The dynamics of low-temperature fluids, such as superfluid helium 4, is an open scientific problem. The experimental study of similarities and differences between quantum (superfluid) and classical (viscous) flows is specifically an active research field, which already led to significant progress in our phenomenological understanding of the underlying physics. It also revealed that a comprehensive theoretical description is still missing, as, for example, in the case of the observed behaviour of moving bodies in quantum flows. The work aim is to derive the existence theory for the weak solution of a relevant system of equations based on the Landau model of superfluid helium 4 and appropriate numerical schemes to solve these equations.