correlation of DJIA *log*-volatilities to decrease with increasing difference in the sampling resolution as well as for the Standard and Poor's dataset. It is again true as can be seen in Figure 11. In this plot the colour of given square denotes the correlation of the log-volatilities with sampling resolution marked on the x axis and on the y axis. This figure is by construction diagonally symmetrical. Once again it is indisputable that the correlations of volatilities measured over shorter time steps are much higher than correlations of those measured with longer steps (see Figure 13).

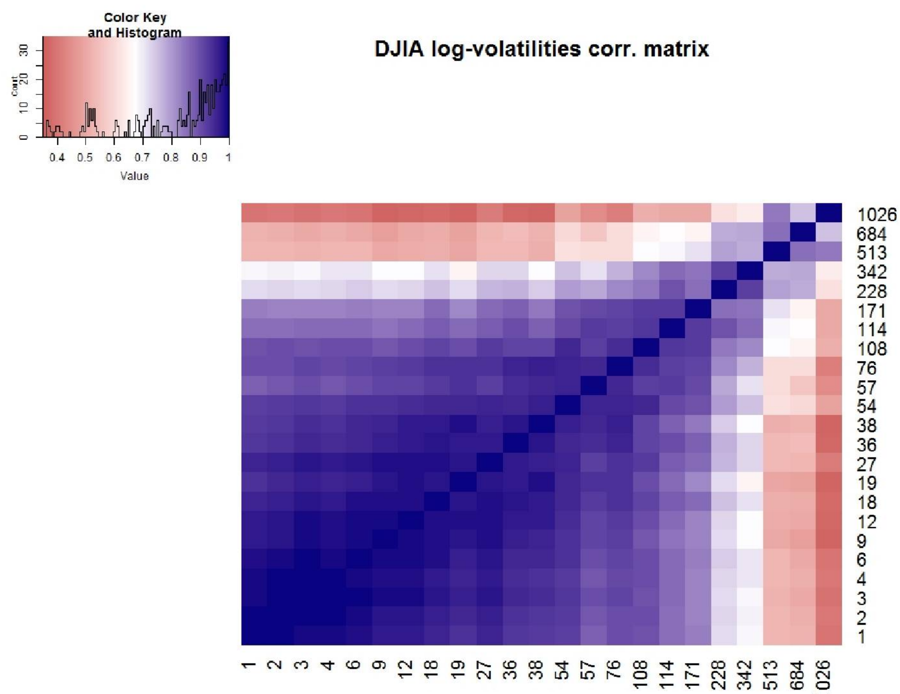

Figure 13: DJIA log-volatilities correlation matrix. The color denotes the correlation of respective log-volatilities

As for the Standard & Poor's dataset we plotted how the volatilities evolved throughout the examined period (see Figure 14). This gives us the first impression of how our scale will capture the market development between 3/2016 and 3/2017. The colour denotes the volatility with specific sampling resolution (rows) in specific week (columns). We can detect enlarged value of the whole volatility vector in week 17 of the examined period, which corresponds to the sharp decline in prices at the end of June 2016. We can also see that in week 36 the long term investors show enlarged volatility. This could be linked with the US presidential elections after which the prices went upward. The short term investors show slightly increased volatility as well.

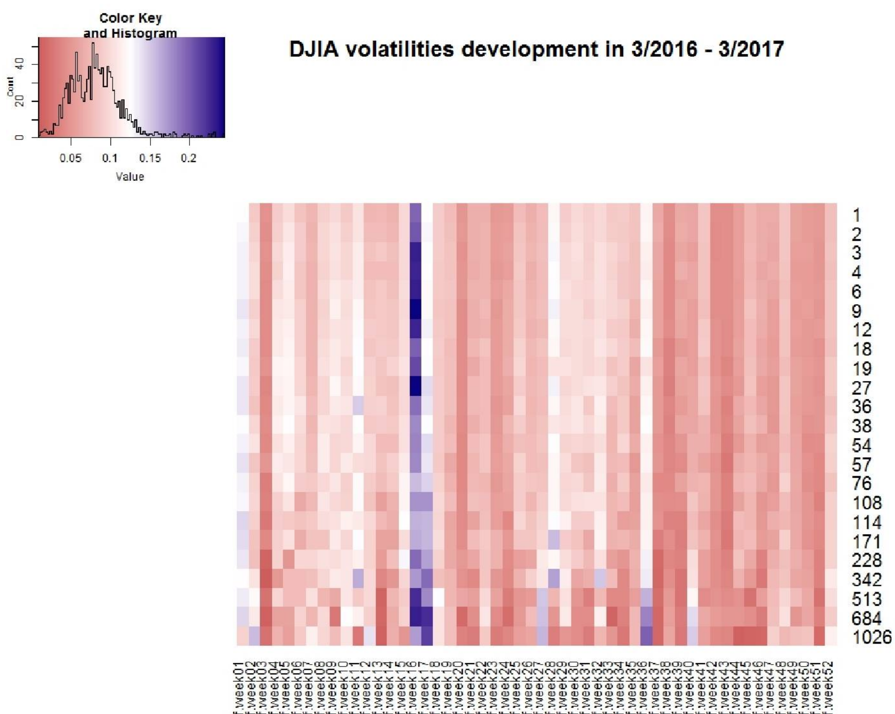

Figure 14: DJIA volatilities development between 3/2016-3/2017. The color denotes the value of volatility calculated with respective granularity in respective week

4.4 Principal Component Analysis

We decided to use variance reduction technique in order to lower the computational complexity. The computational complexity stemming from the use of Monte Carlo technique could effectively prevent usage of our tool on large datasets, as it would not be possible to obtain the results fast enough to use them for setting policies. In perfect case, the scale should be able to capture market energy with no bigger than one week delay, which should allow the policy makers to incorporate the information into the policies reacting to the market development.

We also employ the variance reduction technique in order to find optimal weighting scheme for our volatilities. As mentioned earlier, we are not able to build on the weighting scheme used by Zumbach et al. as the underlying theory is not presented in detail. Therefore we will make use of technique called Principal Component Analysis.

Principal Component Analysis was invented by Karl Pearson (Pearson, 1901). It is an statistical procedure that employs an orthogonal transformation in order to reduce the number of dimensions of the data. In layman's terms it basically rotates the axis in order to capture the maximum variance (for a nice, interactive visualization see *http://setosa.io/ev/principal-component-analysis/*). The output are so-called Principal Components. These are the rotated axis numbered by the amount of variance they capture (for illustration on 2D example see Figure 15).

The number of Principal Components is always equal or lower than the minimum of the number of the original dimensions and the number of dimensions. The Principal Components are ordered by the amount of variance they capture. The real challenge is to choose the number of Principal Components to use in further analysis. The approaches differ, however, we decided to take components that contribute to overall variance more than an average component does. In our case, this means taking two components, which indicates strong dependencies amongst our data.

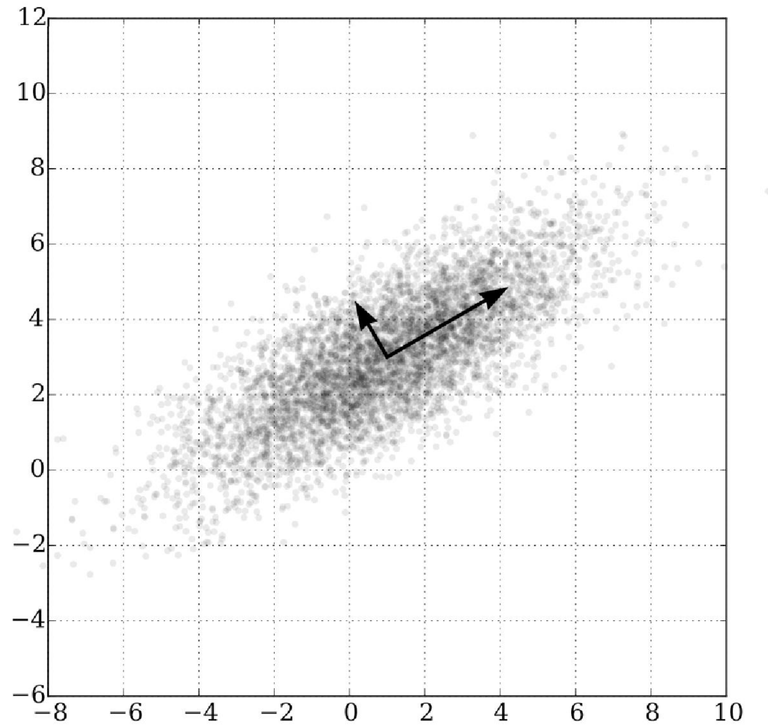

Figure 15: PCA illustration on 2D example (the arrows denotes the new axes, the so-called Principal Components)

Conducting Principal Components Analysis consists of 5 consecutive steps. The first is data preparation. It is desired to have centered data, which can be obtained by subtracting mean for each variable. The resulting variables have zero mean. Some also argue that the data should be scaled, however we decided not to scale as the observations of volatilities do not differ dramatically between sampling resolutions and therefore no variable should be distinctly more significant than others. Moreover we won't loose the information about different ranges of values in volatilities calculated in different sampling resolutions. The second is calculating the covariance matrix. Some again advise to use correlation, however we sticked with covariance matrix for the same reasons for which we did not scale our data. The covariance matrix should preferably have large diagonal values as these correspond to strong *signal*. On the other hand, the non-diagonal values should be low as these present the redundancy in data, in other words *the noise*. After this it is needed to calculate the eigenvectors and eigenvalues of the covariance matrix. Based on these it is possible to choose the Principal Components which one would further use. The final step is to compute the new dataset (the observations are still the same, however, they are expressed on different axis - the Principal Components), which can be done by multiplication of the transposed eigenvectors with transposed centered data. We used the function *prcomp* from R package {stats}.

Our covariance matrix shows strong noise for both datasets, nevertheless we will use the variance reduction technique and in section results we will evaluate its performance and assess, whether this technique should be used and if so, then under what circumstances.

4.5 Monte Carlo simulations

In this thesis, we used a Monte Carlo simulation to generate random numbers from unknown underlying multivariate normal distribution. After computing the mean of each volatility and covariance matrix for these volatilities, we used a Monte Carlo simulation to draw a million random observations from estimated distribution. We specified that the underlying marginal probabilities are normal (for justification see section *4.3.1 Normality of log-volatilities*) and their mutual dependency (captured in the variance-covariance matrix). We obtained a million vectors of volatility observations and for each actual measurement of volatility, we calculated a ratio of cases when this value was higher then the relevant simulated volatility in hypothetical vectors and cases when this value was lower then the relevant simulated volatility in hypothetical vectors. By computing this ratio, we got the probability of non-exceeding measured volatility value for each volatility measurement. These we later used for construction of the Scale of Market Movements. The same mechanism was applied after employment of the

Principal Component Analysis on the two vectors of Principal Components we decided to keep and on their variance-covariance matrix. Analogically, we computed the probability of non-exceeding these values and these probabilities were later used for construction of the scale.

In our case, we needed to estimate 23-dimensional multivariate normal distribution. Estimating of higher-dimensional distributions is usually a very complex computational process. In order to ease the technical demands on hardware, we decided to use a simplifying transformation proposed by Alan Genz (Genz, 1992). In his algorithm, three consequential transformation were used. At first, Cholesky decomposition of the covariance matrix was used to transform the original integral into an integral over a unit hypercube. The second transformation simplifies the original integrand and the third one puts integral into a constant limit form and forces priority ordering on the integration variables based on their importance, so that the variable which has most dependent factors is computed first. For details on this algorithm please see (Genz, 1992). For the generation of random numbers from estimated underlying multivariate normal distribution we used function *rmvnorm* from R package {mvtnorm}.

5 Results

In this section we will present our findings and we will link the development of the values on our scale to real-world market events. We will assess whether the variance reduction technique improved the scale performance and whether incorporating of market volume improved the scale's performance. We will present the results in two sections, one for each dataset we used.

5.1 S&P 500 dataset

Our S&P 500 dataset captures the time period between January 2015 and December 2015. The year 2015 was the worst since 2008 for S&P 500 as it had declined by 0.7%[4]. The S&P 500 was dragged down by significant falls in oil and materials sector. In the same time, the stocks of so-called 'FANGs[5])' had risen in price and saved the S&P 500 from more significant fall.

Due to the construction, our scale attained positive as well as negative values. In order to ease the interpretation we shifted the scale by 4 points upwards. Before using variance reduction technique, our scale ranged from 0.9 to 6.2, the median was 1.4. As we predicted, the highest value was attained in week 34.

In week 33 and week 34, the S&P 500 fell by almost 11 percentage points, which was the worst two week performance since 2011. The dramatic events at the market were probably driven by three factors. Firstly, the China's government reported that its manufacturing activity was in July 2015 the lowest in six years. This led to concerns, whether the previous announcement about 7% growth of China's economy was credible. Secondly, 10 minutes from July meeting of FED was released. This footage raised concerns about

[4]https://www.ft.com/content/70a80b66-aedf-11e5-993b-c425a3d2b65a

[5]the high-performing tech companies – Facebook, Amazon, Netflix and Google(now Alphabet Inc.

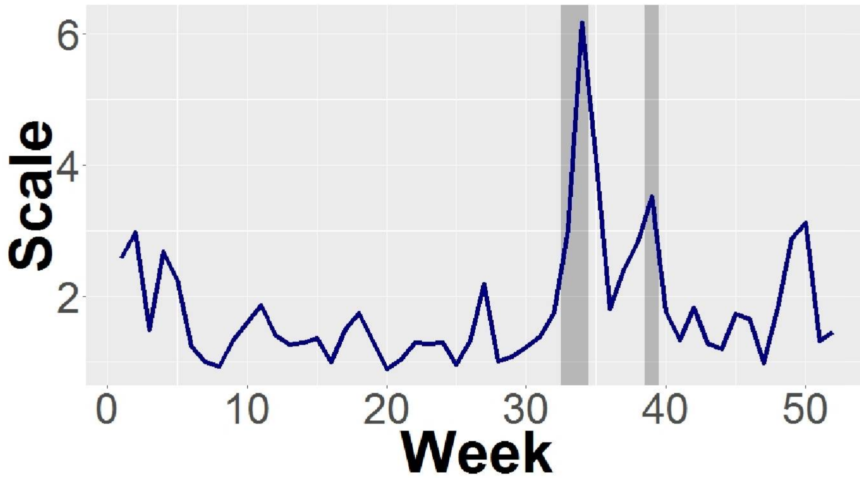

Figure 16: Scale of Market Movements for S&P 500 in 2015

raising interest rates, which brought uncertainty to the market. Lastly, the oil prices dropped below the key level of $40 per barrel. A year ago, the barrel was worth $100.

In week 39, the measurement on our scale shows value 3.52. This is still pretty far from the median, however, it is not as high value as we obtained for week 34. The stock price development did not experience any large jump, but rather a sequence of smaller sharp moves. In this week, Nike published a report about its sales which pushed Nike's stocks up. However, not long after the China's government reported that the profits in Chinese industrial companies drop 8.8% from the year-ago level, which lowered the oil prices and this further negatively influenced stock prices of energy companies. Another event that influenced the market was an announcement by two members of the FED council who once again suggested that the interest rate may hike in coming weeks. Even though the overall price move was not as rapid, our scale shows high value, as the market energy was rather high in this week. We believe that it was high due to mixed signals coming from different directions to which the investors had to react.

After employment of the variance reduction technique, our scale looked very different (see Figure 17). The scale now clearly does not capture any shocks that occurred in 2015. We believe that the variance reduction technique did not work very well, as our covariance matrix had not have large values on the diagonal and small values otherwise. Therefore we believe our

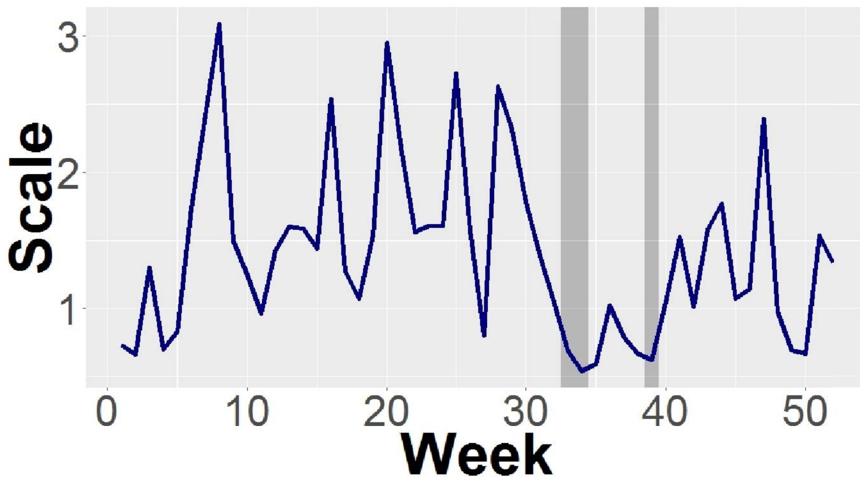

Figure 17: Scale of Market Movements after using PCA for S&P 500 in 2015

log-volatilities structure had a lot of noise, which the PCA was not able to separate from the signal and as a consequence it performed poorly. We conclude that using PCA for this dataset is not appropriate and that the US regulatory bodies need to find another way how to reduce the computational complexity of the task.

5.2 Dow Jones Industrial Average

Our DJIA dataset captures the time range between 3/2016-3/2017. In this period, two important events occurred - firstly, on the 24th of June it was announced that the British voted leave in the Brexit referendum. Secondly, after heated campaign, the US presidential election was won by Donald Trump. This two acts were perceived as events that may influence further direction of US and EU politics.

Our scale correctly captured the Brexit vote in week 16 and assigned to this event value 5.04. This is by far the highest value in the whole period and it suggests that this event was of much higher importance for the US stock

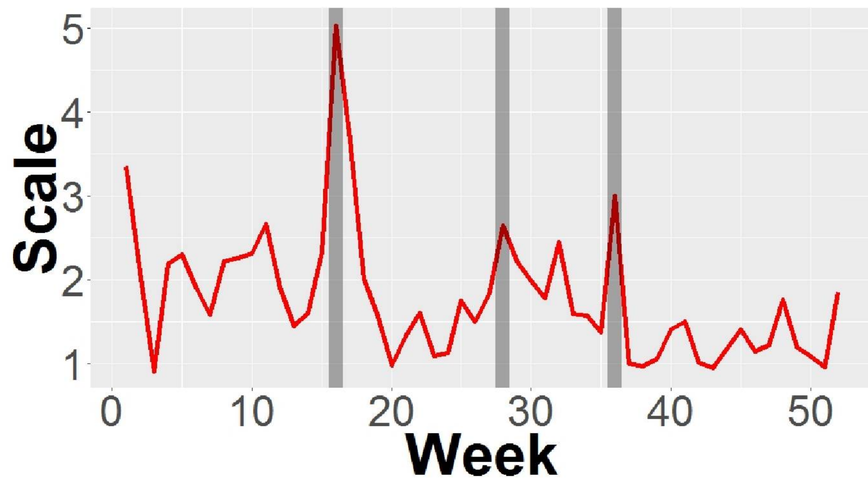

Figure 18: Scale of Market Movements before using PCA for DJIA without volume

market than the US presidential election. The markets expected that the economic growth of UK would slow down during the negotiations to leave the EU.

In the week 28, there is another spike. Once again, this spike on our scale is not connected to some significant sharp move upwards or downwards, but rather to a sequence of smaller events. In this week, the Samsung issued an official note for Galaxy Note 7 users which said that they should stop using their smartphones due to concerns over the battery. In the same week Apple introduced its new iPhone 7 and the demand after iPhones spiked. Also, East Coast had to face gas shortages and price hikes after a leak in a crucial pipeline had caused the closure of this pipeline.

The US presidential election was held on the 8th of November, which corresponds to week 36 in our dataset. We can see a sharp spike of market energy in this week, followed by a period with relatively low market energy. It is clear that the market agents reacted to the end of uncertainty. However, we can see that value on our scale is much lower than the one attained after the 'Leave' vote, which indicates that the US stock market was influenced more by the Brexit referendum than by presidential elections in US.

After incorporating market volume into the scale (see Figure 19), the

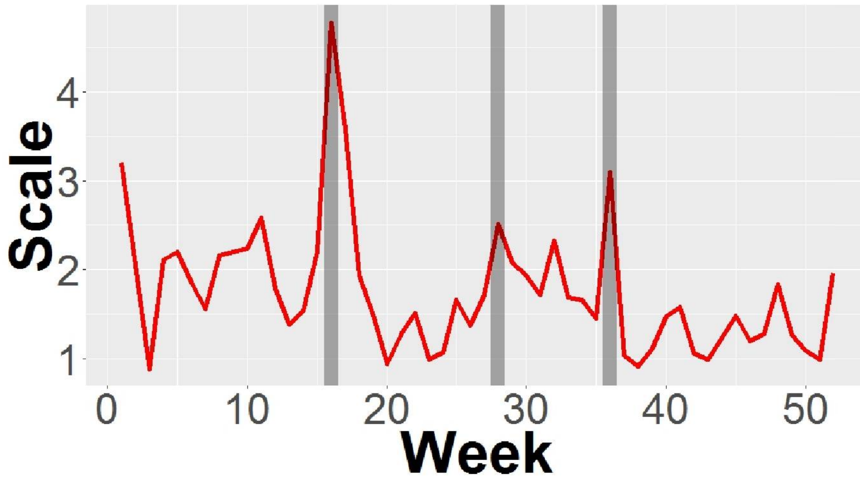

Figure 19: Scale of Market Movements before using PCA for DJIA with incorporated trading volume

scale did not significantly change. As we stated before, the traded volume was rather stable with one sharp move upwards between the 13th of October and the 20th of October. Therefore the scale after this adjustment is a bit flatter in the left half and a bit sharper in the right half. Nevertheless, the spikes did not changed.

After employment of PCA same thing as by the S&P 500 happened. The scale does seem to follow patterns in the data at all and as such is of no use. We believe this confirms our belief that this technique is not appropriate for this kind of data.

5.3 Evaluation of the approach

Our approach towards constructing a scale for measuring market movements was based on the using volatilities calculated over differing time ranges. We took those as proxies for behavior of different market agents based on evidence in literature (Zumbach et al., 1999), (Maillet et al., 2003). We found evidence that the volatilities are *log*-normally distributed, therefore we used logarithms of them and using a Monte Carlo simulation we obtained probabilities of exceeding the measured values of volatility. These we turned

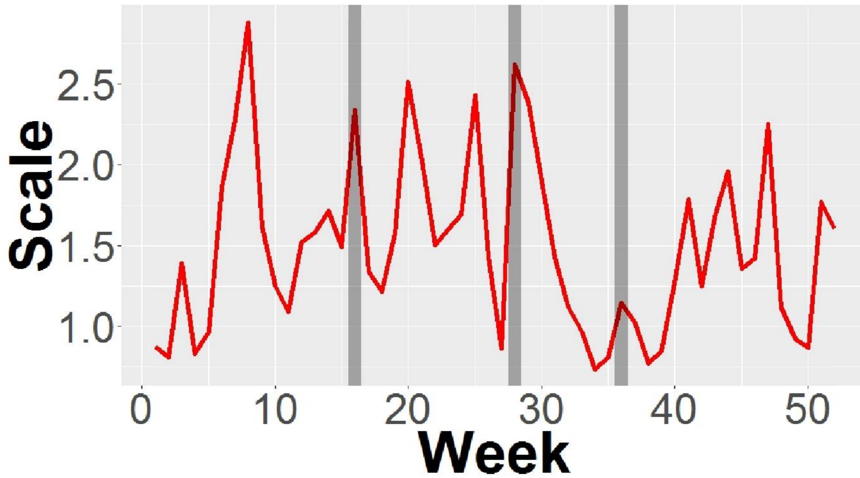

Figure 20: Scale of Market Movements after using PCA for DJIA with incorporated trading volume

into a so-called Scale of Market Shocks, which should allow for quantifying market energy. We also tried to reduce the computational complexity of the task by employing a variance reduction technique, however this technique proved itself to be of no use for this kind of data.

The main drawback of this approach is the relativity of the output. It is necessary to compute this scale for longer period and than compare the values with each other. There is not a way how to objectively assess what a value on the scale means without proper comparison. Nevertheless, the scale is not useless, as it can still uncover interesting information such as the 'Leave' vote influenced the US stock market more than US presidential election or that the weeks after the US presidential election were rather calm for US stock market.

The main advantage lies in the easy comparability of shocks captured. It does not really matter whether the crash gradually takes on severity or it is one sharp move, the values assigned by the scale may be directly compared and proper policies can be implemented. We also incorporated the market volume into the measure which adds another information to the construction of the scale.

6 Conclusion

In this thesis we aimed to create a measure of market shocks for US stock market. This measure should not have been able to predict the crisis, but rather allow for proper assessment of the severity of the crisis. We used data from Dow Jones Industrial Average and S&P 500 and we incorporated the information about market volume from DJIA dataset. We found out that market energy in 2015 was generally lower than in the period between 3/2016-3/2017. We also found out that the shock in August 2015 was greater than any other in examined periods. We were able to compare the shocks after the United Kingdom European Union membership referendum and after the US presidential election (both in 2016) and conclude that the shock after the referendum was more severe.

We tried to employ Principal Component Analysis for reduction dimensions in our data and to reduce the computational complexity, however this method did not yield good results as the noise in the time series is not easily separable from the signal we care for. We did not either confirm or reject that the market volume adds valuable information into the construction of the scale as the volume was rather stable during the examined period and therefore did not bring any additional information.

For consequent research we recommend to quantify longer time frame of market data on our scale. This would allow for comparison of the shocks the market experienced during this longer time frame and assessment of the steps taken by US regulatory bodies. This would lead to recommendation of policies for specific situations that may occur in the future. It would be also valuable to test our scale on time period with volatile market volume to assess the contribution brought by this variable.

It would also be interesting to quantify different stock markets on our scale and compare the average market energy and stability of these markets. This would lead to another variable that could be incorporated into calculation of risk.

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Table 3: SMM for DJIA before PCA without volume

Week	SMM value	Week	SMM value
Week 01	3.350	Week 27	1.821
Week 02	2.093	Week 28	2.648
Week 03	0.908	Week 29	2.215
Week 04	2.181	Week 30	1.988
Week 05	2.299	Week 31	1.783
Week 06	1.906	Week 32	2.445
Week 07	1.577	Week 33	1.593
Week 08	2.213	Week 34	1.568
Week 09	2.262	Week 35	1.375
Week 10	2.309	Week 36	2.995
Week 11	2.661	Week 37	0.995
Week 12	1.907	Week 38	0.967
Week 13	1.444	Week 39	1.049
Week 14	1.601	Week 40	1.398
Week 15	2.318	Week 41	1.502
Week 16	5.036	Week 42	1.003
Week 17	3.676	Week 43	0.940
Week 18	2.008	Week 44	1.174
Week 19	1.559	Week 45	1.408
Week 20	0.976	Week 46	1.141
Week 21	1.332	Week 47	1.220
Week 22	1.604	Week 48	1.761
Week 23	1.091	Week 49	1.199
Week 24	1.122	Week 50	1.084
Week 25	1.746	Week 51	0.957
Week 26	1.494	Week 52	1.857

Table 4: SMM for DJIA before PCA with volume

Week	SMM value	Week	SMM value
Week 01	3.198	Week 27	1.715
Week 02	2.051	Week 28	2.514
Week 03	0.885	Week 29	2.083
Week 04	2.109	Week 30	1.93
Week 05	2.197	Week 31	1.714
Week 06	1.856	Week 32	2.324
Week 07	1.560	Week 33	1.682
Week 08	2.162	Week 34	1.654
Week 09	2.199	Week 35	1.452
Week 10	2.234	Week 36	3.092
Week 11	2.582	Week 37	1.038
Week 12	1.787	Week 38	0.909
Week 13	1.387	Week 39	1.105
Week 14	1.539	Week 40	1.467
Week 15	2.197	Week 41	1.58
Week 16	4.780	Week 42	1.056
Week 17	3.586	Week 43	0.99
Week 18	1.930	Week 44	1.229
Week 19	1.494	Week 45	1.476
Week 20	0.940	Week 46	1.193
Week 21	1.269	Week 47	1.274
Week 22	1.507	Week 48	1.838
Week 23	0.992	Week 49	1.259
Week 24	1.066	Week 50	1.091
Week 25	1.654	Week 51	0.99
Week 26	1.371	Week 52	1.961

Table 5: SMM for DJIA after using PCA without volume

Week	SMM value	Week	SMM value
Week 01	0.918	Week 27	0.918
Week 02	0.826	Week 28	2.761
Week 03	1.428	Week 29	2.529
Week 04	0.859	Week 30	1.956
Week 05	1.012	Week 31	1.484
Week 06	1.912	Week 32	1.183
Week 07	2.302	Week 33	0.919
Week 08	2.945	Week 34	0.700
Week 09	1.662	Week 35	0.767
Week 10	1.294	Week 36	1.108
Week 11	1.125	Week 37	0.981
Week 12	1.623	Week 38	0.820
Week 13	1.652	Week 39	0.806
Week 14	1.785	Week 40	1.211
Week 15	1.573	Week 41	1.695
Week 16	2.465	Week 42	1.189
Week 17	1.373	Week 43	1.593
Week 18	1.268	Week 44	1.870
Week 19	1.642	Week 45	1.294
Week 20	2.606	Week 46	1.358
Week 21	2.123	Week 47	2.155
Week 22	1.600	Week 48	1.074
Week 23	1.761	Week 49	0.879
Week 24	1.782	Week 50	0.864
Week 25	2.562	Week 51	1.709
Week 26	1.563	Week 52	1.523

Table 6: SMM for DJIA after using PCA with volume

Week	SMM value	Week	SMM value
Week 02	0.809	Week 28	2.621
Week 03	1.391	Week 29	2.378
Week 04	0.831	Week 30	1.899
Week 05	0.967	Week 31	1.427
Week 06	1.862	Week 32	1.125
Week 07	2.277	Week 33	0.971
Week 08	2.878	Week 34	0.738
Week 09	1.615	Week 35	0.810
Week 10	1.252	Week 36	1.144
Week 11	1.092	Week 37	1.024
Week 12	1.521	Week 38	0.771
Week 13	1.586	Week 39	0.850
Week 14	1.715	Week 40	1.270
Week 15	1.491	Week 41	1.783
Week 16	2.339	Week 42	1.252
Week 17	1.339	Week 43	1.677
Week 18	1.219	Week 44	1.958
Week 19	1.574	Week 45	1.356
Week 20	2.509	Week 46	1.420
Week 21	2.022	Week 47	2.251
Week 22	1.504	Week 48	1.121
Week 23	1.602	Week 49	0.923
Week 24	1.693	Week 50	0.870
Week 25	2.426	Week 51	1.767
Week 26	1.435	Week 52	1.608

Table 7: SMM for S&P 500 before using PCA

Week	SMM value	Week	SMM value
Week 01	2.580	Week 27	2.183
Week 02	2.975	Week 28	1.012
Week 03	1.490	Week 29	1.077
Week 04	2.685	Week 30	1.22
Week 05	2.257	Week 31	1.384
Week 06	1.236	Week 32	1.741
Week 07	0.996	Week 33	3.016
Week 08	0.930	Week 34	6.182
Week 09	1.339	Week 35	4.112
Week 10	1.597	Week 36	1.817
Week 11	1.873	Week 37	2.411
Week 12	1.409	Week 38	2.831
Week 13	1.261	Week 39	3.518
Week 14	1.299	Week 40	1.773
Week 15	1.365	Week 41	1.338
Week 16	0.993	Week 42	1.832
Week 17	1.489	Week 43	1.285
Week 18	1.747	Week 44	1.203
Week 19	1.317	Week 45	1.731
Week 20	0.900	Week 46	1.649
Week 21	1.041	Week 47	0.981
Week 22	1.293	Week 48	1.833
Week 23	1.278	Week 49	2.880
Week 24	1.289	Week 50	3.117
Week 25	0.956	Week 51	1.325
Week 26	1.324	Week 52	1.449

Table 8: SMM for S&P 500 after using PCA

Week	SMM value	Week	SMM value
Week 01	0.734	Week 27	0.806
Week 02	0.660	Week 28	2.632
Week 03	1.301	Week 29	2.316
Week 04	0.701	Week 30	1.777
Week 05	0.829	Week 31	1.392
Week 06	1.736	Week 32	1.048
Week 07	2.420	Week 33	0.691
Week 08	3.093	Week 34	0.542
Week 09	1.499	Week 35	0.591
Week 10	1.247	Week 36	1.026
Week 11	0.966	Week 37	0.789
Week 12	1.420	Week 38	0.668
Week 13	1.600	Week 39	0.625
Week 14	1.588	Week 40	1.05
Week 15	1.440	Week 41	1.524
Week 16	2.538	Week 42	1.016
Week 17	1.274	Week 43	1.579
Week 18	1.072	Week 44	1.77
Week 19	1.535	Week 45	1.074
Week 20	2.953	Week 46	1.146
Week 21	2.180	Week 47	2.395
Week 22	1.559	Week 48	0.976
Week 23	1.610	Week 49	0.694
Week 24	1.606	Week 50	0.671
Week 25	2.729	Week 51	1.534
Week 26	1.574	Week 52	1.34

Table 9: Dow Jones Industrial Average Components

Ticker	Company
MMM	*3M*
AXP	*American Express*
AAPL	*Apple*
BA	*Boeing*
CAT	*Caterpillar*
CVX	*Chevron*
CSCO	*Cisco*
KO	*Coca-Cola*
DIS	*Disney*
DD	*E I du Pont de Nemours and Co*
XOM	*Exxon Mobil*
GE	*General Electric*
GS	*Goldman Sachs*
HD	*Home Depot*
IBM	*IBM*
INTC	*Intel*
JNJ	*Johnson & Johnson*
JPM	*JPMorgan Chase*
MCD	*McDonald's*
MRK	*Merck*
MSFT	*Microsoft*
NKE	*Nike*
PFE	*Pfizer*
PG	*Procter & Gamble*
TRV	*Travelers Companies Inc*
UTX	*United Technologies*
UNH	*UnitedHealth*
VZ	*Verizon*
V	*Visa*
WMT	*Wal-Mart*