REPORT ON LACHMAN'S MASTER THESIS

Referee: Dr. Manish Mishra
Designation: Assistant Profssor of Mathematics
Affiliation: Indian Institute of Science Education and Research - Pune, India
Suggested grade: 1

Comments

Bruhat and Tits, in the 1970s and 80s, developed the theory of affine buildings. To a reductive group G defined over a local field F, they associated a polysimplicial complex $\mathcal{B}(G)$ satisfying certain strong regularity conditions and which has a $G(F)$ action. To each $x \in \mathcal{B}(G)$, they also attached a smooth connected affine group scheme $\underline{G_x}$ over the ring of integers $\mathcal{O}_F$ of F with generic fiber G and such that the stabilizer of x in $G(F)$ is $\underline{G_x}(\mathcal{O}_F) \cap G(F)_0$ where $G(F)_0$ is a certain subgroup of $G(F)$ containing all open normal subgroup (it is the kernel of the Kottwitz homomorphism).

When G is a classical group, $\mathcal{B}(G)$ has a very concrete description in terms of the standard representaiton V of G. It can be realised as an equivalence class of a set of norms on V. Equivalently, it can be realised as a set of graded lattice chains on V satisfying certain conditions and the group schemes $\underline{G_x}$ are realised as stabilisers of these lattice chains in V.

The author makes a careful study of the lattice chain model of the Bruhat-Tits building in the case $G = \mathrm{SL}_n$ and $F = \mathbb{Q}_p$. After defining an abstract simplicial complex, he states the axioms of a building. He then gives the construction of a building using lattice chains. In §4, he proves several of the axioms of buildings for this construction.

The novel part in this thesis are certain explicit formulas. The author obtains an explicit formulas for graph distance between two points when $n = 2$. For this, he works with certain matrix representations of points and apartments of the building. To obtain the formula for arbitrary n, he defines certain numbers m_i. If A and B are the matrix representatives of the two points, then m_i is the minimum of the valuation of the determinant of the collection of matrices obtained from A and B, by taking i rows from A and $n - i$ rows from B. The graph distance is then given by $m_0 + m_d - (m_{1+}m_{d-1})$.

The author also gives formulas for gallery distance between two points and gallery distance between two chambers. For this, he works with representatives which are *generalised permutation matrices*. In the last section, he generalises the distance formulas to the case of three vertices.

The lattice chain model of Bruhat-Tits building was extended by Jiu-Kang Yu and Wee Teck Gan to groups of type G_2 in their 2003 Bull. Soc. math. France paper. One possible direction of generalization of the results in this thesis would be compute such formulas in this case.

Except for minor English errors at various places, this thesis is quite well written. All the proofs are given and details are not skipped. A better written introduction,

explaining the statements of the main results, would have benefitted this thesis though. The explicit formulas obtained in this thesis are a useful addition to the existing literature. I would therefore strongly recommend this thesis to be accepted as a Master's thesis and I suggest the grade 1 for this thesis.