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External Referee Report on doctoral thesis
"Special Graph Classes and Algorithms on Them" by Martin Pergel
August 26, 2008

It is my pleasure to referee Martin Pergel's doctoral thesis. My impression is positive and I recommend acceptance of the thesis. The breadth and importance of the new scientific results of Chapters 2, 3, and 4 exceed the requirements for a doctoral thesis and demonstrate the author's ability to do creative scientific work.

The notions of complicacy of polygon circle graphs, the subtree overlap NP-completeness reductions, and the idea of sandwiching are ideas that will have impacts in other intersection graph studies. The results of Chapter 3 settle some questions that have been open for several years. In addition, the decomposition of PC graphs and studies of unique representability may lead to other polynomial time algorithms for those graphs.

Although the writing is variable some parts of the thesis are well-written while some are not – the results certainly warrant acceptance.

Chapter 1 contains some definitions and general background. While there are some inaccuracies (eg. P_n is incorrectly defined as a path of length n, *minimum* and *maximum* are interchanged on page 4, and the word *convex* is missing from the definition of CONV graphs), the chapter provides an essential introduction to the area of the thesis. I would have liked to have seen a glossary of all of the graph classes mentioned in Chapter 1. Throughout the thesis, there is a persistent grammatical error, specifically, the English articles *the* and *a* are missing in many places.

In Section 2.1, the *complicacy* of a polygon circle graph G is defined to be the minimum k such that G is the intersection graph of convex k-gons in a circle. The section gives proofs of the following results: (1) the maximum complicacy of any graph on n vertices is $n - \log n + o(\log n)$; and (2) for any fixed k, it is NP-complete to determine whether a given graph G has complicacy at most k, even if it is known to be at most $k + 1$. There is a small error in the caption of Figure 2.2, specifically, $\ell = 2$ should be $\ell = 1$. But all of the results in this section are very interesting, well-written, and a pleasure to read.

Section 2.2 is on the subject of CONV graphs, that is, the intersection graphs of convex polygons in the plane. It is shown that Cartesian coordinates do not provide a polynomial-sized certificate for the existence of a CONV representation of a given graph. The result is proven by showing that any convex polygon representation of a particular graph gadget is constrained in a certain way, and then giving a construction of a sequence of CONV graphs for which every CONV representation requires size exponential in the size of the graph. The construction makes use of properties of the previously mentioned graph gadget, and the existence of SEG graphs requiring exponential sized representations. (The thesis is lacking a reference for the existence of such SEG graphs.) This section is quite challenging to read; in particular, I found that the proof of Lemma 10 lacked some justifications.

Section 2.3 considers the hardness of recognizing some subclasses of subtree overlap graphs. Specifically, the results of Section 2.3 are: (1) it is NP-hard to recognize 3-SOGs; (2) given tree T of maximum degree at least 3 and graph G, it is NP-complete to determine whether G is the overlap graph of subtrees in tree T' where T' is obtained from T by subdividing edges; and (3) it is NP-hard to recognize k-SOGs, for any $k > 2$. The description of the construction on page 35 is inconsistent with the caption of Figure 2.11, and with the later arguments in points 1, 4, 5, and 6. I believe that the results of this section are true, but the write-up is lacking the level of detail to be convincing.

The sandwiching idea of Chapter 3 allows for results of the form: No polynomially recognizable graph class B can be sandwiched between A and C unless P=NP. Section 3.1 shows that it is NP-complete to decide whether a graph is the intersection graph of homothetic copies of a fixed convex polygon. The proof involves a complicated reduction from E3-NAE-SAT(4) which produces a graph that is an intersection graph of homothetic convex polygons or a graph that is not a PDISK graph. Thus, no polynomially-recognizable graph class can be sandwiched between HOM-k-GONs and PDISKs unless P=NP. Sections 3.2 and 3.3 show the NP-completeness of recognizing polygon circle graphs and interval filament graphs, with the conclusion that no polynomially recognizable class can be sandwiched between those two classes unless P=NP. The results of Chapter 3 are substantial and important in the area of intersection graphs. Sections 3.2 and 3.3 settle interesting questions that have been open for several years.

Chapter 4 gives a polynomial time recognition algorithm for polygon circle graphs of girth at least 5, based on an interesting decomposition of those graphs. On the other hand, it is shown that it is NP-hard to recognize SEG or PSEG graphs of high girth.

I enjoyed the descriptions of the open problems of Chapter 5.

Questions:

1. In Figure 1.9, why is there a box around 1-CROSS (PSEG)? Which of the graph classes in the figure are of key importance to the thesis? ... of key importance in the study of intersection graphs?
2. Each section of Chapter 2 considers the complexity of determining whether an intersection representation of a particular type or size exists for a given graph. Discuss whether or not the representation problems considered in Chapter 2 and elsewhere in the thesis are known to be in NP or other complexity classes.
3. Show that the problem of Theorem 20 is in NP.
4. In Chapter 4, you show that restricting to high girth "helps" with the recognition of polygon circle graphs. What other restrictions might lead to polynomial time recognition algorithms?
5. In Lemma 54, you show that certain restricted polygon circle graphs have a unique PC representation. Do you know of any other uniquely representable classes of PC graphs? Any conjectures?

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