

Abstract: In Riemannian geometry, the fundamental fact is that there exists a unique torsion-free connection (called the Levi-Civita connection) compatible with the Riemannian metric  $g$ , i.e. having the property  $\nabla g = 0$ . In projective geometry, the class of covariant derivatives defining the geometry is fixed and all these covariant derivatives have the same class of (non-parametrized) geodesics. Old (and non-trivial) problem is to find whether these curves are geodesics of a (pseudo-)Riemannian metric. Such projective structures are called metrizable. Surprisingly enough, U. Dini and R. Liouville found in 19th century that the metrizability problem leads to a system of linear PDE's. In the last years, there were several papers dealing with these problems. The projective geometry is a representative example of the so called parabolic geometries (for full description, see the recent monograph by A. Čap and J. Slovák). It was realized recently that the corresponding linear metrizability operator is a special example of the so called first BGG operator. The flat model of projective geometry is the (real) projective space.

In this more general context, the metrizability problem for (pseudo-)Riemannian geometries is naturally generalized to the sub-Riemannian situation. In the recent preprint, D. Calderbank, J. Slovák and V. Souček are discussing the classification of (real) *irreducible* parabolic geometries for which the linearisation method can be applied. A part of the classification is the case of complex simple Lie algebras considered as real Lie algebras.

The aim of this thesis is to formulate the linearisation method in a full generality and to classify completely the cases of complex simple Lie algebras where the linearisation method is applicable. In Sect. 2, there is a summary of description of invariant differential operators on parabolic geometries and comments how to use it for real cases. A general discussion of the linearisation method is contained in Sect.3. The classification result for the case of complex simple Lie algebras is presented in Sect.5. Some examples of explicit solutions are contained in Sect. 6. There are several Appendices summarizing results used in the thesis.