Review of the PhD Thesis
Time-Domain Modelling of Global Barotropic Ocean Tides
by David Einšpigel

Modelling of ocean tides is a classical problem with important theoretical and practical applications. Traditional approaches are based on solving linearized equations in the frequency domain. On the contrary, the thesis is devoted to solving this problem in the time domain. This enables to incorporate in a straightforward natural way non-linearities of ocean dynamics and assimilation of satellite altimetry data.

The thesis consists of introduction, six chapters, conclusion and two appendices. The brief introduction starts with selected references to several applications of tide models, development of their observations and numerical modelling. Approximately half of the introduction is then devoted to an overview of the main problems solved in the thesis and its structure.

The first chapter deals with mathematical formulation of the shallow water equations in the geographical coordinates. I think that the final system of equations is correct; nevertheless, I have several questions.:

Q1 *What do you mean by the sentence on p. 7: "Moreover, the complicated nature of the equations restricts possibilities of their numerical solution."*

Q2 *Are any parts of your derivation new or did you just unify several ideas that can be found elsewhere?*

Q3 *Both the radial component of the Coriolis force and contribution of the radial velocity to the Coriolis force are neglected in a general form of the momentum equations (1.7)-(1.9). Why?*

Q4 *What is the meaning of the boundary condition (1.10) in 3-D? (The coast line is a set of zero measure, thus implying that the normal to the boundary is not defined.)*

Q5 *The coefficients A_H, A_V and A_δ in (1.20) are assumed to be constant but they are related to velocities that are time-variable quantities. What is the physical justification of this assumption?*

Q6 *Could you comment on validity of the relations $W = \epsilon U$, $P = U^2$ on p. 16?*

Q7 *On p. 17 you define the Rossby number by $R_o = U/(fL)$, and thus R_o is latitude-dependent due to the Coriolis factor f. According to the definition of f on p. 9, $f = 0$ on the equator and $R_o \to \infty$. Is this reasonable?*

Q8 *Section 1.6, which summarizes the shallow water equations, is missing boundary conditions. Could you add them?*

The second chapter deals with an application of finite differences to the shallow water equations. The author employs the staggered Arakawa C-grid in space and a combination of the Adams-Bashforth and Adams-Moulton time-stepping. He demonstrates an excellent accuracy of the method as to preserving the total mass and energy (if eddy viscosity is neglected) in "the tsunami experiment", documented in Figs. 2.2 and 2.3. My questions are:

Q9 *Could you explain "the blue singularity" in the left bottom panel in Fig. 2.3?*

Q10 *The boundary conditions are missing again. How do you employ them into the used FD scheme?*

The next chapter yields a nice review of ocean tides that are used as the source term in numerical models. Compound tides, that exist due to non-linear couplings, are also mentioned. In this context, the following is not clear:

Q11 *How do you discriminate compound tides generated by tidal forcing from similar waves, that are also a result of non-linearities, but which are generated by non-tidal forcing?*

The chapter four then describes a purely hydrodynamical model developed by the author and called DEBOT-h. First, the author mentiones the fact that self-attraction and loading (SAL), must be taken into account. Although the full SAL effect can be computed iteratively in each time step via an integro-differential equation, DEBOT-h incorporates a simple scalar approximation of the SAL effect. This results in a simple "reduced gravity".

Q12 *Could you provide more details on the validity of the approximation (4.4)? Is it the same in continental shelf regions with high tides and in deep oceans?*

Second, the author takes into account energy conversion of barotropic tidal currents into baroclinic waves. It is performed by means of the relation (4.5). There are two parameters, κ and N_b. I would like to know

Q13 *What do you mean by "N_b is the observed buoyancy frequency at the seabed"? You solve the problem in the time domain but (4.5) seems to be written in the frequency domain. How do you implement this relation into your code that is written in the time domain? How is "the tunable parameter" κ tuned? Are posterior estimates based on numerical experience the only estimates available?*

The thesis then continues with results of parameter studies, summarized in Sections 4.3 and 4.4. These results are very interesting. They demonstrate author's deep experience with difficulties of numerical modelling, employed approximations of the SAL effect and energy conversion mentioned above.

Q14 *There are substantial differences in the relative RMS differences between DEBOT-h and data in the European shelf and other shallow regions. What are the reasons of such a different accuracy of your model in different shallow regions?*

Q15 *Could you explain the nature of numerical instabilities that occur when eddy viscoity is neglected?*

Chapter 5 is devoted to the DEBOT-a model which enables to assimilate data from satellite altimetry. I think that this model is the main achievement of the thesis. Other state-of-art assimilative models are based on linearised equations solved in the spectral domain. They require pre-processing of data when obtaining their spectral representation. On the contrary, the non-linear model of David Einšpigel assimilates data successively in time by weightening between the data and the state of the hydrodynamical model.

Q16 *Is there any trade-off between the weightening parameter w and assimilation time interval ΔT? Could you demonstrate such a trade-off in a* *systematic way? For instance, can you provide a "2-D map" for variables w and ΔT for at least one of the tidal constituents (i.e., unify Figs. 5.2 and 5.3 for more systematic combinations of w and ΔT)?*

The author then studies an accuracy of his model and shows that it is similar to accuracies of other available models. A large number of numerical examples is presented in Appendix B.

In chapter 6, modelling of full tides also including non-linear compound tides is briefly discussed. In the last section, David Einšpigel recapitulates his thesis, and mentions possible future applications of his model DEBOT. One of them is tsunami wave propagation, which inspires my last question:

Q17 *What do you think about possible application of your model for tsunami warning, where calculations and data assimilation must be performed in a relatively short real time?*

Summary: The thesis is written on a high sophisticated intellectual level, but it still is in a clear readable form employing precise mathematical constructions. It presents results of an extensive work and I appreciate that it is written in a monographic style. Both theoretical and numerical results achieved clearly show author's ability for a creative scientific work. I would also like to emphasize the fact that the author has opened a new branch of research at our department. I strongly recommend David Einšpigel to be awarded the PhD degree.

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